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Liquid Storage Tanks Under Vertical Excitation

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ABSTRACT

Until recently, the hydrodynamic effects on liquid storage tanks induced by an earthquake excitation were basically treated for the horizontal component of the earthquake. Recent studies, however, showed that the hydrodynamic effects due to the vertical component of an earthquake may be significant. In these studies the tank is assumed to be fixed at the bottom. This paper is concerned with the hydrodynamic behavior of liquid storage tanks induced by vertical earthquake input excitation. First, the fluid-tank system is treated as a fixed-base system and a simple formula is obtained for the coupled fluid-structure natural frequency. Second, additional interaction effects due to the foundation flexibility on the fluid-tank system are investigated. It is concluded that the foundation flexibility may have a significant effect on the hydrodynamic behavior of the liquid storage tanks under a vertical ground shaking.

NOMENCLATURE

c_o	:	velocity of sound in the fluid
D	:	membrane stiffness
E	:	modulus of elasticity
F_h	:	hydrodynamic loading
H	:	tank height
I_o	:	modified Bessel function of zero order
J_o	:	ordinary Bessel function of zero order
$[K_m]$	:	2x2 stiffness matrix defined in Eq. 8.2
m_m	:	fluid apparent mass
$[M_m]$	:	2x2 mass matrix defined in Eq. 8.2
p_o	:	fluid static pressure
q	:	pressure fluctuation about p_o
R	:	tank radius
S_o	:	function defined in Eq. 5.3
(x,θ,r)	:	cylindrical coordinate system
$\ddot{x}_g$	:	vertical earthquake acceleration
u	:	longitudinal displacement component
w	:	radial displacement component
β_m	:	parameter defined in Eq. 8.3
ρ	:	tank wall mass density
ρ_o	:	fluid mass density
ν	:	Poisson's ratio
ω_m	:	frequency in rad/sec

INTRODUCTION

The damages observed in liquid storage tanks during earthquakes such as those due to the Alaska 1964 earthquake, raised a serious concern among engineers regarding the significance of the fluid dynamic effects on the design of liquid storage tanks. Many studies on this subject appeared in the literature during the last thirty years which deal with the dynamics of the fluid-tank systems due to seismic excitations.

In the early treatments of this problem an equivalent mechanical model was proposed by Housner (1) which simulates the fundamental sloshing frequency by a spring-mass unit. This model is based on the assumption that the tank wall is rigid. Veletsos (2,3) and Yang (4) showed that the hydrodynamic effects in a flexible tank may be much higher than those associated with a rigid tank. In subsequent studies the wall flexibility was incorporated into the dynamic equations of motion of the fluid-tank systems. In these studies the coupled fluid-shell frequencies are obtained by application of appropriate boundary conditions to the continuum fluid equations so that the flexibility of the wall is accounted for. The usual procedures used for this purpose are based on a coupling of the fluid modes with the bending type of modes of the tank i.e., cantilever beam type of modes. Clearly, this approach cannot be used for tanks which have significant modes of the ring type, i.e., cos and sin modes. On the other hand, if the wall flexibility is accounted for by using the ring type of modes, the results cannot be used for tanks which have significant modes of the bending type. Simple hand calculations made by the author for a test model containing liquid which is going to be tested in a foreign country showed that the ring types of modes are the predominant ones. The shell bending mode appears at a high frequency, i.e., 20 Hz. Subsequent finite element calculations verified these results. It is doubtful that this test model can be used to simulate seismic failures of prototypes that are known to be of the flexural beam behavior type.

Haroun (5) obtained natural frequencies of the coupled fluid-tank system by using discrete equations for the motion of the shell according to the finite element approach and continuum equations for the fluid motion. Based on this approach, a fluid added mass matrix was developed. These studies were then used by Housner and Haroun (6) to propose design procedures for liquid storage tanks. In a later work by Haroun (7) the effects of the foundation deformability on the fluid-tank system were investigated for horizontal inputs. Appropriate boundary conditions were applied at the tank bottom to account for the interaction forces from the foundation. The solutions presented are based on impedances given by Richard (8). Clearly these results are applicable to surface founded tanks. Furthermore the interaction law employed cannot predict lifting-off effects which may be important for surface founded liquid storage tanks subjected to strong earthquake horizontal inputs. In general, lifting-off effects are expected to further "soften" the fluid-tank system.

