UCRL- 91875
PREPRINT
CONF-8410210--10

QUARK MATTER TRANSITION IN THE BARYON RICH DOMAIN

George F. Chapline
Lawrence Livermore National Laboratory
University of California
Livermore, CA 94550

Invited talk at the
7th High Energy Heavy Ion Study, GSI Darmstadt,
October 8, 1984

December 1984

QUARK MATTER TRANSITION IN THE BARYON RICH DOMAIN

UCRL--91875

DE85 007978

George F. Chapline
Lawrence Livermore National Laboratory
University of California
Livermore, CA 94550

Invited talk at the
7th High Energy Heavy Ion Study, GSI Darmstadt,
October 8, 1984

ABSTRACT

The implications of nuclear matter equation of state measurements for a quark matter phase transition are discussed. The possibility of detecting such a phase transition by looking for changes in the pattern of collective flow associated with heavy ion collisions is pointed out.

In this talk I will suggest some reasons why it is important to measure the equation-of-state of compressed nuclear matter, and will describe some simple ways to deduce the consequences of a particular equation of state. The main focus of my remarks will be the idea that at sufficiently high baryon number densities nuclear matter makes a transition to a Fermi gas of quarks. Indeed, it was just a little over eight years ago in a paper contributed to the 3rd High Energy Heavy Ion Study that I suggested that possibly the most interesting reason for studying relativistic heavy ion collisions is to look for the transient formation of quark matter. The same idea independently occured to Arthur Kerman about the same time, and our ideas are documented in a Lawrence Livermore Laboratory report.[1] Our suggestions were inspired by the brilliant observation of Collins and Perry[2] that as a consequence of asymptotic freedom the quarks in nuclear matter will be deconfined at very high baryon number densities. We note in this connection that at very high baryon number densities the interaction between quarks is characterized by a coupling strength[3]

$$\alpha_c = \frac{\pi}{22 - 4N} \quad \frac{1}{\ln(k_F/\Lambda_F)}$$

where N is number of different kinds of quarks, k_F is the Fermi momentum and Λ_F is a constant.

Whether the transition from compressed normal nuclear matter is to quark matter is smooth or discontinuous is unknown at the present time. However, if the equation-of-state of compressed nuclear matter is fairly stiff then it follows from the Maxwell construction (Fig. 1) and the fact that the equation -of-state of quark matter is fairly soft[4] that the transition is a first order phase transition. Indeed, using a typical phenomenological equation of state (e.g. Bethe-Johnson equation-of-state) one obtains a phase transition at zero temperature like that shown in Fig. 2. In Table I we list the baryon number densities where the phase transition occurs at zero temperature for several theoretical nuclear matter equations of state. It can be seen that the phase transition typically occurs in the density regime of 3-15 times the density of ordinary nuclei. At finite temperatures the transition would be expected to occur at somewhat lower baryon number densities but be qualitatively the same as the zero temperature case.

As a matter of historical interest I might mention that I was first motivated to suggest[5] that the transition from normal nuclear matter to quark matter is a first order phase transition by the fact that Mott transitions in cold materials are in all known cases first order transtitions. Whereas the ionization of atoms resulting from a rise in temperature is a smooth transition, the delocalization of electrons in cold materials resulting from an increase in pressure apparently is always accompanied by a phase transition.

Whether the actual equation of state of hot compressed nuclear matter is similar to any of the equations of state referred to in Table I remains to be seen. However, the determination of the equation of state of compressed nuclear matter at densities up to a few times that in ordinary nuclei might be considered to be the most important objective of Becalac research. Preliminary indications[6] are that the equation of state is fairly stiff and would lead to a first order phase transition.

