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ANALYSIS OF MAGNETIC REFRIGERATION WITH EXTERNAL REGENERATION*

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1. INTRODUCTION

Several recent developments in magnetic refrigeration (MR) have been concerned with the temperature range from 4 K to 20 K [1-5]. In addition to use for basic research, the applications for such devices include the cooling of superconducting magnets for NMR imaging and other purposes and the cooling of infrared and other types of sensors. The requirements for cooling to T < 10 K usually involve rejecting heat to a hot sink at 20-22 K; such a sink is likely to be a LH_2 bath. Thus, one finds very attractive the idea of efficient magnetic refrigeration directly from a hot sink at the LN_2 bath temperature of 77 K down to <10 K because this avoids the hazards of LH_2 handling and because of other reasons to become apparent. Such 10-77 K MR systems do not exist at the present time. Meanwhile, commercially available mechanical refrigerators [6] which use He gas as the working fluid can provide cooling from 300 K down to ~4.2 K but they are noteriously inefficient [7] and relatively expensive.

The potential application of a 20-77 K MR system for efficient reliquifaction of cold H_2 vapor is discussed in a recent report [8] in which several ideas for MR systems were considered. While MR systems with regeneration and with cyclic operations roughly analogous to Stirling and Brayton cycles were studied qualitatively, it was realized that a quantitative analysis of cyclic operations must be performed in order to understand system behavior. The purpose of the present paper is to report the results of numerical studies [9] of a MR system which uses an external regenerator composed of several isolated stages. We use the abbreviation REGMR to denote such systems.

We discuss in Section 2 the concept and cyclic operations of REGMR models with 5 to 10 stages per regenerator. In order to facilitate the numerical analysis we use linearized entropy-versus-temperature functions to represent the working magnetic material (MM). These functions provide a close approximation to the properties of real GdNi close to the Curie temperature T_0 of 70.5 K but are less accurate representations at lower temperatures. Details of the numerical analysis are given in Section 3 along with results for the efficiency of the REGMR models studied. In Section 4 we give a summary in which the remarkable properties of REGMR systems disclosed by the study are discussed.

2. REGENERATIVE MAGNETIC REFRIGERATION MODEL

We show in Fig. 1 a schematic of a REGMR with a regenerator having five separate regenerator stages. The ith stage is a periodic heat storage unit (similar to a heat exchanger) having mass M_i. It is assumed that He gas circulated by the blower carries heat from the MM to the particular mass M_i when valve v_i is opened. Similarly, one has access to a cold heat exchanger (CHEX) of mass M_x when the valve v_c is opened and to a hot heat exchanger (HHEX) at the hot sink temperature T_H when the valve v_H is opened.

In practice, one would have to take into account the mass of the light-weight manifold system, the valves and blower and the exchange gas in this system because this constitutes an addenda. Similarly, hydrodynamic effects of the gas and heat-transfer irreversibilities will give rise to additional inefficiencies which should be small in an optimized design. However, in this theoretical

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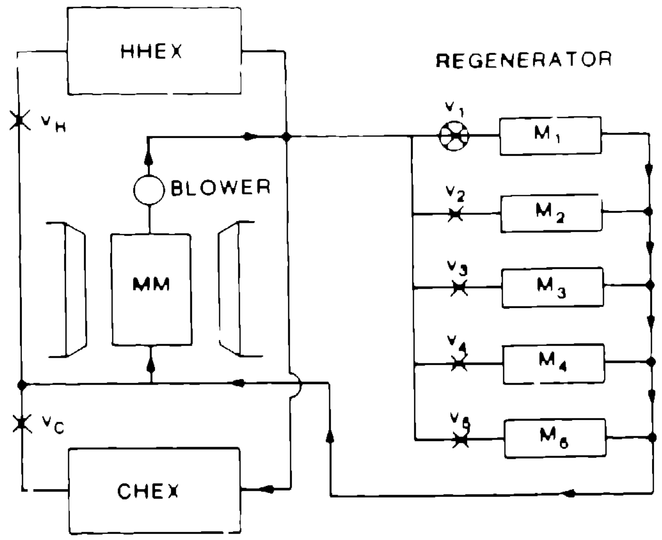

