UCRL-85825 PREPRINT

Conf-810602--33

THE ELECTROMAGNETIC VELOCITY GAUGE: USE OF MULTIPLE GAUGES, TIME RESPONSE, AND FLOW PERTURBATIONS

L. M. Erickson C. B. Johnson N. L. Parker H. C. Vantine R. C. Weingart R. S. Lee

Seventh International Detonation Symposium Annapolis, Maryland June 16-19, 1981



April 8, 1981



UCRL--85825

DE82 005702

3

THE ELECTROMAGNETIC VELOCITY GAUGE: USE OF MULTIPLE GAUGES, TIME RESPONSE, AND FLOW PERTURBATIONS

L. M. Erickson, C. B. Johnson, N. L. Parker, H. C. Vantine, and R. C. Weingart Lawrence Livermore National Laboratory P. D. Box 808, L-368 Livermore, CA 94550

and

R. S. Lee Kansas State University Manhattan, KS 66502

We have developed an in-situ electromagnetic velocity (EMV) gauge system for use in multiple-gauge studies of initiating and detonating explosives. We have also investigated the risetime of the gauge and the manner in which it perturbs a reactive flow. We report on the special precautions that are necessary in multiple gauge experiments to reduce lead spreading, simplify targe fabrication problems and minimize cross talk through the conducting explosive. Agreement between measured stress records and calculations from multiple velocity gauge data give us confidence that our velocity gauges are recording properly. We have used laser velocity interferometry to measure the gauge risetime in polymethyl methacrylate (PMMA). To resolve the difference in the two methods, we have examined hydrodynamic and material rate effects. In addition, we considered the effects of shock tilt, Ī٥ electronic response and magnetic diffusion on the gauge's response time.

1. INTRODUCTION

We have developed an in-situ electromagnetic velocity (EMV) gauge system for use in multiple-gauge studies of initiating and detonating explosives. We have also investigated the response time of the gauge and the manner in which it perturbs a reactive flow. Dur gauges are made from anodized aluminum foils with an active element 8 mm long x 3 mm wide x 25 µm thick. The gauge is oriented in an experiment so that motion of the active element cuts field lines of an externally-applied magnetic field. When the element is oriented perpendicular to the field direction, the induced emf is uBL, where u is the gauge velocity, B the magnetic field intensity and L the effective length of the gauge.

Multiple velocity gauges have been used in an explosive by Cowperthwaite and Rosenberg(1,2) and Vantine, et al(3), Rosenberg brought the gauge

DISCLANIER ----

Discloping preventing metabolic of work concerning on upper of the United State Government, Notices no, Diversities Government ware align you have a new of the United State Government, Notices no, Diversities and an annual set right in this to receive shall be the concernent of upper and annual set right in this to receive shall be the concernent of the concernent of the set of the concernent of the set of the concernent of the concernent of the set of the concernent of the concernent in concernent of the set of the concernent of the end executive concernent of the concernent of the set of the concernent set of executive concernent of the concernent of the set of the concernent set of executive concernent of the set of the concernent of the set of the set of the the set of the set of the set of the set of the concernent of the set of the the set of the concernent of the set of the the set of the the set of the set of the set of the set of the concernent of the set of the the set of set of the the set of the the set of the the set of the the set of the the set of the the set of the se

"Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-ENG-48,"

leads out of the back (i.e. downstream side) of the explosive target so that the leads would not be spread by rare-factions. For cast explosives, he developed an emplacement method in which 0.15 mm thick aluminum gauges were embedded in the target during casting. For pressed explosives, bringing the leads out of the back required machining and difficult assembly. We have found that bringing the leads out of the side of the target is more convenient for multiple gauge applications. We counteract the effect of "lead spreading", caused by side rarefactions, by using flat-sided projectiles to generate the input shock wayes.

A further advantage of using side leads is that it allows us to use brittle, anodized aluminum gauges whose leads cannot be bent to exit from the back of the target. We anodize the jauges because we have used 12 gauges in experiments where the spacing between gauges is 2 mm and find it necessary to

"insulate the gauges to prevent crosstalk. This is conveniently done by anodizing the 25 µm aluminum gauge to a depth of a few µm.