In order to estimate the beam energies needed to produce baryon number densities like those shown in Table I one may use the simple shock wave model illustrated in Fig. 3. Here the compressed material is at rest in the center of mass system and the velocity $\beta = \sqrt{\gamma_{CM}^2 - 1}/\gamma_{CM}$. In this frame of reference the equations for conservation of energy, momentum, and baryon number take the form

$$e_2 \beta_S = \gamma_{CM}^2 \, e_1 (\beta + \beta_S) \tag{1a}$$

$$P_2 = \gamma_{CM}^2 \, e_1 (\beta + \beta_S) \tag{1b}$$

$$n_2 \beta_S = \gamma_{CM} \, n_1 (\beta + \beta_S) \tag{1c}$$

Dividing (1a) by (1c) one obtains

$$\frac{e_2}{n_2 mc^2} = \gamma_{CM} \tag{2}$$

i.e. the energy per baryon in the compressed state is simply γ_{CM}. Dividing (1b) by (1c), and assuming that the equation-of-state of nuclear matter has the form

$$p = (\nu - 1)(e - nmc^2) \quad . \tag{3}$$

we find that

$$\frac{n_2}{n_1} = \frac{\nu}{\nu - 1} \gamma_{CM} + \frac{1}{\nu - 1} \tag{4}$$

It should be noted that the baryon number compression is a linear function of γ_{CM}. Also Eq.(4) provides a simple estimate of how the stiffness ν of the equation-of-state affects the maximum baryon number density that one can attain in a central collision.

As another application of Eq's.(1) one can suppose that the compressed material is quark matter whose equation of state is of the form

$$p = \frac{1}{3}(e - 4B) \quad , \tag{5}$$

where B is the "bag constant." In this case, the baryon number compression is given by,

$$\frac{n_2}{n_1} \simeq 4\gamma_{CM} - \frac{3}{\gamma_{CM}}\left(1 - \frac{B}{e_1}\right) \tag{6}$$

For center of mass kinetic energies per baryon in the range 2 - 5 GeV the corresponding baryon number densities lie in the range 12 - 24 times normal nuclear density. As can be seen from Table I this is very likely within the range of baryon number densities expected just above a quark matter phase transition.

Before going on to discuss how one might detect such a phase transition in heavy ion collisions I would like to mention that there is a cloud on the horizon regarding the possible existence of quark matter phase transition. In the last two years a new paradigm has emerged for nuclear physics, based on the non-linear meson theory of Skyrme.[7] Witten has shown[8] that Skyrme's soliton solution is a fermion with baryon number = 1. This new paradigm allows one to calculate nucleon-nucleon forces and the equation-of-state of compressed nuclear matter. The somewhat suprising outcome of initial calculations[9] for nuclear matter is that the equation of state is very soft--in fact, almost identical to that for quark matter; i.e. $e \sim n^{4/3}$ for large n. Therefore, there is no first order phase transition to quark atter. One apparent defect of the Skyrme model for a nucleon is that it is not consistent with asymptotic scaling, i.e. the existence of free quarks at small distances. Whether the addition of quarks to the Skyrme model (for example, in the form of a chiral bag) will restore the quark matter phase transition remains to be seen.

At this point it should also be remembered that there is one constraint on the equation of state of compressed nuclear that follows from observational astrophysics: namely, the maximum mass of a neutron star. The general relativistic equilibrium condition for uniform density star is[10]:

$$p/e = \xi(x) \tag{7}$$

where x is related to the mass M and radius R of the star by $\sin^2 x = 2GM/c^2R$ and

$$\xi = \frac{3\cos x}{9/2\cos x - \sin^3 x/(x - \sin x \cos x)} - 1 \quad .$$

As shown by Chandrasekhar[11] any star becomes unstable when the adiabatic index $\gamma \equiv \frac{d\ln P}{d\ln n}$ falls below a certain critical value, and he gave some numerical estimates of this critical adiabatic index for several values of x. Using the uniform density model Michael Nauenberg and I were able to derive an accurate analytic approximation for the critical adiabatic index[10]:

$$\gamma_c \simeq (\xi + 1) \left\{ 1 + \frac{(3\xi + 1)}{2} \left[\frac{\xi + 1}{6\xi} \tan^2 x - 1 \right] \right\} \tag{8}$$

