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Dynamic Analysis of Systems Having Large Damping Variations

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ABSTRACT

In the earthquake response analysis of structures in which the damping characteristics between the elements varies significantly the standard mode superposition method cannot be used. Several approximations have been proposed that allow the application of the modal superposition method for cases in which the damping matrix is not orthogonal with respect to the modal shapes. The most commonly used approximation is based on a composite damping value which is employed in the modal equations. This value is a weighted average of the damping values of the individual components of the structural model. In this paper an investigation of the errors introduced by the composite damping in the response of simple structures is presented. The results given in the paper can be used for benchmarking the approximations in more complex systems for which composite damping solutions are employed.

NOMENCLATURE

c_i	:	damping coefficient
c_{ij}	:	elements of modal damping matrix
$[C]$	:	system damping matrix
$H_{y_i}(\omega)$	:	transfer functions
$H_i(\omega)$	:	modal transfer functions
$\{H(\omega)\}$	:	vector of transfer functions
k_i	:	stiffness coefficient
$[K]$	:	stiffness matrix
m_i	:	mass
$[M]$	:	mass matrix
q	:	mass ratio
u_i	:	system absolute displacements
x_g	:	free-field input
y_i	:	system displacements relative to base
γ_i	:	participation factors
ξ_i	:	uncoupled damping ratio
ξ_i	:	composite damping ratio
$\Pi(\omega)$	:	complex function defined in Eq. 7.3
$[\Phi]$	:	modal shape matrix
$[\Delta]$	:	matrix defined in Eq. 12
ω_i	:	uncoupled frequencies
ω_{0_i}	:	modal frequencies
ω	:	frequency variable

INTRODUCTION

The dynamic analysis of linear structures in the time domain is conventionally done using the solution of the corresponding undamped eigenvalue problem (i.e., mode shapes/ frequencies). Results are then obtained by some combination of the modal responses using modal damping. There are many practical cases, however, in which the system damping matrix cannot be diagonalized with the same undamped mode shapes that diagonalize both the mass and stiffness matrices. The soil-structure interaction problem is a typical case of this kind. More generally, such damping matrix behavior is encountered in structural models having individual components with large differences in damping. In such cases it is required to combine low with high damping. Two basic problems are encountered First, one single value must be assigned to each mode which reflects effects from significantly different energy dissipation. Second, since such a damping matrix cannot uncouple the equations of motion, off-diagonal terms are usually ignored.

Various approaches have appeared in the literature for the treatment of this problem. Comparison between them is rather controversial. The most common one is based on some weighted average damping[1] usually referred to as composite damping. A more sophisticated approach to this problem is based on matching transfer functions[2]. It is hardly used in practice because of its numerical involvement. Of course the problem can be attached directly using complex eigenvalues/vectors[3] and thus avoiding the approximations introduced by the composite damping. Such a solution, however, does not have the advantage of the classical modal superposition. In practice, it is desirable to use composite damping in conjunction with the undamped modes, even though this approach does not give, in general, accurate results.

This paper presents a comprehensive appraisal of the degree of the approximation due to composite damping. Theoretical and numerical evaluations are given. Transfer functions for simple dynamic systems are given and compared to those generated using composite damping. It is shown that errors become larger as the difference in damping between components increases.

SYSTEM DEFINITION

To simplify the presentation of the subsequent treatment the system considered consists of two mass-spring units in series. The parameters are m_i, c_i, k_i; $i=1,2$ where m,c and k denote mass, damping and stiffness coefficients respectively. Viscous damping is considered. The mass m_2 is connected to a moving system which is excited by x_g. The system is shown in Fig. 1. A two-story building under an earthquake excitation can be represented by such mathematical model.

The absolute displacements of m_1, m_2 are:

$$u_1 = y_1 + x_g$$
$$u_2 = y_2 + x_g \tag{1}$$

where y_1, y_2 are the displacements of m_1, m_2 with respect to the moving system. The uncoupled parameters are:

$$\omega_i^2 = \frac{k_i}{m_i}$$
$$\xi_i = \frac{c_i}{2\sqrt{k_i m_i}} \tag{2}$$

for $i = 1,2$.

