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TITLE A SYSTEMS ANALYSIS APPROACH TO PROBABILISTIC MODELING
OF FAULT TREES

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SUBMITTED TO 8th International Conference on "Structural Mechanics in
Reactor Technology," Brussels (Belgium)

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A

A SYSTEMS ANALYSIS APPROACH TO PROBABILISTIC
MODELING OF FAULT TREES

by

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Abstract

A method of probabilistic modeling of fault tree logic combined with stochastic process theory (Markov modeling) has been developed. Systems are then quantitatively analyzed probabilistically in terms of their failure mechanisms including common cause/common mode effects and time dependent failure and/or repair rate effects that include synergistic and propagational mechanisms. The modeling procedure results in a state vector set of first order, linear, inhomogeneous, differential equations describing the time dependent probabilities of failure described by the fault tree. The solutions of this Failure Mode State Variable (FMSV) model are cumulative probability distribution functions of the system. A method of appropriate synthesis of subsystems to form larger systems is developed, and applied to practical nuclear power safety systems.

1. Introduction

Nuclear reactor power technology development has widely used the fault tree as a tool for assessing safety, reliability, and risk. A fault tree depicts the occurrence of basic events (initiators or causes) that cause undesirable intermediate, and finally, top events representing system or component failures, where these events are modeled stochastically. The initiators (roots) of the fault tree pass through an interconnected (branched) system of Boolean OR and AND gates to which respectively apply the fourth and fifth axioms of

order, linear, inhomogeneous, differential equations describing the time dependencies of failure probabilities of failure described by the fault tree. The solutions of this Failure Mode State Variable (FMSV) model are cumulative probability distribution functions of the system. A method of appropriate synthesis of subsystems to form larger systems is developed, and applied to practical nuclear power safety systems.

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2. Method of Analysis

Using the disjoint property of Markov states S_i ; $i = 0, 1, \dots, m$, a set of Adjoint states \hat{S}_i ; $i = 0, 1, \dots, m$ was formulated comprising successive unions of the S_i in which all combinations of occurrences of basic events were depicted. The \hat{S}_0 state was chosen to represent S_m , the occurrence of all n basic events ($m = 2^n - 1$). The \hat{S}_m state was chosen to represent Ω or the union of all of the S_i . The intermediate \hat{S}_i ; $i = 1, 2, \dots, m-1$ represent all of the combinations of occurrences of any one, any two, etc. basic events. There is a transformation matrix \underline{E} for the probability state vector transformation equation ($\hat{\underline{P}}(t) = \underline{E} \underline{P}(t)$). The transformation \underline{E} is one-to-one and an m^{th} -order Markov model of the form:

$$\dot{\underline{P}}(t) = \underline{A} \underline{P}(t), t > 0 ; \underline{P}(0) \tag{1}$$

is transformed to the Adjoint state model

$$\dot{\hat{\underline{P}}}(t) = \hat{\underline{A}} \hat{\underline{P}}(t), t > 0 ; \hat{\underline{P}}(0) \tag{2}$$

by the similarity transformation

$$\hat{\underline{A}} = \underline{E} \underline{A} \underline{E}^{-1} . \tag{3}$$

Three generic fault trees each having two failure modes (inputs to the top gate) comprising two, three, or four statistically independent (S-independent) initiators together with common cause and/or common mode S-independent initiators were developed. These are shown in Fig. 1. In addition to the common cause/common mode events that result in S-dependent failure modes we included time dependent, synergistic failure-repair rate S-dependencies between these modes. We also developed a propagational failure rate S-dependency for a three-identical-component model. The fourth, eighth, and sixteenth order Markov and Adjoint state models are formulated. Using generalized state variable simulation models drawn from modern control system theory, a new model called the Failure Mode State Variable (FMSV) inhomogeneous model was formulated and found to have a general mathematical form. For example, the state variable analog simulation general form and four different two component models are shown in Fig. 2. A subsystem fault tree synthesis methodology was developed where lifetime cumulative distribution functions (l.cdf's) of the subsystem top event occurrence probabilities are curve fit with single term decaying exponential functions

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3. Applications and Results

Engineered safety systems in nuclear reactor technology are analyzed by the FMSV method. A simplified reactor shutdown (SCRAM) system (Bartholomew [3]) was computer simulated, and the λ cdf's were calculated (Fig. 3). A more detailed system fault tree (Fig. 4a) discussed by Caldarola and Wickenhauser [4], and comprising 30 initiators (some of which are common cause/common mode) was computer simulated using the generic fault tree models for subsystem portions (Figs. 4b, 4c). Approximate failure mode and top event λ cdf's were calculated assuming no repair mechanisms. The approximate failure mode and top event failure rates are listed in Table I.

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A. Conclusions

1. The FMSV method gives time dependent solutions of generic fault trees more directly than does the probability expansion of minimal cut sets.
2. The FMSV method for two initiators with common cause and having three kinds of failure-repair rate coupling mechanisms is readily computer simulated by generalized state variable techniques. A three identical component "jump" failure rate dependency can also be included.
3. The fault tree synthesis method utilizing generic subsystem fault trees is a practical approximate alternative to minimal cut set expansion for large fault trees, and retains engineering modeling interpretation and control of components, subsystems, and complete systems reliabilities.

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B

- [1] Arley, K. and D. R. Buch, Introduction to the Theory of Probability and Statistics, (John Wiley and Sons, 1950) pp. 13-16.
- [2] Shooman, M. L., Probabilistic Reliability: an Engineering Approach (McGraw-Hill, NY, 1968) pp. 61-67.
- [3] Bartholomew, R. J., "A State Space Method of Fault Tree Analysis with Applications" (Ph.D. Dissertation, University of New Mexico, July 1984) LA-10298-T, December 1984, Chapter 5, pp. 87-94.
- [4] Caldarola, L. and A. Wickenhauser, "Recent Advancements in Fault Tree Methodology at Karlsruhe," in Nuclear Systems Reliability and Risk Assessment, J. B. Fussell and G. R. Burdick, Eds. (SIAM, 1977) pp. 518-542.

- ... 1987 pp. 87-94.
- [3] Bartholomew, R. J., "A State Space Method of Fault Tree Analysis with Applications" (Ph.D. Dissertation, University of New Mexico, July 1984) LA-10298-T, December 1984, Chapter 5, pp. 87-94.
- [4] Caldarola, L. and A. Wickenhauser, "Recent Advancements in Fault Tree Methodology at Karlsruhe," in Nuclear Systems Reliability and Risk Assessment, J. B. Fussell and G. R. Burdick, Eds. (SIAM, 1977) pp. 518-542.