Given the existence of a critical adiabatic index it is a simple matter to show that the mass of a cold star is bounded by[10]:

$$M \leq \frac{1}{2} \left(\frac{3}{8\pi}\right)^{1/2} \left(\frac{c^2}{G}\right)^{3/2} \left[\frac{1 - (\xi_c + 1)/\gamma_c}{e_1 - P_1/V_c^2} \right]^{1/2} \sin^3 x_c \quad , \tag{9}$$

where V_c is a critical value for the speed of sound such that $(1 + 1/\xi_c)$ $V_c^2 = \gamma_c$. The values of e_1 and P_1 are <u>measured</u> values of the energy density and pressure. In the case of neutron stars one would use the energy density and pressure of compressed nuclear matter corresponding to the highest baryon number density for which experimental values are confidently known. An example of how this scheme works is shown in Table II. The parameter V_c is actually unknown, but in any case is less than c, yielding a maximum mass of approximately $3M_\odot$. The important point for us is that neutron star masses $\simeq 1.5M_\odot$ have been observed,[12] so that one has a significant constraint on the equation of state of compressed nuclear matter.

As a particular application of this constraint the pure Skyrmion equation-of-state appears to be ruled out. However, an equation of state which is Bethe-Johnson-like up to a few times normal nuclear density and Skyrmion-like at higher densities would be marginaly consistent with the observed neutron star masses.[9] Therefore, at the present time neutron star masses do not be themselves tell us whether a first order quark matter phase transition occurs.

It is interesting, though, in this connection to inquire whether quark matter might occur inside neutron stars and whether stars made primarily of quarks might exist.[13] The range of possibilities, assuming that the energy density of nuclear matter $e = an^2 + mnc^2$, is shown in Fig. 4. One conclusion is that free quarks will not occur at the center of neutron stars unless Λ_F is at the lower end of its range of plausible values.

corresponding to a phase transition at less than 10 times normal nuclear density. As for pure quark stars one finds from Eq.(5) that $\gamma_c \approx 2.26$ and from Eq.(6) that $M \lesssim 0.1\, \Lambda_F^{-2} M_\odot$, so that typically quark stars are less massive than neutron stars. Quark stars also have a minimum mass; corresponding to the curve in Fig. 4. Indeed, for plausible values of a and ν quark stars cannot exist at all. In any case, we disagree with the recent suggestion of Witten that Jupiter-sized chunks of quark matter might exist in nature.

Let us now turn to the question of how one would defect a first order quark phase transition during relativistic heavy ion collisions. The question of how one would detect the transient presence of quark matter in the baryon rich region during the collision of two heavy nuclei has been previously discussed by Hörst Stöcker.[14] I would like to suggest that one should look for the effect of a first order phase transition on the flow of nuclear matter.

As a test of this idea a colleague, Alex Granik, and I have studied how a first order phase transition affects the deflection of a relativistic flow by an oblique shock wave. As is well known,[15] supersonic flow around a sharp pointed object involves a conical shock wave (Taylor and Maccoll, 1933); while supersonic flow around a blunt object involves a detached shock, which is normal to the flow in front of the object and trails away at the Mach angle at large distances from the object (See Fig. 5). Locally, the deflection of flow by the blunt object may be modeled as the deflection of the flow by an oblique shock wave. As one varies the inclination of the shock with respect to the direction of incoming flow then the deflection of the flow by the oblique shock will increase from zero at normal incidence, reach a maximum and then decrease again. As is evident from Fig. 5 the maximum deflection by an oblique shock will be a measure of the maxumum deflection by a blunt object, and therefore (hopefully) indicative of the deflection one would obtain in a realistic treatment of heavy ion collisions. (Since there is a stagnation point directly in front of the blunt body, we have in mind that the frame in which the blunt body is at rest can be identified with the center of mass system for heavy ion collisions.)