Figure 1. Schematic of a magnetic refrigerator with an external regenerator with five stages. The magnetic material MM is shown between the pole pieces of a magnet. A blower circulates He gas through the MM and elements of the circuit. Gas flow is always in the same direction through the elements, as indicated by the arrows. The figure shows valve v_1 open and the gas communicating heat between the MM and stage 1 mass M_1. The cycle is defined by the following sequence of operations and valve opening and closing: Magnetization, v_H, v_1, v_2, v_3, v_4, v_5; Demagnetization, v_C, v_5, v_4, v_3, v_2, v_1. When this cycle is repeated many times, heat may be removed from the cold heat exchanger (CHEX) and rejected to the hot heat exchanger (HHEX).

study, we neglect these losses because the initial goal is to determine the operating characteristics of ideal REGMR systems. Subsequently, when a particular REGMR has been deemed to be feasible, we can then study the degradation of its performance by taking into account the above irreversitibilities.

The sequence of operations of a REGMR having five regenerator stages is described in the caption of Fig. 1. In the computer program developed for the analysis of various REGMR systems, we could choose arbitrarily the following parameters: $5 \leq n \leq 10$, where n is the number of regenerator stages; M, the mass of the magnetic material MM (usually 1 kg); M_i, the mass of the material of the ith regenerator stage; M_x, the mass of the material of the CHEX; T_H, the temperature of the HHEX or hot sink; T_0, the Curie temperature of the MM; N, the number of repetitive cycles of operation (usually N = 200); R, the heat capacity coefficient for the regenerator material; and X, the heat capacity coefficient for the CHEX material. These latter heat capacities are assumed to be linear in T over a small range near the Curie temperature T_0. R and X were chosen to give the internal energy of copper exactly at T_0.

We used a Hewlett Packard HP-87 XM and programming was done entirely in HP-Basic. This software development for the problem was further facilitated by the used of a linearized model for the entropy-versus-temperature functions of the magnetic material.

The entropy model assumed for the magnetic material is shown in Fig. 2. For the case of zero magnetic field, the entropy $S(T) = MAT$ when $T < T_0$ and $S(T) = M(B_0 + B_1T)$ when $T > T_0$. The specific heat $C(T) = MAT$ when $T < T_0$ and $C(T) = B_1T$ when $T > T_0$; the internal energy $U(T) = MAT^2/2$ when $T < T_0$ and $U(T) = M(AT_0{}^2 + B_1T^2 - B_1T_0^2)/2$ when $T > T_0$.

In the high constant magnetic field H, the MM model properties are as follows:

