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A Neoclassical Theory of Durable Good Diffusion

Jerry R. Jackson
David L. Kaserman

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A NEOCLASSICAL THEORY OF DURABLE GOOD DIFFUSION

Jerry R. Jackson* and David L. Kaserman

Department of Energy
Office of Conservation and Solar Applications
and
Energy Information Administration

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*Present address: Charles River Associates, Boston, MA 02116

OAK RIDGE NATIONAL LABORATORY
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ABSTRACT

Existing studies that deal with the diffusion of durable good innovations have been justifiably criticized for their common lack of an explicit testable theory of new product growth. This paper attempts to remedy this situation by providing a theoretical model of market penetration of new durable goods that is derived from the basic assumption that potential users of the new intermediate product attempt to minimize the discounted costs of production over time. The resulting model defines a time path of short-run equilibrium market shares that are determined by the cost characteristics (capital cost and operating and maintenance expenses) of both the new innovation and the equipment that it is designed to replace, the age distribution of the existing capital stock, and the growth rate of the adopting sector.

This model is shown to exhibit several attractive features lacking in existing models of the diffusion process. First, it yields a number of testable hypotheses, some of which have received indirect empirical support in previous studies on the subject. Second, it is operational in the absence of historical data on the market experience of the new good under investigation. And third, it is capable of generating, on the basis of such *ex ante* information, the complete range of functional forms used in prior models to represent the relationship between market share and elapsed time since introduction of the new innovation. These features render the model inherently superior to existing studies for the analysis of emerging products and frontier technologies for which market data are not yet available.

I. INTRODUCTION

The dynamic process whereby a new product increases its share of a given market over time has received increasing attention over the past two decades following the appearance of the seminal papers by Griliches (1957) and Mansfield (1961). Over this period, a variety of models that describe the time path of market share growth following the introduction of a new innovation have been examined in attempts to develop forecasting tools that may be used for investment planning and related activities.[1] While the empirical results obtained with such models have often been quite good by forecasting standards, the theoretical foundation that supports these efforts has never been fully developed.[2] As a result, the current literature on this subject exhibits two major related shortcomings, both of which can be traced to the lack of an explicit testable theory of the market penetration process. First, a concensus concerning the precise causal mechanism that generates specific observed diffusion paths remains elusive at the present time. And second, without exception, existing forecasting models require the use of historical data on the market experience of the given innovation in order to be implemented.[3] Together, these shortcomings severely limit the applicability and, therefore, the general usefulness of the tool kit that has come to be known as technological forecasting.

The purpose of this paper is to derive an explicit microeconomic theory of durable good diffusion from the basic maintained hypothesis that potential users of a new innovation attempt to minimize the discounted costs of producing a given output over time. That is, we assume that purchasers of a given set of substitutable durable goods have the objective of dynamic cost minimization, whether such purchasers are households (for consumer durables) or firms (for producer durables.)[4]

The resulting model will be seen to exhibit several attractive features. First, it is founded upon a clearly defined hypothesis of potential user optimization. Second, it is structural in nature, with explicit channels of causation depicted. Third, it is completely operational in the absence of historical data concerning the market experience of the new good under investigation. And fourth, it receives considerable empirical support from the results of numerous past studies on the subject. These features render this model inherently superior to existing studies for the analysis of expected diffusion of anticipated innovations and currently emerging products for which market data are not yet available, thereby significantly expanding the range of potential applications of market penetration analysis.

The report is organized as follows: Section II summarizes the prior literature concerning the diffusion of new innovations. Three basic classes of market penetration functions are described, and some early empirical evidence relating to the causal factors that influence the rate of adoption of new goods over time is presented. Section III describes a conceptual framework for viewing the market penetration process within the traditional demand and supply model of microeconomic theory. This section also lists several assumptions that are employed in the analysis presented in this report (most of which are not crucial to the theory that is derived but serve to simplify the presentation and highlight the basic process involved). Section IV presents a new theory of durable good diffusion that is derived from the basic maintained hypothesis of dynamic cost minimization and explores the role of investment in the adopting sector (both for replacement and expansion) in determining the time path of new product growth. Section V examines the impact of new product performance uncertainty on the time path of diffusion generated by the model derived in the preceding section. Section VI presents some simulation results obtained with this model that demonstrate the manner in which market penetration forecasts can be derived. Finally, Section VII summarizes the analysis and describes some potentially fruitful areas for future research.

II. A BRIEF SYNOPSIS OF THE DIFFUSION LITERATURE

Analysis of the dynamic process through which the market adjusts to the appearance of a new product has traditionally proceeded in two distinct stages. First, a market penetration function that expresses the percentage market share (in terms of either sales or equipment in place) of the new good in a given market as a function of time since introduction has been fit to the historical data for a particular innovation. Then, the slope parameter of this estimated relationship has been employed as the dependent variable in a second stage of estimation to explain the causal forces that influence the speed of adjustment either across innovations or across separate markets (either geographical or industrial) for a given innovation. The following two sections describe the kinds of approaches taken and the results obtained in carrying out these two stages of analysis.

A. Market Penetration Functions

With regard to the first stage of estimation, three basic classes of market penetration functions have been employed with varying degrees of success in the literature. Although none of these functions has been founded upon an explicit theory of producer or consumer optimization, the first two classes do postulate specific behavioral assumptions that lead directly to the estimating equations.

First, what has come to be called the "modified exponential" market penetration function is founded upon the assumption that the instantaneous rate of growth of the market share of the new product depends solely upon the remaining distance to the maximum attainable share, i.e.,

$$\frac{dP(t)}{dt} = a[L - P(t)], \qquad (2.1)$$

where P(t) is market share at time t, L is the upper limit of market share (≤ 1), and a is a constant to be estimated.[5] Assuming that the process starts at the origin (which is assured by measuring time from the moment at which the innovation is introduced) the solution to this differential equation is

$$P(t) = L[1 - \exp(-at)], \qquad (2.2)$$

the graph of which appears as Fig. 2.1. In order to estimate the parameter a, Eq. (2.2) may be transformed to

$$\ell n\left[\frac{L - P(t)}{L}\right] = -at, \qquad (2.3)$$

where the left-hand side is calculated from observations on P(t) and prior estimates of L.[6] A basic feature of this class of penetration functions is that the rate of increase in market share over time declines monotonically. The highest growth rate is attained immediately upon introduction of the new product.

The second class of penetration functions that is founded upon an explicit behavioral assumption is the logistic.[7] Here, it is postulated that the instantaneous rate of growth of market share is proportional to the product of the remaining distance to the maximum attainable share and the currently attained share, i.e.,

$$\frac{dP(t)}{dt} = aP(t)[L - P(t)]. \qquad (2.4)$$

The solution to this differential equation is

$$P(t) = L[1 + b \exp(-aLt)]^{-1}, \qquad (2.5)$$

where b is a constant depending upon the initial conditions. In order to estimate the parameter a, Eq. (2.5) may be transformed to

$$\ell n\left[\frac{P(t)}{L - P(t)}\right] = \ell n(1/b) + aLt, \qquad (2.6)$$

where, again, prior estimates of L are required.[8] The graph of the logistic penetration function is shown in Fig. 2.2. Two basic features of this class of penetration functions stand out. First, the function increases at an increasing rate (positive first and second derivatives) during the early stages of new product growth and then increases at a decreasing rate (positive first derivative and negative second derivative) during the later stages. And second, the function is symmetric about the point of inflection at P(t) = L/2.[9]

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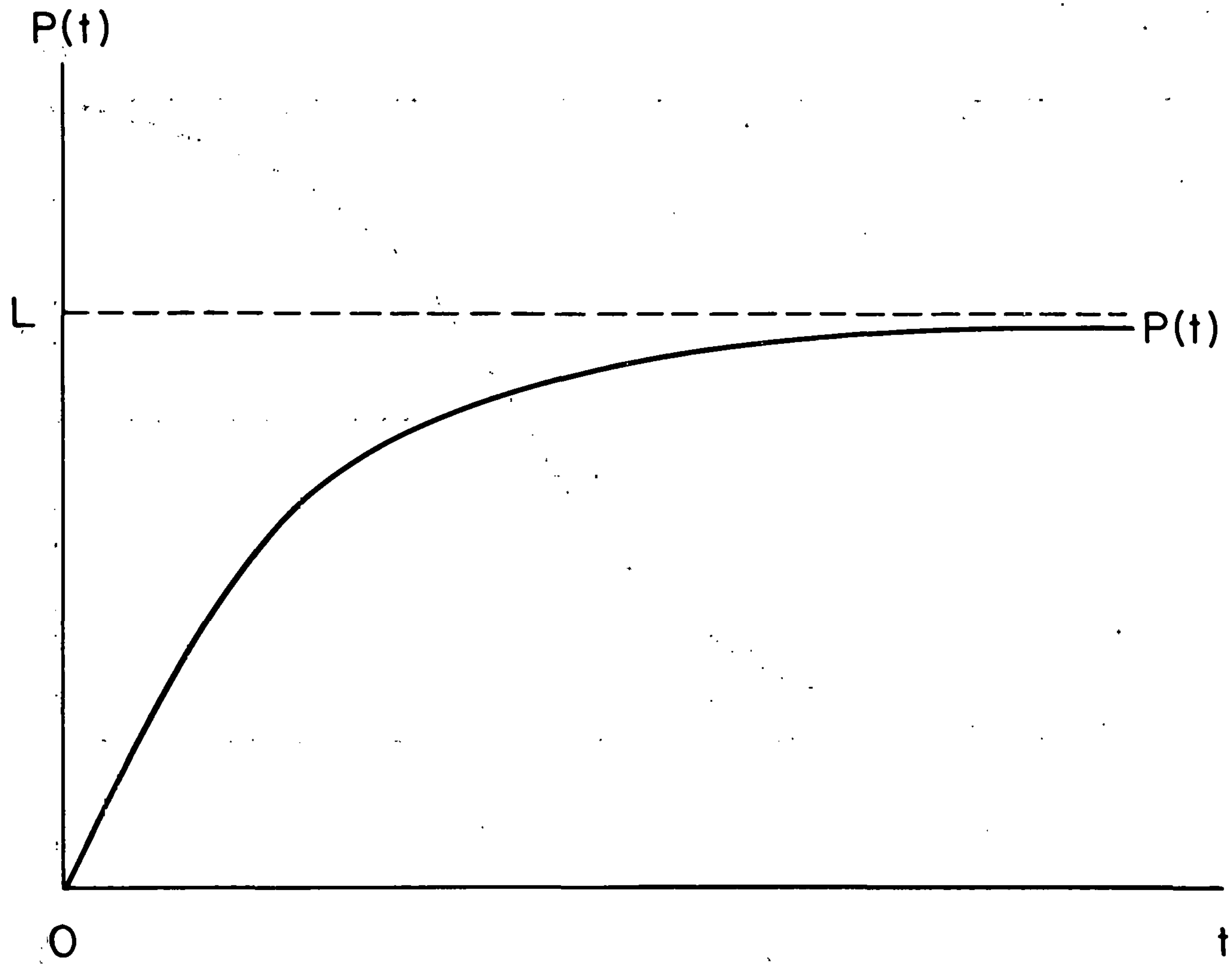

Fig. 2.1. Modified exponential penetration function.

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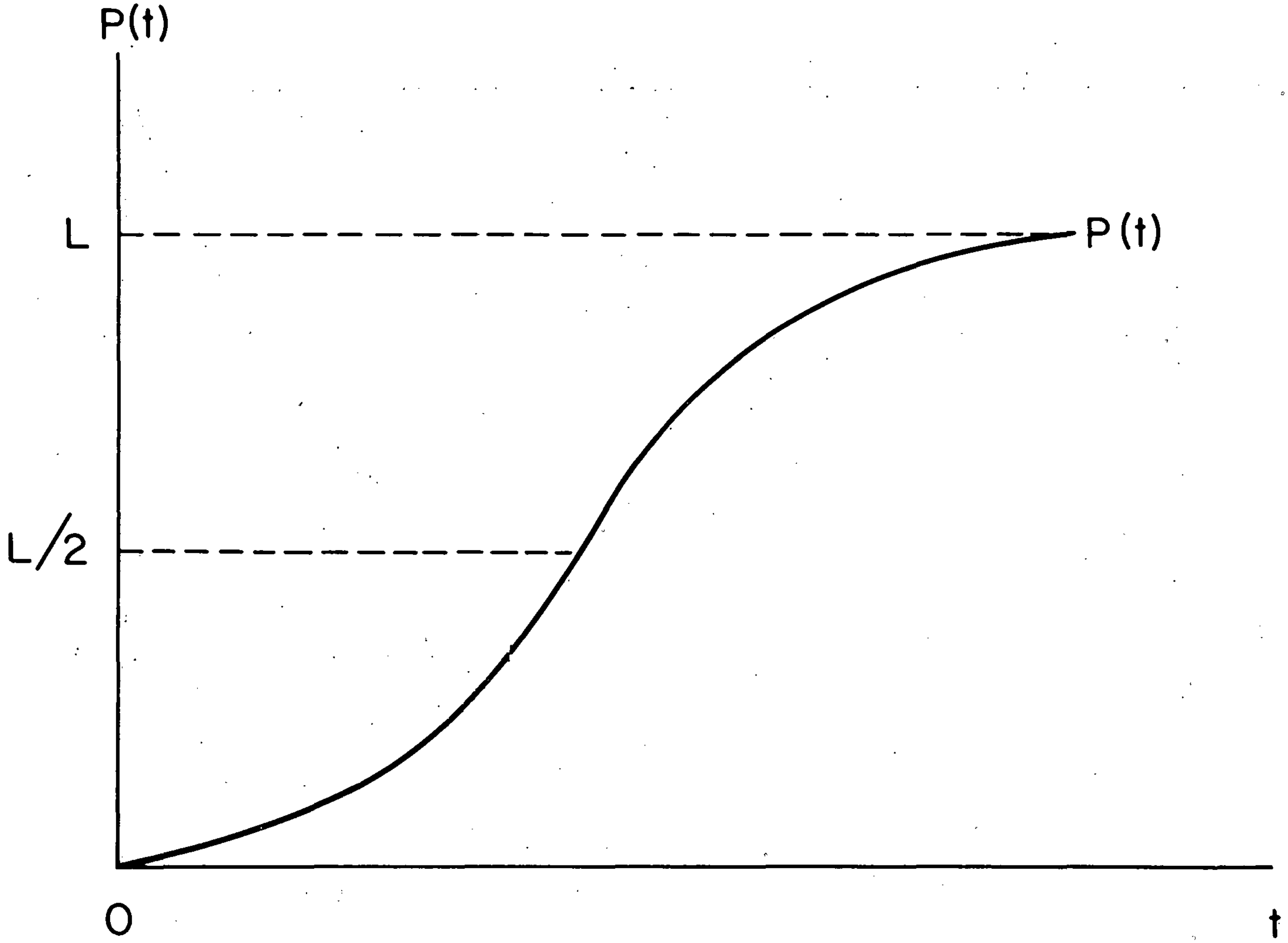

Fig. 2.2. Logistic penetration function.

Finally, the third major class of market penetration functions is not based upon any common behavioral assumption, although certain individual models in this class do postulate specific hypotheses concerning the dynamic process involved. Rather, the rationale for this class of functions rests upon the empirical observation that many (if not most) market penetration curves observed in practice exhibit an asymmetric S-shape with the upper portion of the curve elongated.[10] Figure 2.3 depicts such a curve. Two skewed S-shaped functions that have appeared in the literature are the Gompertz curve given by

$$P(t) = La^{(b^t)}, \quad 0 < a, b < 1, \qquad (2.7)$$

and the cumulative lognormal given by

$$P(t) = L\int_0^t \frac{1}{(2\pi\sigma^2)^{1/2}\theta} \exp\left\{-\frac{1}{2\sigma^2}\left[\log(\theta) - \mu\right]^2\right\} d\theta. \qquad (2.8)$$

The basic feature of this class of functions is a positive first derivative with a second derivative that changes from positive to negative at some $P(t) < L/2$.[11] Depending upon the location of the point of inflection, the skewed S-shaped curve can approximate either the modified exponential or the logistic.

B. Factors Affecting the Rate of Market Penetration

Given an estimate of the time path of market share growth (which is, in essence, nothing more than a concise method for describing the observed growth path) several studies have proceeded to investigate various hypotheses concerning those factors that may be expected to influence the rate of market penetration over time. This second stage of analysis has generally been carried out by specifying and estimating a functional relationship between the estimated value of the slope parameter of a given penetration function (usually the logistic) and a vector of explanatory variables.