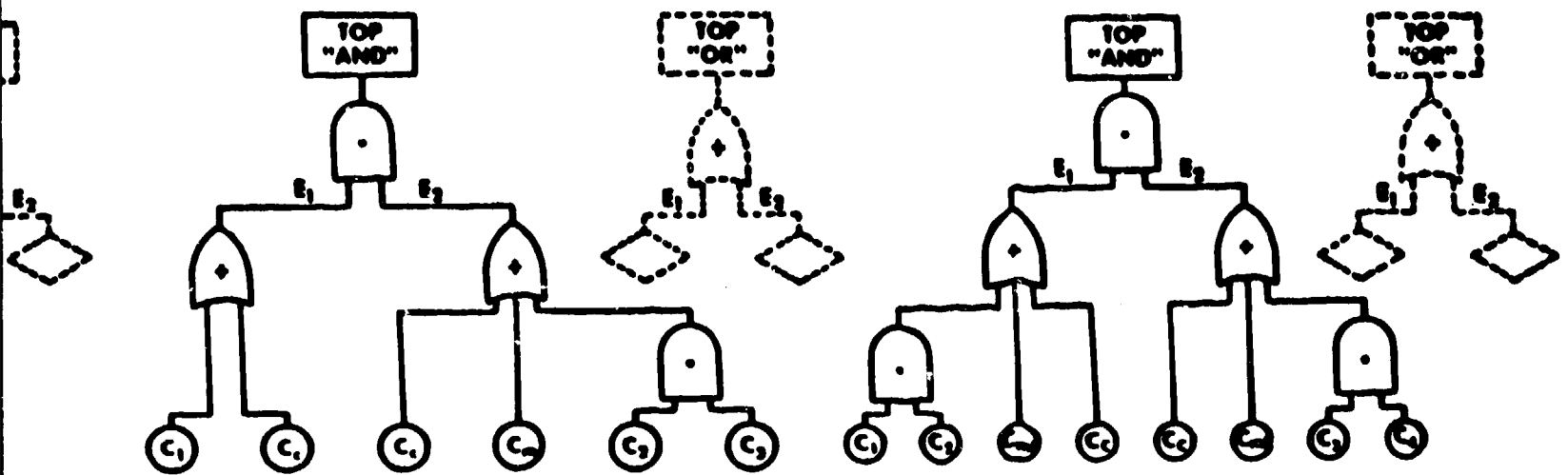
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GENERIC FAULT TREES

THREE COMPONENT

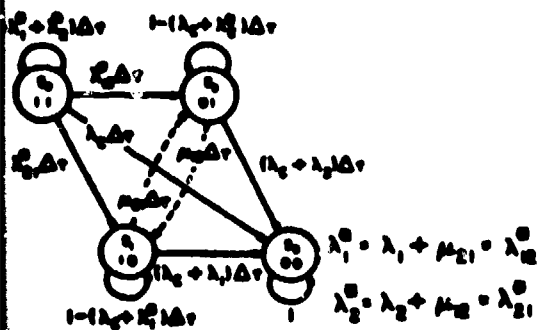
FOUR COMPONENT



C_{mi} = COMMON CAUSE, C_{mi} = i th COMMON MODE, E_i = i th EFFECT

MODELS

EXAMPLE

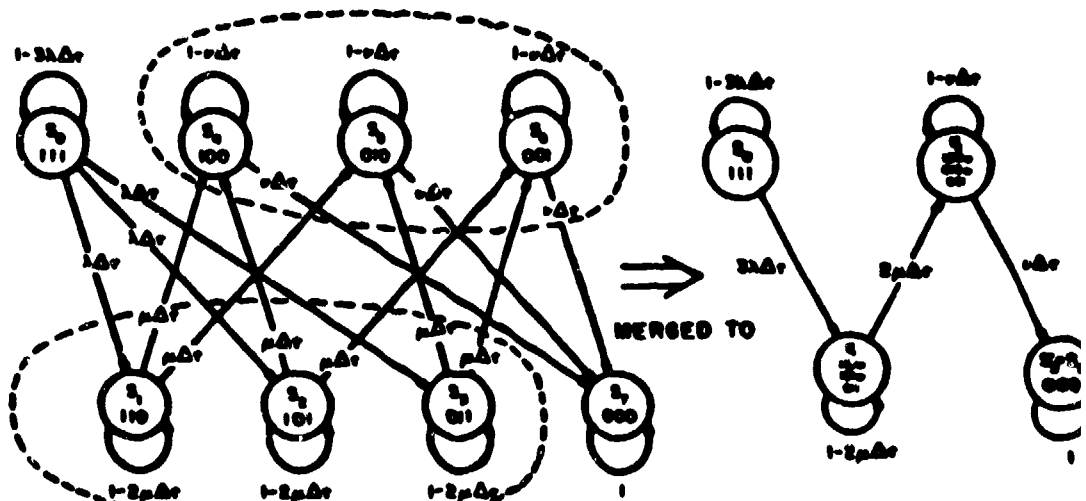


(2) Internal synergistic failure-repair

State	Component 1	Component 2
S_0	1	1
S_1	1	0
S_2	0	1
S_3	0	0

1 = FUNCTIONAL
0 = FAILED

MERGING OF MARKOV MODEL STATES



MERGED TO

DEGRADATION $\Rightarrow \lambda < \mu < 0$

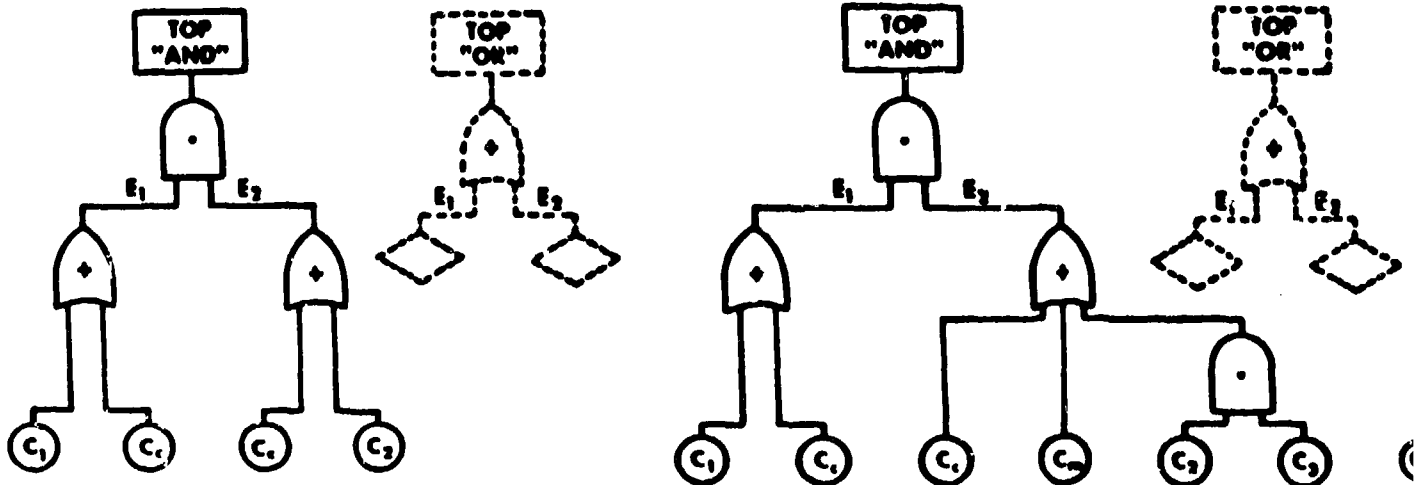
HEALING $\Rightarrow \lambda > \mu > 0$

ic fault trees and Markov models for subsystems synthesis of large systems.