In a reference frame where the oblique shock wave is at rest (Fig. 6) the energy and momentum conservation equations across the shock take the form

$$(P_2 + e_2)\,\gamma_2^{\,2}\beta_{2n} \;=\; e_o\gamma_1^{\,2}\beta_{1n}$$

$$P_2 + (P_2 + e_2)\,\gamma_2^{\,2}\,\beta_{2n}^{\,2} \;=\; e_o\,\gamma_1^{\,2}\,\beta_{1n}^{\,2} \qquad (10)$$

$$(P_2 + e_2)\,\gamma_2^{\,2}\,\beta_{2n}\beta_{2t} = e_o\,\gamma_1^{\,2}\,\beta_{1n}\,\beta_{1t}$$

where P_2 and e_2 are the pressure and energy density behind the shock, β_{1n} and β_{2n} are the velocities normal to the shock, and β_{1t} β_{2t} are the velocities parallel to the shock. In writing these equations we have assumed that $P_1 = o$ and $e_1 = e_o$, where $e_o = 0.15\ \mathrm{GeV/fm}^3$ is the energy density of cold normal density nuclear matter. If we assume that the material behind the shock is quark matter described by a bag model equation of state, $P = \frac{1}{3}(e - 4B)$, then Eqs. (10) reduce to a quadratic equation for β_{2n}/β_{1n} whose solution is[16]

$$(\beta_{2n}/\beta_{1n})_{\text{quark}} \;=\; \frac{2}{3}\left(1 + \frac{B}{e_oU_{1n}^{\,2}}\right) - \left[\frac{4}{9}\left(1 + \frac{B}{e_oU_{1n}^{\,2}}\right)^2 - \frac{1}{3}\left(1 + \frac{1}{U_{1n}^{\,2}}\right)\right]^{1/2}, \qquad (11)$$

where $U_{1n} = \beta_{1n}\gamma_1$. If the material behind the shock is compressed nuclear matter described an equation of state of the Bethe-Johnson form (3) then one obtains a cubic equation for the ratio β_{2n}/β_{1n}.

Given a solution for β_{2n}/β_{1n} as a function of the shock inclination angle $\emptyset$, one can use the geometric identity $\beta_{2n}/\beta_{1n} = (\beta_{2x} - \beta_{2y}\cot\emptyset)/\beta_1$ to calculate the "shock polar," i.e. the curve $\beta_{2y}/\beta_1 = f(\beta_{2x}/\beta_1)$. In the case where the material behind the shock is quark matter with the bag model equation of state (Eq. (5)) then the shock polar is a strophoid[16]:

$$y^2\left[(1 - x) + \frac{4B}{3e_oU_1^{\,2}}\right] + (1 - x)^3 - \left(\frac{2}{3} - \frac{4B}{3e_oU_1^{\,2}}\right)(1 - x)^2$$

$$+ \frac{1}{3U_1^{\,2}}\left(1 - \frac{4B}{e_o}\right)(1 - x) \;=\; o \qquad (12)$$

In the ultrarelativistic limit $U_1 \rightarrow \infty$ the shock polar, Eq. (12), becomes a circle with radius $\frac{1}{3}$. A typical shock polar for quark matter along with the corresponding shock polar for nuclear matter is shown in Fig. 7. the tangent lines originating at the origin determine the maximum angle of deflection in the two cases. As indicated by the shaded area the maximum angle of deflection is significantly greater in the case of quark matter than in the case of nuclear matter. Some numerical results for the maximum angles of deflection and corresponding shock inclinations are given in Table 3. These calculations suggest that in the case of heavy ion collisions there will be a significant increase in the center of mass $\langle P_{\perp} \rangle$ for collective flow when one reaches the quark matter phase transition.

Another possibility for detecting a first order phase transition is illustrated in Fig. 8. It was pointed out a long time ago by Hans Bethe[17] that in presence of a first order phase transition there will be some range of shock strengths for which a single shock wave is unstable. This is actually easy to see given the fact that the speed of a shock wave is given by the slope of the chord from the initial to final state on the Hugoniot curve. As is evident from Fig. 8 if the Hugoniot curve has an inflection point at A due to a phase transition, then a shock wave corresponding to the chord AX will travel slower than the one corresponding to chord OA. Thus for final states between A and B a single shock wave is unstable to splitting into two shocks. Some detailed calculations illustrating this effect have recently been carried out by Laszlo Csernai and his colleagues.[18] Because the speed of the leading shock wave is constant for some range of incoming velocities, one expects that some observable quantities, e.g. differential cross-sections will have anomalous behavior for some range of center-of-mass energy.