The observational units employed in this second stage estimation (i.e., the units across which separate market penetration functions are estimated) are either individual geographic or industrial markets for one given innovation [Griliches (1957) and Romeo (1975)] or aggregate

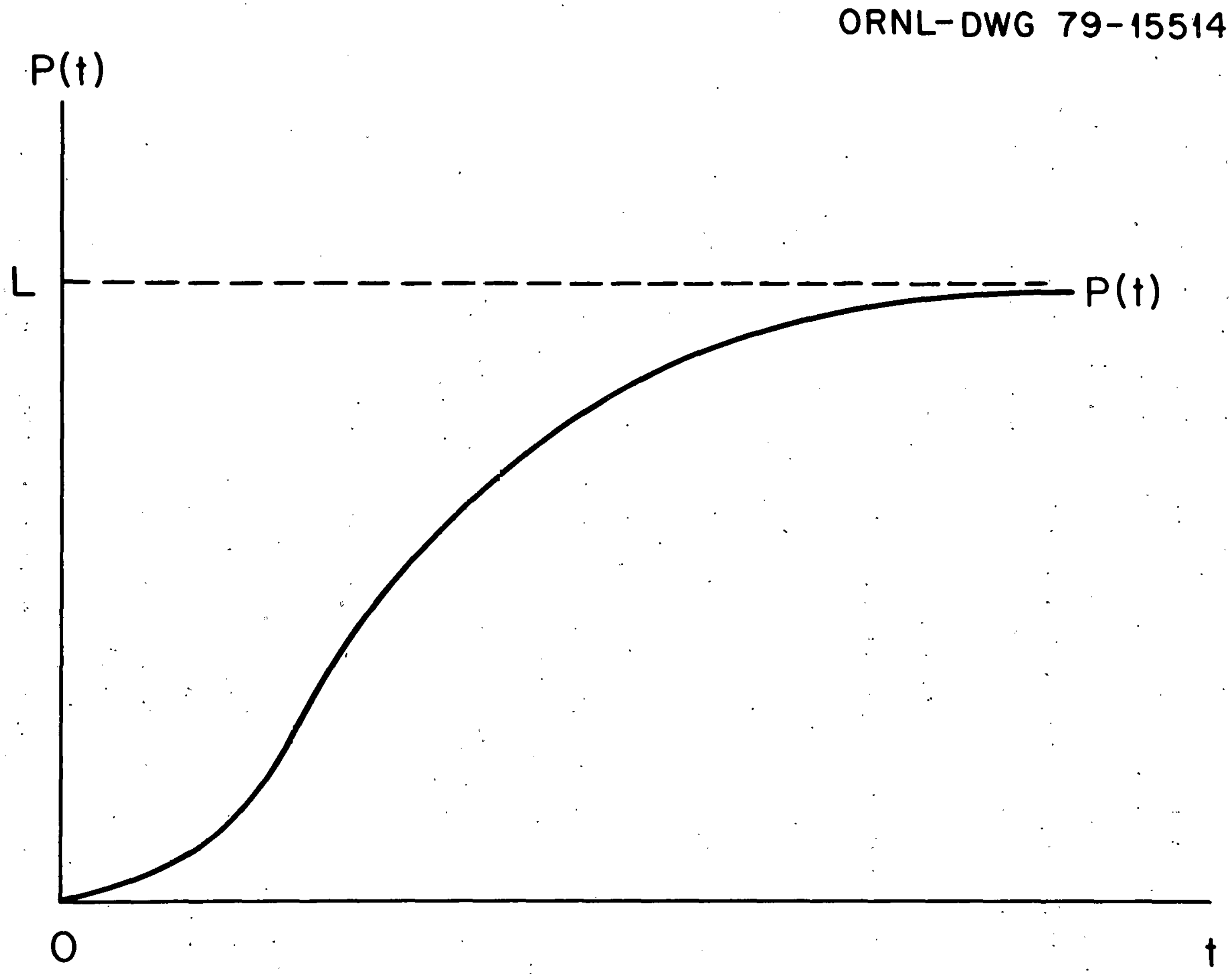

Fig. 2.3. Skewed S-shape penetration function.

markets for a sample of different innovations [Mansfield (1961) and Blackman (1972)]. Across these units, variations in the estimated values of the slope parameters of the penetration functions [e.g., variations in the estimated values of the parameter a from Eq. (2.6) for a sample of innovations] have generally been explained by variations in the measured profitability of introducing the innovation and the size of the investment required for installation. Hypotheses concerning the effects of these variables on the rate of penetration over time are that increases in profitability will accelerate the penetration process and that increases in the size of the required investment will retard it. The rationale often provided for the first hypothesis is obtained by analogy to the biological sciences where it has been found that the speed of response to a stimulus is directly related to the intensity of that stimulus.[12] The second hypothesis is based upon a perceived reluctance to commit a large amount of funds to a new technology and possible difficulties in obtaining financing for relatively costly projects.[13]

The study by Mansfield (1961) illustrates the approach adopted in these investigations and the kinds of results generally obtained. After fitting a logistic penetration function to the data for 12 innovations that were adopted in four industries, the following equation is specified:

$$\hat{a}_{ij} = \beta_{i0} + \beta_1\pi_{ij} + \beta_2 S_{ij} + \mu_{ij}, \qquad (2.9)$$

where π_{ij} is the average profitability of introducing the jth innovation in the ith industry, S_{ij} is the size of the investment required for purchase and installation, $\hat{a}_{ij}$ is the estimated value of the slope parameter of the logistic penetration function for each of the 12 innovations, μ_{ij} is a random disturbance term, and the βs are constants to be estimated. Notice that the intercept of the specified relationship is allowed to vary across the four industries in the sample. This variation is incorporated in order to reflect basic differences among these industries in their inherent inclination to innovate.

As reported by Mansfield, estimation of the parameters of Eq. (2.9) by ordinary least squares yields the following results:

$$\hat{a}_{ij} = \begin{Bmatrix} -.29 \\ -.57 \\ -.52 \\ -.59 \end{Bmatrix} + \underset{(.015)}{.530}\,\pi_{ij} - \underset{(.014)}{.027}\,S_{ij}, \quad r - .997, \qquad (2.10)$$

where, reading from the top, the intercept terms in brackets apply to the brewing industry, the coal industry, the steel industry, and the railroads, respectively. The figures in parentheses are the estimated standard errors of the respective coefficient estimates. The model appears to fit the data quite well and provides empirical support for the hypotheses described above concerning the qualitative influence of π_{ij} and S_{ij} on the observed rate of market penetration. Although the precise causal mechanism through which these effects occur is not made clear, these results do indicate a definite correlation.

Having obtained these results, Mansfield re-estimates Eq. (2.10) incorporating four additional exogenous variables that may be expected to influence the rate of adoption of new innovations. These variables are added to the right-hand side of the estimating equation one at a time.[14] The additional factors included are: (1) d_{ij}, the number of years that typically elapsed before the old equipment was replaced prior to the appearance of the new innovation (to capture the influence of durable fixed equipment); (2) g_{ij}, the annual rate of growth of industry sales during the observational period (to account for use of the innovation in new plants constructed to accommodate industry growth); (3) t_{ij}, the year in which the innovation was introduced (to reflect the increasing efficiency of communications channels over time); and (4) S_{ij}, a binary variable indicating the phase of the business cycle when the innovation was introduced. The results obtained are inconclusive. Although the qualitative effects are in the expected directions, none of the coefficients of these additional variables are statistically significant. Furthermore, inclusion of any of these variables renders the coefficient estimate of S_{ij} insignificant.[15] Consequently, on the basis of this evidence, Mansfield concludes that the primary determinants of the rate of adoption of new innovations are profitability and size of investment.

C. Shortcomings of the Traditional Approach

The seminal papers by Griliches (1957) and Mansfield (1961) provide a useful foundation for the analysis of new product growth by both bringing to light the importance of the market penetration process and describing the time path of diffusion for a variety of innovations. In addition, these papers present initial empirical evidence relating to the determinants of variations in observed growth rates across these innovations. Unfortunately, the approach adopted in these studies inhibits or prohibits a direct application of the methodology employed to many important questions of current concern. More unfortunately, however, subsequent analyses have failed to build upon the foundation provided in these studies and have, instead, continued to apply only slight variations of the original approach. Although such applications have served to substantiate the initial results obtained, confirming both the empirical regularity of the general sigmoid penetration function and the significant role of profitability and size of investment in determining the rate of penetration across innovations, they have failed to refine or extend the methodology employed in these early studies. As a result, the current state of the diffusion literature continues to exhibit certain basic shortcomings that severely limit the range of questions that can be addressed.

The principle shortcoming is that the existing literature does not provide an explicit testable theory of the diffusion process. That is, the estimating equations and empirical hypotheses are not derived from an explicit optimization process wherein potential users of a new innovation attempt to maximize or minimize a given objective function under clearly defined constraints. Instead, specific behavioral assumptions [e.g., the assumptions represented in Eqs. (2.1) and (2.4) above] are employed to generate empirical relationships that appear to agree with a given set of data with no explanation provided as to why one should expect such behavior to be representative of the population in general. Thus, existing models remain essentially descriptive providing little more than *ad hoc* explanations for *ex post* observations. Consequently, the causal links through which variations in exogenous variables lead to anticipated variations in observed patterns of diffusion are never clearly defined.

The descriptive nature of existing diffusion models renders them totally inadequate for long-term forecasting purposes. Such inadequacy stems from three more specific sources. First, restricting the sample to that portion of the potential user population that does, in fact, eventually adopt the innovation under investigation begs the question of forecasting the long-run equilibrium market share that a new product may be expected to attain.[16]

Second, the absence of an explicit theoretical model that is capable of generating a time path of diffusion on the basis of potential user optimization in the presence of exogenous constraints results in a marked inability to select from among the various possible penetration functions in the absence of some historical experience. Given the empirical success enjoyed in particular applications by each of the three basic classes of penetration functions described above, one cannot be confident, on the basis of pre-experience data, that a given innovation will conform to a specific pattern of diffusion that can be represented by one of these functions. Consequently, existing diffusion models require some actual observations on the time path of market share growth in order to establish the appropriate functional form to employ in a given situation.[17] The burden of this requirement renders such models useless in forecasting the market penetration of new products for which market experience data are unavailable. This, in turn, severely limits their applicability to long-range forecasting in areas experiencing rapid technological change.

And third, even if one were willing to make the somewhat heroic assumption that all future market penetration functions would assume the logistic shape and attempt to apply Mansfield's results to emerging products, the incorporation of unexplained industry-specific effects in the empirical model [i.e., allowing the intercept term in Eq. (2.10) to vary across industries] would inhibit forecasting capabilities when the new innovation is directed toward industries for which prior estimates of these effects do not exist. Given anticipated values for π_{ij} and S_{ij} for a given innovation and estimates of β_1 and β_2 in Eq. (2.9), one cannot forecast the future value of $\hat{a}_{ij}$ unless an estimate of β_{i0} exists for the industry in question. Moreover, in the absence of any theory or evidence concerning the determination of variations in β_{i0}, there is little reason to expect such prior estimates to remain valid over time.[18]

Given these problems associated with the traditional approach to the modeling of diffusion, it was decided that a revised methodology should be developed. To be useful in long-range forecasting, such a methodology should exhibit two basic characteristics. First, the model should be founded upon an explicit theory of potential user optimization. And second, the model should be operational in the absence of historical data for the particular innovation under investigation. The following sections are devoted to the derivation of a diffusion model that exhibits these basic characteristics.

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III. CONCEPTUAL FRAMEWORK AND SIMPLIFYING ASSUMPTIONS

A useful framework for the analysis of new product growth is provided by the concept of a series of short-run market share equilibria that approach a stable long-run equilibrium position over time. This market-oriented view of the dynamic process of product innovation was originally suggested by Griliches (1957) and is useful in separating the causal forces involved into supply-side and demand-side phenomena to which existing economic theory may be applied.

The mechanism through which this conceptual framework operates to establish both the time path of diffusion and the upper-limit market share of a new product is shown in Fig. 3.1. In the graph, both the supply curve, $S(t)$, and the demand curve, $D(t)$, for the new product shift outward over time until the long-run equilibrium quantity, $Q(t_n)$, is attained n periods after the introduction of the new innovation. These temporal shifts generate a time path of short-run equilibrium quantities, $P(t)$, which, when divided by the exogenously determined total quantity of competing goods, determines the market penetration function in terms of percentage market shares. If this total quantity is stationary and equal to $Q(t_n)$, then the long-run equilibrium market share of the new product represented in the graph is one. If the total is greater than $Q(t_n)$ [it cannot be less than $Q(t_n)$ by definition], then the long-run equilibrium share is less than one. Short-run equilibrium price, $p(t)$, of course, may rise or fall during the period of adjustment, depending upon the relative shifts in supply and demand over this interval of time. In the graph, equilibrium (market clearing) price declines from $p(t_1)$ to $p(t_n)$ over this period.

In the model developed below, we are concerned with the market penetration of durable good innovations. Consequently, the demand and supply curves in Fig. 3.1 must be interpreted in terms of service flows from installed equipment rather than sales, and the market share that is of interest is the percent of the existing capital stock that embodies the new technology.[19] This stock measure of the extent of diffusion (as opposed to a flow measure in terms of percentage sales of the new good) is the more relevant concept when dealing with durable goods inasmuch as

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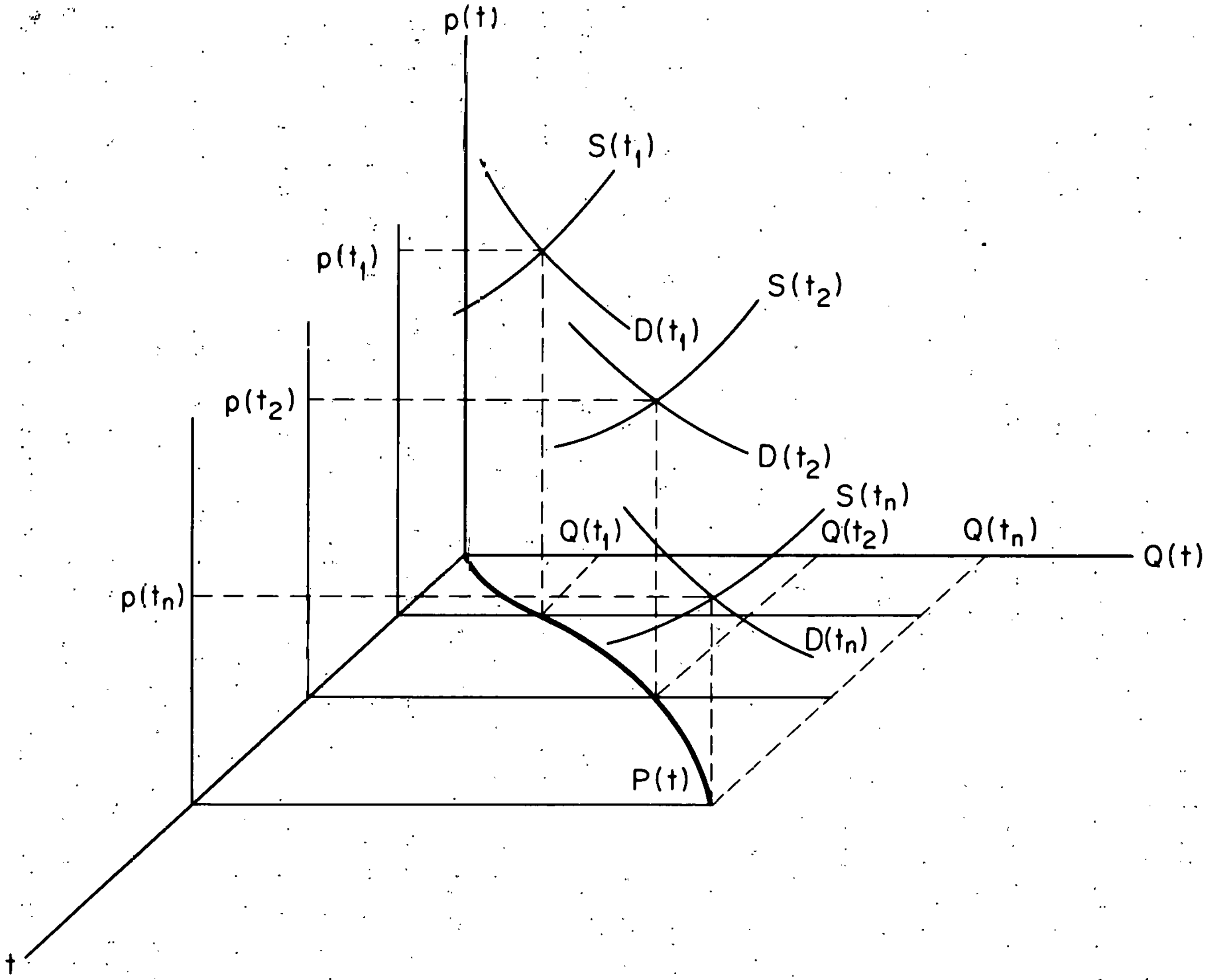

Fig. 3.1. A conceptual framework for analysis of market penetration.

the new technology's potential impact on productivity, energy use, etc., can only become effective as the improved equipment is installed over time. With these interpretive alterations in mind, however, the basic conceptualization depicted in Fig. 3.1 remains valid for the analysis of durable good innovations.

Given this conceptual framework, the derivation of a theory of market penetration becomes a problem in modeling the causal forces that generate temporal shifts in the demand for and supply of services from a new innovation. Outward shifts in the short-run supply curve over time may result from learning-by-doing on the part of the manufacturers of the new product (resulting from both the accumulation of human capital and basic engineering design or organization changes that lower the costs of production), economies of scale in the relevant input supply industries, and entry of new firms into the production of the new good. In the present study, we assume that these potential supply-side influences occur exogenously so that the dynamic adjustment path is endogenously determined by temporal shifts in the demand curve for the new innovation for a given time path of supply prices for the new durable good. In order to simplify our notation, we further assume that, once available on the market, the supply curve of the new product is infinitely price elastic and that this supply curve remains stable in all succeeding periods. This assumption has considerable precedent in the literature on diffusion and allows us to focus our attention on the important determinants of demand-side adjustments.[20]

In order to further simplify the presentation, we also assume that the new innovation is expected to eventually usurp the entire market for the given product class. That is, we assume that the ultimate long-run equilibrium market share of the new good is one. Like the previous assumption, this abstraction has precedent in the antecedent literature.[21] It is not crucial to the analysis but allows us to focus on the causal mechanism that generates the temporal shifts in the demand curve for the new product which, in turn, defines the time path of adjustment.

Finally, in order to focus upon the deterministic causal forces that influence observed diffusion paths (in contrast to the stochastic elements that also influence these paths), we initially depart from the bulk of the existing market penetration literature and assume that potential users of the new innovation make their decisions under conditions of perfect information regarding all parameters relevant to the timing and selection of durable good purchases. This assumption permits us to examine demand-side adjustments in the absence of risk and uncertainty considerations. It is later relaxed in Section V where the effect of new product performance uncertainty on the diffusion of durable good innovations is examined.

Within the confines of these simplifying assumptions, we turn to our examination of the time-dependent demand-side fluctuations that theoretically define the diffusion process.

IV. AN INVESTMENT MODEL OF DURABLE GOOD DIFFUSION

If we define the market share of a new durable good as the percent of equipment in place that embodies the new technology, then a durable good innovation can increase its share of the market in only two ways. First, it can replace previously installed equipment; and second, it can capture new sales that result from overall market expansion. In either case, gross investment in the adopting sector provides the vehicle for new product growth.[22]

The vintage approach and the accelerator principle of conventional investment theory provide the logical framework within which these two basic sources of durable good diffusion can be analyzed. The relative importance of these two sources of investment in determining the time path of short-run market share equilibria will, obviously, vary from one durable good innovation to another, but both sources should play some role in the market penetration of all such innovations.

