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EFFECT OF MAGNETIC BENDING ON THE HIGH-FREQUENCY STABILITY OF THE ELMO BUMPY TORUS*

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ABSTRACT

We have investigated the high frequency stability of the ELMO Bumpy Torus (EBT) device when the wave vector has a finite component along the magnetic field lines.

1. Introduction

Present EBT devices are characterized by the presence of hot electron rings whose poloidal precession frequency is of the same order as the ion cyclotron frequency. Since the rings are usually located near the outer radius, one expects a certain amount of coupling between the hot electron magnetic curvature drift waves and the basically electrostatic high frequency surface waves.[1,2,3] The compressional Alfvén cavity modes may be involved in this coupling process since their frequencies can be comparable with those of the above two waves.

Detailed studies of the EBT high frequency stability problem have been carried out both analytically and numerically in the flute limit ($K_{\|} = 0$ and $\omega \gtrsim \Omega_{ci}$, where $K_{\|}$ is the component of the wave vector parallel to the magnetic field lines and Ω_{ci} is the ion cyclotron frequency). It was shown that two types of modes can exist: (1) the hot electron interchange mode, which determines the maximum allowable ratio of the hot electron density to the ion density, and (2) the high frequency compressional Alfvén mode, which sets an upper limit on the warm plasma beta. In this work we generalize the previous results by allowing for a finite $K_{\|}$, which leads to magnetic field line bending.

Our analysis is related to the study of electromagnetic modes in a plasma-filled cylindrical waveguide where it is known that four basic types of modes can exist: (1) the fast magnetosonic cavity modes, which are global modes satisfying the proper boundary conditions at both the plasma-vacuum and vacuum-metal wall interfaces; (2) the surface modes, which are usually localized in the neighborhood of the plasma-vacuum interface; (3) the discrete global Alfvén waves; and (4) the MHD continuum "modes," related to the condition $\omega^2 = K_\parallel^2 V_A^2$ being satisfied at any point across the radial density profile.[4)] The latter are not true eigenfunctions in the MHD limit ($E_\parallel = 0$) and usually serve to indicate the presence of a higher order coupling process.

2. Model and Analysis

Our basic model will remain that of an infinite cylindrical cavity with a uniform axial magnetic field B_0 that points in the z-direction. The hot electron rings are represented by an infinitely long, circularly symmetric annulus. The present analysis is limited to the high poloidal mode number limit, which allows using a slab model with the x-axis pointing in the direction of the inhomogeneity. A detailed numerical analysis of the more general cylindrical case will be left for a future publication.[5)] We assume that the background plasma is cold and that the hot electrons are monoenergetic perpendicular to the magnetic field lines and cold in the parallel direction. The parallel component of the perturbed electric field is neglected because of the high electron parallel conductivity, but all other electric and magnetic components are expected to be present. Perturbed quantities are assumed to vary as $\phi(x)\exp[-i\omega t + iky + iK_\parallel z]$. Our basic system of equations is the equation of motion for the cold particles, the Vlasov equation for the hot electrons, the quasi-neutrality condition, and the radial and axial components of Ampère's law, together with the constraint $E_\parallel = 0$. After a straightforward but lengthy treatment we arrive at the following dispersion relation:

$$\left(\frac{a/2\varepsilon}{\nu/q + b_1} + a - \frac{\nu^2 - \delta\nu}{\nu^2 - 1}\right) E_x - \frac{1}{k}\left(\frac{K_\parallel^2 V_A^2}{\omega\Omega_{ci}} + \frac{\nu}{\nu^2 - 1}\right)\frac{\partial E_x}{\partial x} =$$

$$i\left(\frac{\alpha\delta b_1}{\nu/q + b_1} + \frac{K_\parallel^2 V_A^2}{\omega\Omega_{ci}} + \frac{\nu - \delta\nu^2}{\nu^2 - 1}\right) E_y - \frac{i}{k}\left(\frac{\alpha/2\varepsilon}{\nu/q + b_1} + \alpha - \frac{\nu^2}{\nu^2 - 1}\right)\frac{\partial E_y}{\partial x} \qquad (1)$$

and

$$\left(\frac{k^2 V_A^2}{\omega\Omega_{ci}}\frac{\nu/q - 1}{\nu/q + b_1} + \frac{K_\parallel^2 V_A^2}{\omega\Omega_{ci}} + \frac{\nu}{\nu^2 - 1}\right) E_x = \left(\frac{\alpha/2\varepsilon}{\nu/q + b_1} + \alpha - \frac{\nu^2}{\nu^2 - 1}\right) iE_y -$$

$$\frac{i}{k}\left(\frac{k^2 V_A^2}{\omega\Omega_{ci}}\frac{\nu/q - 1}{\nu/q + b_1}\right)\frac{\partial E_y}{\partial x} . \qquad (2)$$

The indices i and h denote the ion and hot electron populations, respectively: $\alpha(x) = N_h(x)/N_i(x)$; $\nu = \omega/\Omega_{ci}$; the Alfvén speed $V_A = B_o^2/(\mu_o N_i M_i)^{1/2}$; M_i is the ion mass; $\varepsilon = L_n/R_c$; $\delta = 1/kL_n$; $L_n = N_i/|\partial N_i/\partial r|$; $b_1 = -b = \beta_h/2\varepsilon - 1$, where b is the hot electron poloidal drift frequency normalized to its value in the vacuum field; $\beta_h = 2\mu_o N_i T_{\perp,h}/B_o^2$, and $q = kT_{\perp,h}/eB_o R_c \Omega_{ci}$. Expecting that the unstable modes are localized in the outer part of the ring, we assumed that all components have similar density profiles (see Fig. 1).