In conclusion, let me emphasize the potential advantages of a colliding beam machine for studying the quark matter phase transition in the baryon rich region. These include 1) with variable center-of-mass energy one is considerably less sensitive to uncertainties in the baryon density where the phase transition occurs; 2) changes in the pattern of collective flow would be easier to see in the center-of-mass system; 3) signatures visible at low luminosity would be very dramatic evidence of the phase transition. Finally, I would like to suggest that the demonstration of a quark matter phase transition is more than a parlor trick--in particular, it would probably go a long way towards distinguishing models like the bag model and the Skyrmion model and therefore be of genuine scientific value.

ACKNOWLEDGEMENTS

Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under contract number W-7405-ENG-48.

REFERENCES

1. G. F. Chapline and A. K. Kerman, Lawrence Livermore National Laboratory report UCRL-80737 (January, 1978).
2 J. C. Collins and M. J. Perry, *Phys. Rev. Lett.* 34, 1353 (1975).
3. G. Chapline and M. Nauenberg, *Phys. Rev.* D16, 450 (1977).
4. E. V. Shuryak, *Phys. Reports* 61, 71 (1980).
5. G. Chapline and M. Nauenberg, *Nature* 264, 235 (1976).
6. R. Stock, et al., *Phys. Rev. Lett.* 49, 1236 (1983);
   J. J. Molitoris, et al., *Phys. Rev. Lett.* 53, 899 (1984).
7. T. H. R. Skyrme, *Nucl. Phys.* 31, 556 (1962).
8. E. Witten, *Nucl. Phys.* B223, 422 (1983).
9. M. Kutshera, C. Pethick, and D. G. Ravenhall, *Phys. Rev. Lett.* 53, 1041 (1984).
10. M. Nauenberg and G. Chapline, *Astrophys. J.* 179, 277 (1973).
11. S. Chandrasekhar, *Phys. Rev. Lett.* 12, 114 (1964).
12. P. C. Joss and S. A. Rappaport, in Accretion Driven Stellar X-ray Sources (Cambridge Univ. Press, 1984).
13. G. Chapline and M. Nauenberg, *Ann. of the New York Academy of Sciences* 302, 191 (1977).
14. H. Stöcker, *Nucl. Phys.* A418, 587c (1984).
15. R. Courant and K. O. Friedrichs, Supersonic Flow and Shock Waves, Interscience Publishers, Inc., New York (1948).
16. G. Chapline and A. Granik, to be published.
17. H. Bethe, Office of Scientific Research and Development Report No. 545 (1942).
18. H. W. Barz, L. P. Csernai, B. Kämpfer, and B. Lukacs, University of Minnesota preprint (1984).

Table I

	Bethe-Johnson VH			Pandharipande-Smith			Causality limit		
Λ_F (MeV)	200	300	400	200	300	400	200	300	400
P_T (10^{35} dynes cm^{-2})	10.2	24.0	36.8	-	4.1	10.9	-	3.6	7.3
n_1 (fm^{-3})	1.49	2.15	2.58	-	0.7	1.1	-	0.47	0.59
n_2 (fm^{-3})	1.77	3.61	5.07	-	1.3	3.0	-	1.26	2.68
ρ_1 (10^{15} gm cm^{-3})	3.34	5.60	7.39	-	1.8	3.0	-	0.90	1.30
ρ_c (10^{15} gm cm^{-3})	3.31			1.1			1.6		

Table II

Maximum Mass of a Neutron Star ($M_{\odot}$)

$V_{s,}$ may/C	$P_o = 5.10^{14}$ g/cm^3 $P_o = 7.10^{33}$ dyn cm^{-2}	$P_o = 1.10^{15}$ g cm^{-3} $P_o = 5.10^{34}$ dyn cm^{-2}
1.0	3.6	2.6
0.75	3.0	2.2
0.50	2.0	1.6
0.25	0.7	unstable

Astrophysical Journal 179, 277 (1973).

Table III.

Maximum deflection (δ_m) of flow by an oblique shock wave as a function of incoming kinetic energy (E_1). Also shown is the shock angle (ϕ_m), corresponding to the maximum deflection.