In the model derived below, we shall treat new investment as exogenous and focus primary attention on replacement investment as the important driving force behind new product market share growth. Since the adoption of new, more efficient, durable goods will generally lead to a fall in the relative price of the final product that is produced with these durable goods which will, in turn, lead to an increase in the quantity of the final product demanded and, consequently, an increase in the rate of growth of the capital stock in the adopting sector, some part of the new investment component will, in fact, be endogenous to the diffusion process. Our decision to refrain from endogenizing this particular component and to emphasize replacement investment in the model at this point stems from two considerations. The first of these was our own perception of the relative importance of these two sources of market share growth in a typical diffusion process.[23] And the second was a desire to keep the model as simple as possible at this stage of development. Later refinements of the model will treat a portion of new investment as endogenous, but, for the moment, it will remain exogenous to the system.

With this choice of emphasis in mind, we turn to our derivation of a theory of durable good diffusion.

A. A Generalized Penetration Function

It will be useful to adopt the following notation:

t_0 = the point in time at which the new durable good innovation is introduced;

t = continuous time since introduction of the new good;

K(t) = the size of the capital stock in the adopting sector at time t; and

R(t) = the cumulative percent of the original capital stock in place at time t_0 that has been replaced with the new equipment at time t.

Our simplifying assumption that the long-run equilibrium level of market penetration is equal to one implies that the new innovation is, in the absence of fixed capital considerations, cost effective in all potential applications. That is, the new durable good completely dominates previously existing competing products in the market place. Therefore, under this assumption, all machines purchased as a result of growth in the total capital stock in the adopting sector will embody the improved technology, and the stock of the new durable good in place at time t that results from cumulative new investment will be

$$S_N(t) = K(t) - K(t_0). \qquad (4.1)$$

Generally, we would expect that $K(t) \geq K(t_0)$ so that $S_N(t) \geq 0$, but this need not always be the case.

As for the second potential source of market share growth, the stock of new equipment in place at time t that results from cumulative replacement investment over the t_0,t time interval will be

$$S_R(t) = R(t)K(t_0). \qquad (4.2)$$

Since $0 \leq R(t) \leq 1$ by definition and $K(t_0) \geq 0$, we must have $S_R(t) \geq 0$.

Summing these stocks of the new durable good that are in place at time t and dividing by the total stock of installed capital equipment, we obtain the percentage market share or level of market penetration of the new innovation at time t as

$$P(t) = [S_N(t) + S_R(t)]/K(t). \tag{4.3}$$

Substituting from (4.1) and (4.2),

$$P(t) = [K(t) - K(t_0) + R(t)K(t_0)]/K(t) \tag{4.4}$$

$$= 1 - [1 - R(t)]K(t_0)/K(t). \tag{4.5}$$

Finally, the percentage growth of the total capital stock in the adopting sector over the t_0,t time interval is, by definition,

$$G(t) = \frac{K(t) - K(t_0)}{K(t_0)} . \tag{4.6}$$

Therefore,

$$\frac{K(t_0)}{K(t)} = \frac{1}{1 + G(t)} , \tag{4.7}$$

which, when substituted into (4.5), gives our generalized market penetration function in terms of percentage cumulative replacements and percentage growth in the total capital stock as

$$P(t) = 1 - \frac{1 - R(t)}{1 + G(t)} . \tag{4.8}$$

As noted above, we treat G(t) as exogenous to the model at this point and focus our attention on the economic relationships involved in the determination of R(t). This will be the subject of the next two subsections of the report. Following this, we will return to equation (4.8) to examine some important properties of the generalized P(t) function and to explore some comparative statics of the relationships that are derived.

B. The Individual Replacement Criterion

Consider a production process that extends over a finite period of time and that employs a given durable good in the manufacture of some final product. This product may be a good or service that is sold on the market (if the durable is a producer good) or a commodity that is both produced and consumed within the household (if the durable is a consumer good). Assume that the decision maker controlling the production of this final product has the objective of minimizing the present value of the total costs of production over some given period of time. Further, assume that the relevant planning horizon extends over a sufficient time interval to make replacement of the given durable good necessary at one or more points.

The decision of whether and when to replace a given piece of installed equipment is an economic one that, within our assumed objective of dynamic cost minimization, depends upon the relative cost characteristics of the new innovation and the existing equipment.[24] In order to examine this decision process, it will be useful to adopt the following notation:

- F_i = capital (or fixed) cost of the ith durable good inclusive of the cost of installation, where i denotes temporal ordering;
- $c_i(t)$ = operating and maintenance (or variable) cost of production using the ith durable good, which is assumed to increase monotonically with elapsed time since installation;
- t_i^* = optimal age to replacement of the ith durable good;
- T = total length of the planning horizon; and
- r = continuous rate of discount.

The actual cause behind increasing variable cost over time for installed equipment is not important to our analysis at this point. Such cost increases may stem from physical deterioration of the equipment in place, secular increases in the price of complementary inputs (e.g., fuel), or increasing obsolescence due to technological change (if costs are considered to be in relative terms).[25] The second category, input price increases, may provide an important incentive for replacement investment related to energy using durable goods, even in the absence of technological change.

Using the above notation, we may write the present value of the total costs of production over the 0,T time interval as

$$C = F_1 + \int_0^{t_1^*} c_1(t)e^{-rt}dt + e^{-rt_1^*}\left[F_2 + \int_0^{t_2^*} c_2(t)e^{-rt}dt\right]$$

$$+ e^{-r(t_1^* + t_2^*)}\left[F_3 + \int_0^{t_3^*} c_3(t)e^{-rt}dt\right] + \cdots + e^{-r\sum_{j=1}^{n-1} t_j^*}\left[F_n + \int_0^{t_n^*} c_n(t)e^{-rt}dt\right], \quad (4.9)$$

since the ith machine in the n machine sequence must be purchased at time $t = \sum_{j=1}^{i-1} t_j^*$. Defining $t_0^* = 0$, this expression may be written as

$$C = \sum_{i=1}^{n}\left\{e^{-r\sum_{j=0}^{i-1} t_j^*}\left[F_i + \int_0^{t_i^*} c_i(t)e^{-rt}dt\right]\right\}. \quad (4.10)$$

Given knowledge of the relevant parameters F_i and r and the functions $c_i(t)$, i = 1,...,n, the decision maker selects the optimal ages to replacement, t_i^*, that minimize C.

The first-order conditions necessary for such minimization are given by

$$\frac{\partial C}{\partial t_i^*} = c_i(t_i^*)e^{-r\sum_{j=1}^{i} t_j^*} - re^{-r\sum_{j=1}^{i} t_j^*}\left[F_{i+1} + \int_0^{t_{i+1}^*} c_{i+1}(t)e^{-rt}dt\right]$$

$$- re^{-r\sum_{j=1}^{i+1} t_j^*}\left[F_{i+2} + \int_0^{t_{i+2}^*} c_{i+2}(t)e^{-rt}dt\right] - \cdots \quad (4.11)$$

$$-re^{-r\sum_{j=1}^{n-1} t_j^*}\left[F_n + \int_0^{t_n^*} c_n(t)e^{-rt}dt\right] = 0, \; i = 1,\ldots,n-1.$$

The optimal age to retirement of the last machine in the sequence, t_n^*, is determined by the exogenously assigned length of the planning horizon, T, and the condition that

$$T = \sum_{i=1}^{n} t_i^*. \tag{4.12}$$

Equation (4.11) may be rewritten as

$$c_i(t_i^*) = r \sum_{j=i+1}^{n} \left\{ e^{-r \sum_{k=i+1}^{j-1} t_k^*} \left[F_j + \int_0^{t_j^*} c_j(t) e^{-rt} dt \right] \right\}, \tag{4.13}$$

where $\sum_{k=i+1}^{j-1} t_k^* = 0$ when $j = i + 1$. This necessary condition for dynamic cost minimization implies that *replacement of an installed piece of equipment should occur at the age at which the variable cost of operation using the existing machine has risen to equality with the rate of discount times the sum of the total life cycle costs of production over the remainder of the planning horizon discounted back to the time of replacement.* Since r is the value of a dollar per unit of time to the decision maker, r times the discounted sum of the life cycle costs of operation over the remainder of the planning horizon is the marginal opportunity cost of delaying installation of the new equipment one time period; that is, it is the cost of having shifted the cost stream back (earlier) one unit of time by having not delayed replacement one more period. Optimality then requires that this cost should be equal to the cost of operation using the installed equipment at the moment at which replacement occurs.

If $c_i(t)$ is monotonically increasing in t, equation (4.13) may be solved for t_i^*, the optimal age to replacement of the ith machine.[26] The solution is given by

$$t_i^* = c_i^{-1} \left\langle r \sum_{j=i+1}^{n} \left\{ e^{-r \sum_{k=i+1}^{j-1} t_k^*} \left[F_j + \int_0^{t_j^*} c_j(t) e^{-rt} dt \right] \right\} \right\rangle, \tag{4.14}$$

$$i = 1, \ldots, n-1,$$

where $c_i^{-1}(\cdot)$ is the inverse function of $c_i(t)$ and is, therefore, monotonically increasing.

Notice that the determination of t_i^* from equation (4.14) requires knowledge of $t_{i+1}^*, t_{i+2}^*, \ldots, t_n^*$. Thus, in calculating the optimal ages to replacement of the n machines in the sequence, the decision maker must solve the entire system of n-1 equations given in (4.15) plus the additional relationship given in (4.12) simultaneously. This simultaneous characteristic of the solution to the equipment replacement decision problem was originally pointed out by Terborgh (1949, p. 57):

> "It is evident from the foregoing that the predictive requirements of replacement analysis extend far beyond the forecasting of the future performance of presently available machines. The analyst must appraise also a series of machines *not now in existence*. If it is permissible so to describe the potentiality of devices still unborn, these machines are ghosts.
>
> Ghosts though they be, it is impossible successfully to exorcise them. For since the choice between living machines can be made only by reference to the machines of tomorrow, the latter remain, whether we like it or not, an indispensable element in the calculation. It may be said, indeed, without too much exaggeration, that the appraisal of the ghosts involved is the heart of the replacement analysis. No replacement theory, no formula, no rule of thumb that fails to take cognizance of these ghosts and to assess their role in the play can lay claim to rational justification."

Terborgh's ghosts obviously impose a substantial information problem on the decision maker pondering the replacement issue. Determination of the optimal age to replacement of an existing machine requires knowledge of all future equipment prices, operating costs, and technological alternatives over the remainder of the planning horizon. A later section of this report will examine the impact of such uncertainties on the timing of equipment replacements; but, for present purposes of theoretical development, we will assume that this information burden is overcome by the decision maker. In practical situations, however, it is clear that predictions, forecasts, expectations, and blind guesses will play an important role in the replacement decision process.

C. The Aggregate Time Path of Cumulative Percentage Replacements

Equation (4.14) defines the optimal age to replacement for the ith machine in the sequence of n machines employed by an individual decision maker in producing some level of final output over the 0,T planning horizon. Without loss of generality, we may suppose that the durable good innovation that we are interested in is introduced on the market at time t_0, where $\sum_{j=1}^{i-1} t_j^* \leq t_0 \leq \sum_{j=1}^{i} t_j^*$. That is, we assume that the new product becomes available after installation of the ith machine in the sequence but before installation of the i + 1th machine in the sequence for all individuals in the population of potential adopters. Therefore, $c^{-1}(\cdot)$ in equation (4.14) will represent the inverse of the function that relates variable costs of operation to elapsed time since installation for the equipment in place at t_0; F_{i+1} and $\int_0^{t_{i+1}^*} c_{i+1}(t)e^{-rt}dt$ will represent the capital cost and the discounted life-cycle operating cost of the new durable good that is to replace the installed equipment;[27] and F_{i+k} and $\int_0^{t_{i+k}^*} c_{i+k}(t)e^{-rt}dt$, $k = 2,\ldots,n-i$, will represent the cost characteristics of all later planned installations.

Then, given some estimate of the optimal age to replacement of the existing equipment with the new innovation, t_i^*, we need only consider the age structure of the aggregate stock of installed machines at the point in time at which the new innovation is introduced in order to determine the fraction of the original (old technology) equipment that will have been replaced with the new durable good in any following time period.[28] That is, the distribution of the ages of the installed equipment at time t_0 provides the necessary information for aggregating individual replacement decisions into a consistent replacement investment function.

Letting $f_{t_0}(A)$ denote the relative frequency distribution of the ages of machines in the existing stock at t_0, we will have

$$\int_0^{\infty} f_{t_0}(A)dA = 1, \tag{4.15}$$

since no machine can have a negative age. It is not necessary to place any restrictions on this distribution at this point except that it be confined to the positive real line.[29]

At time t_0, all installed machines of age t_i^* or greater must be replaced with the new innovation for discounted cost to remain at a minimum. Therefore, the fraction of existing equipment that should be replaced immediately when the new durable good becomes available, which we denote $R(t_0)$, is

$$R(t_0) = \int_{t_i^*}^{\infty} f_{t_0}(A)dA \geq 0. \tag{4.16}$$

If the introduction of the new innovation extends the optimal age to replacement of existing equipment (i.e., if $t_i^* > t_{i-1}^*$), then some machines that would have ordinarily been due for replacement will be kept in operation beyond their "normal" lifetime of t_{i-1}^*. In this case, there will exist no machines of age t_i^* or greater, and $R(t_0) = 0$. Also, $R(t)$ will remain at zero until time $t = t_i^* - t_{i-1}^*$ (where, recall, we are measuring t from t_0, i.e., $t = 0$ at t_0). But if the cost characteristics of the new good act to lower the optimal age to replacement of existing equipment (i.e., if $t_i^* < t_{i-1}^*$), then some fraction of the existing stock will be replaced immediately with the appearance of the new durable good, and $R(t_0) > 0$.

For time periods following the introduction of the new innovation (i.e., for $t > 0$), the cumulative fraction of the original stock of equipment in place at t_0 that will optimally be replaced with the new innovation will increase over time at a rate that is dependent upon the value of t_i^* and the shape of the $f_{t_0}(A)$ distribution. Cumulative percentage replacement will, then, be given by

$$R(t) = \int_{t_i^*-t}^{\infty} f_{t_0}(A)dA \geq 0. \tag{4.17}$$

Together, equations (4.15) and (4.17) imply that the replacement process will have run to completion t_i^* units of time after the introduction of the new innovation since

$$R(t_i^*) = \int_{t_i^*-t_i^*}^{\infty} f_{t_0}(A)dA$$

$$= \int_0^{\infty} f_{t_0}(A)da \qquad (4.18)$$

$$= 1.$$

Figures 4.1 and 4.2 depict the aggregate time path of cumulative replacements generated by this model. In both figures, $f_{t_0}(A)$ assumes the same approximate lognormal distribution with A_0 denoting the age of the oldest machine in the stock. In Fig. 4.1, $t_i^* < A_0$, so $R(t_0) > 0$. $R(t)$ then increases from this initial value according to the cumulative distribution of $f_{t_0}(A)$ to a value of one t_i^* units of time after introduction of the new product. In Fig. 4.2, $t_i^* > A_0$, so $R(t_0) = 0$ and remains at that value until time $t = t_i^* - A_0$. Then, $R(t)$ increases according to the cumulative distribution of $f_{t_0}(A)$ to a value of one at time $t = t_i^*$. Since the $f_{t_0}(A)$ distribution is the same in these two figures, the considerably delayed growth of $R(t)$ in Fig. 4.2 is entirely due to the increased value of t_i^* between these graphs which, in turn, must be due to a divergence in the cost characteristics exhibited by the two hypothetical innovations (i.e., $F_{i+1} + \int_0^{t_{i+1}^*} c_{i+1}e^{-rt}dt$ must be smaller in Fig. 4.1).

D. The Structural Penetration Function

The results of the two preceding subsections may now be used to write our generalized market penetration function in terms of its underlying structural components. Substituting equation (4.17) into (4.8), we obtain

$$P(t) = 1 - \left[1 - \int_{t_i^*-t}^{\infty} f_{t_0}(A)dA\right]\frac{1}{1 + G(t)} . \qquad (4.19)$$

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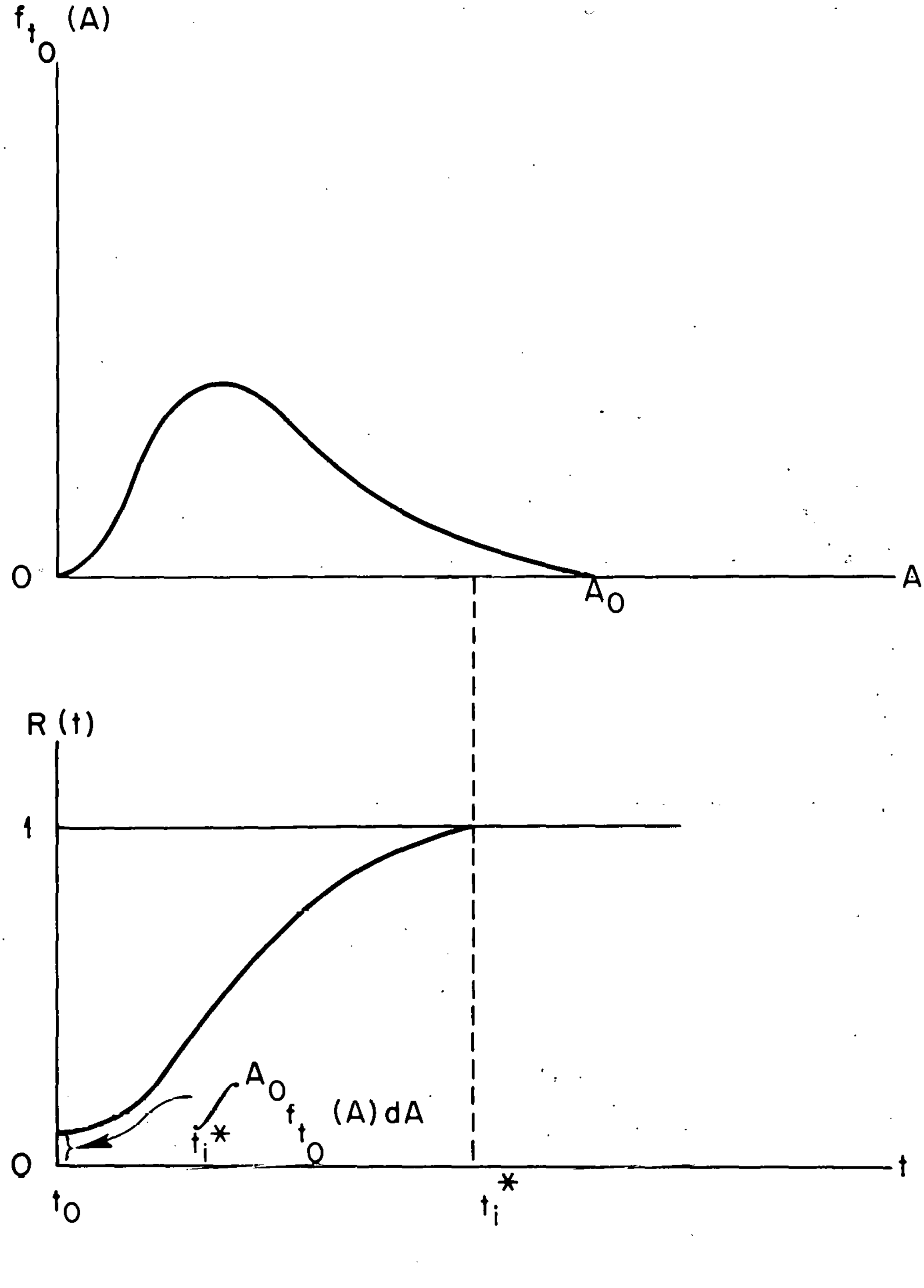

Fig. 4.1. Time path of R(t) with $t_i^* < A_0$.