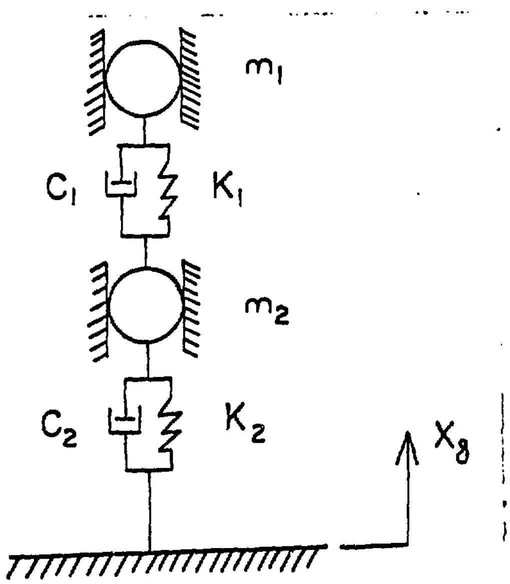

Fig. 1 - Two degree-of-freedom model

The following mass ratio is introduced

$$q = \frac{m_1}{m_2} \tag{3}$$

Based on the above, the equations of motion of the system in terms of y_1, y_2 can be set using the parameters

$$\text{mass} \quad : \quad [M] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \tag{4a}$$

$$\text{damping} \quad : \quad [C] = \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix} \tag{4b}$$

$$\text{stiffness:} \quad [K] = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \tag{4c}$$

TRANSFER FUNCTIONS

Let $H_{y_1}(\omega)$, $H_{y_2}(\omega)$ represent the transfer function between the excitation x_g and the displacements y_1, y_2 of the system. These functions may be obtained by setting

$$\ddot{x}_g(t) = e^{i\omega t}$$
$$y_1 = H_{y_1}(\omega)\, e^{i\omega t} \tag{5}$$
$$y_2 = H_{y_2}(\omega)\, e^{i\omega t}$$

Substitution of Eq. 5 into the equations of motion of the system gives

$$\{H(\omega)\} = \begin{bmatrix} -\omega^2 m_1 + i\omega c_1 + k_1 & -i\omega c_1 - k_1 \\ -i\omega c_1 - k_1 & -\omega^2 m_2 + i\omega(c_1+c_2) + (k_1+k_2) \end{bmatrix}^{-1} \begin{Bmatrix} m_1 \\ m_2 \end{Bmatrix} \tag{6}$$

where ω is the frequency variable.

After performing the necessary manipulations in Eq. 6 it is obtained

$$\Pi(\omega)H_{y_1}(\omega) = \omega^2 - i\omega \left[2\xi_1\omega_1(q+1) + 2\xi_2\omega_2\right] - \left[\omega_1^2(q+1) + \omega_2^2\right] \tag{7a}$$

$$\Pi(\omega)H_{y_2}(\omega) = \omega^2 - i\omega\, 2\xi_1\omega_1(q+1) - \omega_1^2(q+1) \tag{7b}$$

where

$$\begin{aligned} \Pi(\omega) = {} & \omega^4 \\ & - i\omega^3 \left[2\xi_1\omega_1(q+1) + 2\xi_2\omega_2\right] \\ & - \omega^2 \left[\omega_1^2(q+1) + 4\xi_1\xi_2\omega_1\omega_2 + \omega_2^2\right] \\ & + i\omega \left[2\xi_1\omega_1\omega_2^2 + 2\xi_2\omega_1^2\omega_2\right] \end{aligned} \tag{7c}$$

$$+\omega_1^2\omega_2^2$$

$$i = \sqrt{-1}$$

From Eqs. 7 it may be seen that the transfer function $H_{y_1}(\omega)$, $H_{y_2}(\omega)$ depend on the uncoupled parameters given by Eq. 2 as well as the mass ratio q. This solution for transfer functions will be referred subsequently to as exact solution, to reflect the fact that the actual damping matrix [C] (Eq. 4b) was used.