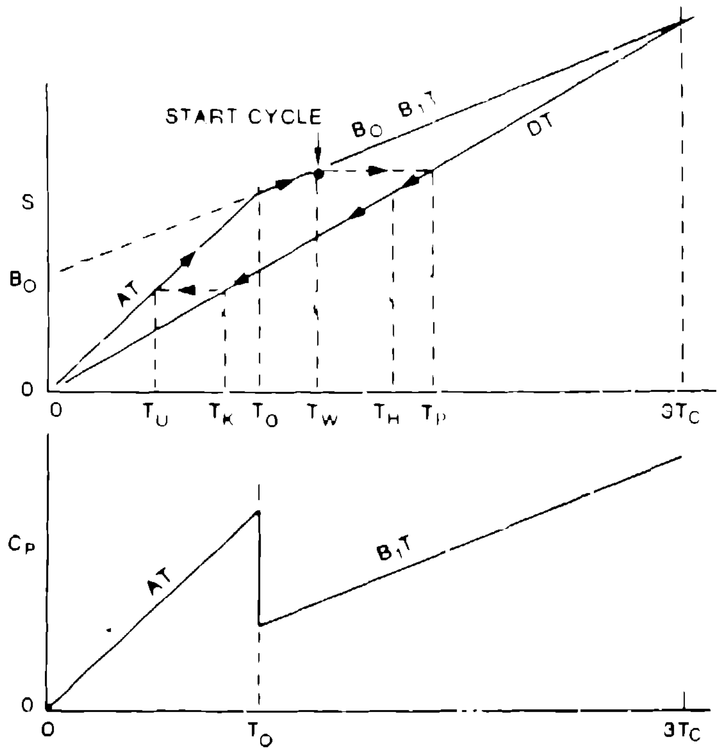

Figure 2. Entropy-versus-temperature model assumed for the REGMR analysis. At zero magnetic field, $S(T) = AT$ when $T < T_O$ and $B_O + B_1T$ when $T > T_O$. At high magnetic field, $S = DT$. The two model curves intersect at $3\ T_O$; thus, all coefficients can be expressed in terms of $S(T_O) = AT_O$ and ΔT_m, the adiabatic temperature change at $T = T_O$. The lower curve gives the zero-field specific heat C_p as a function of temperature for the magnetic material MM.

$S(T) = MD\ T$; specific heat $C(T) = C_H(T) = MD\ T$; enthalpy $E(T) = MDT^2/2$. When $T = T_O$ at zero field, $S(T) = MA\ T_O$. For the same entropy at high field, $S(T_m) = MD\ T_m$. Thus, $T_m = A\ T_O/D$. We thus define the adiabatic temperature change in going from zero to high field by $\Delta T_m = T_m - T_O$. When we input ΔT_m as an arbitrary parameter in the computer program, then $T_m = T_O + \Delta T_m$. Usually, we set $\Delta T_m = 8$ K in agreement with the adiabatic ΔT observed at T_O for GdNi when the the field is increased from zero to 8 T. Coefficient D is then $A\ T_O/T_m$.

The internal energy of the ith regnerator stage is $U_i = M_iR\ T_i^2/2$. We assume that this is equal to the Debye internal energy of copper when $T_i = T_O$. Similarly, we assume $U_X = M_XXT_X^2/2$ for the material of the CHEX. Assuming copper is also the CHEX material, we let $R = X = 2.28$ J/kg-K^2.

3. NUMERICAL ANALYSIS

At the start of the jth cycle, the MM is in zero field at a warm temperature $T_W(j)$. When the MM is magnetized adiabatically, its temperature increases to the peak value for the cycle,

$$T_p(j) = T_W(j) + \Delta T_H(j) \quad . \tag{1}$$

where ΔT_H can be calculated from the model of Fig. 2 for any starting temperature $T_W(j)$. We find $\Delta T_H = T_W(T_m/T_O - 1)$ when $T_W < T_O$ and $\Delta T_H = \Delta T_m[3/2 - (1/2)T_W/T_O]$ when $T_W > T_O$. Valve v_H is now opened and the MM is cooled by the HHEX from $T_p(j)$ to T_H. The amount of heat rejected to the HHEX in this step is given by enthalpy change

$$Q_H(j) = E(T_p(j)) - E(T_H) = M\ D\ [T_p(j)^2 - T_H^2]/2. \tag{2}$$

energy with the #1 regenerator stage. Both come to a final temperature which we denote $Z_1(j)$. The assumption of energy conservation for this process then leads to the relation,

$$Z_1(j)^2 = [M\ D\ T_H^2 + M_1 R\ T_1(j)^2]/(M\ D + M_1 R)\ , \qquad (3)$$

where $T_1(j)$ is the initial temperature of stage #1. Closing valve v_1 and opening valve v_2 then leads to the relation,

$$Z_2(j)^2 = [M\ D\ Z_1(j)^2 + M_2 R T_2(j)^2]/(M\ D + M_2 R). \qquad (4)$$

Continuing this process, we find for a REGMR having k regenerator stages,

$$Z_k(j)^2 = [M\ D\ Z_{k-1}(j)^2 + M_k R T_k(j)^2]/(M\ D + M_k R). \qquad (5)$$

The next step is to demagnetize. This results in the adiabatic temperature decrease,

$$\Delta T_k(j) = Z_k(j) - T_u(j) = Z_k(j)(1 - T_c/T_m)\ , \qquad (6)$$

and leaves the MM on the zero-field curve of Fig. 2 at $T_u(j)$. When valve v_k is closed and valve v_c is opened, the MM interacts with the CHEX with the result,