The above studies, however, dealt with seismic effects on fluid-tank systems which are subjected to the horizontal component of the earthquake ground motion. The need for research work in the area of the fluid-structure interaction phenomenon induced by the vertical component of an earthquake input was clearly identified in Ref. 9 by a committee which revised the state-of-the-art methods available in 1980. Recently, Veletsos (10) presented results for liquid storage tanks under vertical earthquake inputs which are based on the work made by Kumar (11). It is concluded that for a strong vertical earthquake input the hydrodynamic effects may be of the same order as the hydrostatic effects. Moreover, it is shown that for flexible tanks these effects may be substantially higher than those of the rigid tanks. At the same time Haroun (12) presented solutions to this problem by using axisymmetric ring type finite elements for the shell whereas the fluid was treated with an added mass matrix. Based on this approach, he obtained results for the fluid-tank natural frequencies. Finally, in both of these studies on the vertical earthquake response of liquid storge tanks it is assumed that the foundation is rigid. Thus possible interaction of the fluid-tank system with the foundation is neglected.

The work presented in this paper is concerned with the dynamics of liquid storage tanks when subjected to a vertical component of an earthquake input. Some theoretical aspects of the problem are given together with numerical solutions. First, liquid storage tanks are considered as fixed base systems, i.e., their foundations are rigid. Simple equations for the fundamental coupled fluid-structure frequency are presented which consider the general case of a compressible fluid. Furthermore, the influence of the foundation flexibility on the coupled fluid-tank system motion due to a vertical earthquake excitation is examined. It is concluded that the interaction effects from the foundation may be important in the treatment of liquid storage tanks under vertical earthquake inputs.

FLUID-TANK SYSTEM DEFINITION

A circular cylindrical tank is considered. The geometry of the tank is defined by the radius R, height H and wall thickness t. The tank material is defined by the modulus of elasticity E, Poisson's ratio ν and mass density ρ.

The tank is fully filled with a compressible fluid which is nonviscous and has mass density ρ_o. The static internal pressure in the tank is p_o and the velocity of sound in the fluid is c_o. The equations of motion of the fluid-tank system are referred to a cylindrical coordinate system with x-axis along the longitudinal axis of the tank. The system is subjected to a vertical component of an earthquake motion which is applied at the circular base. Under this condition, the axisymmetric breathing coupled motion is of interest. Although this is the only type of motion considered here, it is recognized that antisymmetric motions are also possible, since generally speaking neither the tank is perfectly circular nor the input excitation is perfectly vertical. It is known that in the case of a dry shell, imperfections could affect the vibration characteristics of the shell (13). For the case of a liquid-filled shell, experiment has shown that even though the liquid was excited in a symmetric mode, the shell responded by flexural modes of short wavelength (14). This was attributed later to the out-of-round type of imperfections. Firth (15) conducted a theoretical study in order to explain the development of short-wavelength flexural modes in an acoustically excited shell containing a fluid. In this study, Green's functions are derived from the fluid-shell coupled equations in which the shell radius incorporates distortions of small magnitude.

Tank Motion

The tank is treated as an elastic circular cylindrical shell with constant thickness. With reference to the cylindrical coordinate system (x,θ,r) the axisymmetric condition requires that all derivatives with respect to θ vanish and thus the shell motion is described by the equations (16)

$$\frac{\partial^2 u}{\partial x^2} + \frac{\nu}{R}\frac{\partial w}{\partial x} - \frac{\rho t}{D}\frac{\partial^2 u}{\partial t^2} = 0 \qquad (1)$$

$$\frac{\nu}{R}\frac{\partial u}{\partial x} - \frac{t^2}{12}\frac{\partial^4 w}{\partial x^4} + \frac{RP}{2D}\frac{\partial^2 w}{\partial x^2} - \frac{w}{R^2} - \frac{\rho t}{D}\frac{\partial^2 w}{\partial t^2} = \frac{F_h}{D} \qquad (2)$$

where

u,w = longitudinal and radial displacement components (w in the negative r direction)

F_h = hydrodynamic loading acting on the shell

D = membrane stiffness $Et/(1-\nu^2)$

A solution of the associated homogeneous problem for the freely supported end condition is

$$u_m = A_m \cos\frac{m\pi x}{H} \sin \omega_m t$$

$$w_m = B_m \sin\frac{m\pi x}{H} \sin \omega_m t \qquad (3)$$

where A_m and B_m are constant coefficients and $m/2$ is the number of axial wavelengths. According to this formulation the effect of the internal pressurization p_o is incorporated. Also inertia forces in both the longitudinal and radial directions are included.