COMPOSITE DAMPING SOLUTION

A set to two classical modes can be obtained for the system in the last section by using the matrices [M] and [K] and neglecting the damping matrix [C]. Let ω_{o_1}, ω_{o_2} represent the modal frequencies for the first and second mode respectively. Their values are obtained from the known frequency equation

$$\omega^4 + \left[\omega_1^2(q+1)+\omega_2^2\right] \omega^2+\omega_1^2\omega_2^2 = 0 \tag{8}$$

The corresponding mode shapes are

$$[\phi] = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \tag{9}$$

and are assumed to be normalized to the mass matrix, i.e.,

$$[\phi]^T[M][\phi] = [I] \tag{10}$$

Let ξ_1, ξ_2 represent the composite modal damping for the first and second modes respectively, then

$$\bar{\xi}_i = \omega_{o_i} \frac{\sum_{j=1}^{2} k_j \xi_j \frac{1}{\omega_j} \Delta_{ji}^2}{\sum_{j=1}^{2} k_j \Delta_{ji}^2} \tag{11}$$

where i=1,2 and matrix [Δ] is

$$[\Delta] = \begin{bmatrix} \phi_{11}-\phi_{21} & \phi_{12}-\phi_{22} \\ \phi_{21} & \phi_{22} \end{bmatrix} \tag{12}$$

The composite damping given by Eq. 11 is associated with the diagonal terms c_{11}, c_{22} of the modal damping matrix, i.e.,

$$\bar{c}_{11}=2\bar{\xi}_1\omega_{o_1}=\{\phi_1\}^T[c]\{\phi_1\};\bar{\xi}_1= \frac{1}{\omega_{o_1}} m_1\omega_1\xi_1\Delta_{11}^2+m_2\omega_2\xi_2\Delta_{21}^2 \tag{13}$$

Using Eq. 11

$$\bar{\xi}_1 = \omega_{o_1} \frac{m_1\omega_1\xi_1\Delta_{11}^2+m_2\omega_2\xi_2\Delta_{21}^2}{k_1\Delta_{11}^2+k_2\Delta_{21}^2} \tag{14a}$$

and considering the orthogonality of the modes with respect to the stiffness matrix

$$\omega_{o_1}^2 = k_1\Delta_{11}^2 + k_2\Delta_{21}^2 \tag{14b}$$

it can be seen that Eqs. 13 and 14 are identical. A similar result can be obtained for ξ_2.

Based on the above, modal solutions can be obtained using ξ_1, ξ_2. Off-diagonal terms c_{12}, c_{21} of the modal damping matrix $[\phi]^T[C][\phi]$ are therefore neglected to have uncoupled equations. This approximation is inevitable since classical modes are assumed. Using the modal equations, the transfer functions between input excitation x_g and displacement components y_1, y_2 of the system are

$$\{H(\omega)\} = [\phi] \{\bar{H}(\omega)\}$$

where

$$\bar{H}_1(\omega) = \frac{\gamma_1}{-\omega^2+i\omega2\bar{\xi}_1\omega_{o_1}+\omega_{o_1}^2} \tag{15}$$

$$\bar{H}_2(\omega) = \frac{\gamma_2}{-\omega^2+i\omega2\bar{\xi}_2\omega_{o_2}+\omega_{o_2}^2}$$

DAMPING VARIATIONS

The composite damping values ξ_1,ξ_2 were computed as functions of the system parameters. Systems with uncoupled frequency ratios in the range of 0.1 - 10 were considered. The damping of the individual components of the system was chosen to represent situations which reflect large damping variations. Figure 2 shows the composite damping of both modes as a function of the system frequency and mass ratio for ξ_1 = 2% and ξ_2 = 30%. The mass ratios are ranging from 0.01 to 1.

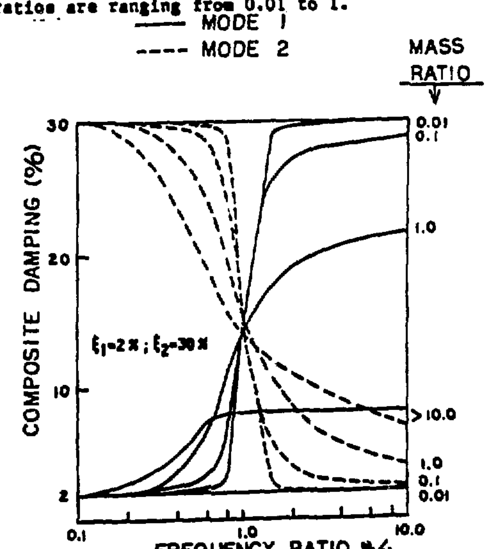

Fig. 2 - Composite damping as function of system parameters. Component damping: ξ_1=2%, ξ_2 = 30%.