$$T_x(j+1)^2 = [M\ A\ T_u(j)^2 + M_x X\ T_x(j)^2]/(M\ A + M_x X), \qquad (7)$$

where $T_x(j+1)$ is now the new temperature of the CHEX. It will remain at this value until it again encounters the MM in the next cycle unless there is another heat input to the CHEX from the cold source to be refrigerated. We can simulate a heat load in the calculations by simply changing $T_x(j+1)$ to $T_x(j+1) + \Delta T_x$, where ΔT_x is a small but arbitrary increment. The energy input corresponding to ΔT_x is given by

$$Q_k(j+1) = M_x X[\Delta T_x\ T_x(j+1) + \Delta T_x^2/2]. \qquad (8)$$

After heat exchange between the MM and the CHEX, the MM is at the jth cycle temperature,

$$V_k(j) = T_x(J+1) \qquad (9)$$

which is smaller than $Z_k(j)$, the current temperature of the regenerator mass M_k.

To start the warm-up half of the cycle, valve v_c is closed and v_k is opened, and heat is exchanged between the MM and M_k. In again interacting with the kth stage, the temperature of M_k is slightly reduced to $t_k(j)$ while that of the MM is slightly increased to $V_{k-1}(j) = t_k(j)$, according to,

$$t_k(j)^2 = V_{k-1}(j)^2 = [M\ A\ V_k(j)^2 + M_k R\ Z_k(j)^2]/(M\ A + M_k R). \qquad (10)$$

Valve v_k is now closed and v_{k-1} is opened. Thus, we have $t_{k-1}(j) = V_{k-2}(j)$, etc. Continuing the warm-up half of the cycle, we come to,

$$t_2(J)^2 = V_1(j)^2 = [M\ A\ V_2(j)^2 + M_2 R\ Z_2(j)^2]/(M\ A + M_2 R). \qquad (11)$$

Finally, closing v_2 and opening v_1, we come back to the starting point with the result that $t_1(j) = V_0(j) = W(j)$, according to,

$$t_1(j)^2 = W(j)^2 = [M\ A\ V_1(j)^2 + M_1 R\ Z_1(j)^2]/(M\ A + M_1 R). \qquad (12)$$

Notice that the new temperature of the regnerator mass M_1 and of the MM are not equal to the inital value $T_1(j)$ at the start of the jth cycle, unless the condition of "cyclic equilibrium" has been achieved.

... $T_j(j+1)$ for the set $\{M_i\}$, i = 1 to k, of regenerator stage masses, the temperatures $T_X(j + 1)$ for the CHEX, and the warm temperature $T_W(j + 1) = W(j)$.

The computer program was developed to carry out the operations indicated in Eqs. (1)-(12) for each cycle. The final results for a jth cycle served as the inital conditions for the (j + 1)th cycle. After each major loop of 200 cycles, the program halted and the data could be tested for cyclic equilibrium. The most sensitive test of cyclic equilibrium was to compare the internal energy change of the k-regenerator stages during the cool-down one-half cycle with that of the warm-up one-half cycle. When these two energy totals agreed to within one part in 100,000, we assumed that cyclic equilibrium had been substanially achieved. The number of cycles required to reach cyclic equilibrium was found to vary with the number k of regenerator stages for a particular REGMR and with the mass assumed for each stage.

In all of the runs, we assumed the same stage mass for each stage, i.e., $M_i = M_R = f\ M$, for i = 1 to k, and M = MM mass; f was varied as follows: 2.5, 5, 10, 20. The number of cycles required to reach cyclic equilibrium varied dramatically with k. For example, with f = 5, k = 5, N was about 450 cycles. However, when f = 5, k = 10, N was about 1200 cycles. In the case f = 20, k = 10, equilibrium was reached only after 3600 cycles. When f = 2.5 and k = 5, equilibrium was reached in about 200 cycles.

The temperature $T_X(j)$ of the CHEX started out at T_H at j = 1 and approached its cyclic equilibrium value asymptotically, its lowest temperature, and was essentially flat for any number of cycles greater than those mentioned above.

