

TM-1490

Tracking Results Using a Standard Cell Lattice

Norman M. Gelfand Fermi National Accelerator Laboratory P.O. Box 500, Batavia, Illinois 60510

October 1987



This is a summary of results obtained by tracking a single particle through a lattice composed of a r.f. cavity and standard FODO cells. lattice also includes two The families of sextupoles for controlling the chromaticity. The parameters of the cells, i.e. their length and phase advance, closely resemble those of the Fermilab Main Ring or the Tevatron. We therefore have a model lattice which is but without the similar to that of those accelerators straight sections present in the actual machines. It is hoped that the simplified model used will exhibit the salient features of the actual accelerator but will be simpler to understand.

The model machine contains 774 dipoles (the number in the Tevatron) and the high order multipoles (n>1) measured at the Fermilab MTF for the Tevatron magnets were used in the tracking calculations. The exclusion of the measured skew quadrupole moments of the dipoles means that there is no linear coupling in our model. The Tevatron magnets have fields that fall off sharply at a radius of approximately 1 in. This shape is peculiar to the Tevatron magnets (as well as any other "cos theta" magnets) and to study more general kinds of behavior it would not be desirable to have

this sharp fall off. For this reason only the measured sextupole and octupole (both normal and skew) moments were used in the tracking.

The ordering of the magnets in the lattice determines the strength of the harmonic driving terms. The ordering used for the nominal case corresponds to the "shuffled" order used in the actual Tevatron where the magnets are positioned in the lattice to reduce several of the low order driving terms.

I. Fourier Transform Analysis

In the tracking code (Tevlat) a particle was placed at an initial point in phase space and with an initial value of dp/p and was tracked around the test lattice. As the particle returned to the starting point the current phase space coordinates were recorded. Thus a turn (or alternately a time) sequence of phase space coordinates was The Fourier transform of this sequence was developed. computed using a fft routine to determine the tune spectrum of the motion. Figure 1 shows the spectrum in the case where the initial dp/p was 0.0% and without synchrotron The sextupole families were set to give the oscillations. lattice zero chromaticity. The spectrum is rich. There are components corresponding to the linear tunes in x and y as well as harmonic components corresponding to the non-linear contributions from the sextupole and octupole moments.

 $\mathbf{2}$

In the tracking code the values of both x and x' are available at the end of each turn. With (x, x') and a knowledge of the values of beta and alpha at the starting amplitude $(=(\mathbf{x}^2+(\beta_{\mathbf{x}}\mathbf{x}^2+\alpha_{\mathbf{x}}\mathbf{x})^2)^{1/2}$ point the х canbe calculated at the end of each turn. If this were simply a linear lattice, without coupling the amplitude would be the same turn to turn. The variation of the amplitude therefore reflects the deviation from the simple linear machine. The x and y amplitudes, as a function of turn, are plotted in Figure 2a. Also plotted in Figure 2a is the square root of the sum of the squares of the x and y amplitudes. The The Fourier amplitudes are obviously notconstant. transform (Figure 2b) of the amplitude shows the same frequencies are present in the amplitudes as are present in the coordinates. This is not surprising. What is interesting and possibly useful, in studying the behavior of a real machine, is that the amplitudes of the linear tunes are much smaller relative to the other lines than when the coordinates are analyzed.

In addition to the lines corresponding to the harmonics of the tunes and the non-linear elements seen in the fourier transform of the coordinates there is also very obviously a very long period oscillation of the amplitude.

The question naturally arises whether the particular behavior seen, particularly of the amplitudes, is simply due to the ordering of the magnets or is a more general feature Accordingly the order of the magnets was of the model. permuted and the tracking was repeated. The amplitude is plotted as a function of turn for 10 permutations (Figure 3). The distributions certainly differ. There is also apparent in several of the distributions, the long period oscillation in the amplitudes seen in our original ordering. The period of these very slow oscillations differs from the permutation to the next and is therefore not likely to be due to the structure of the lattice but instead it reflects the ordering of the magnets. The lines in the fourier transforms of these amplitude-turn distributions have similar positions but the strength of the lines differ.

The long term oscillations are an unexpected result of the tracking for which no explanation was obvious to me. The tracking was therefore repeated first only the sextupole moments and then only with the octupole moments. When the amplitudes from the case with only sextupoles (Figure 4) are studied no long term oscillations are apparent. For the amplitudes, in the case where the only high order moments are the octupole moments (Figure 5a), there is a suggestion of a long term oscillation with ever a greater period than

before. It is crucial to recognize that these last tracking results were obtained with the chromaticity sextupoles adjusted to give zero chromaticity. When the tracking is repeated with the chromaticity sextupoles turned off (the lattice now has it natural chromaticity of cx - 20, cy - 20) there is no suggestion of a long term oscillation (Figure 5b). I feel it is proper at this point to conclude that the long term oscillation is the result of interaction between the sextupole and octupole moments in the lattice.

The effect of the synchrotron oscillations on the motion has been studied. Figure 6a shows the amplitude variation in the case of an off momentum particle and with synchrotron oscillations. There is a long period variation in the amplitudes in this situation too. The variation in the x and y amplitudes, both with and with out synchrotron oscillations, seem to be "in phase". The distributions in Figure 6a are with the chromaticity sextupoles set to give zero chromaticity. Since we have seen that the long term oscillations are sensitive the strength of these tosextupoles I have varied them by instructing the program to set the chromaticity to cx = +10.0, cy = -10.0. The tracking was then repeated. In general the distributions are similar to those with zero chromaticity (Figure 6b) but the distribution resulting from the sixth permutation is

anomalous showing a change in the amplitude and in the period of the long term oscillation after approximately 1000 turns (Figure 6c).