We have observed that rise times of velocity gauges in explosives are often longer than simple theory would predict. Jacobs and Edwards discussed EMV gauge response time, at the Fifth Detonation Symposium(4), but at that time no experimental technique was time no experimental available which would permit a simultaneous comparison of the gauge signal with the gauge velocity measured by another technique. We have used laser velocity interferometry to make such a comparison. We interpret the data by considering the effects of shock tilt, electronic response, rate effects in polymethyl methacrylate (PMMA) and diffusion on gauge magnetic the response time.

We investigated the dynamic interaction of a reactive shock wave with multiple gauges by using the records from a multiple velocity gauge experiment to calculate the pressure in the flow along a particle path. The calcuated pressure agreed well with the pressure measured directly by a manganin gauge.

11. EXPERIMENTAL

A. LLHL Gauge

Our gauges were developed with several criteria in mind. They should lend themselves to installation in multiple gauge configurations so that measurements can be made at a variety of depths in the reactive flow. They should offer minimum perturbation to the flow, and should not interact electrically with each other in the conductive medium of the reaction products. In addition, we considered it important that gauges lend themselves to easy installation in a target assembly, and that the finished assembly be readily evaluated in terms of flatness, adhesive thickness, and precise gauge location and orientation.

Aluminum was chosen as the gauge material. It is a good electrical conductor and is a reasonably good shock impedance match to most high explosives. In addition, it is readily insulated electrically by anodizing the surfaces.

Our gauge configuration is shown in Figure 1. Gauges are etched from 25 µm aluminum using a standard photo etch process for aluminum. Several gauges

2



target assembly

magnetic field



25 µm-thick aluminum gauge

Figure 1. Exploded view of a typical multiple gauge installation showing gauge orientation. Flat edges on flyer are provided to minimize lead spreading effects from edge rarefactions. Dimensional details of the gauge are shown in the sketch at the bottom of the figure. <u>с</u>

are etched simultaneously, side-by-side and they are tied together by leaving a tie strip of aluminum at the terminal end which ties the legs of each gauge together, and provides a great deal of stability to the legs while gauges are being glued in place in a target assembly.

Except for the terminals, the gauges are anodized on all surfaces to provide electrical insulation. The anodization process is carried out in a bath of 15 wt. percent sulfuric acid in water at room temperature, at a current density of about 120 A/m² for 15 minutes. Gauges are subsequently sealed in a boiling water bath for 15 minutes. The process results in an anodized coating about 0.004 mm thick.

Gauges are installed in the assembly by conding them individually to discs of explosive using a low-viscosity, compatible epoxy. A flat glass plate holds the gauge in place and a release sheet of thin Mylar presents, adherance to the glass. Modest pressure is applied to keep the gauge in place, while bonding, and excess epoxy is removed before it is fully set. Discs of explosives are finally bonded together using the same epoxy to form the completed experiment.

> B. Set-up for Risetime Measurements

It was desirable to conduct experiments using two independent methods to determine particle velocity and more importantly, gauge risetime. We designed an experiment to permit simultaneous measurement of flow velocity by an EMV gauge and by a fabry-Perot velocity interferometer, using light reflected from the gauge surface. The use of the Fabry-Perot velocimater required a transparent target. PMHA was selected as the target material. The laminated target contained two 25 µm, gauges one aluminum and one copper. The gauge surface was optically orepared to achieve diffuse reflection. A typical experimental arrangement is shown in Fig. 2. 1

A shock wave was produced in the target from the impact projectile accelerated by a light gas gun. The gun has a length of 18.3 m and an inside bore diameter of 101 mm. It is capable of providing a maximum projectile velocity of 1.0 km/s using a helium gas breech and 2.0 km/s using a powder breech. Projectile velocity is measured within 0.5 percent using a 180 kV OC flash x-ray system. As the projectile leaves the gun's muzzle, it interrupts a light beam to provide triggering for the first of two x-ray tubes. The second tube is triggered similarly, resulting in two projectile images on a single radiograph. Flyer tilt is determined using an array of crystal pins mounted in the target holder. Signals from these pins are recorded on raster scopes for later analysis.

The magnetic field is supplied by an electromagnet having flat pole faces 0.20 m in diameter spaced 0.33 m apart. The two coils each consist of 256 turns of No. 4 ANG square copper conductor driven by a current regulated power supply. The two coils are connected in series, and a current of 78 A provides a magnetic field of 0.1 tesla at the target position. Coils and pole pieces are protected by non-magnetic stainless



steel covers 12 mm thick. The coils and poles are mounted on 19 mm thick steel plates which mount in the experimental chamber. This configuration provides a field that is uniform to 0.5 percent throughout the 20 cm³ volume occupied by the gauges in a typical experiment. The field remains stable with no evidence of coil overheating in the vacuum for periods of up to 20 minutes. No means of external cooling is provided.