GENERIC FAULT TREES

TWO COMPONENT

THREE COMPONENT



$C_i \equiv$ i th CAUSE, $C_c \equiv$ COMMON CAUSE, $C_{mi} \equiv$ i th COMMON MODE

GENERIC MARKOV MODELS

TWO COMPONENT EXAMPLE

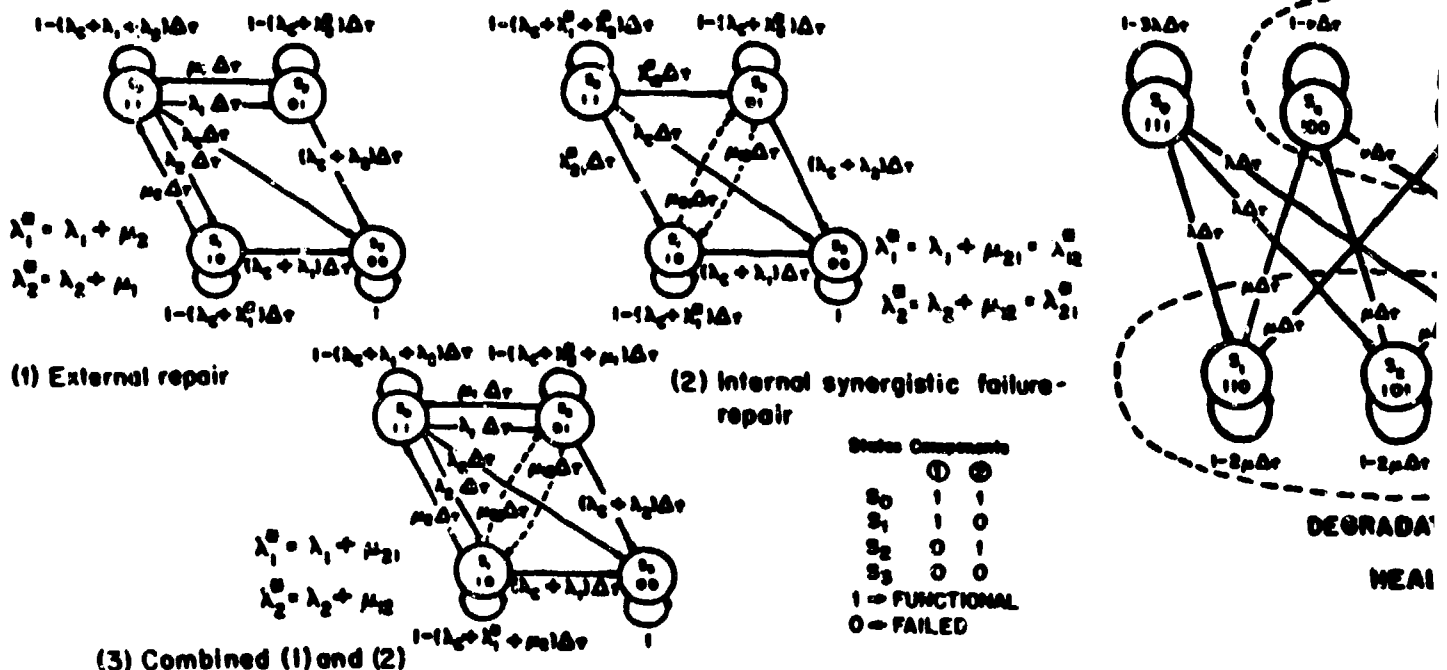


Figure 1. Generic fault trees and Markov models for subsystems synthesis

TRANSFORMATION FOR INTERNAL SYNERGISTIC FAILURE-REPAIR RATE COMPLEX

TWO COMPONENT EXAMPLE

Let $\lambda_1, \lambda_2, \mu_1, \mu_2$ be given with $\lambda_1 + \lambda_2 = \mu_1 + \mu_2$

$$E = \begin{bmatrix} -(\lambda_1 + \lambda_2 + \mu_1) & 0 & 0 & 0 \\ \lambda_1 & -(\lambda_2 + \mu_1) & \mu_2 & 0 \\ \lambda_2 & \mu_1 & -(\lambda_1 + \mu_2) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E^{-1} = \begin{bmatrix} \lambda_1 + \lambda_2 + \mu_1 & 0 & 0 & 0 \\ \lambda_1 & -(\lambda_2 + \mu_1) & \mu_2 & 0 \\ \lambda_2 & \mu_1 & -(\lambda_1 + \mu_2) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E^{-1} = \begin{bmatrix} \lambda_1 + \lambda_2 + \mu_1 & \lambda_2 & \lambda_1 & \lambda_2 \\ \mu_1 & -(\lambda_2 + \mu_1) & 0 & \lambda_1 + \mu_2 \\ \mu_2 & 0 & -(\lambda_1 + \mu_2) & \lambda_2 + \mu_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$E^{-1} = \begin{bmatrix} \lambda_1 + \lambda_2 + \mu_1 & \lambda_2 & \lambda_1 & \lambda_2 \\ \mu_1 & -(\lambda_2 + \mu_1) & 0 & \lambda_1 + \mu_2 \\ \mu_2 & 0 & -(\lambda_1 + \mu_2) & \lambda_2 + \mu_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

THE FAILURE MODE STATE VARIABLE MODEL FOR SYNERGISTIC FAILURE-REPAIR RATE COMPLEX

TWO COMPONENT EXAMPLE

$$Q(t) = P \left\{ \text{both } C_1 \text{ and } C_2 \text{ occur or } C_2 \text{ occurs in } [t, t+\Delta t] \right\} = \tau_{21}(t)$$

$$Q(t) = P \left\{ C_1 \text{ or } C_2 \text{ occurs in } [t, t+\Delta t] \right\} = \tau(t)$$

$$Q(t) = P \left\{ C_1 \text{ or } C_2 \text{ occurs in } [t, t+\Delta t] \right\} = \tau(t)$$

$$Q(t) = \tau(t)$$

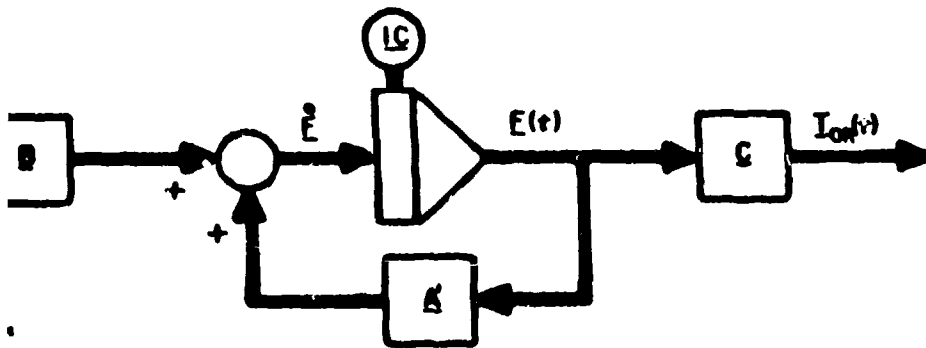
Thus, the FMSV model of the form $E = E + G U(t), E(0) = U(0)$ is:

$$E = \begin{bmatrix} -(\lambda_1 + \lambda_2 + \mu_1) & 0 & \mu_2 \\ 0 & -(\lambda_2 + \mu_1) & \mu_2 \\ \lambda_2 & \lambda_1 & -(\lambda_1 + \mu_2) \end{bmatrix} U = \begin{bmatrix} \lambda_2 + \mu_1 & 0 & 0 \\ \lambda_1 + \mu_2 & 0 & 0 \\ \lambda_1 & 0 & 0 \end{bmatrix} U(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$E^{-1} = \begin{bmatrix} \lambda_1 + \lambda_2 + \mu_1 & 0 & 0 \\ \lambda_1 & -(\lambda_2 + \mu_1) & \mu_2 \\ \lambda_2 & \mu_1 & -(\lambda_1 + \mu_2) \end{bmatrix}$$

$$U(t) = [1 \ 1 \ -1] E(t) + [0 \ 0 \ 0] U(0)$$

FAILURE MODE STATE VECTOR (FMSV) MODELS



FORM

(1) INTERNAL SYNERGISTIC FAILURE-REPAIR

(2) COMBINED (1) AND (2)