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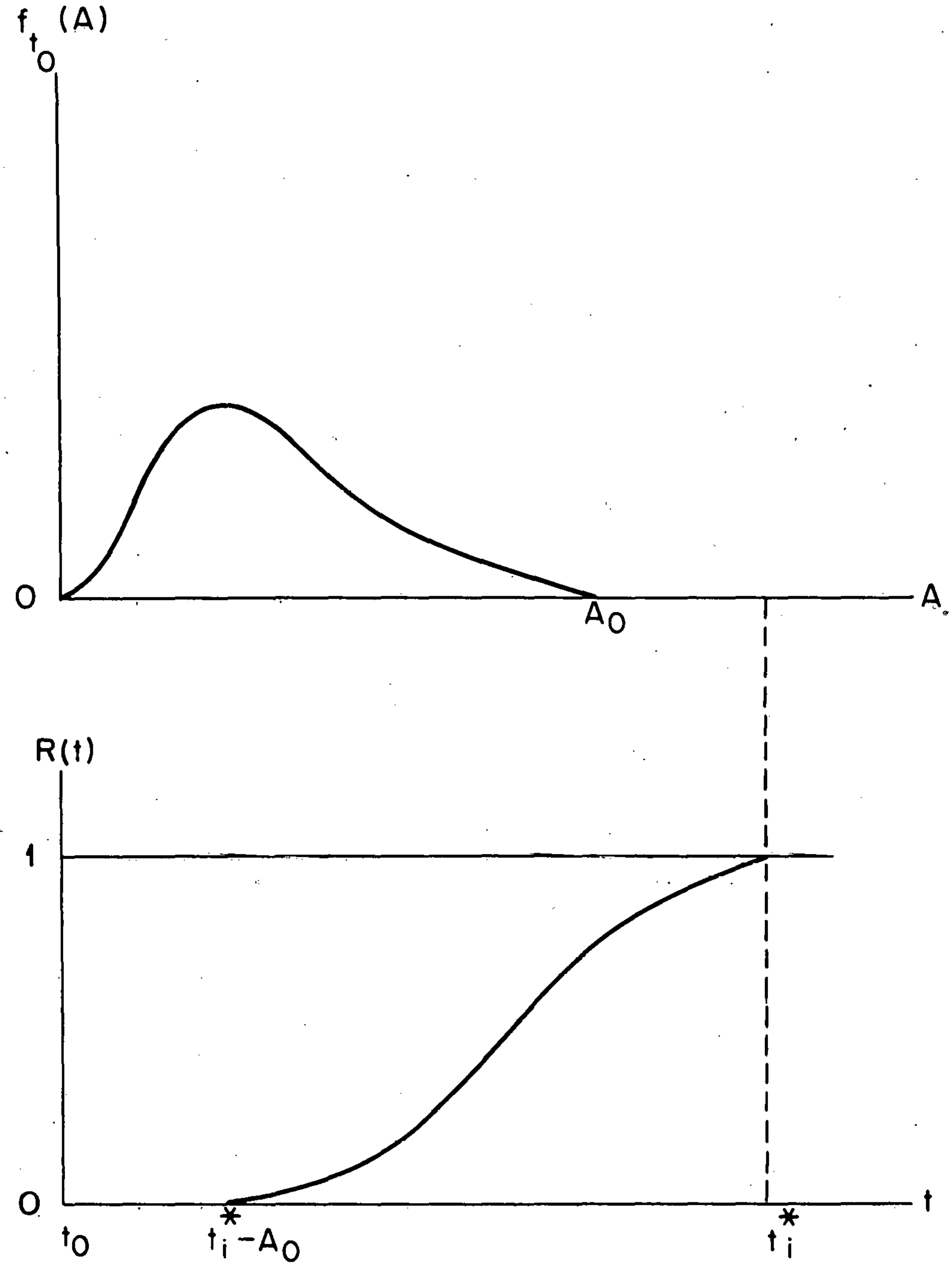

Fig. 4.2. Time path of R(t) with $t_i^* > A_0$.

Then, substituting (4.14) for t_i^* in this expression, we have

$$P(t) = 1 - \left[1 - \int_{c_i^{-1}\left\langle r \sum_{j=i+1}^{n} \left\{ e^{-r \sum_{k=i+1}^{j-1} t_k^*} \left[F_j + \int_0^{t_j^*} c_j(t)e^{-rt}dt\right]\right\}\right\rangle - t}^{\infty} f_{t_0}(A)dA\right]^{\frac{1}{1 + G(t)}} \quad (4.20)$$

as our final structural equation.

This expression clearly demonstrates the functional dependence of the percentage penetration of a new durable good on (1) the capital cost and discounted life cycle operating cost of the new innovation; (2) the discount rate; (3) the rate of increase over time in the variable cost of operation using the installed equipment; (4) the age distribution of the existing capital stock; (5) the percentage growth of the total stock of equipment in place in the adopting sector; and (6) time. Clearly, the relative importance of these various factors will vary from one durable good innovation to another (or, for a given innovation, from one adopting sector to another); but as long as the decision makers involved attempt to minimize the discounted cost of operation, these variables will influence the time path of diffusion of the new product.

In addition to portraying the above structural relationships, the market penetration function given in (4.20) exhibits several properties that are in agreement with both *a priori* expectations and prior empirical observations concerning the time path of new product growth. First, since the total capital stock in the adopting sector cannot increase instantaneously, at the time at which the new innovation is introduced the percentage market share of the new good should be equal to the percent of installed equipment that is immediately replaced [i.e., $P(t_0)$ should equal $R(t_0)$]. This property is easily verified. At t_0, $K(t) = K(t_0)$, so, from (4.6), $G(t) = 0$. Substituting this into (4.8) and simplifying, we have

$$P(t_0) = R(t_0). \quad (4.21)$$

Thus, our generalized penetration function fulfills this initial condition.[30]

Second, under our simplifying assumption that the long-run equilibrium market share of the new durable good is one, all new investment undertaken as a result of capital stock expansion will embody the new technology. In that case, we should expect the long-run equilibrium market share to be attained at that point in time at which the replacement process has run to completion (i.e., at $t = t_i^*$). Using equation (4.18), we have from (4.8) that

$$P(t_i^*) = 1, \tag{4.22}$$

so this terminal condition is also satisfied.

Third, in the absence of exogenous influences such as the appearance of additional innovations, the market share of the new good should increase monotonically over time from its initial value at t_0 to its long-run equilibrium value at t_i^*. That is, $\partial P(t)/\partial t$ should be nonnegative throughout. From (4.8), we have

$$\frac{\partial P(t)}{\partial t} = \frac{[1 + G(t)] \frac{\partial R(t)}{\partial t} + [1 - R(t)] \frac{\partial G(t)}{\partial t}}{[1 + G(t)]^2} . \tag{4.23}$$

From (4.17), $\partial R(t)/\partial t = f_{t_0}(t) \geq 0$, and $\partial G(t)/\partial t \geq 0$ by assumption. Then, since $0 \leq R(t) < 1$, $\partial P(t)/\partial t \geq 0$. Thus, $P(t)$ is nondecreasing in t.

In addition to fulfilling these prior conditions, the generalized market penetration function given in equations (4.8), (4.19), and (4.20) has the important property of being amenable to forecasting techniques prior to or at the time of introduction of the new innovation. All components of the structural equation are either observable or predictable at t_0 or before: t_i^* may be calculated via expression (4.14) from engineering estimates of the capital cost and life cycle operating cost of the new good under investigation and the rate of increase in the operating cost of the installed equipment; $f_{t_0}(A)$ is observable at t_0 or may be predicted prior to that time; and $G(t)$ may be predicted from growth forecasts for the adopting sector. Consequently, one can apply this diffusion model to new products which have not yet been introduced on the market.

Clearly, significant empirical difficulties are likely to be encountered in obtaining precise estimates of the various structural components. The appropriate value for t_i^* will vary across the stock of installed equipment to the extent that this stock is not perfectly homogeneous. Also, Terborgh's ghosts of technology future will return to haunt us in any calculation of t_i^*, even within a homogeneous capital stock. The age distribution of the installed stock of capital equipment, while potentially observable at t_0, is seldom available in the published data and will also suffer from nonhomogeneity of the machines in place. And, finally, predictions of the percentage growth of the capital stock in the adopting sector will be subject to forecasting uncertainties.

Despite these empirical problems of implementation, however, consideration of the available alternatives leaves little doubt concerning the usefulness of the above approach. The perennial problem encountered in market penetration analysis concerning the inability to select an appropriate functional form to represent the relationship between percentage market share and elapsed time since introduction of a new innovation on the basis of pre-experience information can be overcome through use of the structural model derived above.[31]

E. Some Comparative Static Results

The penetration function given in expression (4.20) has been derived from our maintained hypothesis that potential users of a new durable good innovation behave in accordance with the objective of dynamic cost minimization. This theory of durable good diffusion implies the existence of certain causal relationships between the market share of the new product at time t and the values of the exogenous variables that enter on the right-hand side of the P(t) function. Such implied relationships constitute the qualitative hypotheses of the model and are briefly examined in this section of the report.

First, it is easily shown from equation (4.8) that *increases in the percentage growth of the capital stock in the adopting sector lead to increases in the level of market penetration.* From (4.8), we have

$$\frac{\partial P(t)}{\partial G(t)} = \frac{1 - R(t)}{[1 + G(t)]^2} \geq 0, \tag{4.24}$$

where the inequality holds in all periods in which R(t) < 1 (i.e., prior to $t = t_i^*$). Thus, market penetration can be expected to proceed more rapidly in markets that are experiencing growth.

Next, it can be shown that *increases in the capital cost or the discounted life-cycle operating cost of the new durable good will lead to decreases in the level of market penetration.* To show this, we note that

$$\frac{\partial P(t)}{\partial F_{i+1}} = \frac{\partial P(t)}{\partial R(t)} \frac{\partial R(t)}{\partial t_i^*} \frac{\partial t_i^*}{\partial F_{i+1}} . \tag{4.25}$$

Then, from equation (4.8),

$$\frac{\partial P(t)}{\partial R(t)} = [1 + G(t)]^{-1} > 0. \tag{4.26}$$

From equation (4.17),

$$\frac{\partial R(t)}{\partial t_i^*} = - f_{t_0}(t_i^*) \leq 0. \tag{4.27}$$

And from equation (4.14),

$$\frac{\partial t_i^*}{\partial F_{i+1}} = r \frac{\partial c_i^{-1}(x)}{\partial x} > 0, \tag{4.28}$$

where we have defined

$$x = r \sum_{j=i+1}^{n} \left\{ e^{-r \sum_{k=i+1}^{j-1} t_k^*} \left[F_j + \int_0^{t_j^*} c_j(t) e^{-rt} dt \right] \right\}, \tag{4.29}$$

so that, from (4.14), $t_i^* = c_i^{-1}(x)$ and $\partial c_i^{-1}(x)/\partial x > 0$ since $c_i^{-1}(\cdot)$ is monotonically increasing. With this definition, expression (4.28) is obtained from the chain rule, $\partial t_i^*/\partial F_{i+1} = [\partial c_i^{-1}(x)/\partial x] \cdot \partial x/\partial F_{i+1}$. Then, substituting equations (4.26), (4.27), and (4.28) into equation (4.25), we have

$$\frac{\partial P(t)}{\partial F_{i+1}} = - [1 + G(t)]^{-1} f_{t_0}(t_i^*) r \frac{\partial c_i^{-1}(x)}{\partial x} \leq 0. \tag{4.30}$$

Also, from (4.29), we see that

$$\frac{\partial x}{\partial \int_0^{t_{i+1}^*} c_{i+1}(t) e^{-rt} dt} = \frac{\partial x}{\partial F_{i+1}} = r, \tag{4.31}$$

so that,

$$\frac{\partial P(t)}{\partial \int_0^{t_{i+1}^*} c_{i+1}(t) e^{-rt} dt} = \frac{\partial P(t)}{\partial F_{i+1}} \leq 0. \tag{4.32}$$

Thus, the lower the capital cost or the discounted life-cycle operating cost of the new innovation, the faster the penetration process can be expected to proceed.

Finally, while it is not possible to fully explore the implications of variations in the parameters of the $f_{t_0}(A)$ distribution without imposing a specific assumption about the form of the distribution, we can show from our general model that *increases in the age of the oldest machine in the stock of equipment in place at* t_0 *will lead to increases in the level of market penetration.* Letting A_0 denote the age of the oldest machine in the capital stock, we may rewrite equation (4.17) as

$$R(t) = \int_{t_i^*-t}^{A_0} f_{t_0}(A) dA, \tag{4.33}$$

since $f_{t_0}(A) = 0$ for all $A > A_0$. Then,

$$\frac{\partial R(t)}{\partial A_0} = f_{t_0}(A_0) > 0. \tag{4.34}$$

Since $\partial P(t)/\partial A_0 = [\partial P(t)/\partial R(t)] \cdot \partial R(t)/\partial A_0$, equations (4.26) and (4.34) imply that

$$\frac{\partial P(t)}{\partial A_0} = f_{t_0}(A_0)[1 + G(t)]^{-1} > 0. \tag{4.35}$$

Intuitively, this result implies that a concentration of old equipment in the capital stock of the adopting sector will facilitate new product growth through accelerated replacements.

Expressions (4.24), (4.30), (4.32), and (4.35) represent testable hypotheses that have been derived from our neoclassical theory of durable good diffusion. Obviously, similar hypotheses concerning the influence of changes in the discount rate and the rate of increase in the variable cost of operation using installed (old technology) equipment could also be derived if specific functional forms were adopted to represent the generalized functional relationships employed in the model. Those familiar with existing empirical studies of the market penetration process will recognize shades of these various hypotheses throughout much of the published literature on this subject (e.g., Mansfield's (1961) study contains hypotheses concerning the influence of the size of the initial investment and profitability of adoption of a new innovation that may be easily interpreted in terms of expressions (4.30) and (4.32)]. Other representations of these hypotheses may be found in the extant literature, but a thorough testing of all the implications of this theory must await the collection of more complete data.

V. THE IMPACT OF PERFORMANCE UNCERTAINTY ON NEW PRODUCT GROWTH

All results derived thus far have been obtained under the assumption that decision makers faced with the option of replacing existing equipment with the new durable good have perfect information concerning the time paths of the operating and maintenance costs of all future machines. Such information requires exact knowledge of the technological and performance characteristics of these machines and all future input prices (i.e., Terborgh's ghosts must be known with complete certainty). Clearly, this amount of information will not be available in practice, and, as a result, the potential adopter of a new durable good innovation will be forced to make the equipment replacement decision under conditions of imperfect information. In this section, we briefly examine the effects of new product performance uncertainty on the optimal age to replacement of existing equipment and, through this, the impact of imperfect information on the time path of new product growth.

A. The Decision Maker's Objective Function Under Uncertainty

Our analysis of the replacement decision under uncertainty is conducted in an expected utility framework. This approach has its origins in the game theoretic work of von Neumann and Morgenstern (1944). Its subsequent adoption in the analysis of choices involving risk stems from the classic paper by Friedman and Savage (1948).

In the present context, we specify a utility function for the decision maker that is defined in terms of the present value of either profits (if the new innovation is a producer durable) or net value (if the new innovation is a consumer durable). In either case, the argument of the decision maker's utility function is assumed to be given by

$$\pi = \int_0^T V(t)Q(t)e^{-rt}dt - \tilde{C}, \tag{5.1}$$

where V(t) is interpreted as either the per unit output price (for producer durables) or the per unit implicit value (for consumer durables) at time t, and Q(t) is the quantity of output produced by the durable good at time t. Thus, the first term on the right-hand side of expression (5.1) is the discounted total revenue or total value of the output stream from the durable good over the 0,T planning horizon. The second term on the right-hand side, $\tilde{C}$, is defined as

$$\tilde{C} = \sum_{i=1}^{n} \left\{ e^{-r \sum_{j=0}^{i-1} \tilde{t}_j^*} \left[F_i + \int_0^{\tilde{t}_i^*} c_i(t,\mu_i) e^{-rt} dt \right] \right\}, \qquad (5.2)$$

where the $\mu_i, i = 1,\ldots,n$, are independently distributed random variables with associated densities $f_i(\mu_i)$ that describe the decision maker's subjective probability distributions over the uncertain operating and maintenance costs of all future machines. The $\tilde{t}_i^*, i = 1,\ldots,n$, are the optimal ages to replacement of the n machines in the sequence, and all other variables are defined as in the preceding section of this report. Thus, $\tilde{C}$ represents the present value of the total costs of production over the 0,T planning horizon, which is now random because of the uncertainty surrounding the performance characteristics of future machines. Given this randomness in $\tilde{C}$, π will also be random.