From Fig. 2 it may be seen that for frequency ratios smaller than one, the composite damping of the first mode is smaller than that of the second mode. Furthermore, the composite damping associated with the first mode approaches to the value of 2% at low frequency ratios. For the same frequency range, the second mode damping approaches the 30% value. For frequency ratios greater than one the first mode is more damped than the second. Some exception appears to be the case with large mass ratio, i.e., equal to 10. Similar observations can be made from Fig. 3 in which larger differences in the damping of the system components is used, i.e., $\xi_1 = 2\%$, $\xi_2 = 50\%$. In this case, at the low frequency range the first mode approaches to 2% whereas the second mode approaches the 50% value.

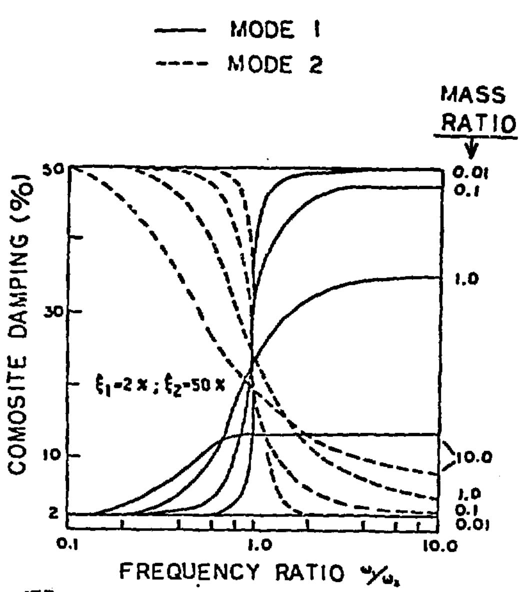

Fig. 3 – Composite damping as function of system parameters. Component damping: ξ_1=2%, ξ_2=50%.

SYSTEM MAXIMUM AMPLIFICATIONS

The maximum modulus of the transfer function with respect to the frequency variable ω reflects the maximum amplification of the system. The approximation introduced by the composite damping on these amplifications can be assessed by comparing the two solutions given previously. Particularly, this approximation can be computed from the difference between the exact solution given by Eq. 7 and the composite damping solution given by Eq. 15. This difference is expressed in percent of the deviation from the exact solution. Results are shown in Figs. 4 and 5 for the two

Figure 4 is shown the case with system damping ξ_1=2%, ξ_2=30%. From Fig. 4 it can be seen that for system frequency ratios in the vicinity of one the composite damping gives unconservative results for small mass ratios. Also from Fig. 4 it may be concluded that the composite damping predictions are generally conservative for second DOF of the system. By comparing Figs. 4 and 5 it can be seen that as the damping contrast between the components of the system becomes larger the unconservatism for DOF-1 increases whereas the conservatism of DOF-2 predictions increase. Furthermore, for higher mass ratios the approximations associated with composite damping solutions are generally small.

MULTI-FREQUENCY INPUTS

For multi-frequency inputs the system amplifications are sensitive to the frequency distribution of the system transfer functions. Thus the composite damping approximations cannot be assessed on the basis of maximum amplification only. The difference between the transfer functions representing the exact and composite damping solutions should be viewed in terms of its variation with respect to the frequency variable ω. Transfer functions for both these solutions are shown together in Figs. 6 to 9. The system parameters are: mass ratio 0.01 component damping ξ_1=2%, ξ_2=50%. System frequency ratios are 1,2,3 and 4. The difference between the dotted and the solid line represents the approximation due to composite damping for the various components of a given multi-frequency input. From these figures it can be concluded that composite damping solutions underestimate the response of the first degree-of-freedom of the system for the frequency components of the input which fall within the frequency range of the maximum difference between the two solutions. The approximation for the second degree-of-freedom, however, is usually less and on the conservative side.

It should be pointed out that Figs. 6 to 9 have been selected from the results of a parametric evaluation in which a wide variation of system parameters are considered. For low system frequency ratios the difference between the exact and composite damping solutions is small. Similarly, small deviations are associated with high system frequency ratios. Furthermore, for small damping contrast between the components of the system the two solutions are close. Similarly, small deviations are associated with systems characterized by higher mass ratios.