E_1 (GeV/A)	Nuclear Matter δ_m(°)	ϕ_m(°)	Quark Matter (B/e_0 = 0.45) δ_m(°)	ϕ_m(°)
1.6	15.5	45.3	27.1	58.9
2.0	14.2	43.4	27.8	59.9
3.0	16.0	46.0	28.8	60.0
4.0	14.2	44.2	29.2	60.0
5.0	12.8	42.9	29.4	60.0

FIGURE CAPTIONS

Fig. 1. Maxwell construction, illustrating how a hard nuclear matter equation of state and a soft quark matter equation of state will lead to a first order phase transition.

Fig. 2. First order phase transition resulting from a typical phenomenological equation of state for nuclear matter.

Fig. 3. One dimensional model for the shock compression of nuclear matter.

Fig. 4. Equation-of-state parameters that allow quark stars to exist.

Fig. 5. Supersonic flow around a sphere.

Fig. 6. Coordinate system for an oblique shock wave at rest.

Fig. 7. $\gamma_1 = 3$ shock polars for nuclear matter and quark matter: shaded area indicates increase in deflection that would accompany quark matter phase transition.

Fig. 8. Instability of a single shock wave where the Rankine-Hugoniot curve has an infection point.

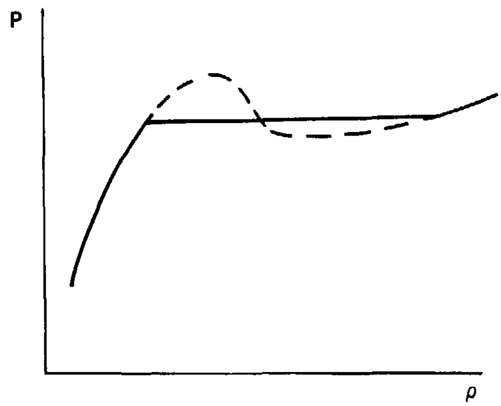
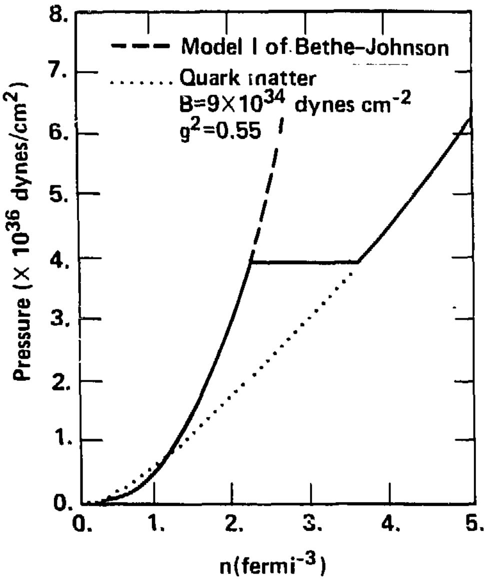


Fig. 1



Model I of Bethe–Johnson
Quark matter
B=9×10^34 dynes cm^-2
g^2=0.55
Pressure (× 10^36 dynes/cm^2)
n(fermi^-3)
0. 1. 2. 3. 4. 5. 6. 7. 8.
0. 1. 2. 3. 4. 5.

Fig. 2

Shock compression of nuclear matter

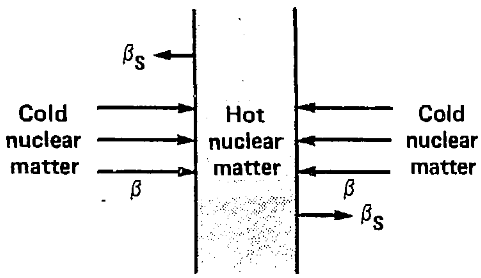
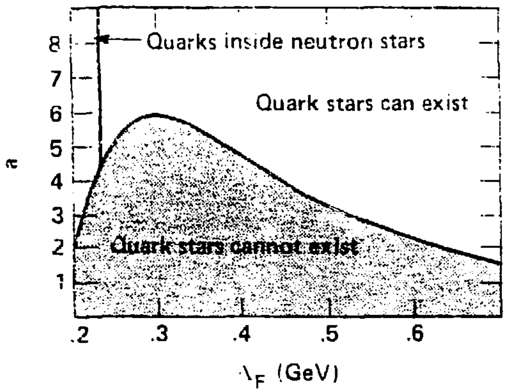