We show in Fig. 3 the asymptotic lowest temperature T_X of the CHEX achieved after reaching cyclic equilibrium for a number of REGMRs. Each data point corresponds to a separate refrigerator with no heat load. Refrigeration is from a HHEX sink temperature of 75 K down to that shown in the figure. These results suggest that a REGMR may be able to produce a temperature change $(T_H - T_X)$ that is 4 to 5 times larger than the adiabatic temperature change ΔT_m at T_0 with a single MM stage.

The insets of Fig. 3 show the dependence of the lowest temperature achieved as a function of regenerator stage mass for 5- and 10-stage REGMRs. The shapes of these inset curves indicate that there is not much advantage in using stage masses much greater than 10 M.

The thermodynamic cycle of Fig. 2 consists of a clockwise path as indicated by the arrows. The path connects the coordinates $[T_W,S(T_W)]$, $[T_p,S(T_p)]$, $[T_H,S(T_H)]$, $[T_k,S(T_k)]$, and $[T_u,S(T_u)]$. However, a point is missing from Fig. 2. When there is a heat load, we must add a point $[T_X,S(T_X)]$, which falls just above T_u on the AT curve. This point represents the heat absorbed by the MM after interacting with the CHEX when it has a heat load, as given by Eq. (8). At cyclic equilibrium, the amount of this heat load turns out to be exactly equal to

$$Q_L = M\ A\ (T_x^2 - T_u^2)/2, \qquad (13)$$

and agrees with Eq. (8). The heat rejected to the HHEX is determined by the enthalpy change in going from T_p to T_H on the DT curve. This is given by,

$$Q_H = M\ D(T_p^2 - T_H^2)/2. \qquad (14)$$

Adiabetic magnetization takes us from the point $[T_W,S(T_W)]$ to the point $[T_p,S(T_p)]$. Thus, using the standard relation $dE = T\ dS - u_0 H\ d\bar{M}$, where E is the enthalpy, we have, when dS = 0, a prescription for determining the magnetic work done in magnetizing from T_W to T_p in Fig. 2. Because of the discontinuity in the zero-field curve, two expressions are needed for this magnetic work, e.g.,

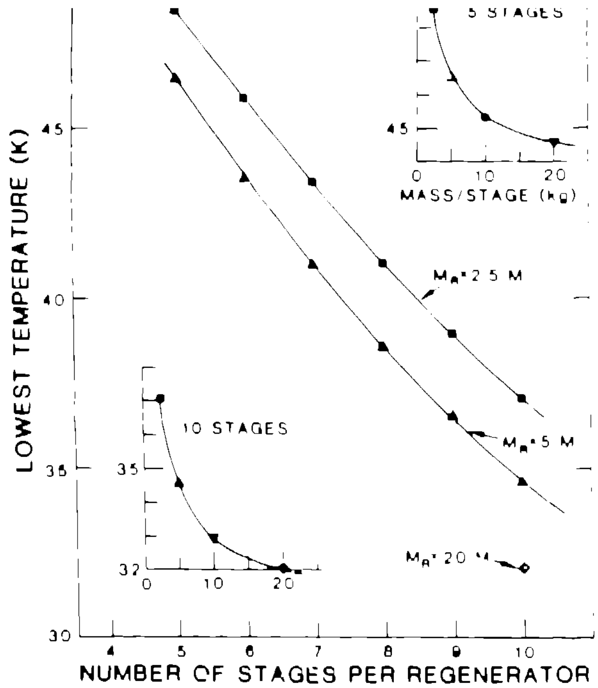

Figure 3. Lowest temperature reached by REGMRs upon achieving cyclic equilibrium versus the number of stages per regenerator. $T_H = 75$ K, $T_o = 70.5$ K, mass M of MM = 1.0 kg. Squares: mass of each regenerator stage = 2.5 M. Triangles: M_R = 5.0 M. Diamond: M_R 20 M. Upper inset: Temperature versus mass per stage for 5-stage REGMR. Lower inset: Temperature versus mass/stage for 10-stage REGMR. These data represent zero heat load at the CHEX.