CONCLUSIONS

Based on the results presented in this paper it may be concluded that composite damping solutions can under- or over-estimate the response depending on the system parameters. For low and high system frequency ratios the approximation is usually small for all mass ratios and damping variations considered. The approximation becomes considerable for system frequency ratios close to one when the mass ratio is small and the differences in component damping is high. Under these conditions, one of the degree-of-freedom of the system is overestimated whereas the other is underestimated. Differences as high as eighty percent are presented for the case of a simple system. For more complex systems the results presented here may be used to identify possible cases of concern when composite damping solutions are employed.

degrees-of-freedom (DOF's) of the system. DOF-2 corresponds to the mass attached directly to the moving system.

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NOTICE

This work was performed under the auspices of the U.S. Nuclear Regulatory Commission, Washington, DC. The findings and opinions expressed in this paper are those of the authors, and do not necessarily reflect the views of the United States Nuclear Regulatory Commission or organizations of authors.

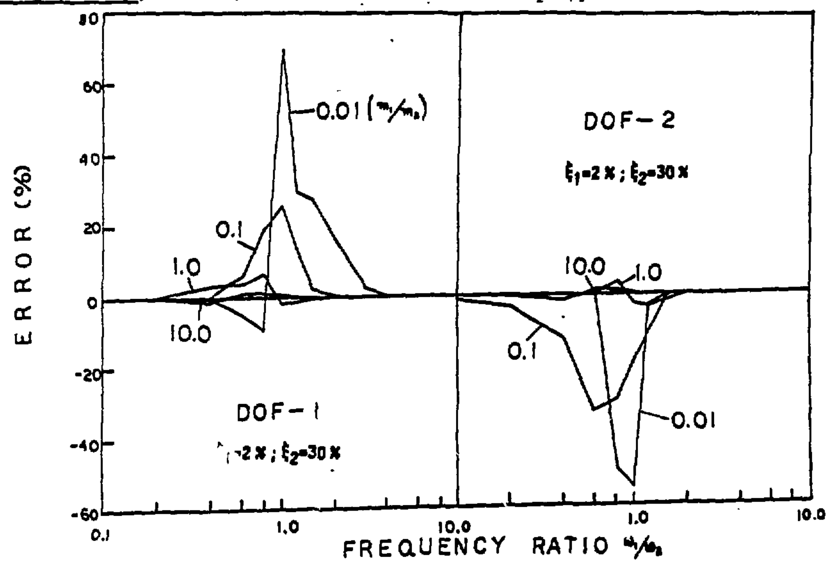

Fig. 4 - Composite damping approximations. Case: ξ_1 = 2%, ξ_2 = 30%.

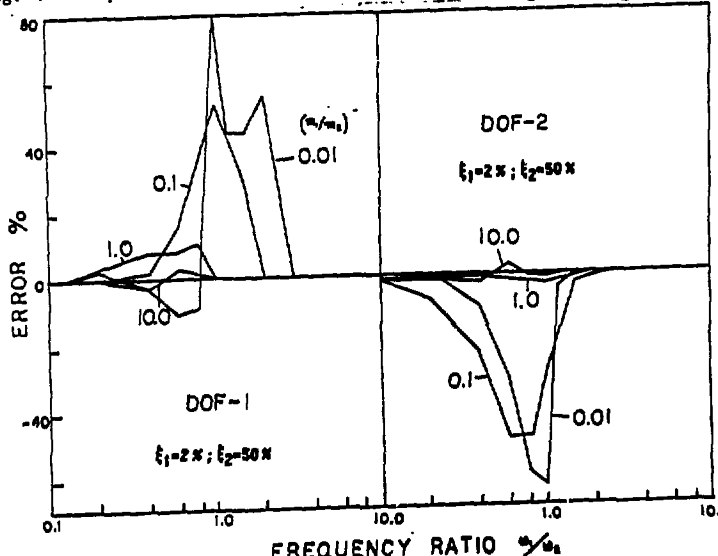

Fig. 5 - Composite damping approximations. Case: ξ_1 = 2%, ξ_2 = 50%.