We shall refer to π as the discounted net value function and assume that the individual charged with the responsibility for making the replacement decision has a von Neumann-Morgenstern utility function that is given by

$$U = U(\pi), \qquad (5.3)$$

where this function is assumed to be smooth and twice differentiable, with $U' > 0$ and $U'' \gtrless 0$ (where U' denotes the first derivative of U with respect to π).

As shown by Friedman and Savage (1948), the behavior of the individual decision maker in an uncertain environment is determined by the shape of this utility function. If this function is concave, as in Figure 5.1, the individual is said to be risk averse. Such an individual will refuse all actuarially fair gambles, because the increment in utility from a gain in discounted net value from $E(\pi)$ to π_2 is less than the decrement in utility suffered from an equivalent loss in discounted net value from $E(\pi)$ to π_1. In general, concavity of the utility function implies that $U[E(\pi)] > E[U(\pi)]$, which, in turn, implies a willingness to pay some positive amount for the opportunity to exchange an uncertain

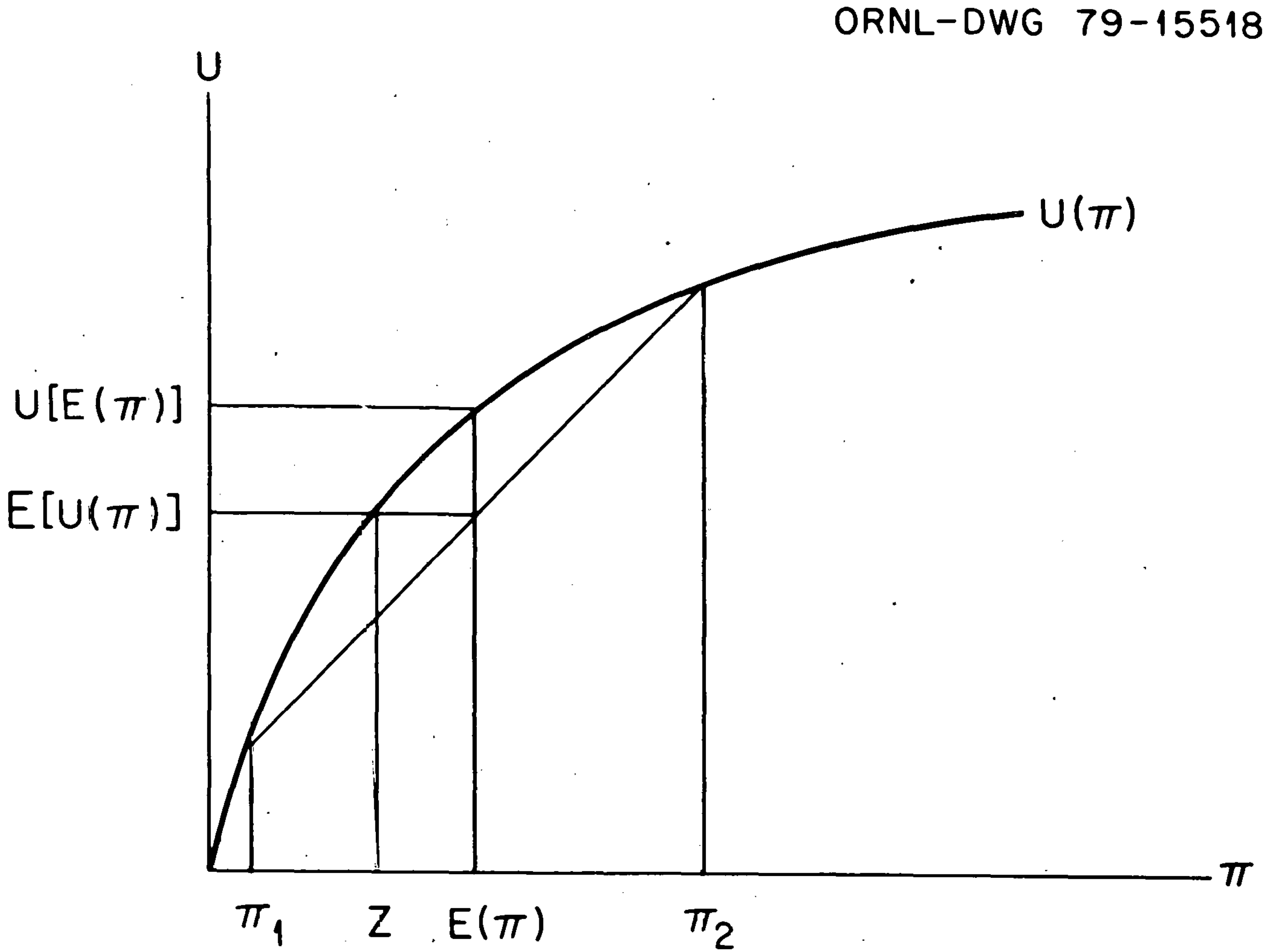

Fig. 5.1. The utility function of a risk-averse individual.

outcome with a given expected net value, say $E(\pi)$, for a certain outcome with a known net value equal to that expected net value. The maximum amount that the individual is willing to pay for the avoidance of the risky situation is referred to as a "risk premium," which we denote by ρ, and is given in the graph by

$$\rho = E(\pi) - z > 0, \tag{5.4}$$

where

$$z = U^{-1} \left\{ E[U(\pi)] \right\} . \tag{5.5}$$

In contrast, convexity of the utility function implies a willingness to pay for the opportunity to exchange a certain outcome for an uncertain outcome with equivalent expected value and, hence, a negative risk premium (i.e., $\rho < 0$). Individuals characterized by such a utility function are said to have a preference for risk.

And finally, linearity of the utility function implies indifference between a certain outcome and an uncertain outcome with the same expected value, which, in turn, implies a $\rho = 0$. Such individuals are said to be risk neutral.

The individual's attitude toward risk is, therefore, reflected in the sign of the second derivative of U with respect to π, with $U'' < 0$ implying risk aversion, $U'' = 0$ implying risk neutrality, and $U'' > 0$ implying a preference for risk. As will be seen below, these alternative attitudes toward risk are important in determining the individual's behavior regarding the timing of equipment replacements in the presence of uncertainty.

In deterministic microeconomic theory, the decision maker is assumed to maximize profit (in the theory of the firm) or utility (in the theory of demand). Under uncertainty, however, profit and/or utility is random; and, consequently, the appropriate objective function becomes the expected value of the utility of profit or net value. Therefore, we assume that potential adopters of a new durable good innovation make their equipment replacement decisions in a manner consistent with the maximization of

$$E[U(\pi)] = E\left[U\left(\int_0^T V(t)Q(t)e^{-rt}dt - \sum_{i=1}^{n}\left\{e^{-r\sum_{j=0}^{i-1}\tilde{t}_j^*}\left[F_i + \int_0^{\tilde{t}_i^*} c_i(t,\mu_i)e^{-rt}dt\right]\right\}\right)\right]. \tag{5.6}$$

The following section examines the implications of such behavior for the optimal timing of equipment replacements.

B. The Optimal Age to Replacement

Assuming that the new durable good is introduced on the market during the operating lifetime of the ith machine in the sequence so that the subscript i refers to the installed equipment and i + 1 refers to the new product that is to replace it, the optimal age to replacement is determined by finding a value for $\tilde{t}_i^*$ that maximizes $E[U(\pi)]$ over the remainder of the 0,T planning horizon. Since the operating and maintenance costs of the installed equipment are observable at each point in time, the random term, μ_i, is assumed to disappear in the variable cost function of the equipment in place. Then, the first-order condition necessary for the maximization of the expected utility of net value is given by setting

$$\frac{\partial E[U(\pi)]}{\partial \tilde{t}_i^*} = E\left[U' e^{-r\tilde{t}_i^*}\left(-c_i(\tilde{t}_i^*) + r\sum_{j=i+1}^{n}\left\{e^{-r\sum_{k=i+1}^{j-1}\tilde{t}_k^*}\left[F_j + \int_0^{\tilde{t}_j^*} c_j(t,\mu_j)e^{-rt}dt\right]\right\}\right)\right] \tag{5.7}$$

equal to zero. Doing so, dividing through by $e^{-r\tilde{t}_i^*}$, and rearranging yields

$$E(U')\left[-c_i(\tilde{t}_i^*) + r\sum_{j=i+1}^{n}\left\{e^{-r\sum_{k=i+1}^{j-1}\tilde{t}_k^*}F_j\right\}\right]$$

$$+ rE\left[(U')\left(\sum_{j=i+1}^{n}\left\{e^{-r\sum_{k=i+1}^{j-1}\tilde{t}_k^*}\left[\int_0^{\tilde{t}_j^*} c_j(t,\mu_j)e^{-rt}dt\right]\right\}\right)\right] = 0. \tag{5.8}$$

Then, since the expectation of the product of two random variables is equal to the product of the expectations of these variables plus the covariance between them, this expression may be rewritten as

$$c_i(\tilde{t}_i^*) = r\sum_{j=i+1}^{n}\left\{e^{-r\sum_{k=i+1}^{j-1}\tilde{t}_k^*}\left[F_j + E\left(\int_0^{\tilde{t}_j^*} c_j(t,\mu_j)e^{-rt}dt\right)\right.\right.$$
$$\left.\left. + \frac{cov\left(U', \int_0^{\tilde{t}_j^*} c_j(t,\mu_j)e^{-rt}dt\right)}{E(U')}\right]\right\}. \tag{5.9}$$

This expression represents the uncertainty equivalent to equation (4.13) of the preceding chapter and provides the optimum decision rule for equipment replacements when the performance characteristics of future machines are uncertain.

The condition states that the existing equipment should be replaced with the new innovation at that point in time at which the variable costs of production using the installed machine has risen to equality with the discount rate times the discounted sum of the capital costs and expected life-cycle variable costs of all future machines plus the covariances between the marginal utility of net value and the discounted life-cycle variable costs of these future machines divided by the expectation of the marginal utility of net value. More simply, the condition states that the decision maker should delay the replacement of existing equipment until the marginal cost of further delay is equal to the marginal opportunity cost of immediate replacement, where the latter now includes an additional factor that reflects the decision maker's subjective valuation of the riskiness of the performance of future machines.

Solving equation (5.9) for the optimal age to replacement, we have

$$\tilde{t}_i^* = c_i^{-1}\left\langle r\sum_{j=i+1}^{n}\left\{e^{-r\sum_{k=i+1}^{j-1}\tilde{t}_k^*}\left[F_j + E\left(\int_0^{\tilde{t}_j^*} c_j(t,\mu_j)e^{-rt}dt\right) + \frac{cov\left(U', \int_0^{\tilde{t}_j^*} c_j(t,\mu_j)e^{-rt}dt\right)}{E(U')}\right]\right\}\right\rangle \tag{5.10}$$

which is the uncertainty analog of equation (4.14) of the preceding section. From this expression, we can see that the impact of new product performance uncertainty on the timing of equipment replacements depends upon the signs and magnitudes of the additional terms involving the covariances between the marginal utility of discounted net value and the discounted life-cycle variable costs of future machines. The signs of these additional terms depend, in turn, upon the decision maker's attitude toward risk.

Since the denominator of these additional terms (the expectation of the marginal utility of discounted net value) is positive by assumption, the overall signs will be the same as the signs of the covariance terms in the numerators. If the decision maker is risk averse (i.e., if $U'' < 0$), these covariances will be positive. This is because increases in the discounted life-cycle variable costs of future machines will lead to decreases in discounted net value, which, with a concave utility function, will lead to increases in the marginal utility of discounted net value. By an analogous argument, these covariance terms will be negative if the decision maker exhibits a preference for risk (i.e., if $U'' > 0$) and zero if she is risk neutral (i.e., if $U'' = 0$). Given this relationship between the signs of the covariance terms and the decision maker's attitude toward risk, we can see from equation (5.10) that, *in the presence of uncertainty, the optimal age to replacement of existing equipment will be higher than (lower than) (the same as) in the deterministic case examined in the preceding chapter if the individual exhibits risk aversion (risk preference) (risk neutrality).*

This result provides a utility interpretation of the descriptive classifications often found in the literature (e.g., Bass, 1969) wherein individuals adopting new goods are categorized into an arbitrary number of groups depending upon the timing of their adoption decisions (e.g., early adopters, late adopters, laggards, etc.). Here, we can see that, for a given age of existing capital equipment in place at the time the new product is introduced, risk preferring individuals will be the first to install the new durable good, with risk neutral individuals following next and risk averse individuals adopting last. Thus, prior classificatory schemes may be viewed as segmentations of the continuum of risk premiums that exist in the population at large.

C. The Effect of Uncertainty on the Market Penetration Function

If there exists a distribution of risk preferences in the population of potential adopters of a new innovation, we should then expect to observe a time path of cumulative replacements that exhibits much the same shape as that generated in the preceding chapter even in the absence of any variation in the ages of installed equipment. Therefore, such a distribution is a sufficient condition for the generation of market penetration functions that correspond to the traditional functions employed in the literature on diffusion.

Given both a distribution of ages of installed capital equipment and a distribution of attitudes toward risk, the diffusion path that is generated will reflect a combination of the two effects, and the relative importance of each will vary from one innovation to another depending upon the technological, economic, and informational forces at work. This multiplicity of possible explanations of observed behavior creates an identification problem that may be quite difficult to sort out empirically.

The extent to which this composite market penetration function will differ from that described in the preceding chapter depends upon the decision makers' risk preferences and the extent of perceived uncertainty surrounding the performance of future machines. Assume that the subjective probability distributions over the discounted life-cycle variable costs of future machines are identical across all individuals. Then, if all individuals have identical and concave utility functions (i.e., if everyone is identically risk averse), $\tilde{t}_i^* > t_i^*$ and $\tilde{P}(t) < P(t)$, where $\tilde{P}(t)$ denotes the market penetration function that incorporates the influence of new product performance uncertainty, so that the effect of incorporating uncertainty is to shift the penetration function downward by some fixed amount at each point in time. If, on the other hand, all individuals have identical and convex utility functions (i.e., if everyone is a risk taker), $\tilde{t}_i^* > t_i^*$ and $\tilde{P}(t) > P(t)$, so that the opposite effect occurs. If, on the other hand, some individuals have a convex utility function while others have a concave utility function or if individuals differ in their perceptions of the $f_i(\mu_i)$ distributions, $\tilde{P}(t)$ will lie above $P(t)$ over the early part of its range and below it over the latter part of its range. Finally, if, as often postulated in past studies of diffusion, the degree of uncertainty surrounding the performance of future machines is reduced with cumulative aggregate experience (i.e., if the variances of the $f_i(\mu_i)$ distributions decline over time), then $\tilde{P}(t)$ will approach $P(t)$ over time, and the impact of uncertainty on market share growth will erode with cumulative experience.

Thus, the introduction of uncertainty in the presence of nonlinear risk preferences can shift the market penetration function in a variety of ways. And, since utility functions are not observable, it is impossible to directly incorporate the effects of uncertainty in forecasts of new product growth. It is conceptually feasible, however, to separate the effects of uncertainty from the deterministic causes of new product growth for those innovations for which complete historical data are available. By forecasting the growth path of a new durable good on the basis of the model presented in the preceding chapter and comparing the predicted path with the actual, one can (in theory) isolate the influence of uncertainty from the deterministic elements involved in the growth process. In this fashion, a rough idea of the qualitative direction and quantitative significance of the effects of imperfect information on the growth of new products can be gained. If sufficient data are available to carry out this exercise for a variety of innovations, one could then proceed to model the uncertainty component of the process separately, incorporating such factors as advertising, demonstrations, product complexity, etc., in an attempt to develop a forecasting tool that could be applied in conjunction with the deterministic model to generate predictions of the composite market penetration function. The data requirements of such an exercise, however, are not likely to be met.

In the absence of an explicit model of the uncertainty component of the diffusion process, one is left with two equally unattractive alternatives for empirical implementation of the model. The first is to assume that decision makers possess perfect information or, equivalently, possess unbiased information and are risk neutral and proceed on the basis of the deterministic model derived in the preceding chapter. The second is to make *ad hoc* adjustments in the value of t_i^* in order to reflect the potential effects of uncertainty. Clearly, both of these options are less than satisfactory. But, until a structural model of the uncertainty component becomes available, such approaches will be unavoidable.

VI. SOME SIMULATION RESULTS

As with any theory of behavior, there exists a variety of empirical tests that may be used to verify the adequacy of the above model of durable good diffusion. Such tests include: (1) the performance of simple simulations of the model in the absence of actual data in order to compare the implied responses of the dependent variable to variations in the values of the structural components with prior information and expectations; (2) the prediction of past events for which complete data are available in order to examine the forecasting performance of the model; and (3) statistical testing of the causal hypotheses that are implied by the model (such as those derived in Section IV above). Because of data, time, and space limitations, however, we subject the model only to the first (and least demanding) of these tests at the present time.

The penetration function given in equation (4.19) was simulated under the assumptions that the capital stock in the adopting sector grows over time at an exponential rate and that the age structure of the existing stock of equipment in place at t_0 is distributed lognormally.[32] Thus, it was assumed that

$$K(t) = K(t_0)e^{\delta t}, \qquad (6.1)$$

where δ is the rate of growth of the capital stock over time, and

$$f_{t_0}(A) = \frac{1}{A\sigma\sqrt{(2\pi)}} \exp\left\{-\frac{1}{2\sigma^2}(\log A - \mu)^2\right\} dA, \qquad (6.2)$$

where μ and σ^2 are the mean and variance of the corresponding normal distribution of the variate $Y = \log A$.

Under these assumptions, the penetration function given in equation (4.19) becomes

$$P(t) = 1 - \left[1 - \int_{t_i^*-t}^{A_0} \frac{1}{A\sigma\sqrt{(2\pi)}} \exp\left\{-\frac{1}{2\sigma^2}(\log A - \mu)^2\right\} dA\right] e^{-\delta t}, \qquad (6.3)$$

where A_0 is the age of the oldest machine in the capital stock at t_0.[33] Equation (6.3) was simulated for two values of μ, two values of δ, and two values of t_i^*, yielding a total of eight separate penetration functions. These functions, along with their two associated age distributions are shown in Figs. 6.1-6.10.