ų P

٠.

The fabry-Perot velocimeter consists of a cylindrical lens, a Fabry-Perot interferometer, a spherical lens and an electronic streak Camera. Light from an argon-ion laser operating in the single-frequency, single-longitu-dinal mode near 514.5 nm is focussed directly onto the gauge element with a field lens. Scattered reflected light collected by the field lens is returned as a parallel beam to the velocimeter. The cylindrical lens introduces a small convergence in the horizontal plane. Since, for a given wave length, only certain engles of incident rays passing through the Fabry-Perot interferometer, constructively interfere, a series of dots is formed at the focal plane of the spherical lens. The position of these dots will change with a change in wavelength. The dot motion, from the Doppler-shifted return beam, is recorded with the electronic streak camera[6].

The target, magnet coils and poles, and radiographic film holder are mounted in a 0.74 m diameter cylindrical, steel experimental chamber. At one end of the chamber is the gun's muzzle and at the other, separated by a 0.25 mm Mylar membrane, is a 2.1 m diameter cylindrical catcher tank. The chamber's maximum capacity is a 200 g PBX-9404 equivalent of high explosive.

> C. Set-Up for Lead Spread Experiments

The orientation of the EMV gauge leads in a target is an important experimental consideration. The leads must be oriented either along magnetic field lines or along the direction of material motion. In the former case, the target may be a stack of slabs with leads coming out the side of the target. In the latter, the leads must he brought out the back of the target. Both arrangements are shown schematically in Fig. 3.

When the leads are brought out the back of the target they are not subject to significant transverse flow during the time of experimental interest so

Figure 3. Sketch of a lead spreading experiment. The flat edge of the flyer produces a rarefraction which only stretches the leads (a) while the radial flow from the round edge tends to spread the leads apart (c). Gauge (b) has leads brought out through the back of the target so it is not subject to lead spreading.

that only the active element of the gauge produces a signal. A disadvantage is that shot construction and assembly are more difficult, particularly in experiments where multiple gauges are used.

If the leads are brought out the side, the target for a multiple-gauge experiment consists of a series of discs with the gauges mounted between the discs. Target assembly is easier and dimensional perturbations are two minimized. A potential drawback to this procedure is that the leads come out of the target through the twodimensional flow produced by the edge rarefactions. Gupta(5) has pointed out that this causes "lead spreading" and can give rise to errors in velocity. We sought to remedy the effect of "lead spreading" caused by side rarefraction, by using flat-sided projectiles, Fig. 3. With the flat-sided projectile, edae rarefractions do not propagate radially. but rather along the lead direction. Therefore, the leads do not spread. We experimentally compared the three lead configurations in symmetric impact experiments.

111. DATA AND RESULTS

A. Risetime

A typical velocity-time history for a 25 wm aluminum gauge in a PMMA target shocked to an initial stress of 1.6 GPa is shown in Fig. 4. The risetime, the time necessary for the signal to change from 10 percent to 90 percent of its maximum, is 56 ns. This is much longer than the time response of the measuring system which was determined by injecting a pulse of known risetime from a mercury pulser, which provided a low impedance source in lieu of the gauge. The voltage was then monitored across a 50 ohm terminating resistor. For our measuring circuitry the total electronic risetime was experimentally determined to be less than 2 ns.

The data from the Fabry-Perot velocimeter is also plotted in Fig. 4. In this case, the risetime was measured to be 39 ± 2 ns. To determine the response time of the velocimeter, one must consider Fabry-Perot mirror reflectivity and spacing, and camera slit width, sweep speed and resolution. For this experiment the instrument response was calculated to be 9 ns.

8. Lead Spread

The curves in Fig. 5 show the signals observed with leads brought out



Figure 4. Comparison of ENV gauge and Fabry-Perot experimental results with a hydrodynamic calculation. The data shown is for a 25 um aluminum gauge in PMMA shocked to an initial stress of 1.6 GPa.