(3) IDENTICAL JUMP FAILURE RATE DEPENDENCY AMONG THREE IDENTICAL COMPONENTS

$$E = \begin{bmatrix} -\mu_1 & 0 & 0 \\ \mu_1 & -\mu_2 & 0 \\ -(\lambda_1 + \lambda_2 + \lambda_3) & 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} -(\lambda_1 + \lambda_2 + \mu_1) & 0 & \mu_2 \\ 0 & -(\lambda_2 + \mu_1) & \mu_2 \\ \lambda_2 & \lambda_1 & -(\lambda_1 + \mu_2) \end{bmatrix}$$

$$E = \begin{bmatrix} -(\lambda_1 + \lambda_2 + \mu_1) & \mu_1 & (\mu_1 + \mu_2 - \mu_3) \\ \mu_2 & -(\lambda_2 + \mu_1) & (\mu_1 + \mu_2 - \mu_3) \\ \lambda_2 & \lambda_1 & -(\lambda_1 + \mu_2) \end{bmatrix}$$

$$E = \begin{bmatrix} -3\lambda & 0 & 0 \\ 2\mu & -2\mu & 0 \\ 0 & \nu & -\nu \end{bmatrix}$$

$$E(t) = [E(t) \ Q(t) \ \tau_{21}(t)]^T, U = [U_1 \ U_2 \ U_3]^T, U(0) = [1 \ 0 \ 0]^T$$

$$E(t) = [E(t) \ Q(t) \ \tau_{21}(t)]^T$$

$$U = [U_1 \ U_2 \ U_3]^T$$

$$U(0) = [1 \ 0 \ 0]^T$$

$$B = \begin{bmatrix} (\lambda_1 + \lambda_2) & 0 & 0 \\ (\lambda_1 + \lambda_2) & 0 & 0 \\ \lambda_3 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} (\lambda_1 + \lambda_2) & 0 & 0 \\ (\lambda_1 + \lambda_2) & 0 & 0 \\ \lambda_3 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 3\lambda & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G = [1 \ 1 \ -1]$$

$$G = [1 \ 1 \ -1]$$

$$G = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mu_3 = \lambda_1 + \lambda_2 + \mu_1, \lambda_3 = \lambda_1 + \lambda_2$$

$$\lambda_3 = \lambda_1 + \lambda_2, \lambda_3 = \lambda_1 + \lambda_2$$

For failure defined:

(1) only one of three,

(2) only two of three, or

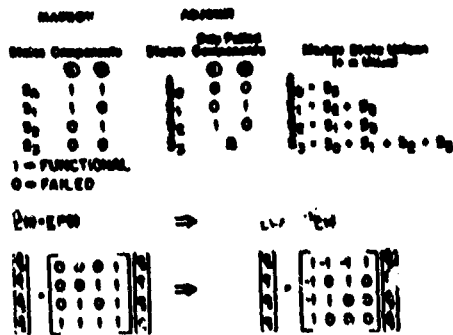
(3) all three,

$$I_{out}(t) = [\tau_1(t) \ \tau_2(t) \ \tau_3(t)]^T \text{ and } U(0) = [1 \ 0 \ 0]^T$$

Component of Failure Mode State Vector (FMSV) models of two components fault trees having statistical dependencies.

MARKOV TRANSFORMATION FORMULATION

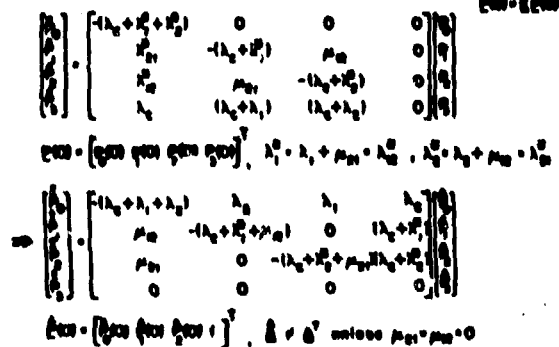
MARKOV - ADJOINT MODEL TRANSFORMATION FOR TWO COMPONENT SYSTEM



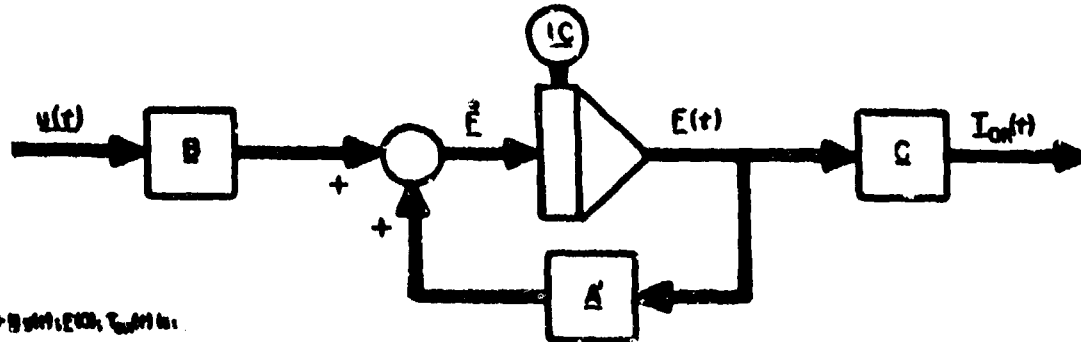
MARKOV TRANSFORMATION FOR INTERNAL SYNERGISTIC FAILURE-REPAIR RATE COMPLEX

TWO COMPONENT EXAMPLE

FOR GEN. I/O OVER TIME $\Rightarrow E(t) = E_0 e^{E t} E_0^{-1} E(t) = 0$, GIVEN E_0 WITH $E(t) = E^{00}$



FAILURE MODE STATE VECTOR (FMSV) MODELS



Thus, the FMSV model of the form $\dot{E} = E E + B u(t) + C I_{on}(t)$ is:

(1) EXTERNAL REPAIR

$$E = \begin{bmatrix} -(\lambda_1 + \lambda_2) & 0 & \mu_1 \\ 0 & -(\lambda_1 + \lambda_2) & \mu_2 \\ \lambda_2 & \lambda_1 & -(\lambda_1 + \lambda_2 + \mu_1) \end{bmatrix}$$

(2) INTERNAL SYNERGISTIC FAILURE-REPAIR

$$E = \begin{bmatrix} -(\lambda_1 + \lambda_2 + \mu_1) & 0 & \mu_1 \\ 0 & -(\lambda_2 + \mu_2) & \mu_{21} \\ \lambda_2 & \lambda_1 & -(\lambda_1 + \lambda_2 + \mu_1) \end{bmatrix}$$