Clearly, the overall behavior of P(t) in these simulations conforms well with the descriptive functions encountered in existing empirical studies of market penetration. In addition, we observe the expected influences of changes in: the age distribution of the existing equipment; the growth rate of the capital stock in the adopting sector; and the optimal age to replacement of the old technology machines. Comparison of Fig. 6.2 with Fig. 6.7, Fig. 6.3 with Fig. 6.8, Fig. 6.4 with Fig. 6.9, and Fig. 6.5 with Fig. 6.10 demonstrates the retarding influence on new product growth of a concentration of recently installed equipment in the adopting sector. Comparison of Fig. 6.2 with Fig. 6.3, Fig. 6.4 with Fig. 6.5, Fig. 6.7 with Fig. 6.8, and Fig. 6.9 with Fig. 6.10 demonstrates the positive effect of overall sector growth on market penetration. And comparison of Fig. 6.2 with Fig. 6.4, Fig. 6.3 with Fig. 6.5, Fig. 6.7 with Fig. 6.9, and Fig. 6.8 with Fig. 6.10 shows the marked impact of variations in the optimal age to replacement (which, from the above analysis, must be due to variations in the cost characteristics of the new innovation or the equipment it is designed to replace.

Although these simple simulation exercises are lacking in explicit empirical content and, consequently, cannot provide the verification required for complete acceptance of the above modeling approach, they do demonstrate the kinds of results that can be generated and suggest a broad agreement with prior empirical studies on the subject. In addition, they lend support to the comparative static results derived in Section IV.E above. Further verification will have to involve empirical testing of derived hypotheses combined with examination of forecating results for those innovations for which historical data exist and is, unfortunately, beyond the scope of this report.

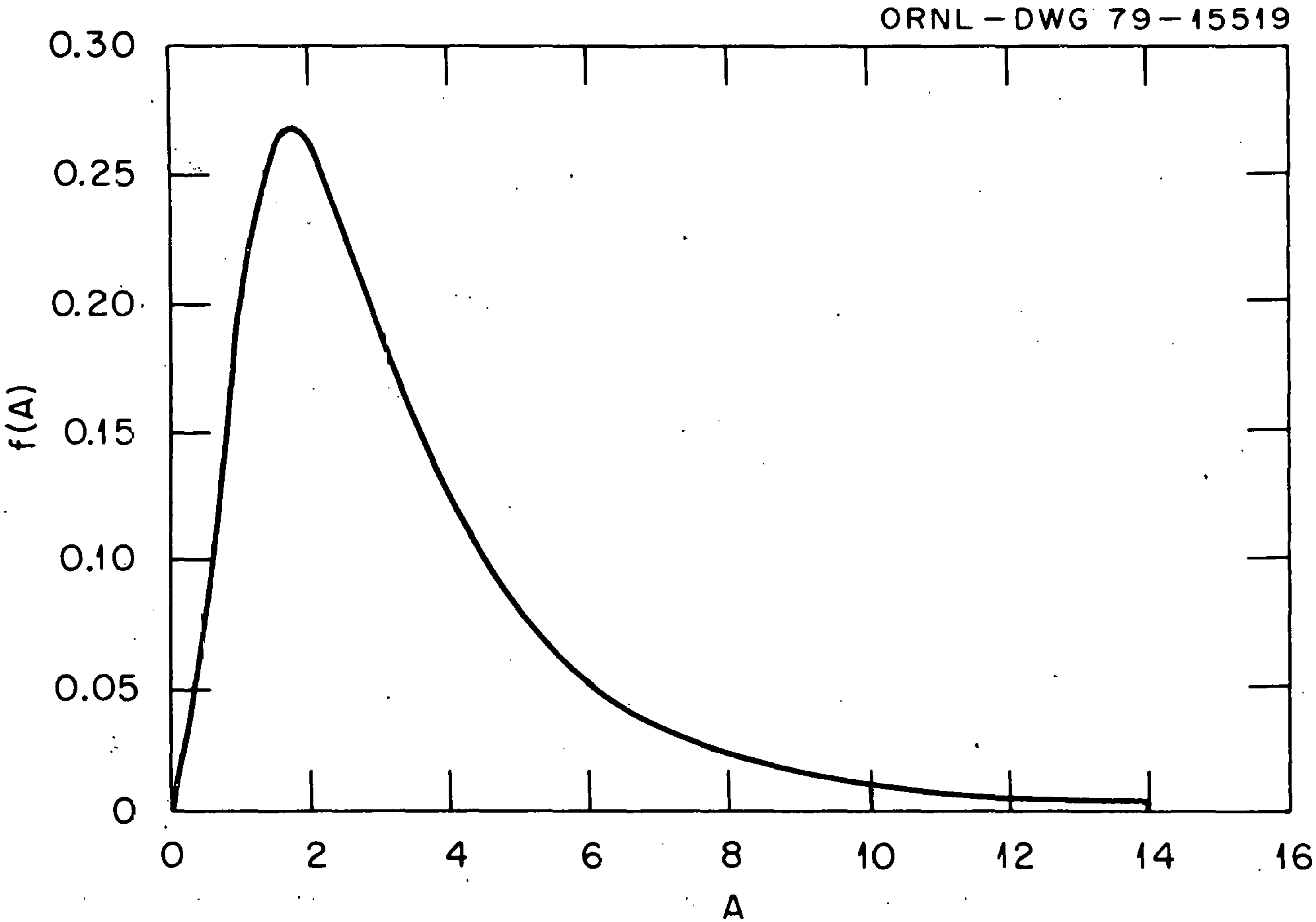

Fig. 6.1. Assumed lognormal distribution of ages of equipment in place, with $\mu = 1$, $\sigma^2 = 0.5$.

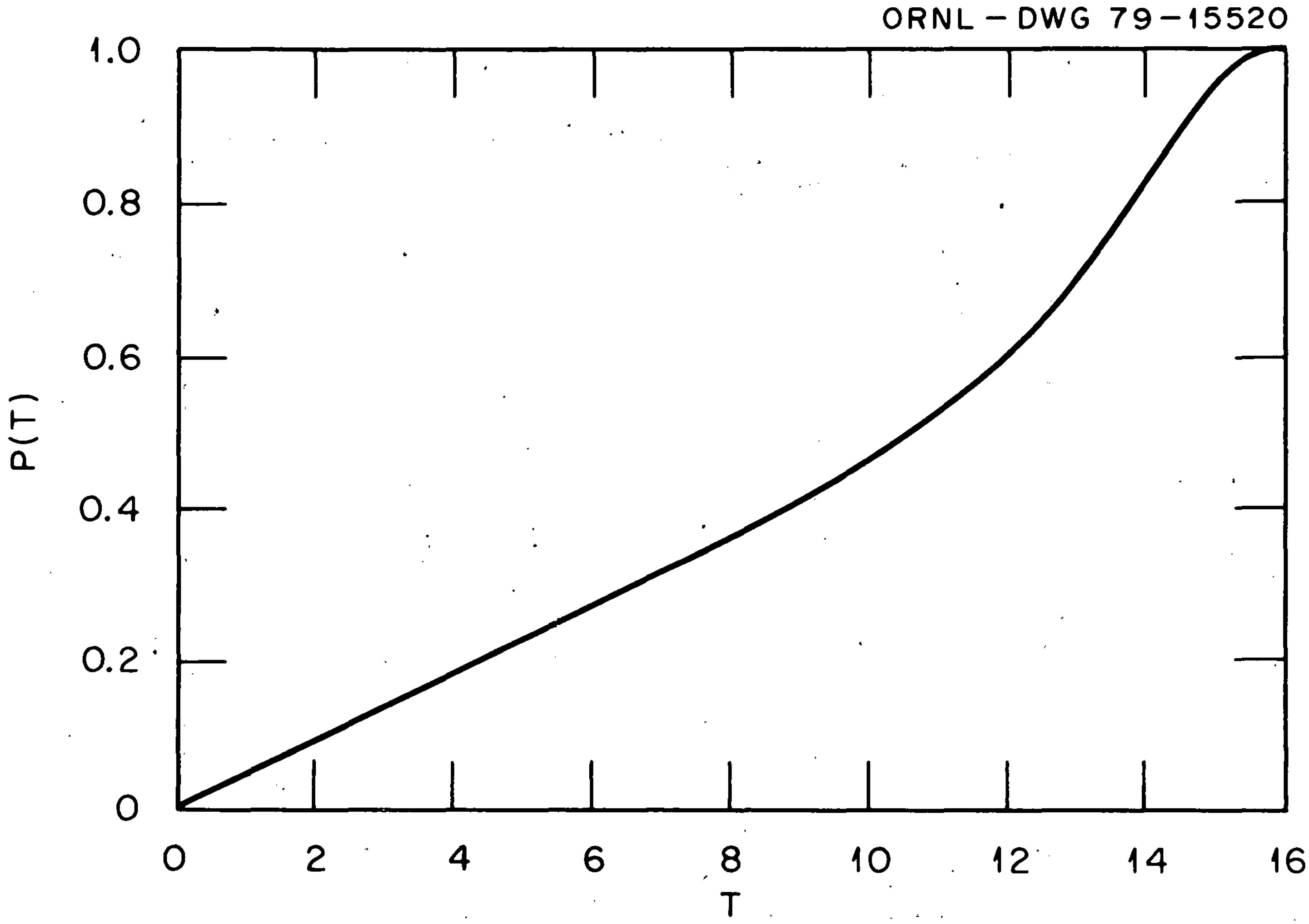

Fig. 6.2. Market penetration function with $t_i^* = 16$, $\delta = 0.05$, $f_{t_0}(A)$ as shown in Fig. 6.1.

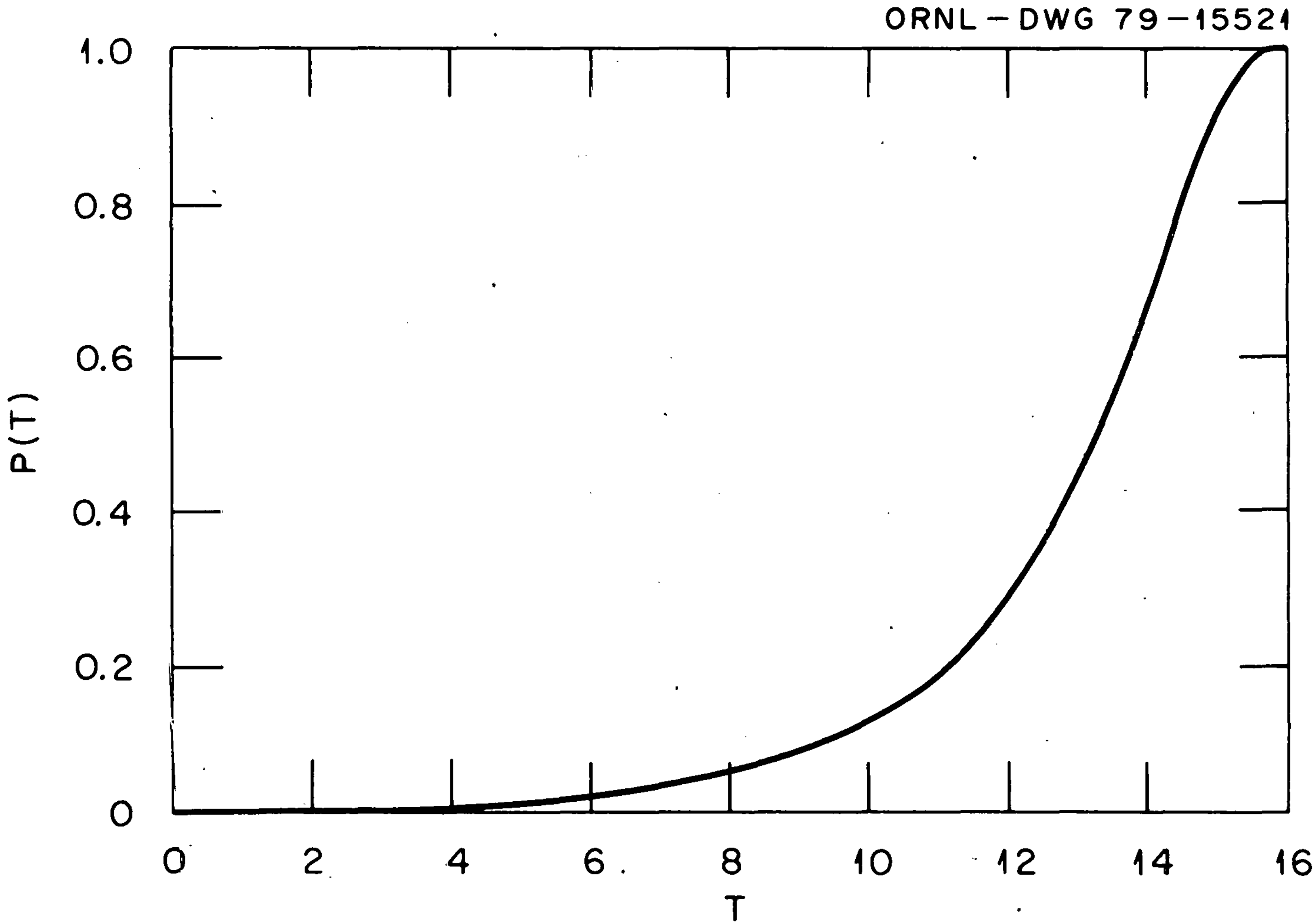

Fig. 6.3. Market penetration function with $t_i^* = 16$, $\delta = 0$, $f_{t_0}(A)$ as shown in Fig. 6.1.

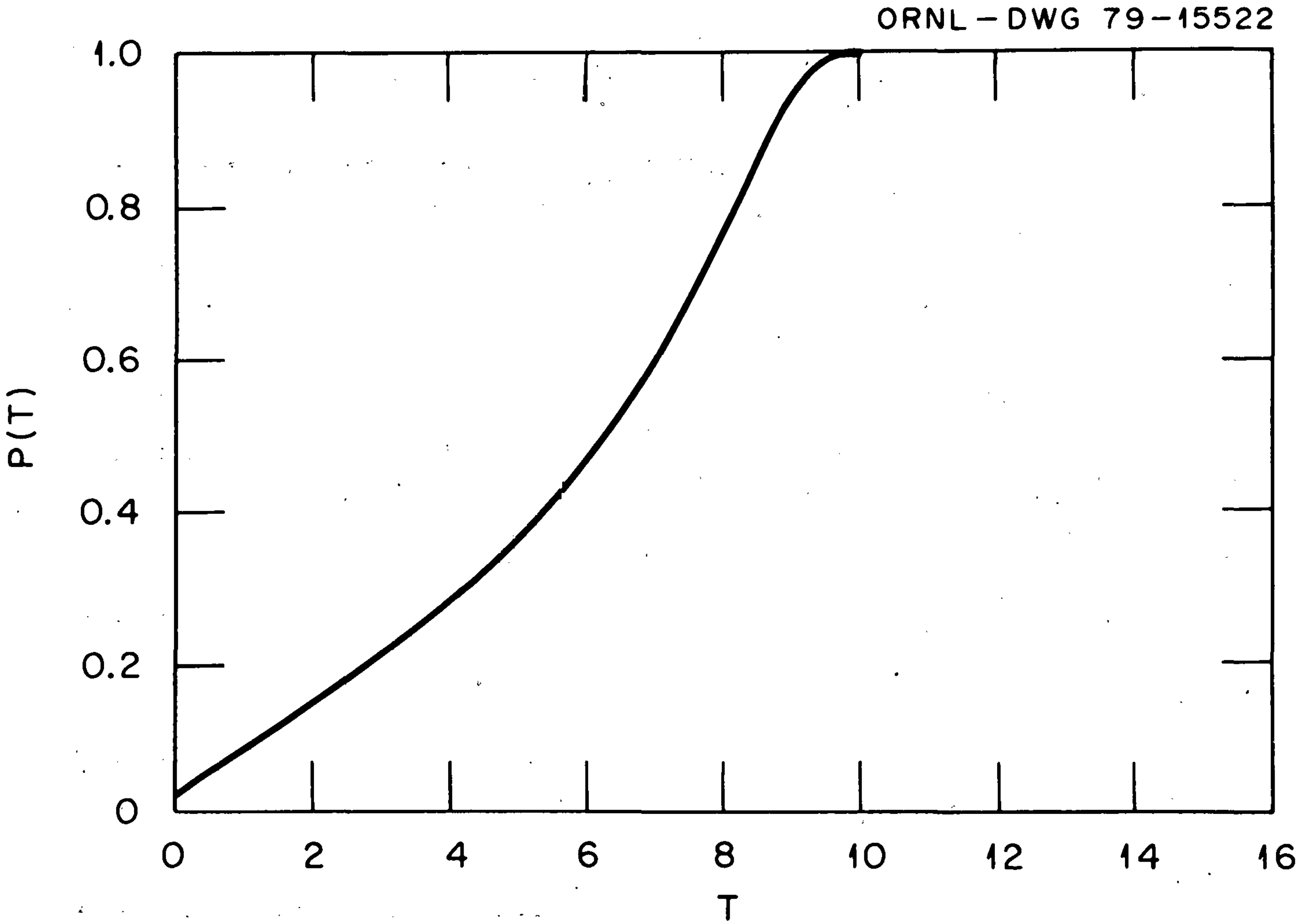

Fig. 6.4. Market penetration function with $t_i^* = 10$, $\delta = 0.5$, and $f_{t_0}(A)$ as shown in Fig. 6.1.