Figure 5. Comparison of EWV gauge output for three gauges arranged in Teflon as depicted in Fig. 3. The linear increase in curve c is due to lead spreading. Curves a and b result from gauges whose leads are not spreading.

The the back and sides of the target. rounding of the initial part of the signals is caused by shock reverberations in the gauge elements and viscoelastic effects in the target, as we discuss elsewhere in this paper. We see that the signal from the gauge with leads brought out the back and thus not subjected to lead spreading, curve b, is similar to the signal from the gauge whose leads were brought out the side and impacted by a flat-sided projectile, curve a. There is no lead spreading. The signal from a gauge with side leads, impacted by a round-sided projectile, is shown in curve c. This signal, however, never reaches a steady state but continues to increase linearly with time. The increase is caused by lead spreading from the tangential velocity components near the edges of the target.

C. Flow Perturbations

Do multiple velocity gauges perturb the reactive flow process? To answer this question, we measured velocity-time histories of a nonsteady initiation wave at four positions (0,2,4,6 mm) in a PBX-9404 explosive target shocked to an initial stress of 2.7 GPa, using embedded electromagnetic velocity gauges. In two companion experiments we measured stress-time histories at 0 mm and 6 mm in a target shocked to 2.7 GPa, using embedded manganin stress gauges. We calculated the stress field from the

velocity gauge records and one of the stress gauge records (0 mm) and compared the calculation with the measured stress record at 6 mm. A high degree of consistency between the measured and calculated stress records would indicate that both the multiple velocity gauges and the pressure gauges are accurately recording the flow in the severe. explosive environment.

Each experimental assembly consisted of a laminated explosive/gauge target located near the muzzle of a 101 mm gas gun. The impact of a Teflon flyer sent a stress wave into the target. Stress measurements were made with insulated foil type manganin gauges.

The data from the velocity gauge experiments are shown in Fig. 6. The velocity records show a nearly constant amplitude shock front followed by a reactive wave.

Analysis of the embedded gauge data proceeds from the consideration of the integrated form of the momentum equation, written in Lagrange coordinates:

$$p(h, t) = p(h_0, t) - \frac{1}{v_0} \int_{h_0}^{h} \left(\frac{\partial u}{\partial t}\right)_{h} dh \qquad (1)$$

where p is normal stress, h is the position at which the stress is eval-



Figure 5. Results for velocity gauges at four positions in a PBJ-9404 explosive target shocked to an initial stress of 2.7 GPa.

uated, h_D is the position from which the integration starts, u is the particle velocity, vo is specific volume and t is time. A discussion of the integration procedure can be found in Ref. 3. Numerical errors in the integration procedure are less than 1 percent(3). In Fig. 7 we compare the computed stress curve at h=6 mm with the experimental value. For this calculation we used velocity records shown in Fig. 6 and a stress record at h=0 mm. We observed a high degree of consistency between the two stress records indicating that both the multiple velocity gauges and the stress gauges are accur-ately recording the flow variables in the explosive environment.

IV. DISCUSSIONS

A. Risetime

1. Non-Magnetic Effects

We considered hydrodynamics, material rate effects and projectile tilt as we investigated the difference in risetimes from the EMW gauge signals and the signals from the fabry-Perot velocimeter. To estimate the hydrodynamic time we calculated the gauge's response using a one-dimensional, Lagrangian, elastic-plastic material hydrodynamic code. The result is plotted in Fig. 4 with experimental data for comparison. The 20 ns rise-



Figure 7. Comparison of a stress curcalculated from the velocity records shown in Fig. 7 and an experimental stress record at a 6 mm position in a FBI-9404 explosive target.

time in the code calculation is due to hydrodynamic time only, that is, the time required for the shock wave to reverberate in the gauge until the gauge reaches the pressure and particle velocity of the host material. All three traces agree, within their respective error parameters, in final particle velocity, but differ in risetime. This confirms that the velocity gauge records to * 2 percent accuracy.

It will be noted in Fig. 4 that the calculated risetime is faster than the risetime observed using the Fabry-Perot velocimeter. We show in the next section that magnetic diffusion time is very small and we believe that the additional risetime is due to viscoelastic effects in the PMMA targets. Steady shock waves with the rounded particle velocity profiles have been observed in PMAA by Barker and Hollenbach(7) and Schuler(8). Schuler's data shows an abrupt rise in particle velocity to about 80 percent of the final level with most of the transition to final velocity occurring over approximately 0.5 usecs. Nunziato et al.(9) using a non-linear viscoelastic model obtained ound agreement with experimental particle velocity waveshapes. We observed considerably less rounding of the signal than was reported by the authors above, but we feel that this is reasonable, since our experiments were done at a much higher stress level.