Fluid Motion

The fluid considered here is compressible and nonviscous. Under these conditions its motion is described by the equation

$$\text{where} \quad \nabla^2 q = \frac{1}{c_o^2}\frac{\partial^2 q}{\partial t^2} \qquad (4)$$

where
q = pressure fluctuation about p_o
∇^2 = Laplacian operator

A solution of this equation is

$$q_m = c_m S_o(\lambda_m \frac{r}{R}) \sin\frac{m\pi x}{H} \sin\omega_m t \quad (5.1)$$

where

$$\lambda_m^2 = \left[\left(\frac{\omega_m}{c_o}\right)^2 - \left(\frac{m\pi}{H}\right)^2\right] R^2 \quad (5.2)$$

and

$$S_o : \begin{cases} J_o & \text{for } \frac{\omega_m}{c_o} > \frac{m\pi}{H} \\ I_o & \text{for } \frac{\omega_m}{c_o} < \frac{m\pi}{H} \end{cases} \quad (5.3)$$

J_o, I_o are ordinary and modified Bessel functions of zero order respectively.

The boundary conditions at the shell wall require that the liquid normal acceleration be equal to that of the tank wall and the liquid pressure be equal to the hydrodynamic load on the tank. These conditions are expressed as

$$F_h = q$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{1}{\rho_o}\frac{\partial q}{\partial r} \quad (6)$$

In view of Eqs. 3 and 5.1 the last equation gives

$$q_m = -\omega_m^2 \rho_o R Q_m W_m \quad (7.1)$$

$$\text{where} \quad Q_m = \frac{S_o(\lambda m)}{\lambda m S_o'(\lambda m)} \quad (7.2)$$

The term $\rho_o R Q_m$ in Eq. 7.1 is the fluid apparent mass $\bar{m}_m$, i.e.,

$$\bar{m}_m = \rho_o R Q_m \quad (7.3)$$

EIGENVALUE PROBLEM

Substituting Eqs. 3 into Eqs. 1,2 and considering the boundary condition expressed by Eq. 7 the following eigenvalue problem is obtained

$$([K_m] - \omega^2 [M_m]) \begin{Bmatrix} A_m \\ B_m \end{Bmatrix} = 0 \quad (8.1)$$

where $[K_m]$, $[M_m]$ are the 2x2 matrices

$$[K_m] = \begin{bmatrix} \beta_m & \frac{\nu}{R}\beta_m \\ \frac{\nu}{R}\beta_m & \frac{t^2}{12}\beta_m^4 + \frac{RP_o}{2D}\beta_m^2 + \frac{1}{R^2} \end{bmatrix} ;$$

$$[M_m] = \begin{bmatrix} t_\rho & \\ & t_\rho + \bar{m}_m \end{bmatrix} \quad (8.2)$$

$$\text{and} \quad \beta_m = \frac{m\pi R}{H} \quad (8.3)$$

If the displacement and pressure wave patterns are

$$u_m = A_m \sin\frac{m\pi x}{H} \sin\omega_m t$$

$$\text{and} \quad w_m = B_m \cos\frac{m\pi x}{H} \sin\omega_m t \quad (9)$$

$$q_m = C_m S_o(\lambda_m \frac{r}{R}) \cos\frac{m\pi x}{H} \sin\omega_m t$$

then the off-diagonal terms in matrix $[K_m]$ change their sign whereas everything else in these equations remain the same.

When the shell inertia due to the radial motion is only, considered, then the first diagonal term in the matrix $[M_m]$ becomes equal to zero and the corresponding eigenvalue problem of Eqs. 8.1 leads to the frequency equation (m=1)

$$(t\rho + \bar{m}_1)\,\omega_1^2 = \frac{Et}{R^2} + \frac{Et^3}{12(1-\nu^2)}\left(\frac{\pi}{H}\right)^4 + \frac{1}{2}RP_o\left(\frac{\pi}{H}\right)^2 \quad (10)$$

which is the same equation given in Ref. 17 with n=0, m=1.