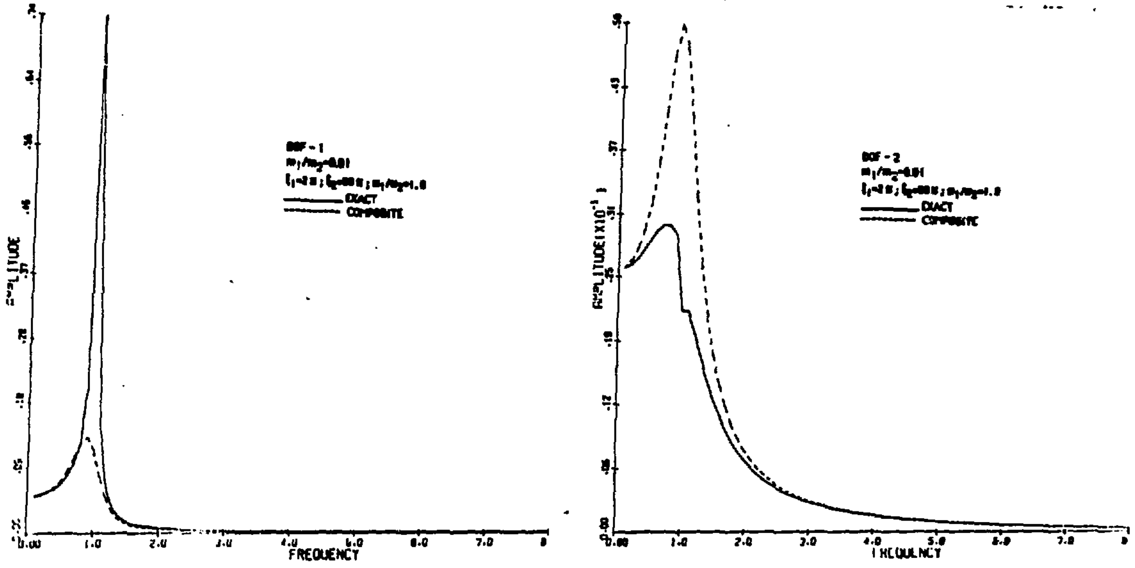

Fig. 6. Transfer functions for system frequency ratio 1.0.

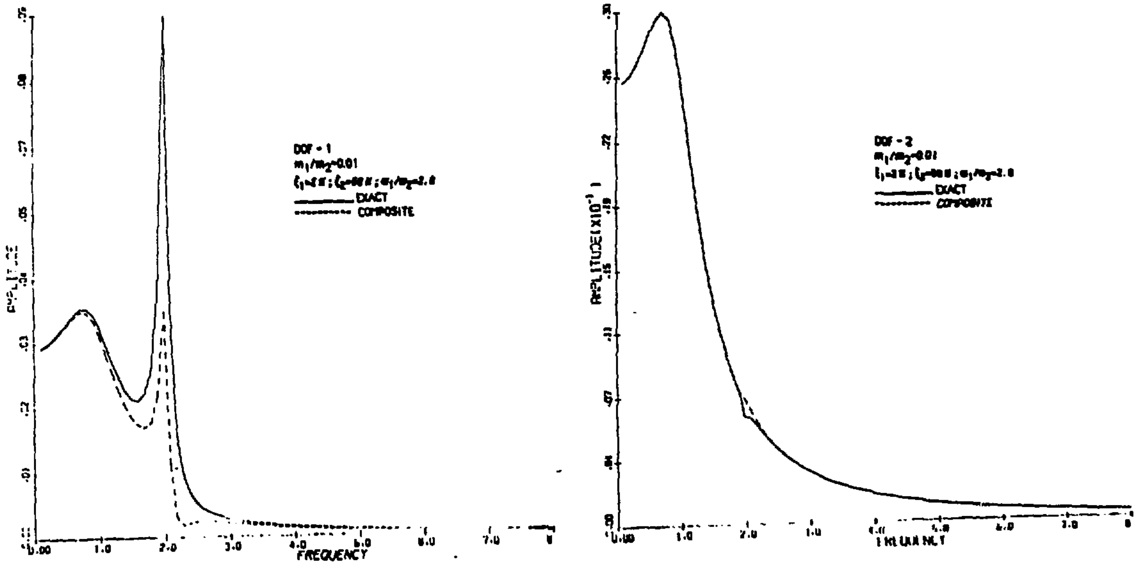

Fig. 7. Transfer functions for system frequency ratio 2.0.