Tilt of the flyer at impact can also have a large influence on gauge risetime, since the shock front will not arrive at all points along the gauge simultaneously. In the experiment shown in Fig. 4, the flyer had 2 milliradians of tilt and was oriented such that the shock wave swept across the gauge element from one end to the other. This contributed a 20 ns ramp to gauge risetime. The effect of tilt on the fabry-Perot result was only 1 as since the laser spot size had a diameter 1/20 of the gauge's effective length.

2. Magnetic Diffusion

An ultimate limitation on the response time of EMV gauges is the time required to establish a steady-state current flow in the gauge and measuring circuit when the gauge experiences a step change in velocity. We may represent the gauge system by an equivalent circuit as shown in Fig. 8, where the gauge is represented by an emf source in series with a variable resistance $R_g(t)$ and the measuring circuit resistance R(w) and inductance L(w) in series with a load resistor R_1 .



Figure 8. Schematic of our gauge system represented by a equivalent circuit.

The response time of the gauge can be estimated by a simple magnetic diffusion calculation. If the gauge undergoes a step change from zero to velocity, U, at time t=0, the initial gauge resistance will be very high, because all of the induced electric field is confined to the surface of the gauge. Because of this initial high resistance, virtually all of the voltage drop in the circuit at t=0 will be observed across the load resistor, Ri. The induced electric field, E, will diffuse into the gauge according to the heat equation.

$$\Delta_5 = \pi^0 \alpha \frac{9t}{9E} \qquad (5)$$

subject to the initial condition that E=0 at t=0. At the surface of the gauge, $E=uB_0$ where B_0 is the external magnetic field. When the electric field has diffused into the gauge to a depth which makes the gauge resistance much less than R_1 , the full induced emf, $V=uB\ell$, will appear across R_1 .

Carslaw and Jaeger(10) give a series solution for Eq. 2 for a semiinfinite slab of thickness 2d as,

$$\frac{E}{aB_0} = 1 - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)} e^{-(2n+1)^2} \pi^2 T/4 \chi$$

$$\cos \frac{(sn+1)\pi t}{2}$$
(3)

2

where $T=t/\nu_{cd}d^2$, $\xi=x/d$ and x is the distance from the center of the slab. Using Eq. 3, we find that for a copper foil of 50 µm thickness, the induced electric field at a depth of 5 µm will reach 50 percent of its final value in 2 ns. The resistance of a 5 µm thick layer on the surface of our gauge element at this time would be .03 ohns, so the full signal would appear across R_g . We feel that a conservative estimate of the magnetic field into

the gauge is less than 2 ns for 50 µm thick Cu. For Al and thinner Cu gauges the response time will be less.

3. Summation of Effects

38.10

٠.

Relating the observed risetimes to the hydrodynamic and electrical effects discussed above is not an easy task. We feel that the instrument response is certainly fast enough to record true gauge signals and that the magnetic diffusion contribution is negligible when compared to hydrodynamic effects, material rate effects or tilt. Tilt is a linear effect which can be easily handled.

We have not measured the rute effect contribution to risetime, but plan to do so indirectly by using a very thin gauge $(1 \ \mu m)$ whose hydrodynamic time will be small (<1 ns}. Then the experimental risetime for a Fabry-Perot velocimeter will be mainly due to PMMA material rate effects.

B. Lead Orientation

If curve c, Fig. 4 is extrapolated back to the shock jump we obtain U=0.479 km/s, in agreement with the extrapolation of curve b and in good agreement with Up obtained from the measured projectile velocity of V=0.951±.003 km/s. The slope of curve c amounts to an increase in relative error 1.4 percent each microsecond (0.007 mm/us²).