Furthermore, if the fluid is assumed to be incompressible i.e., $c_o \to \infty$ and the radial inertia is kept only then for the case of an unpressurized tank ($p_o=0$) the second term of the second diagonal element in $[K_m]$ becomes equal to zero while the first diagonal term in $[M_m]$ vanishes. In this case the eigenvalue problem of Eqs. 8 reduces to

$$(t\rho + \bar{m}_m)\omega_m^2 = \frac{Et}{R^2} + \frac{Et^3}{R^4}\lambda_m^4 \quad (11)$$

It may be noted that this result is the same as in Ref. 18.

Finally, if the tank behaves as a membrane, then the second diagonal term in $[K_m]$ is reduced to $1/R^2$ and the frequency equation is further simplified to

$$(t_\rho + \bar{m}_m)\omega_m^2 = \frac{Et}{R^2} \quad (12)$$

Finally, by substitution of Eqs. 5.2, 7.2 and 7.3 into Eq. 12 with $c_o \to \infty$ and m = 1/2, the coupled frequency given by Eq. 12 can be put in the form

$$\omega_o \left[1 + \frac{\rho_o}{\rho}\cdot\frac{R}{t}\cdot\frac{1}{\lambda}\cdot\frac{I_o(\lambda)}{I_o'(\lambda)}\right]^{-1/2} \quad (13.1)$$

where $\omega_o = \frac{1}{R}\sqrt{\frac{E}{\rho}}$: ring extentional frequency (13.2)

and $\lambda = \frac{\pi R}{2H}$

Eq. 13.1 is basically the simplified formula given by Veletsos (10) for the fundamental coupled frequency. From the solution of Eqs. 8 the sin and cos pressure components (Eq. 5.1, 9) are obtained. The pressure q can be put in series expansion form with the amplitudes determined from the condition at the base

$$\frac{1}{\rho_o}\frac{\partial q}{\partial x} = \ddot{x}_g \qquad (14)$$

NUMERICAL SOLUTIONS FOR FREQUENCIES

Solutions for the coupled fluid-shell mode shapes/frequencies using Eq. 8 are, in general, difficult to obtain. The resulting frequency equation from this eigenvalue problem is not of the standard polynomial type with constant coefficients. Therefore, the coupled frequencies can be obtained numerically by simply computing the crossings of the determinant of matrix $[K_m] - \omega^2[M_m]$ with the frequency axis ω. This is done iteratively by choosing an appropriate frequency step so that the frequencies are determined with sufficient accuracy. The number of iterations required for a given tank depends on the height-to-radius ratio. Higher ratios require less amount of iterations.

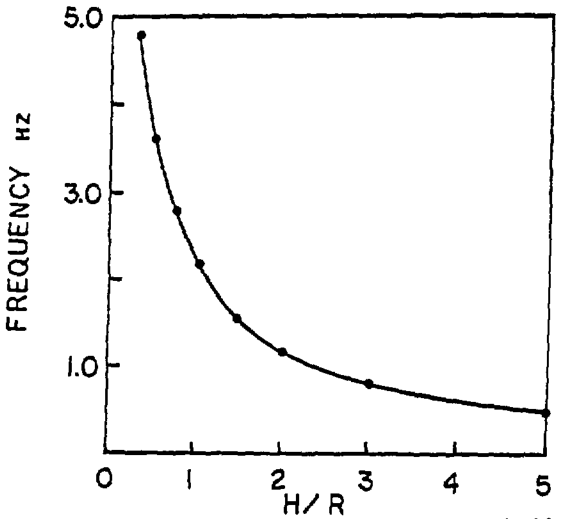

Fig. 1 - Coupled fluid-tank frequencies. Concrete tanks.

Results from application of this procedure to concrete tanks filled with water are shown in Fig. 1. The concrete properties are $E=4\times10^6$ psi, $\nu=0.17$ and $\rho=155$ pcf. From Fig. 1 it may be seen that storage tanks with lower H/R values are associated with higher coupled fluid-shell frequencies. This is in accordance with intuition. Results in Fig. 1 correspond to the case of one axial wavelength.

By inspection of Eqs. 8 it is noted that a set of two coupled frequencies is associated with the m-th displacement wave pattern. The lower is shown in Fig. 1 whereas their ratio (high-to-low) is shown in Fig. 2. It may be noted that for higher H/R values these two frequencies are further apart. Furthermore this ratio is influenced by the internal pressure p_o in the tank. In general, when the internal pressure increases the effect on the low frequency value is rather small. The effect, however, on the high frequency value is usually large.