The local limit of Eqs. (1) and (2) may be easily obtained by ignoring the radial derivatives. This results in a dispersion relation that possesses three distinct branches; the hot electron drift wave, the high frquency surface wave, and the shear Alfvén wave. The first two evolve into two magnetosonic waves propagating in opposite directions along the field lines as $K_\parallel$ is increased. During this process the negative energy branch related to hot electron drift couples to the positive energy surface and shear waves to produce two instabilities.

Turning back to the radial problem, it may be seen from Eqs. (1) and (2) that the coefficient of the highest derivative in the resulting second-order differential equation vanishes when the dispersion relation of the shear wave $[K_\parallel^2 V_A^2 = \omega^2/(1 - \nu^2)]$ is satisfied. The local shear Alfvén waves do not thus have corresponding radially localized eigenfunctions. It is known that a more complete picture can be obtained when one replaces the $E_{\parallel} = 0$ condition by an appropriate second-order differential equation.[6] One may also use the eikonal assumption to solve an inhomogeneous axial problem, as was done in the case of tandem mirrors.[7] A detailed study of this interesting radial problem is beyond the scope of the present work.

The coupling of the hot electron and surface waves can be adequately studied using Eqs. (1) and (2). Localized eigenfunctions may be easily obtained using an appropriate radial shooting code.[8] A WKB analysis can also be performed, resulting in the dispersion relation:

$$\left(\frac{\nu}{q} - 1\right)\left[\frac{k_{\perp}^{2}}{k^{2}}\left(1 + \frac{K_{\parallel}^{2}V_{A}^{2}}{\Omega_{ci}^{2}}\right) - \delta\nu\right] = \frac{\omega^{2}}{k^{2}V_{A}^{2}}\left(b_{1} - \frac{\alpha}{\varepsilon}\right)$$

$$+ \nu\left(\alpha\delta + \frac{\alpha\delta\omega_{ci}^{2}}{2\varepsilon k^{2}V_{A}^{2}} + \frac{K_{\parallel}^{2}}{k^{2}}\delta b_{1}\right), \qquad (3)$$

where all quantities are evaluated at the location where the vacuum magnetic field radial gradient undergoes the largest diamagnetic increase, $k_{\perp}^{2} = k^{2} + k_{r}^{2} = k^{2} + (2n + 1)/L_{n}^{2}$ and $n = 0, 1, 2, \ldots$. We may note that the first term on the left-hand side describes the negative energy wave related to the hot electron drift, while the second term (in brackets) describes the finite $K_{\parallel}$ corrections to the positive energy surface wave. The right-hand side is related to the magnetosonic mode and the hot electron contributions. By solving the above simple quadratic equation for ν we may easily recognize that mode coupling results in a narrow, unstable range in $K_{\parallel}$, outside which stability is again ensured. The results of the shooting code agree quite well with those based on the above WKB expression.

In conclusion, we have described the coupling between the ring drift wave and the background plasma modes due to finite parallel mode numbers. The dispersion relations obtained may be useful in the interpretation of the high frequency fluctuation measurements.

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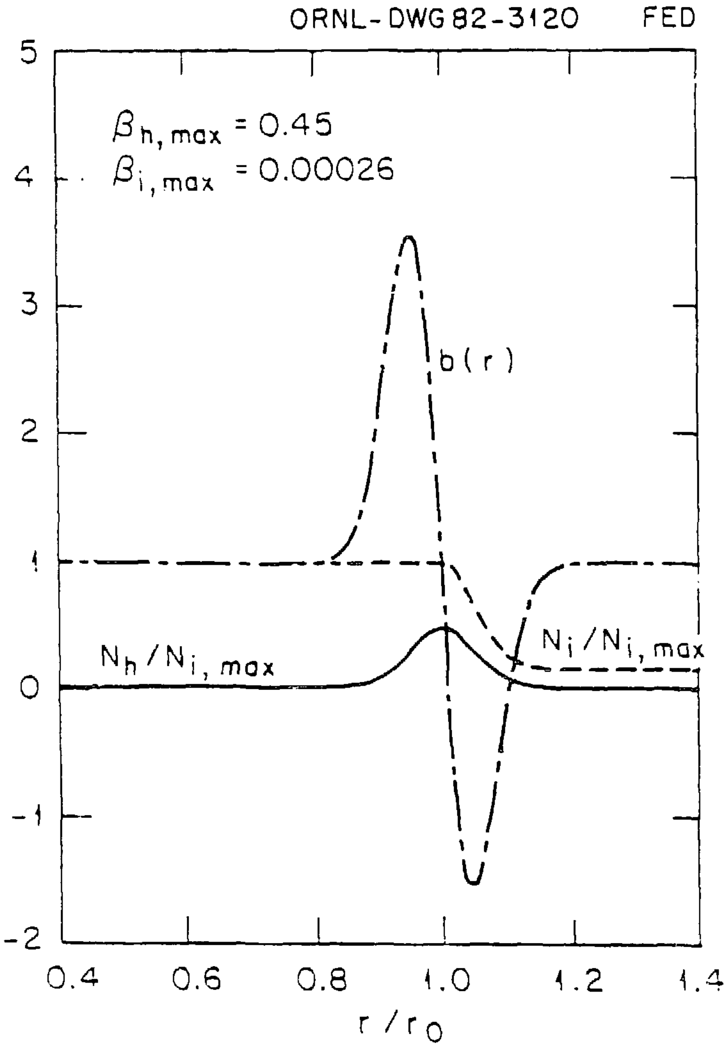

Fig. 1. Normalized radial profiles for the ion density, hot electron density, and hot electron poloidal magnetic drift velocity when enough hot beta is present to create a diamagnetic well.