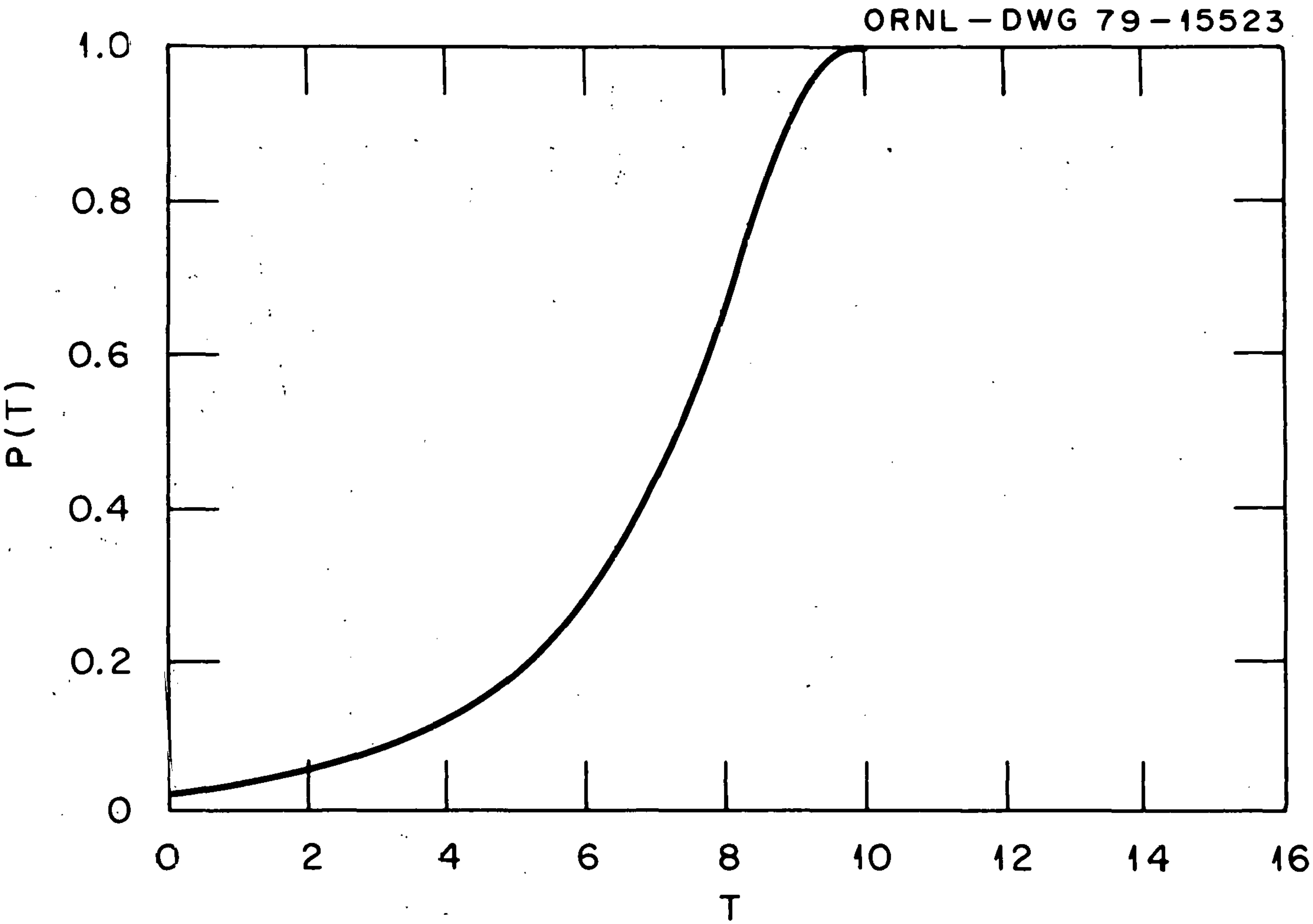

Fig. 6.5. Market penetration function with $t_i^* = 10$, $\delta = 0$, $f_{t_0}(A)$ as shown in Fig. 6.1.

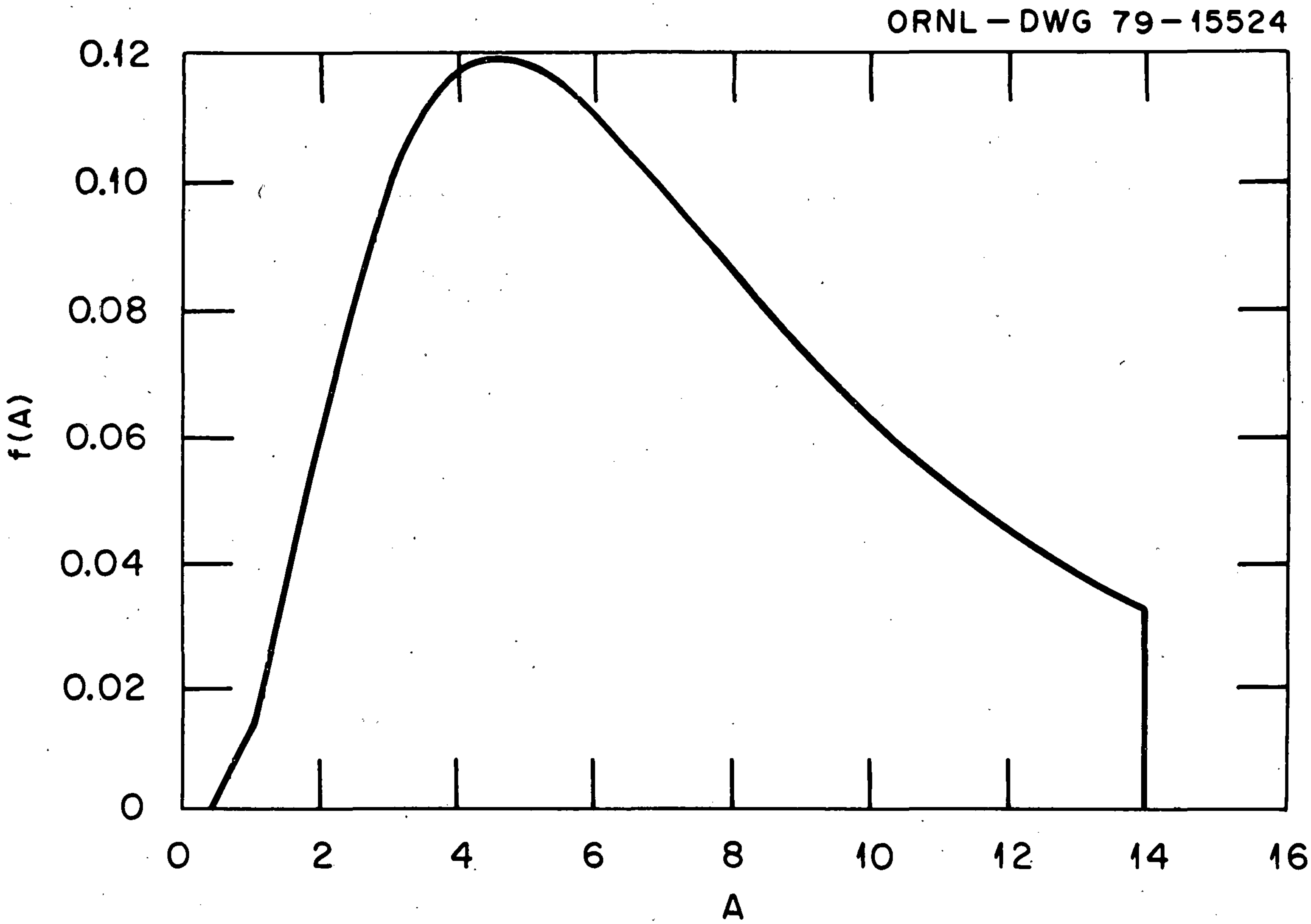

Fig. 6.6. Assumed lognormal distribution of ages of equipment in place, with $\mu = 2$, $\sigma^2 = 0.5$.

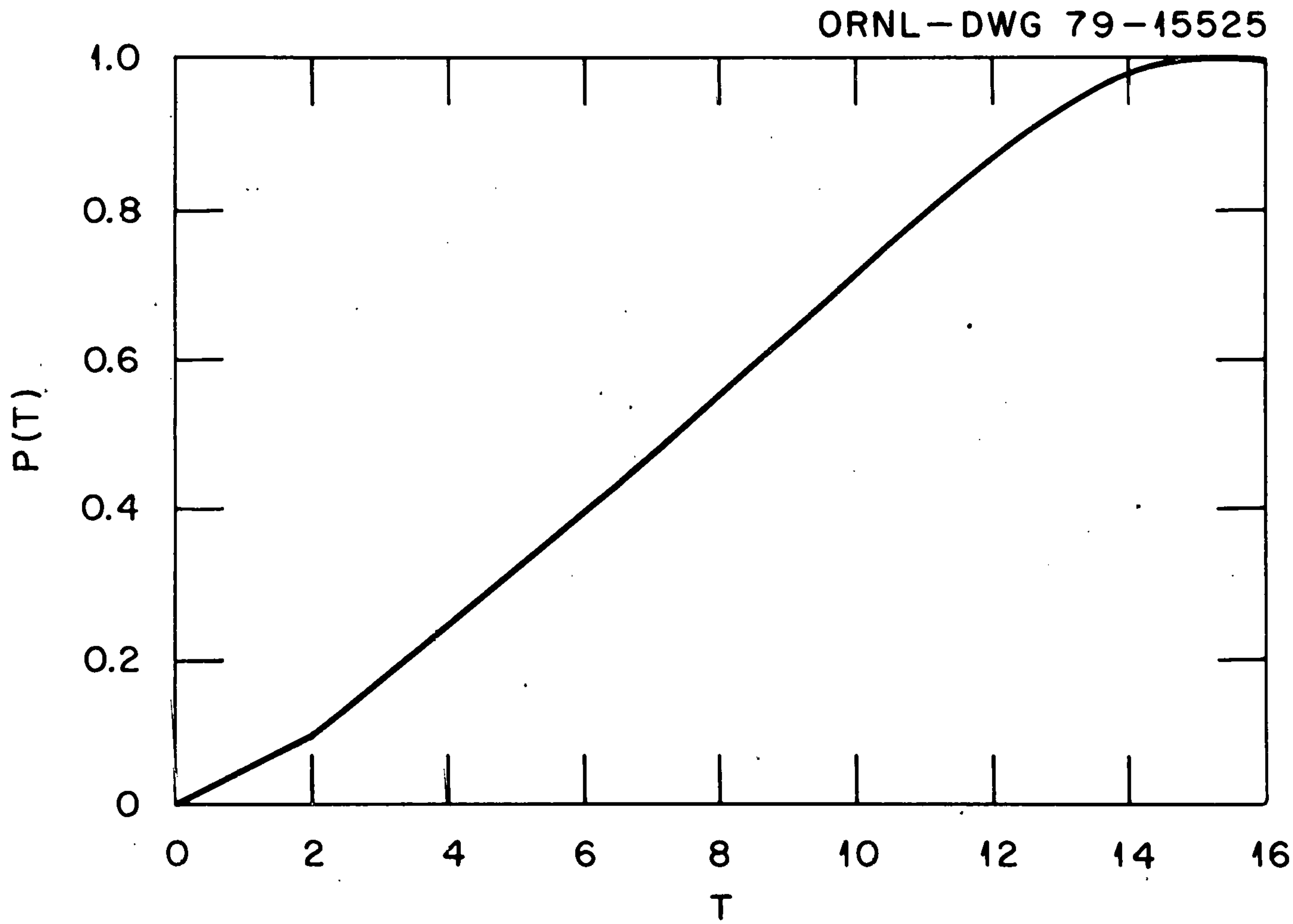

Fig. 6.7. Market penetration function with $t_i^* = 16$, $\delta = 0.05$, and $f_{t_0}(A)$ as shown in Fig. 6.6.

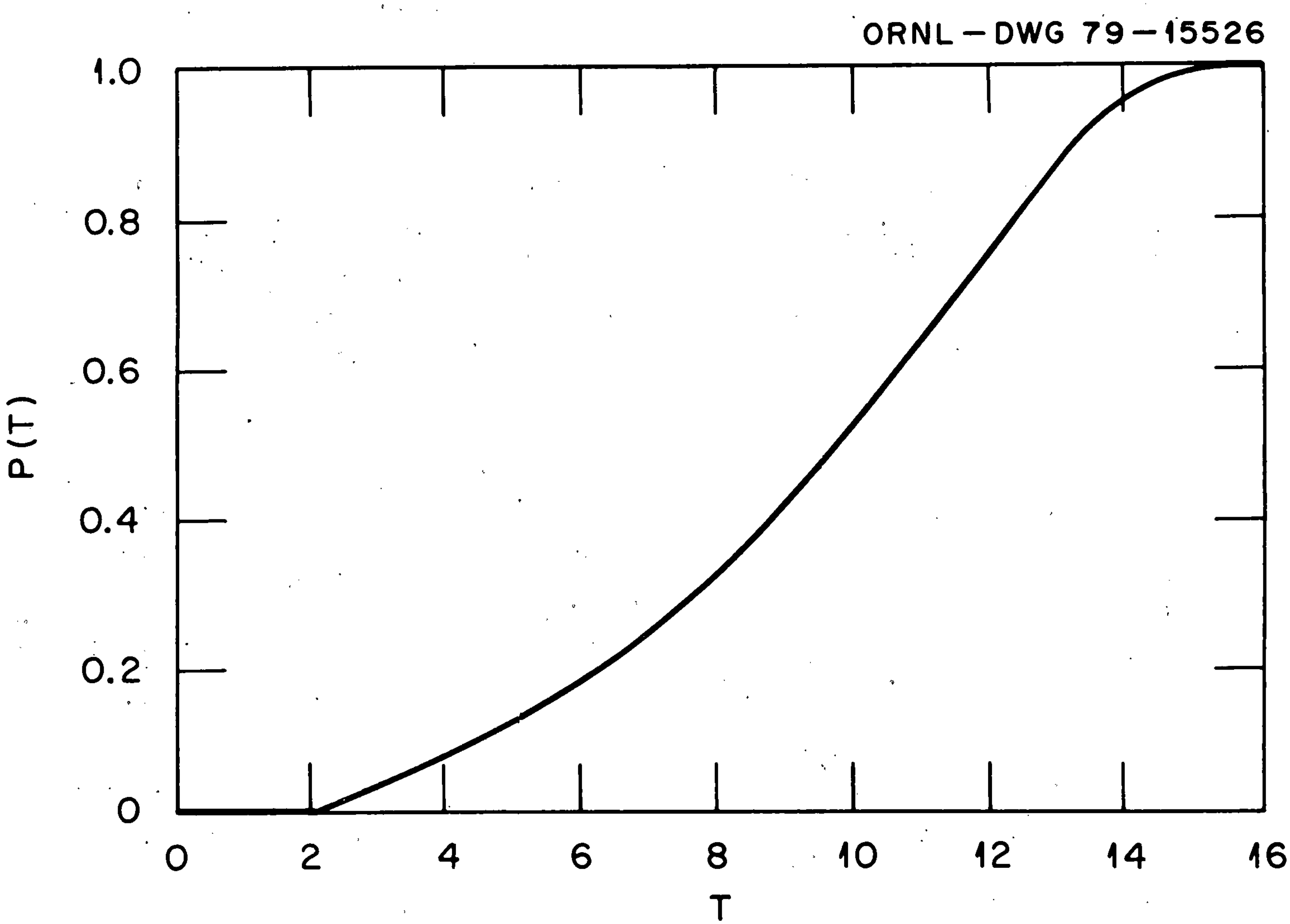

Fig. 6.8. Market penetration function with $t_i^* = 16$, $\delta = 0$, $f_{t_0}(A)$ as shown in Fig. 6.6.

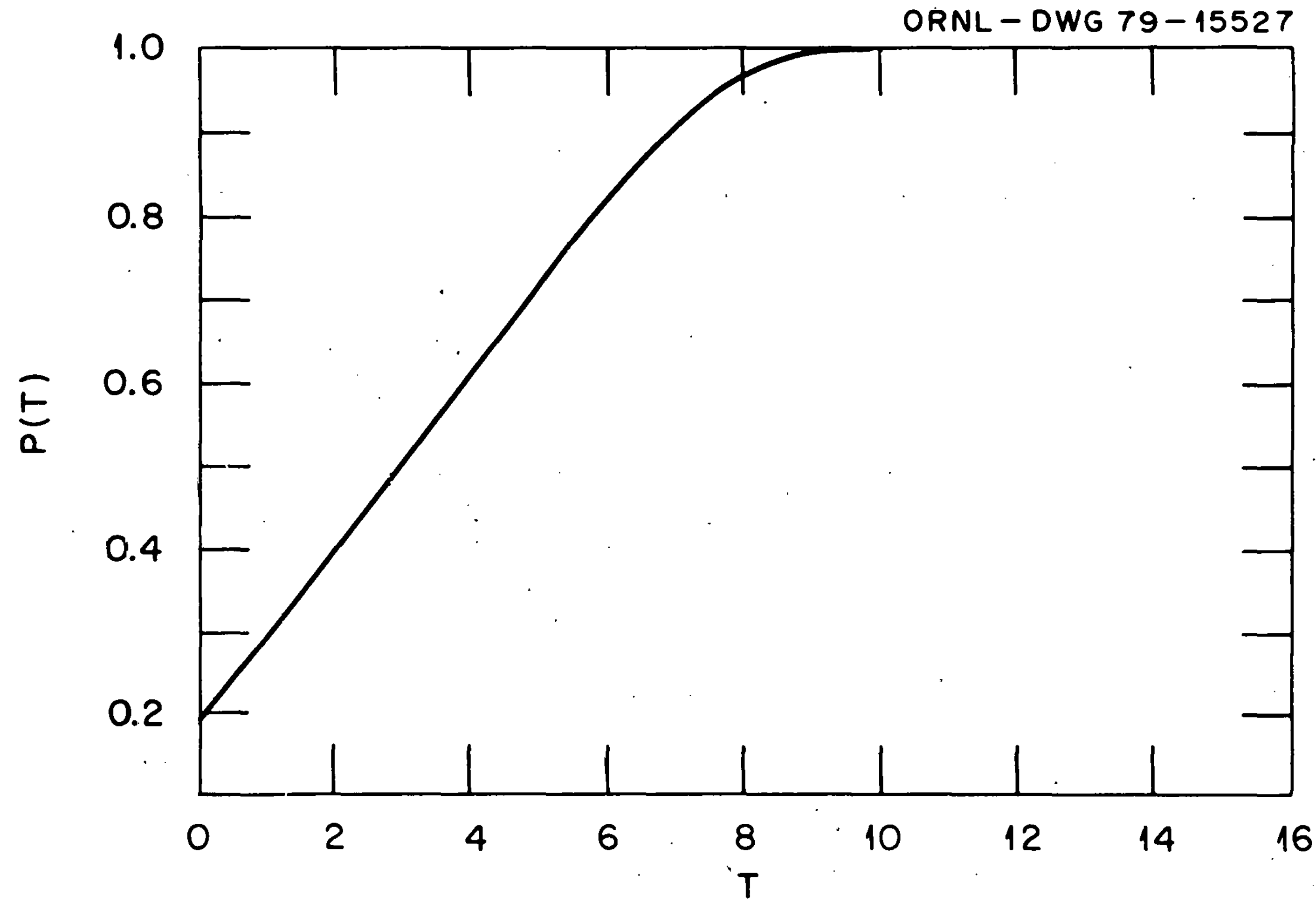

Fig. 6.9. Market penetration function with $t_i^* = 10$, $\delta = 0.05$, and $f_{t_0}(A)$ as shown in Fig. 6.6.