Tank stresses can be obtained as series expansions of the coupled displacement and pressure modes as well as the participation factors associated with the solution of the eigen-problem of Eqs. 8. For practical applications, however, these stresses may be computed by a static analysis using the maximum hydrodynamic wall pressure as static load applied on the tank wall. Therefore, known static solutions for a cylindrical shell with axisymmetric load may be employed. To further simplify the static solution, the hydrodynamic wall pressure distribution may be approximated by a linear, i.e., zero pressure at the surface and maximum pressure at the bottom $\rho H S_a$. The pseudo-acceleration S_a is taken from the vertical earthquake spectrum for the coupled fluid-tank frequencies. The conservatism of this approach, however, should be further investigated with respect to the contribution of higher modes.

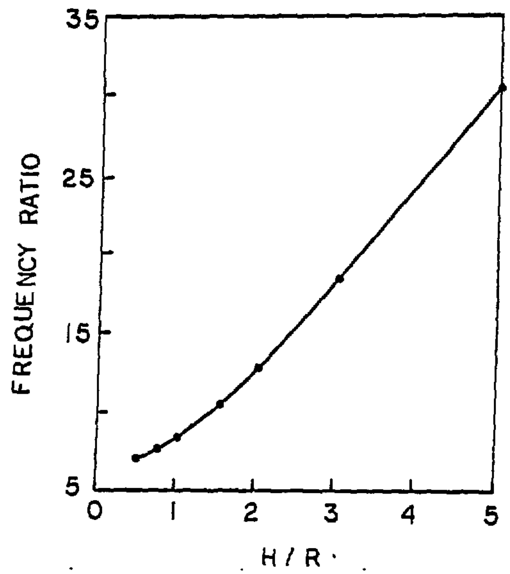

Fig. 2 - Frequency ratio variation. Concrete tanks.

Finally, in the above discussions no mention was made in regard to free-surface effects, i.e., vertical sloshing. Usually such effects in horizontal tank assessments are associated with very low frequencies and thus tend to separate well from the structural frequencies. In the case of horizontal tank motion, the antisymmetric modes are of interest. The later modes produce unbalanced forces which are excerted by the fluid on the tank wall. Based on this, the properties of an equivalent mechanical model are derived and the problem has been simplified to the known spring-mass sloshing unit. For the vertical sloshing problem, however, it is generally difficult to obtain a similar mechanical analog. In general, if the vertical sloshing is produced by symmetric modes then no unbalanced forces are excerted on the tank wall by the liquid. Symmetric sloshing, however, effects the pressure at the bottom of the tank. Furthermore, for the same reasons given in Secton 2, antisymmetric type of sloshing can also be produced in liquid storage tanks under vertical earthquake input.

It may be noted that in both horizontally as well as vertically excited liquid storage tanks the sloshing effects are usually studied under the rigid wall assumption (1,19). Normally, the interaction between the breathing tank modes and the significant sloshing modes is expected to be small. Such interaction, however, has been studied by Liu and Ma (20) using finite element solutions and it was shown that the tank flexibility may affect the sloshing. Although a similar study for tanks subjected to vertical earthquake components is not available, it is expected that this interaction may be also important especially when nonlinear sloshing is considered at the free surface.

FLEXIBLE BASE EFFECTS

In the preceeding sections the fluid-tank system was assumed to be of the fixed-base type. The same assumption has also been made in previous studies on vertically excited tanks. In order to study the effects of the foundation flexibility on the coupled fluid-tank system subjected to a vertical earthquake shaking, some solutions were obtained here numerically using finite elements. For this purpose a surface founded circular cylindrical concrete tank containing water was considered with overall *dimensions:* radius R=120 ft, height H=360 ft and wall thickness t=1.2 ft. The finite element mesh used is given in Fig. 3. It combines the fluid-tank system and the foundation medium in the standard fashion followed by the soil-structure interaction finite element models. A description of the model and some of the results are given below with a minimum reference to known facts from the soil-structure interaction area.