(3) COMBINED (1) AND (2)

$$E = \begin{bmatrix} -(\lambda_1 + \lambda_2 + \mu_1) & \mu_{21} & (\mu_1 + \mu_2) \\ \mu_{21} & -(\lambda_2 + \mu_2) & (\mu_1 + \mu_2) \\ \lambda_2 & \lambda_1 & -(\lambda_1 + \lambda_2 + \mu_1) \end{bmatrix}$$

$$E(t) = (E_{10} \ E_{20} \ E_{30})^T \cdot e^{E t} \cdot \begin{bmatrix} \lambda_1 + \lambda_2 + \mu_1 \\ \lambda_2 + \lambda_1 + \mu_2 \\ \lambda_1 + \lambda_2 + \mu_1 \end{bmatrix}$$

$$B = \begin{bmatrix} (\lambda_1 + \lambda_2) & 0 & 0 \\ (\lambda_1 + \lambda_2) & 0 & 0 \\ \lambda_2 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} (\lambda_1 + \lambda_2) & 0 & 0 \\ (\lambda_1 + \lambda_2) & 0 & 0 \\ \lambda_2 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} (\lambda_1 + \lambda_2) & 0 & 0 \\ (\lambda_1 + \lambda_2) & 0 & 0 \\ \lambda_2 & 0 & 0 \end{bmatrix}$$

$$C = [1 \ 1 \ -1]$$

$$C = [1 \ 1 \ -1]$$

$$C = [1 \ 1 \ -1]$$

$$\lambda_1 = \lambda_1 + \mu_1, \lambda_2 = \lambda_2 + \mu_2$$

$$\lambda_1 = \lambda_1 + \mu_{21}, \lambda_2 = \lambda_2 + \mu_2$$

$$\lambda_1 = \lambda_1 + \mu_1, \lambda_2 = \lambda_2 + \mu_2$$

For letters define:
(1) only one of three,
(2) only two of three,
(3) all three.

$$I_{on}(t) = (I_{10} \ I_{20} \ I_{30})^T \cdot e^{E t} \cdot \begin{bmatrix} \lambda_1 + \lambda_2 + \mu_1 \\ \lambda_2 + \lambda_1 + \mu_2 \\ \lambda_1 + \lambda_2 + \mu_1 \end{bmatrix}$$

Figure 2. Mathematical development of Failure Mode State Vector (FMSV) models for four kinds of two components fault trees having statistical dependent

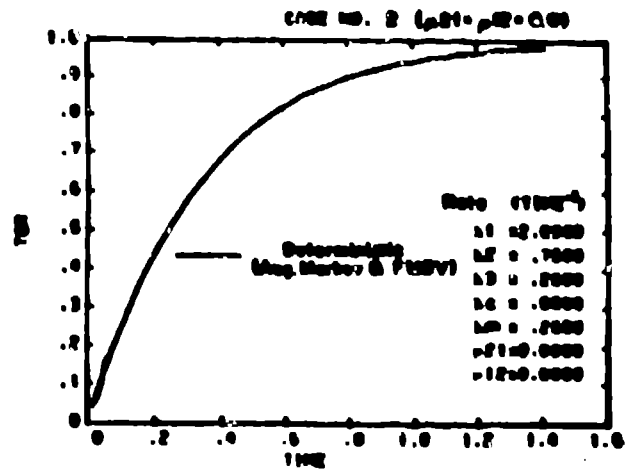
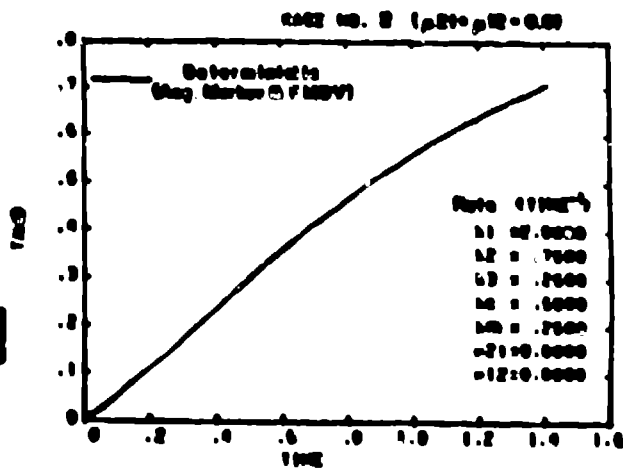
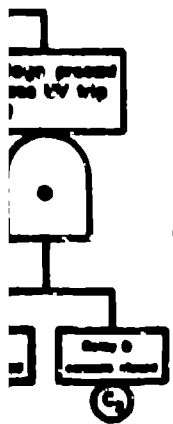
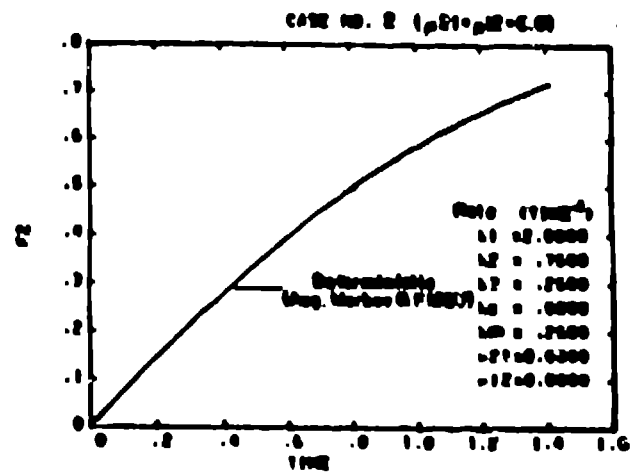
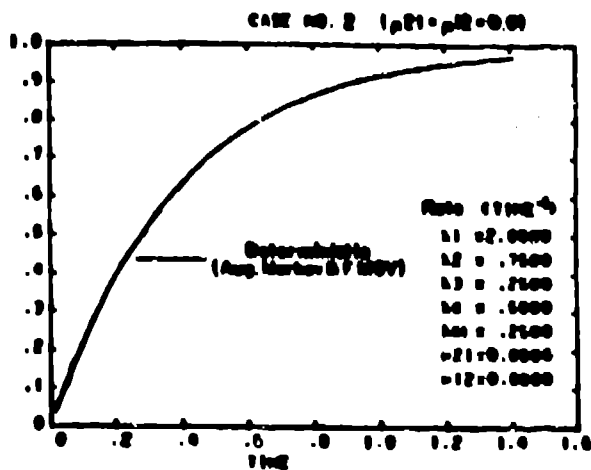
OR SCRAM
 INITIATED BY A
 REACTOR