The details of the amplitude variation depend not only on the ordering of the magnets but also on the initial point in phase space at which we start the tracking. Figures 7a-f are plots of the amplitudes, using the ordering of the magnets of permutation #6, with the chromaticity cx = 10, cy= -10, for six different initial values for (x, x'), all of which have the same x amplitude. Both the period and the size of the amplitude variations change with the different starting points. If we reduce the initial displacement from 7 mm to 6 mm the long term oscillation of the amplitude disappears (Figures 8a-f).

An attempt has been made to calculate the driving terms for the nominal distribution, the anomalous distribution (permutation #6) and for the 1000 random distributions in the hope that one could recognize some property of permutation #6 which would enable one to identify potentially troublesome orderings of magnets. The driving terms from both the sextupole and octupole moments for permutation #6 appear similar to those of the nominal distribution (though, as would be expected, slightly larger in magnitude) but quite typical of the driving terms

calculated from the 1000 random permutations. I would therefore conclude that the driving terms will not be useful in predicting the kind of behavior seen in Figure 3f.

In summary the tracking calculations show unexpected behavior in the amplitudes. These are characterized by long term, apparently periodic, variations in the amplitudes. These variations, both in size and period, depend critically on the initial conditions (being far more prominent at large amplitudes) and on the ordering of the moments in the lattice. I do not have a good understanding of the source of these oscillations but they apparently arise from a conspiracy between the sextupole and octupole moments. The nature of the oscillations also depends on the synchrotron motion.

TABLE I

Nominal Tunes Used:

$$nu-x = 19.42123$$

 $nu-y = 19.38123$

Lattice properties at the starting point of the tracking

beta x = 97.3 mbeta y = 29.2 malpha x = -1.87alpha y = +0.59

r.f. characteristics:

harmonic number (h) = 1113 r.f. phase angle = 30 deg transition gamma = 18.4compaction factor (8 GeV) = -0.00108 Figure Captions

- Figure 1 Amplitude of the Fourier transform of the coordinates. Several of the lines present in the transform of the x coordinate are identified.
- Figure 2 (a) The amplitude as a function of turn for the nominal ordering of the moments.

(b) The amplitude of the Fourier transform of the x amplitude.

- Figure 3 (a)-(j) The amplitude as a function of turn for ten permutations of the order of the moments.
- Figure 4 The amplitude as a function of turn with only sextupole magnets.
- Figure 5 (a) The amplitude as a function of turn with only octupole magnets, cx = cy = 0.0.

(b) The amplitude as a function of turn with only octupole magnets, $cx^2 - 20$.

Figure 6 (a) The amplitude as a function of turn with synchrotron oscillations with cx = cy = 0.0. Nominal distribution.

(b) The amplitude as function of turn with synchrotron oscillations with cx = +10.0, cy = -10.0. Nominal distribution.

(c) The amplitude as function of turn with synchrotron oscillations with cx = cy = 0.0. Permutation #6.

- Figure 7 (a)-(f) The amplitude as a function of turn for the ordering of permutation #6 but with different initial coordinates in phase space. The initial displacement is 7 mm for x and y.
- Figure 8 (a)-(f) The amplitude as a function of turn for the ordering of permutation #6 but with different initial coordinates in phase space. The initial displacement is 6 mm for x and y.



 $\mathbb{C}^{n} \to$











.















FIG.4

:

. . .



.....









FIG.6c





NBNO 400 NO 0000. 4000 00 · N 0 4 · 0 0 a Q Q J-----4 14.24 08.50 1LE \bigcirc . $\sim \sim$ ° ° ~ (() ~ 11 · · · · · · · · · > H H----<u>---</u>+ $\sim \sim$ 00-00 Ш Q - 00 $\oplus \odot$ 69 69 $\vdash \circ$ Ú 1000 1 -- ..l NE 00 Ň $b \sim \infty$ \sim 11 . 1 < > \bigcirc . 1----4 $\psi_{\gamma} <$ Telef)_~ trip co $\sup_{n=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j$ L $\overline{\mathbf{v}_{i}}\sim -\mathbf{u}$ 1. S. 2. L.J -CC $\frac{1}{2} \circ$ (i " 47. SP ≤ 5 2. $\{\cdot\}$ an cr € 5 $\{ < 1 \}$ 1 ile Ch of s star at a star star star star 64.7 94. 94 C B





N 0 N 0 14.24.1 08.50.3 4 \bigcirc Z • • • • • • FIL \bigcirc N • • 0 7 • 0 0 || || • • • 0 > ⊢−−; | | | | $\square \bigcirc \neg \neg \bigcirc \neg \land \neg \land \bigcirc \bigcirc \neg \lor \bigcirc \square$ 87701/22. 87701/23. fc11N **|----**6000 O D \bigcirc NUM= 2 H----- \Box $\frac{1}{2}$ $\times \times \times \times \times$ c $n \mathbb{Z} \mathbb{Z} \mathbb{Z} \mathbb{Q} \mathbb{Q}$ $> \circ \infty$ \bigcirc \square \bigcirc \Box 1 UNNEL. ກ ຕີ $\leftarrow \bigcirc$ $\langle \rangle$ С.J a ЦJ ŝ _____ }---· Ð \bigcirc **1**——4 ന ് a.₹ j $||_{L^{\infty}_{1,1}} \to 0$ $\downarrow \dots \iff$ 20 ЦIJ í.l. \square \sim \times $_{\odot}$ ĊĿ < 0 \geq j. (E. c> < 0 \bigcirc 1 L CT CD L CO 57.58 State 81.50 0617 5411 0611 (201×)040 -/// .0.1 SZ.