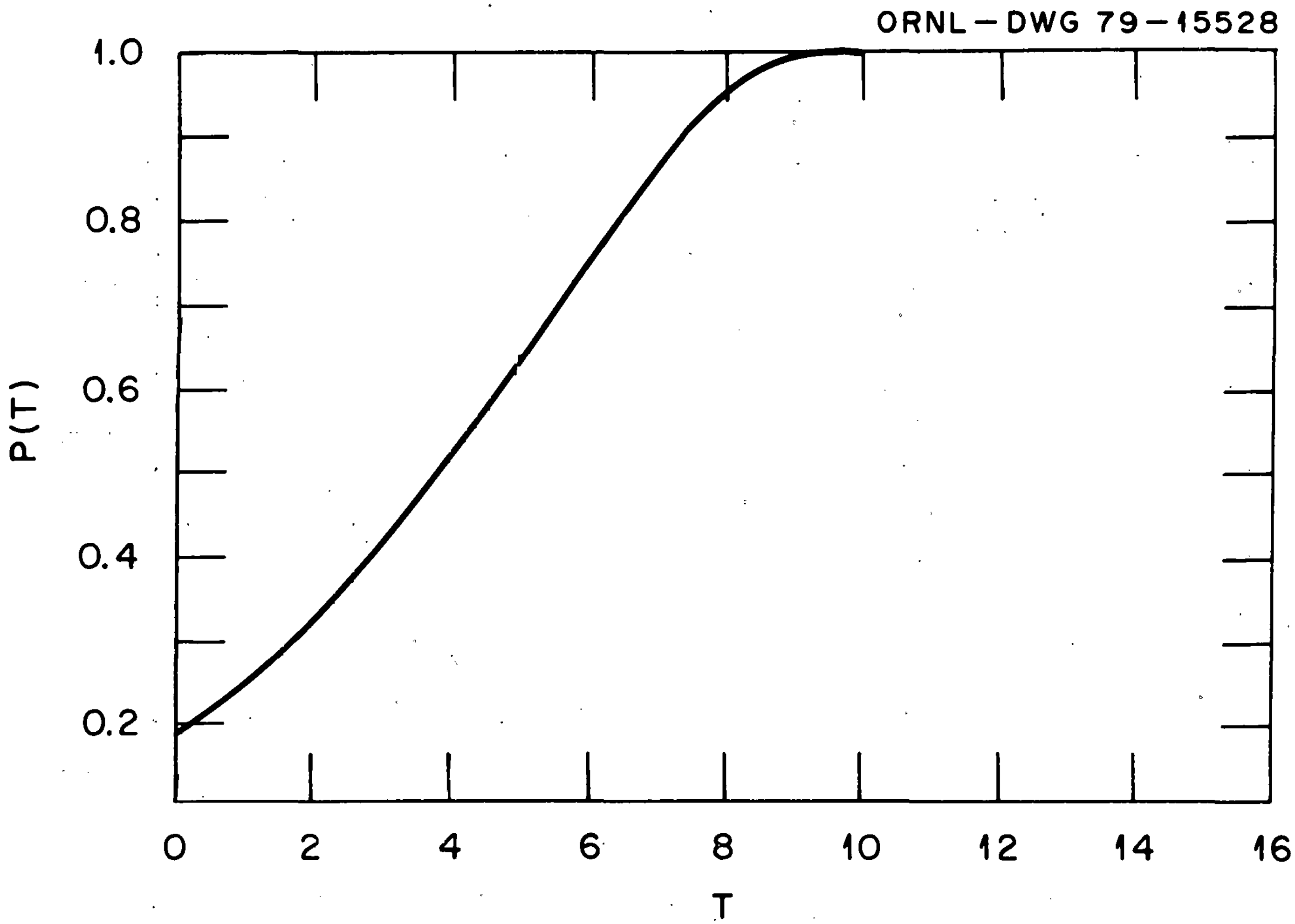

Fig. 6.10. Market penetration function with $t_i^* = 10$, $\delta = 0$, and $f_{t_0}(A)$ as shown in Fig. 6.6.

VII. CONCLUSION

Using the assumption of dynamic cost minimization on the part of potential users of a new durable good, we have derived a model of the market penetration process that exhibits two important advantages over existing diffusion studies. First, the model is operational in the absence of historical data on the actual market experience of the new good in question and is capable of approximating any of the traditional functional forms employed in previous studies to represent observed time paths of market share growth. Given engineering estimates of the cost characteristics of an emerging product, an exogenous estimate of anticipated growth in the adopting sector or industry, and observations on the age distribution and cost characteristics of the existing stock of equipment, the model can forecast the time path of short-run equilibrium market shares for a new product that has not yet appeared on the market. This capability, lacking in previous models, is crucial to the analysis of optimal commercialization and investment decisions concerning frontier technologies for which market data are not yet available.

Second, the theoretical approach developed here contains explicit causal relationships that lead to a variety of testable hypotheses. The inequalities given in expressions (4.24), (4.30), and (4.32) represent such hypotheses, and some of these have already received empirical support in the published literature on market penetration. This structural approach to the analysis of new product growth is additionally attractive in that it facilitates the detailed examination of the impacts of alternative policy measures designed to encourage rapid diffusion of new innovations. For example, market penetration might be accelerated through the use of an investment tax credit for the purchase and installation expense of adopting the new durable good. The impact of such a credit is to reduce the initial capital cost of the new innovation, F_{i+1}, which, in turn, leads to a reduction in the optimal lifetime of installed equipment, t_i^*, which, in turn, leads to an accelerated rate of replacement investment, R(t), which finally translates into an increased level of market penetration, P(t). Other policies such as fuel taxes, loan guarantees, maintenance subsidies, etc., can also be incorporated in the model, and the explicit channels of causation through which they can be expected to operate can be clearly defined.

Despite these advantages over existing studies, there obviously remains much work of both a theoretical and empirical nature in the modeling of new product growth. Some of the more fruitful areas for future research appear to be: (1) including an explicit treatment of the long-run equilibrium market share;[34] (2) making both supply side adjustments and capital stock growth in the adopting sector endogenous to the model; and (3) thorough empirical validation of both the implied hypotheses and derived forecasts. Without minimizing the importance of such future refinements, the present model must stand on its theoretical and practical appeal as a methodology for both explaining and predicting the time path of diffusion of durable good innovations.

FOOTNOTES

1. The list of references provided with this paper represents a partial bibliography of this literature. A review of these and related studies is provided in Hurter *et al.* (1978).

2. This basic lack of a theoretical construct has not escaped the notice of others working in this area. See, e.g., Sahal (1976, p. 242).

3. Blackman *et al.* (1973) develop a methodology that greatly reduces the historical data requirements for implementing the logistic penetration model. But in the absence of a theoretically justifiable technique for choosing from among the various potential market penetration functions (the exponential, the logistic, or the skewed S-shape) on the basis of information available prior to the introduction of the new innovation, the need for such data will remain.

4. This hypothesis is often dismissed out of hand for households with the argument that the initial cost of an investment is weighted much more heavily than the operating cost component of discounted average total costs. This argument is specious, however, because, to the extent that operating costs are considered at all, such observed behavior simply implies a high rate of discount.

5. The modified exponential function has been employed in Fourt and Woodlock (1960), Kelly (1967), and Perry *et al.* (1967). Also, the function employed in Bass (1969) is approximately exponential for certain parameter values.

6. If one is able to observe the entire diffusion process from the time the new product is introduced until the upper limit of penetration is attained, then the sample may be restricted to that portion of the market that eventually adopts the new innovation. In that case, $L = 1$ and the problem of estimating the dependent variable is eliminated. See Mansfield (1961).

7. The logistic penetration function has been the most widely used form in the literature. Studies incorporating the logistic function include: Griliches (1957), Mansfield (1961, 1963), Bain (1962), Haines (1964), Blackman (1971, 1972, 1974), Fisher and Pry (1971), Romeo (1975), Globerman (1975), and Stern *et al.* (1975).

8. Mansfield (1961) arrives at the logistic penetration function by a different route. Dealing with industrial innovations, he expresses the probability that the innovation will be adopted in period t by firms that had not adopted it in period t-1 as a function of: (1) the proportion of firms that have already introduced the new product in period t-1, (2) the profitability of adopting the new innovation, (3) the size of the investment required to incorporate the new product in the production process, and (4) other unspecified variables. Then, taking a Taylor series expansion of this function and assuming that the coefficients of the second and higher order terms in the proportion of previous adopters are zero, a difference equation is obtained to which the logistic provides an approximation.

9. This feature may be seen by inspection of the second derivative of P(t) with respect to t. From (2.4),

$$\frac{d^2P(t)}{dt^2} = a\,\frac{dP(t)}{dt}\,[L - 2P(t)],$$

which equals zero at $P(t) = L/2$. Thus, the point of inflection occurs here.

10. A skewed S-shaped penetration function is employed in Bain (1963), Lekvall and Wahlbin (1973), and Lerviks (1976).

11. The Gompertz curve achieves its maximum rate of growth at $P(t) = 0.37\ L$.

12. See Cattell (1948). In addition, in a world of imperfect information, the greater the profitability of a given innovation the more likely it is that potential users will perceive the benefits of adopting the new product at any given moment in time. This rationale is, in turn, related to the statistical result that the larger the actual difference between two population means, the smaller the sample size required to discern a significant difference. See Griliches (1957, p. 516).

13. See Mansfield (1961, pp. 746-747). For an analysis demonstrating why the per unit cost of capital should be an increasing function of the amount borrowed, see Smith (1971).

14. Obviously, with a sample of only 12 innovations, degrees of freedom difficulties limit the number of variables that may be included simultaneously.

15. The inconclusive nature of some of these results may be due to simple data limitations. For example, if, instead of adding d_{ij} to equation (2.10) as Mansfield does, we substitute it for S_{ij}, we obtain the following results:

$$\hat{a}_{ij} = \left\{\begin{matrix} .28 \\ .55 \\ .54 \\ .56 \end{matrix}\right\} + \underset{(.016)}{.526}\,\pi_{ij} - \underset{(.001)}{.002}\,d_{ij}, \quad r = .997,$$

which provide approximately equivalent empirical support for this alternative model in terms of both goodness of fit and significance of coefficient estimates. Hence, the relatively poor results obtained by incorporating both S_{ij} and d_{ij} simultaneously may be attributable to multicollinearity and/or small sample problems.

16. Those studies that have not restricted their sample to this group have provided only *ad hoc* procedures for estimating the upper limit market share of a new good. Griliches (1957) crudely estimates the value of L in equation (2.6) by plotting P(t) on logistic graph paper and varying L until the resulting graph is approximately linear. Similarly, Bundgaard-Nielsen (1976) varies L until the residual sum of squares from estimating equation (2.6) is minimized. Romeo (1975) simply assumes a particular value for L. Given the importance of this parameter in determining the overall success of a new product, the degree of attention devoted to its estimation in the diffusion literature is surprising.

17. Sahal (1976, p. 242) points out this problem, "It is easy to obtain good *ex post factum* fit to the data on diffusion of technology by means of one or the other form of an S-shaped growth curve. However, the value of such a model is limited. Insofar as it sheds little light on the nature of the underlying mechanism, it is a trivial restatement of facts. The postulate that diffusion of technology envinces an S-shaped pattern is concluded to be inherently unsuitable for *ex ante* prediction unless there is *a priori* justification for choosing a specific form from a wide variety of S-shaped curves that would be appropriate. However, a framework for choosing an appropriate functional form at an early stage in the process of diffusion is lacking."

18. Blackman *et al.* (1973) investigate inter-industry variations in β_{i0} using a factor analysis approach to obtain an "innovation index" for various industrial sectors. This index is then correlated with estimated values of β_{i0} to provide a practical method for forecasting the penetration function of a new innovation in the absence of an adequate historical data base relating to observed market experience of that innovation. This approach, however, serves only to substitute the problem of obtaining prior estimates of the value of the index for the problem of obtaining prior estimates of β_{i0}. In addition, it assumes that all innovations will follow the logistic pattern of growth.

19. This stock measure of market share as an indicator of the extent of diffusion is used extensively in the studies reported in Nabseth and Ray (1974).

20. Griliches (1957) employed this assumption as have most others writing on the subject of diffusion. Peterka (1977) is the primary exception to this rule. According to Nelson *et al.* (1967, p. 105), "Where buyers stand willing and ready, the pace of diffusion clearly is limited by supply factors. But when the growth of demand takes time, as in most cases that have been studied, there is little evidence that bottlenecks on the supply side have been important."

21. See Bain (1962, 1963), Blackman (1971, 1972), Fisher and Pry (1971), Mansfield (1961, 1963a), Oster and Quigley (1977), Peterka (1977), and Stern *et al.* (1975). Empirically, this assumption may be justified by restricting the sample to the set of potential users that eventually adopt the given innovation. Also, inasmuch as upper limit market shares that are less than one must, in theory, be generated by a stable cross-sectional variation while the time path of diffusion must have its source in some form of temporal variation, the two phenomena are analytically separable.

22. Salter (1966, p. 63) emphasized the role of gross investment in translating technical change into productivity gains: "Without gross investment, improving technology that requires new capital equipment simply represents a potential for higher productivity; to realize this potential requires gross investment."

23. At the aggregate level, the annual rate of replacement investment generally exceeds the rate of investment due to market expansion. See Feldstein and Rothschild (1974).

24. The subject of optimal replacement or retirement policy is an old one in the economics literature. See, e.g., Hayek (1941), Hotelling (1925), Lutz and Lutz (1951), and Preinreich (1938, 1940). The implications of this body of theory for the diffusion of innovations, however, has never been fully explored, although David (1969), Mansfield (1961), and Salter (1966) have intimated that such a connection exists.

25. See Feldstein and Rothschild (1974) for a taxonomy of the various types of cost increases that may provide an incentive for equipment replacement decisions.

26. If equipment deterioration follows a "one hoss shay" pattern wherein variable costs remain constant over time until the machine suddenly collapses in a heap of worthless scrap, the replacement timing decision may become trivial. With changing prices and technology, however, variable costs can increase in a continuous fashion even with such discontinuous physical deterioration. Also, a discontinuous jump in variable cost can be closely approximated by a continuous monotonically increasing curve that exhibits a pronounced convexity. Thus, the assumption that $c_i(t)$ is monotonically increasing does not represent a very serious restriction on the applicability of the model.

27. At first blush, one might think that the discounted life cycle costs of operation using the new technology equipment would have to be below the discounted life cycle costs of operation using the old technology equipment for the new innovation to be chosen. This, however, is not the case. Due to the potential difference between the optimal replacement ages t_i^* and t_{i+1}^*, it is entirely possible for $F_{i+1} + \int_0^{t_{i+1}} c_{i+1}(t)e^{-rt}dt > F_i + \int_0^{t_i^*} c_i(t)e^{-rt}dt$ to hold and have the new technology machines represent the discounted cost minimizing choice over a given planning horizon if $t_{i+1} > t_i^*$. Investment alternatives must be normalized to a common investment period before any conclusions can be reached concerning the discounted cost minimizing choice. See Mishan (1976).

28. For simplicity, we have assumed that the new durable good is superior in all potential applications. In practice, however, a technological advancement that is embodied in a piece of durable equipment may be cost effective in only a fraction of the potential applications because of scale considerations. See David (1969). In that case, the long-run equilibrium market share will be less than one, and the model derived here can be applied to that portion of the market that will eventually adopt the new innovations.

29. If prices, technology, and output have remained stable prior to the introduction of the new innovation, there will exist no machines of age greater than t_{i-1}^*, and the $f_{t_0}(A)$ distribution may be confined to the $0,t_{i-1}^*$ closed interval. In general, however, it is not necessary to impose this restriction since the distribution is observable at time t_0.

30. Note that, in general, $G(t) = 0$ implies $P(t) = R(t)$. Thus, for durable good innovations that are adopted by sectors experiencing zero growth in the stock of installed equipment, the market share growth will follow the time path of cumulative percentage replacements. For sectors experiencing positive growth in the stock of installed equipment, $G(t) > 0$, it can easily be shown using (4.8) that $P(t) > R(t)$.

31. This capability of generating forecasts that are both derived from an explicit microeconomic theory of potential user optimization and implemented in the absence of historical data on the market experience of the new product represents a significant improvement over existing models of market penetration. Furthermore, the time path of market share growth generated by expression (4.20) can assume any of the traditional shapes employed in empirical diffusion studies (i.e., the exponential, the logistic, or the skewed sigmoid) depending upon the values of the component variables.

32. These two assumptions are both analytically convenient and empirically relevant. The exponential pattern of growth generates a constant percentage rate of growth and appears to correspond to a number of actual growth processes. The lognormal distribution is convenient because it is confined to the positive real line (age cannot be negative) and also appears to fit a variety of economic variables (e.g., income and firm size). See Aitchison and Brown (1969) for a complete discussion of the properties and applications of the lognormal distribution.

33. The distribution $f_{t_0}(A)$ was truncated at $A_0 = 14$ and normalized to maintain the property that $\int_0^{A_0} f_{t_0}(A)dA = 1$.

34. Analysis of the long-run equilibrium market share of a new product has received some attention outside the traditional diffusion literature. See, e.g., McFadden (1976) and Hausman (1979). These studies making use of a discrete choice modeling approach, however, have not been integrated with the dynamic models of diffusion that are concerned with the time-dependent series of short-run equilibria.

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