**COMPARISON OF RESULTING PERTINENT
 CUMULATIVE DISTRIBUTION FUNCTIONS ESTABLISHED
 EXACTNESS OF EASY MODEL**



Initial condition: $E(0) = Q$

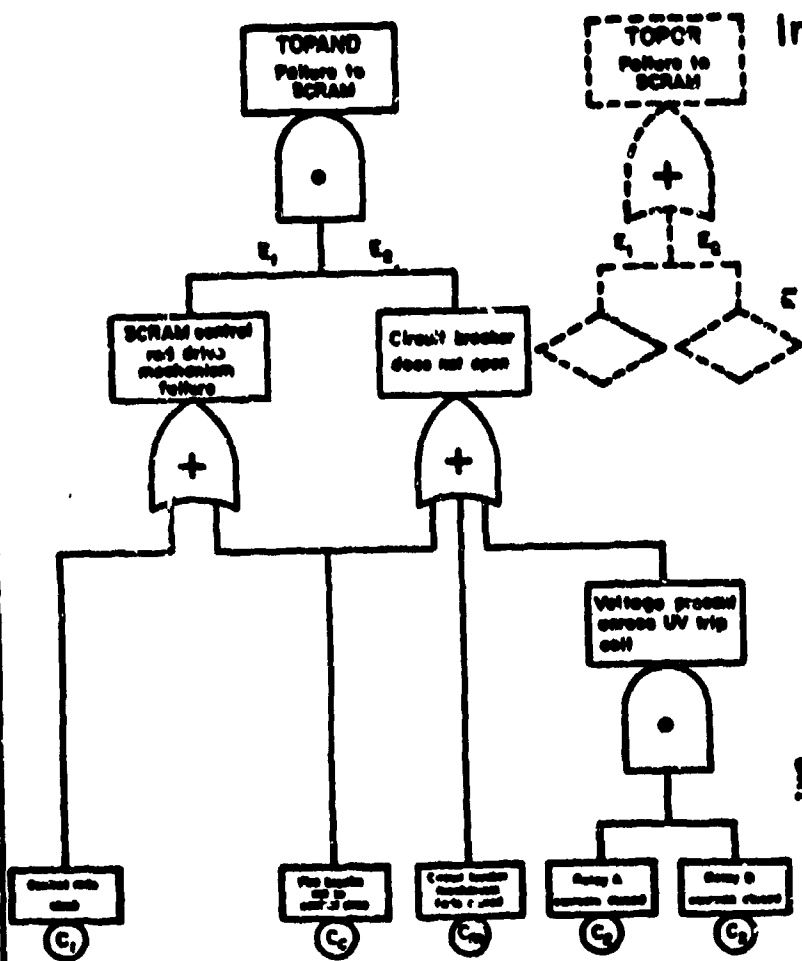
Rate coupling condition: $\mu_{12} = \mu_{21} = 0$



simplified reactor shutdown (SCRAM) system fault tree and resulting time pendent failure probability solutions comparison.

APPLICATION 1. A SIMPLIFIED NUCLEAR REACTOR SCRAM SYSTEM WITH COMMON CAUSE CAN BE REPRESENTED BY A THREE COMPONENT GATEWAY FAULT TREE

COMPARISON OF BINARY CUMULATIVE DISTRIBUTION FUNCTION EXACTNESS OF FAULT TREE



Initial condition: $E(0) = Q$ Rate ca

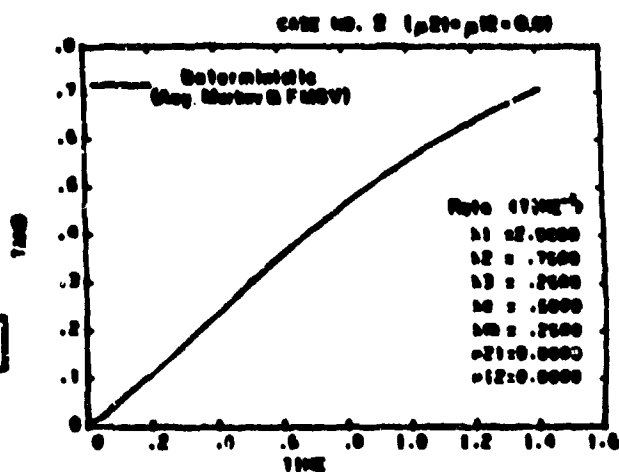
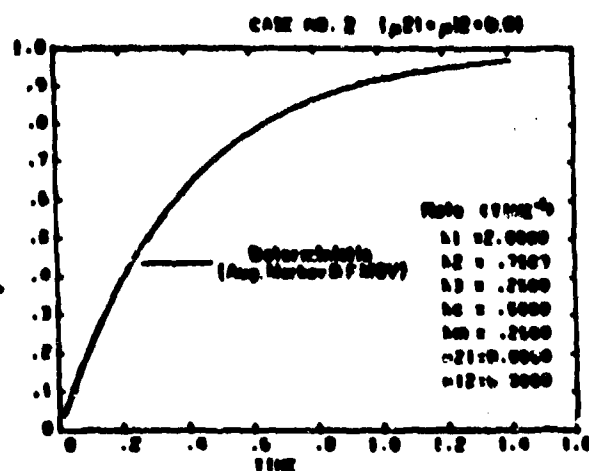
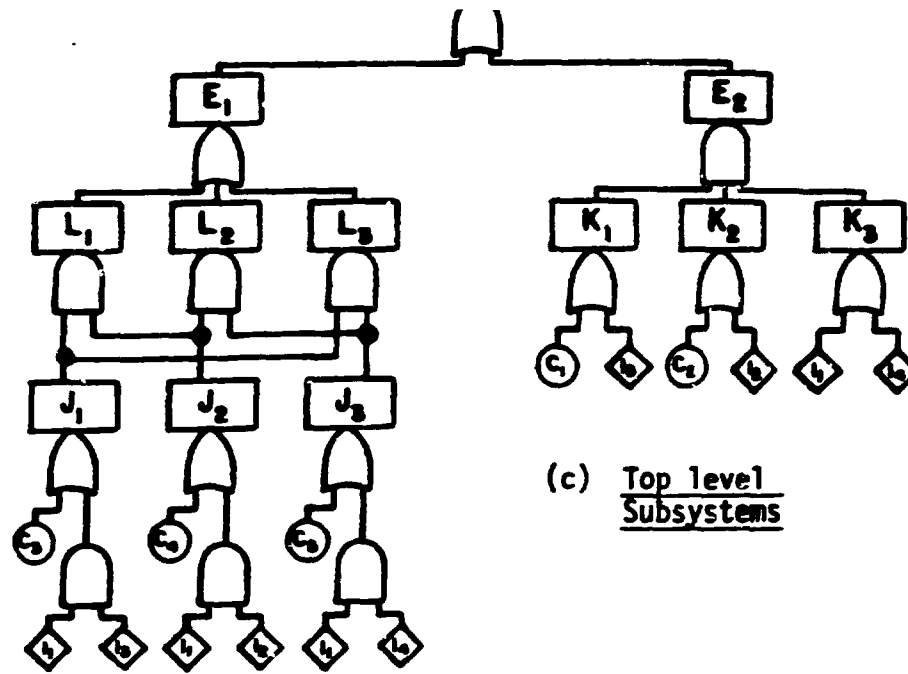
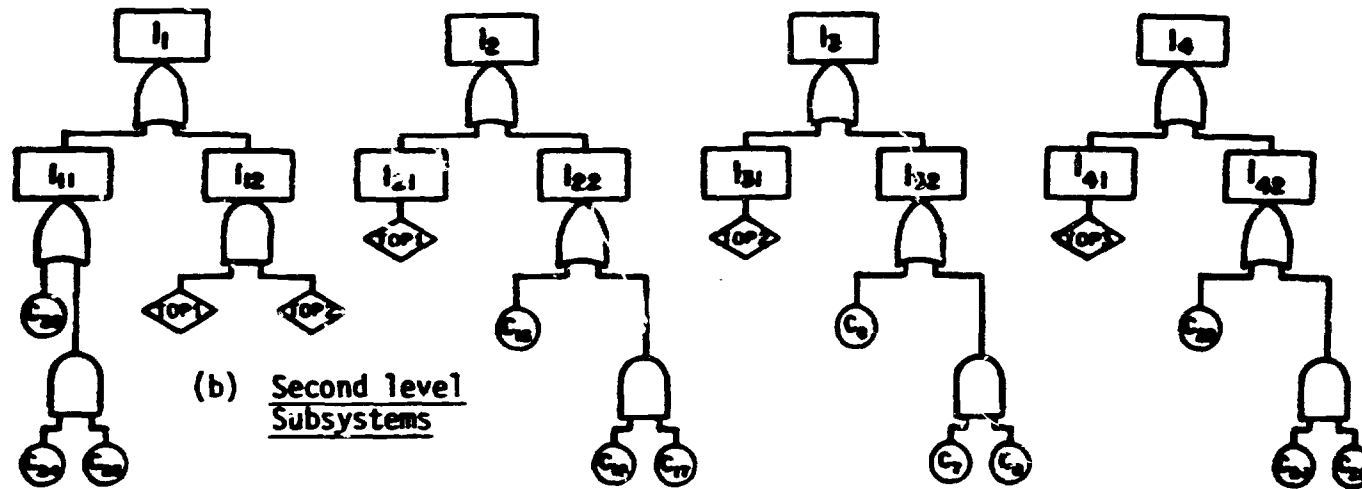