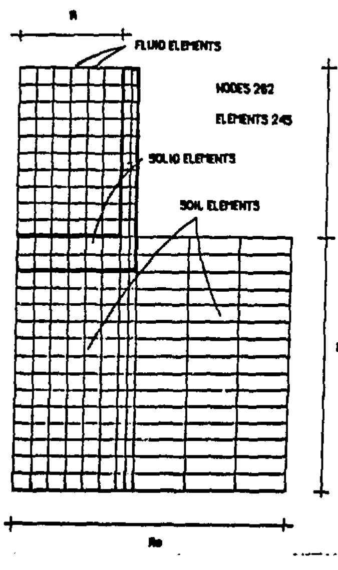

Fig. 3 - Finite element model of the fluid-tank-soil system

The tank wall is made up by two layers of axisymmetric elastic solid elements with properties: $E=4\times10^6$ psi, $\upsilon=0.17$ and $\rho=155$ pcf. The foundation medium considered is also descritized by solid elements which are assigned two types of soil properties associated with S-wave velocities $V_s=500,2000$ ft/sec (corresponding P-wave velocities $V_p=933$ and 3742 ft/sec respectively). For simplicity the mass density and the Poisson's ratio were taken to be the same for both soil types, i.e., $\rho=120$ pcf, $\upsilon=0.30$. The overall dimensions of the soil mesh are $R_0=360$ ft and $H_0=240$ ft. The largest element dimension in the direction of the propagation was kept at least equal to one-fifth of the wavelength of the slowest wave. The fluid was approximated by solid elements using the bulk modulus of the water and a very small value for shear modulus. Thus the fluid-structure interaction effects in this particular problem are not treated by the general procedures used in this area (19,20). Generally speaking, a fluid-structure-soil interaction finite element code is required *for* this type of problems.

The ground surface motion is represented by a synthetic accelerogram having a maximum of 1.0 g. It was synthesized to match a smooth design ground spectrum. The surface motion was deconvoluted down to the rigid bottom boundary of the mesh using a 1-D wave propagation. In this process the three lower natural frequencies of the soil column are 1.0, 3.0, 5.0 Hz and 4, 12, 19 Hz for the first and second types of soils respectively. Using these motions as input to the bottom of the finite element mesh, response computations were done in the frequency domain using the transfer functions of the system.

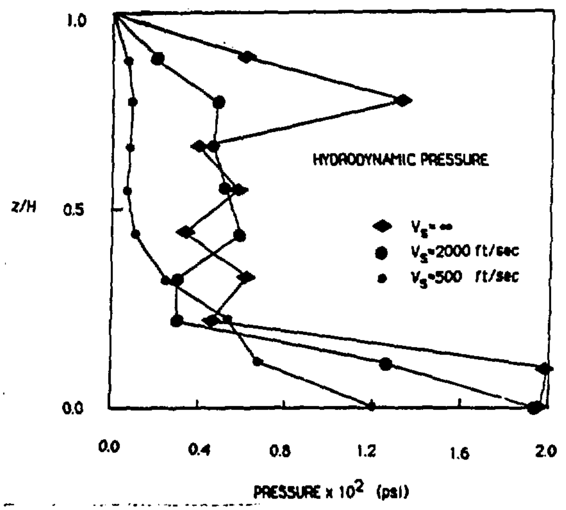

Fig. 4 - Effect of foundation flexibility on hydrodynamic pressure

Hydrodynamic pressure distributions along the tank wall for the rigid and flexible cases are given in Fig. 4. It can be seen that the *hydrodynamic* pressure is effected by the foundation flexibility. For the softest site, i.e., $V_s=500$ *ft/sec*, the hydrodynamic pressure is generally much smaller than that of the rigid base tank. These results indicate that the foundation flexibility can effect the seismic results of liquid storage tanks under vertical earthquake excitation. A parametric study using *different* tanks, i.e., "tall" and "broad" as well as different foundations has been proposed to investigate flexible-base liquid storage tanks under vertical earthquake input. Finally, it may be noted that a linear interaction law was assumed in the above problem at the interface of the tank base and the foundation. A nonlinear interface condition may be more appropriate especially when strong vertical earthquake shaking is considered. Localized nonlinearities of this sort have been addressed in soil-structure interaction problems.

CONCLUSION

A study of liquid storage tanks subjected to vertical earthquake motion was presented. Both fixed base and flexible-base tank cases are considered. Results from a case problem indicated that the hydrodynamic pressure is lowered due to the foundation flexibility. This result of course cannot be generalized and further parametric studies are required to address the additional coupling of the coupled fluid-tank frequencies with interaction frequencies from the foundation.

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NOTICE

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