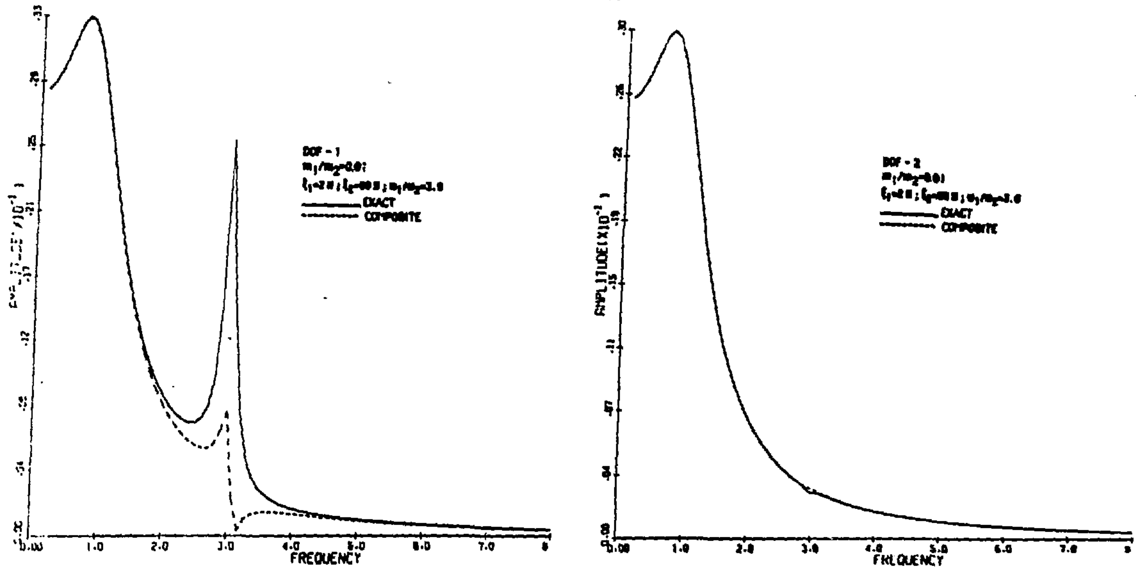

Fig. 8. Transfer functions for system frequency ratio 3.0.

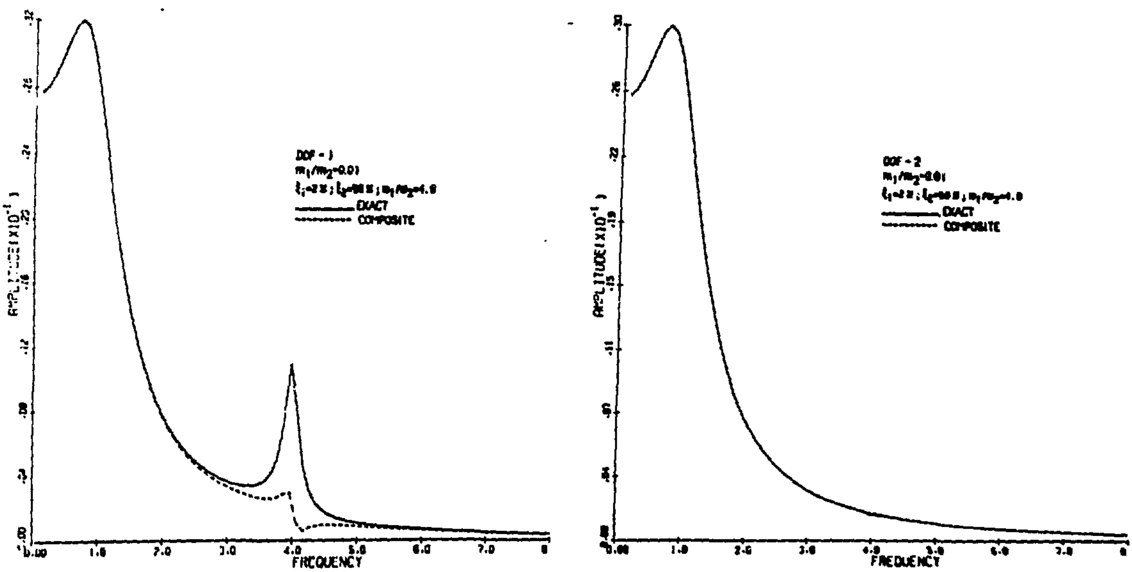

Fig. 9. Transfer functions for system frequency ratio 4.0.