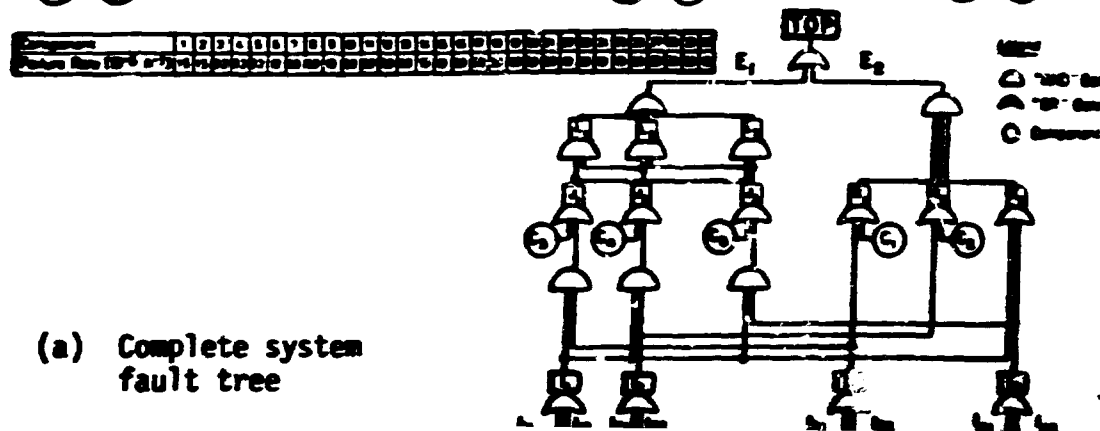
Figure 3. A simplified reactor shutdown (SCRAM) system fault tree and dependent failure probability solutions comparison.



(c) Top level Subsystems



(b) Second level Subsystems



(a) Complete system fault tree

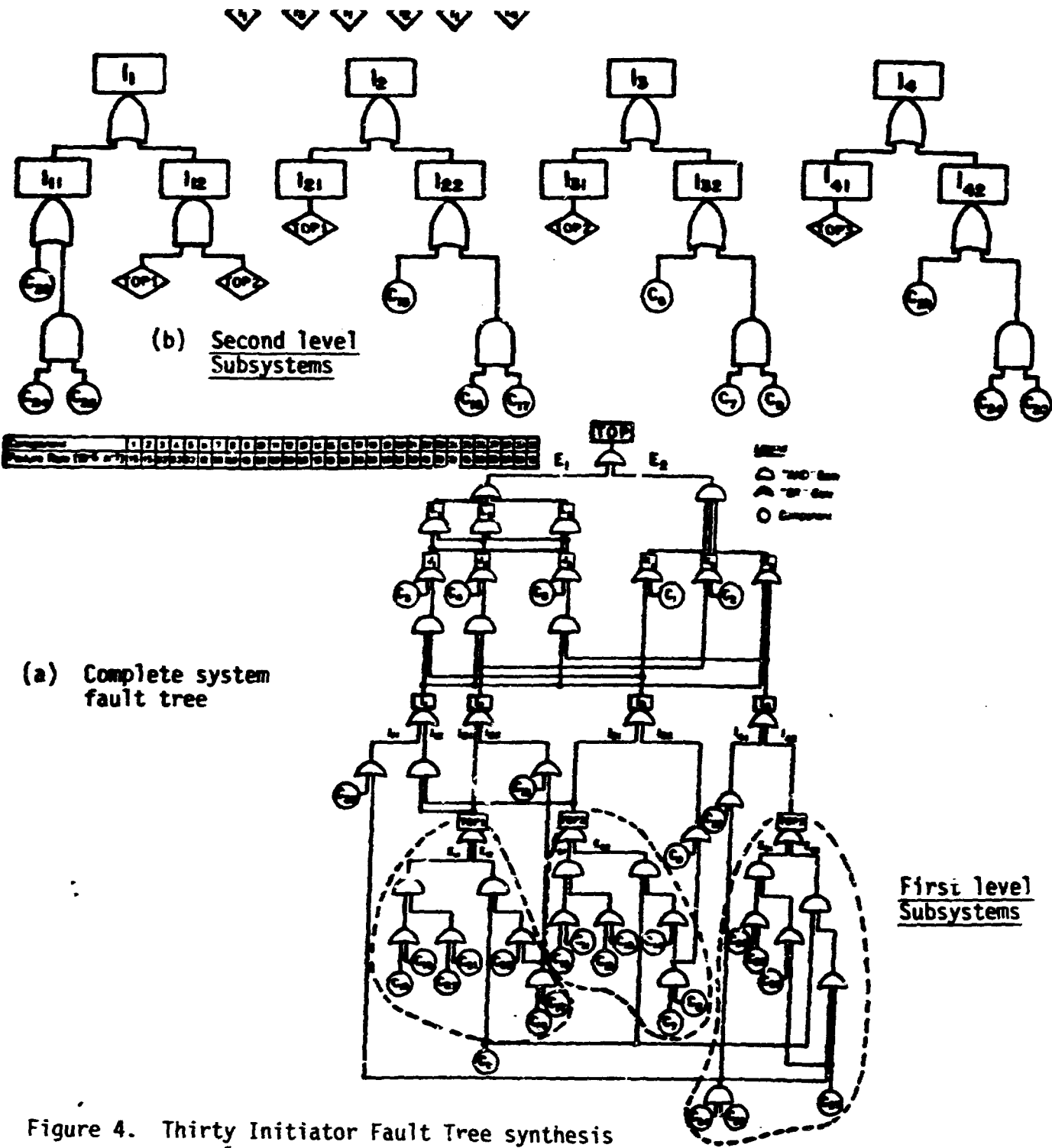


Figure 4. Thirty Initiator Fault Tree synthesis example.