$$W_H = M(D\ T_p^2 - A\ T_w^2)/2\ ;\ T_w < T_o\ , \tag{15a}$$

$$= M[D\ T_p^2/2 - (A\ T_o^2 + B_1 T_w^2 - B_1 T_o^2)/2]\ :\ T_w > T_o \tag{15b}$$

The adiabatic demagnetization process takes us from the coordinate $[T_k, S(T_k)]$ to $[T_u, S(T_u)]$ in Fig. 2. The magnetic work done is,

$$W_k = M(D\ T_k^2 - A\ T_u^2)/2, \tag{16}$$

The net magnetic work done W_{NET} in going around the cycle of Fig. 2 is therefore the difference $W_H - W_k$, which is identical to $Q_H - Q_L$ at cyclic equilibrium.

In the case of an ideal (Carnot cycle) gas refrigerator, the minimum work required to remove Q_L from the cold reservoir at T_x is $W_{min} = Q_L/COP$ where the Carnot COP = $T_x/(T_H - T_x)$. The efficiency of our REGMR relative to the Carnot cycle is therefore $\eta = W_{NET}/W_{min}$. We show in Table 1 the results for a REGMR of 10 regenerator stages and with the mass per stage = 5 M.

Table 1

Efficiency data for a REGMR with 10 regenerator stages with M_R = 5 M. Heat Load Q_L is defined by ΔT of the first column and Eq. (8). W_{NET} and Q_L are in J/cycle. T_H = 75 K.

ΔT	T_x	Q_L	W_{NET}	Carnot COP	η
0	34.4	0.0	450.7	0.848	0.0
0.5	37.2	42.1	439.1	0.957	0.100
1.0	40.1	90.3	425.8	0.1090	0.195
1.5	43.3	145.3	410.4	1.255	0.282
2.0	46.5	207.6	393.0	1.462	0.361

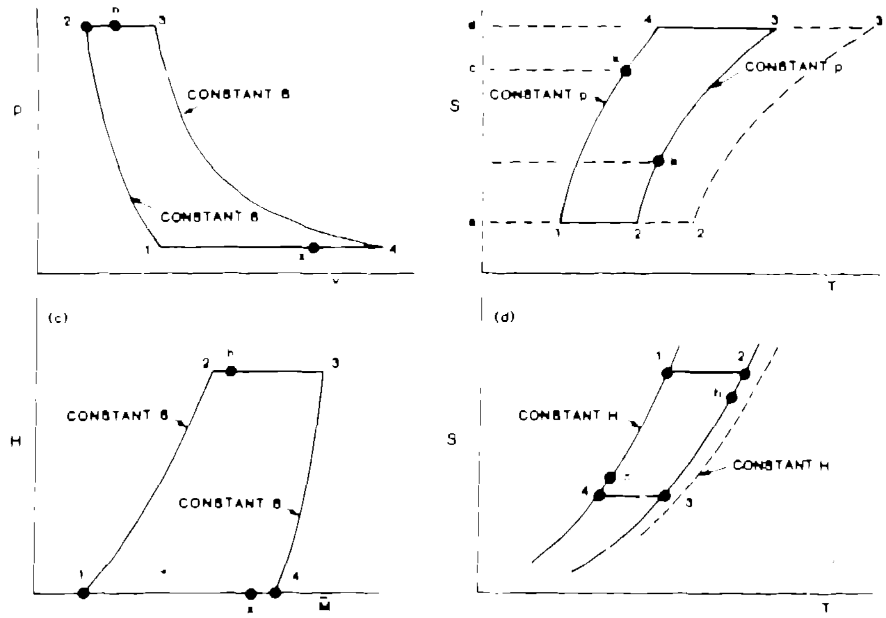

Figure 4. (a): P-vs-V and (b) S-vs-T plots for a Brayton cycle ideal gas refrigerator or engine. (c): $\mu_0 H$-vs-$\bar{M}$ and (d): S-vs-T plots for the cycle of the REGMR.