Fig. 3



Quarks inside neutron stars
Quark stars can exist
Quark stars cannot exist
a
8
7
6
5
4
3
2
1
.2
.3
.4
.5
.6
Λ_F (GeV)

Fig. 4

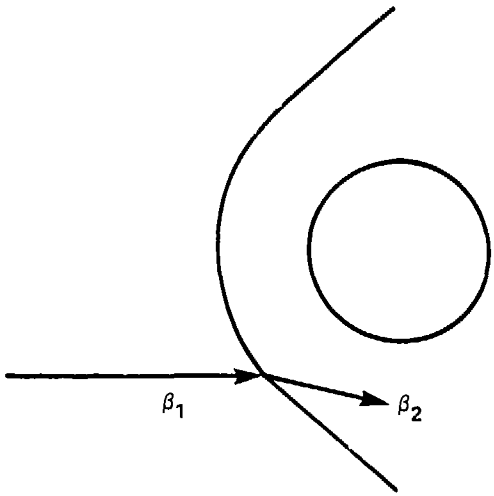


Fig. 5

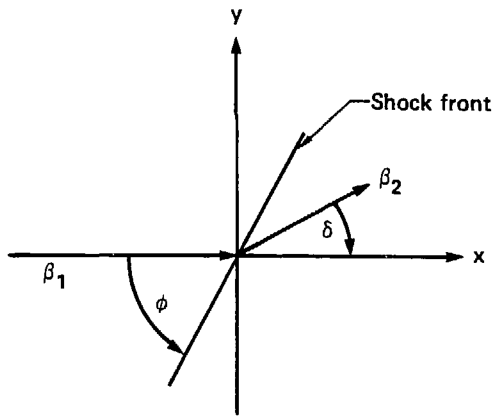


Fig. 6

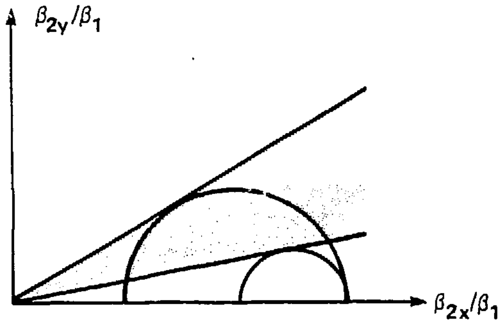
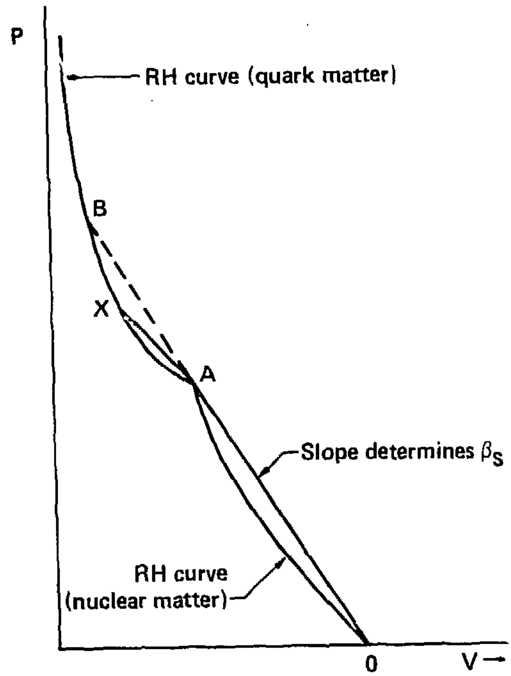


Fig. 7



P
RH curve (quark matter)
B
X
A
Slope determines β_S
RH curve
(nuclear matter)
0
V→

Fig. 8