TABLE I

ANALYSIS AND SYNTHESIS OF THIRTY BASIC EVENT FAULT TREE

SYSTEM	CONTRIBUTING EVENTS	GENERIC FAULT TREES			APPROX. EFFECTIVE OCCURRENCE RATES [10 ⁻⁶ h ⁻¹]
		No. of Components	Kind of Event	Kind of S-Dependencies	
FIRST LEVEL					
Failure Mode E11	(C18 or C19) and (C20 or C21)	2	TOPAND	None	57.47
Failure Mode E12	C9 and (C22 or C16 and C17)	3	TOPAND	CM(C22)	3.16
TOP Event TOP1	E11 or E12	-	TOPOR	---	60.63
Failure Mode E21	(C10 or C11) and (C12 or C13)	2	TOPAND	None	57.47
Failure Mode E22	C9 and (C14 or C7 and C8)	3	TOPAND	CM(C14)	3.16
TOP Event TOP2	E21 or E22	-	TOPOR	---	60.63
Failure Mode E31	(C26 or C27) and (C28 or C29)	2	TOPAND	None	57.47
Failure Mode E32	C9 and (C27 or C24 and C25)	3	TOPAND	CM(C27)	4.70
TOP Event TOP3	E31 or E32	-	TOPOR	---	62.17
SECOND LEVEL (initiators and approximated 1st level TOP events assumed S-independent)					
Failure Mode I11	C30 or C24 and C25	2	TOPOR	CM(C30)	11.18
Failure Mode I12	TOP1 and TOP2	2	TOPAND		27.04
Intermediate I1	I11 or I12	-	TOPOR	---	38.22
Failure Mode I21	TOP1 or				60.63
Failure Mode I22	(C15 or C16 and C17)	3	TOPOR	CM(C15)	
Intermediate I2	I21 or I22	-	TOPOR	---	90.53
Failure Mode I31	TOP2 or				60.63
Failure Mode I32	(C6 or C7 and C8)	3	TOPOR	CM(C6)	
Intermediate I3	I31 or I32	-	TOPOR	---	90.53
Failure Mode I41	TOP3 or				62.17
Failure Mode I42	(C23 or C24 and C25)	3	TOPOR	CM(C23)	
Intermediate I4	I41 or I42	-	TOPOR	---	73.34
TOP LEVEL (initiators, approximated 2nd level I's, and propagated I's through AND gates assumed S-independent)					
Intermediate J1	I1 and I3 or C3	2	TOPAND	CC(C3)	24.33
Intermediate J2	I1 and I2 or C4	2	TOPAND	CC(C4)	24.33
Intermediate J3	I1 and I4 or C5	2	TOPAND	CC(C5)	21.29
Intermediate K1	C1 or I3	2	TOPOR	None	92.03
Intermediate K2	C2 or I2	2	TOPOR	None	92.03
Intermediate K3	I1 or I4	2	TOPOR	None	111.56
2 of 3 L1	J1 and J2	-	TOPAND	None	48.66
2 of 3 L2	J2 and J3	-	TOPAND	None	45.61
2 of 3 L3	J1 and J3	-	TOPAND	None	45.61
Failure Mode E1	J1 and J2 or J2 and J3 or J1 and J3	3	TOPOR	2 of 3	9.06
	K1 and K2 and K3	3	TOPAND	Neglected	19.66

TABLE I

ANALYSIS AND SYNTHESIS OF THIRTY BASIC EVENT FAULT TREE

SYSTEM	CONTRIBUTING EVENTS	GENERIC FAULT TREES			APPROX. EFFECTIVE OCCURRENCE RATES [10 ⁻⁶ h ⁻¹]
		No. of Components	Kind of Event	Kind of S-Dependencies	
FIRST LEVEL					
Failure Mode E ₁₁	(C ₁₈ or C ₁₉) and (C ₂₀ or C ₂₁)	2	TOPAND	None	57.47
Failure Mode E ₁₂	C ₉ and (C ₂₂ or C ₁₆ and C ₁₇)	3	TOPAND	CM(C ₂₂)	3.16
TOP Event TOP1	E ₁₁ or E ₁₂	-	TOPOR	---	60.63
Failure Mode E ₂₁	(C ₁₀ or C ₁₁) and (C ₁₂ or C ₁₃)	2	TOPAND	None	57.47
Failure Mode E ₂₂	C ₉ and (C ₁₄ or C ₇ and C ₈)	3	TOPAND	CM(C ₁₄)	3.16
TOP Event TOP2	E ₂₁ or E ₂₂	-	TOPOR	---	60.63
Failure Mode E ₃₁	(C ₂₆ or C ₂₇) and (C ₂₈ or C ₂₉)	2	TOPAND	None	57.47
Failure Mode E ₃₂	C ₉ and (C ₂₇ or C ₂₄ and C ₂₅)	3	TOPAND	CM(C ₂₇)	4.70
TOP Event TOP3	E ₃₁ or E ₃₂	-	TOPOR	---	62.17
SECOND LEVEL (initiators and approximated 1st level TOP events assumed S-independent)					
Failure Mode I ₁₁	C ₃₀ or C ₂₄ and C ₂₅	2	TOPOR	CM(C ₃₀)	11.18
Failure Mode I ₁₂	TOP1 and TOP2	2	TOPAND	---	27.04
Intermediate I ₁	I ₁₁ or I ₁₂	-	TOPOR	---	38.22
Failure Mode I ₂₁	TOP1 or	-	---	---	60.63
Failure Mode I ₂₂	(C ₁₅ or C ₁₆ and C ₁₇)	3	TOPOR	CM(C ₁₅)	90.53
Intermediate I ₂	I ₂₁ or I ₂₂	-	TOPOR	---	60.63
Failure Mode I ₃₁	TOP2 or	-	---	---	90.53
Failure Mode I ₃₂	(C ₆ or C ₇ and C ₈)	3	TOPOR	CM(C ₆)	62.17
Intermediate I ₃	I ₃₁ or I ₃₂	-	TOPOR	---	90.53
Failure Mode I ₄₁	TOP3 or	-	---	---	62.17
Failure Mode I ₄₂	(C ₂₃ or C ₂₄ and C ₂₅)	3	TOPOR	CM(C ₂₃)	73.34
Intermediate I ₄	I ₄₁ or I ₄₂	-	TOPOR	---	73.34
TOP LEVEL (initiators, approximated 2nd level I's, and propagated I's through AND gates assumed S-independent)					
Intermediate J ₁	I ₁ and I ₃ or C ₃	2	TOPAND	CC(C ₃)	24.33
Intermediate J ₂	I ₁ and I ₂ or C ₄	2	TOPAND	CC(C ₄)	24.33
Intermediate J ₃	I ₁ and I ₄ or C ₅	2	TOPAND	CC(C ₅)	21.29
Intermediate K ₁	C ₁ or I ₃	2	TOPOR	None	92.03
Intermediate K ₂	C ₂ or I ₂	2	TOPOR	None	92.03
Intermediate K ₃	I ₁ or I ₄	2	TOPOR	None	111.56
2 of 3 L ₁	J ₁ and J ₂	-	TOPAND	None	48.66
2 of 3 L ₂	J ₂ and J ₃	-	TOPAND	None	45.61
2 of 3 L ₃	J ₁ and J ₃	-	TOPAND	None	45.61
Failure Mode E ₁	J ₁ and J ₂ or J ₂ and J ₃ or J ₁ and J ₃	3	TOPOR	2 of 3	9.06
Failure Mode E ₂	K ₁ and K ₂ and K ₃	3	TOPAND	Neglected	19.08
TOP	E ₁ or E ₂	-	TOPOR	---	28.16

CM = Common Mode

CC = Common Cause