It is of interest to compare the REGMR cycle of Fig. 2 with that of an ideal gas refrigerator (or engine). For the Brayton-cycle P-vs-V plot in Fig. 4(a), we start at point 1 [$T = T_1$; $P = P(\text{low})$], the refrigerator absorbs heat Q_L from a cold source, and we move to point 4 [T_4, P(low)]. Input work W_{in} causes adiabatic compression to point 3 [T_3, P(high)]. Since $T_3 > T_H$ of the HHEX sink, heat Q_H is rejected to the sink, and we come to point 2 [T_2, P(high)]. Adiabatic expansion now takes us back to the starting point 1 [T_1, P(low)]. Thus, the refrigeration path in Fig. 4(a) is counterclockwise 1-4-3-2-1.

In Fig. 4(c), we assume $\mu_0 H$ is the analog of pressure P and $\bar{M}$ of the MM is analogous to volume V. The fact that PV = energy and $\mu_0 H\bar{M}$ = energy is one reason for this traditional assumption. However, the REGMR refrigeration cycle is clockwise. Starting at point 1 of Fig. 4(c), the first step is adiabatic magnetization (1-2). Next, isofield cooling by the HHEX (2-h); isofield cooling by the regenerator (h-3); adiabatic demagnetization (3-4); zero-field warming of the MM by the CHEX (4-x); and zero-field warming by the regenerator (x-1). The cycle is then (1-2-h-3-4-x-1). We regard the REGMR and Brayton cycles as being equivalent when analogous quantities at corresponding vertices are in one-to-one correspondence. Since this is not found, the above (P-V) and (H-$\bar{M}$) cycles are not equivalent.

Comparison of the S-T cycle plots of Figs. 4(b) and 4(d) shows both cycles are executed clockwise for refrigeration. However, detailed vertex comparison shows anti-correlations. These details are given in a more-complete form in Ref. 9. Accordingly, we come to the conclusion that the REGMR thermodynamic cycle is not equivalent to the Brayton cycle. The conclusion reached in Ref. 9 is that the REGMR cycle is not equivalent to any known thermodynamic cycle and is therefore unique.

4. SUMMARY

The central idea of the magnetic refrigeration systems analysed in this paper (and in Ref. 9) is that of a MR having a regenerator composed of an integral number of separate stages but having only a single magnetic stage. In principle, each stage is thermally isolated from the others, but the stages are accessed by a manifold and valve system (Fig. 1) which allows gas to flow between the MM and the individual stages. The concept therefore is distinctly unlike that of the gas in the manifold is responsible for the actual transfer of heat between elements of the REGMR, it is clear from the cycle description in the caption of Fig. 1 that the fixed mass of the MM is analogous to a fixed mass of a working fluid in a gas refirgerator or engine. Both, in effect, are carried thermally through all parts of their respective systems.

The extensive computer analyses of various REGMR models have been greatly facilitated by the use of the S-vs-T model functions in Fig. 2. Some results of this work are listed below.

- The time required to achieve cyclic equilibrium increases significantly with the number of regenerator stages.
- This time also increases with the mass of the regenerator stages.
- The lowest temperature T_x achieved at the CHEX decreases as the number of stages increases and decreases as the mass per stage increases. See Fig. 3.
- The difference of the mean temperatures between successive stages decreases as the numer of stages increases. It also decreases as the heat load at the CHEX increases.
- The lowest temperature T_x achieved at the CHEX decreases as the temperature T_H of the HHEX decreases.
- The magnetic work of magnetization is the difference $E_h(T_p) - U_o(T_w)$, where E_h is the enthalpy on the high-field curve and U_o is the internal energy on the zero-field curve. That for demagnetization is $- E_h(Z_k) + U_o(T_x)$. The NET magnetic work done is the sum of these two differences.
- The heat rejected to he HHEX is $E(T_p) - E(T_H)$, where E is the enthalpy on the high-field curve. The heat absorbed by the CHEX is $U(T_x) - U(T_u)$, where U is the internal energy on the zero-field curve. The difference in these heats is equal to the NET magnetic work done for the full cycle of Fig. 2.

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