The lead spreading can be reduced significantly by machining flats on the edges of the projectile and orienting the projectile so that at impact the edges of the flats are perpendicular to the lead direction. With flat-sided projectiles the flow in the rarefaction region tends only to stretch the leads instead of separating them. We found that the projectile does not rotate appreciably in our smoothbore gun, so we were able to use this technique. Curve a in fig. 4 shows the signal observed in a Teflon target impacted under the same conditions as curves b and c, except that a flat-sided flyer was used. In curve a the lead spreading produces a slope of only 0.2 percent/us (0.001 mm/us²). The average signal slope we observed from five gauges in three symmetric impact experiments was 0.2 percent/us at a projectile velocity of 0.951±.003 km/s. We also fired a shot at a projectile velocity of 1.9±.038 km/s and observed a slope of .4 percent/us in a single gauge (0.004 mm/us) where we brought the leads out the side. We feel that the technique of using flat-sided projectiles reduces the lead-spreading error to an acceptable value, although we would advise performing experiments to check the error when target configurations or velocity ranges are changed.

V. SUMMARY

We discussed the fabrication of the LENL electromagnetic velocity gauge, and the installation of multiple gauges in explosive targets. We showed that lead spreading could be a source of error and discussed precautions that should be taken to prevent lead spreading. Hc also investigated the risetime of the gauge and showed in general the risetime depends on the details of the particular experiment. A universal risetime figure is not available. Therefore, the use of these gauges to measure fast i.e. nanosecond events requires careful characterization of the system under ob≤ervation. Finally, we showed that multiple velocity gauges do not significantly perturb the reactive flow process. This opens the way to perform a multiple gauge Lagrange analysis using multiple particle velocity gauges.

ACKNOWLEDGEMENTS

The authors thank the 101 mm gun crew, W. Humper, M. Bainter, W. Duguid and those who assembled our shots for careful and conscientious work. This was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory.

"Work performed under the $au_{k+\infty}$, of the U.S. Department of Energy by the Lawrence Livermore Laboratory under contract number W-7405-ENG-48."

ļ

ŝ

R

REFERENCES

- M. Cowperthwaite and J. T. Rosenberg, "Lagrange Gauge Studies of Nonideal Explosives Containing NH4NO3," Final Report prepared for U.S. Army Research Office, Contract DAA G29-76- C-0033, SRI International, Menlo Park, CA 94025, October 1979.
- N. Cowperthwaite and J. T. Rosenberg, "A Multiple Lagrange Gauge Study of the Shock Initiation Process in Cast TNT," Proceedings of the Sixth Symposium (International) on Detonation, Office of Naval Research Report ACR-221, p. 786, 1976.
- H. C. Vantine, W D. Curtis, L. M. Erickson and R. S. Lee, "A Comparison of Stress and Velocity Measurements in PBX-9404 Explosive," Proceedings of the Eighteenth Symposium (International) on Combustion, The Combustion Institute, 1981.
- S. J. Jacobs and O. J. Edwards, "Experimental Study of the Electromagnetic Velocity Gauge Technique," Proceedings of the Fifth Symposium (International) on Detonation, Office of Naval Research, Report ACR-184, p. 413, 1976.

 Y. H. Gupta, D. D. Keough, D. Henley, and D. F. Walter, "Measurement of Lateral Compressive Stresse Under Shock Loading," Applied Physics Letters, Vol. 37, No. 4, p. 395, Aug. 1980. Ť

- H. H. Chau, J. W. Lyle, M. A. Summers, "Simple Velocity Interferometer System," Conference on Laser and Electro-Optical Systems, LINL. Feb. 1978.
- L. H. Barker and R. E. Hollenbach, "Shock-Wave Studies of PMMA, Fused Silica, and Sapphire," Journal of Applied Physics, Vol. 41, No. 10, P. 4208, Sept. 1970.
- K. W. Schuler, "Propagation of Steady Shock Waves in Polymethyl Methacrylate," Journal of Mechanical Physic Solids, Vol. 18, No. 4, p. 277, Sept. 1970.
- 9. Nunziato, Walsh, Schuler and Barker. "Wave Propagation in Non-Linear Viscoelastic Materials," <u>Handbuck</u> <u>Der Physik</u>, Festkorpermechanik <u>JV</u>, pp. 26-48, Springer-Verlog, Berlin, Heidelberg, New York, 1974.
- Carslaw, H. S. and Jaeger, J. C., <u>Conduction of Heat in Solids</u>, p. 100, Oxford Univ. Press, 1959.

9

DISCLAIMER

This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor the University of California nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial products, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorseme of California. The views and opisions of authors expressed herein do not necessarily state or reflect those of the United States Government thereof, and shall not be used for advertising or product endorsement purposes.

Ì