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RECENT RESULTS OF AN INTERNAL TILT MODE CALCULATION IN FRCS

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Abstract

Recent theoretical results on the stability of FRCs to the internal tilt mode are presented. An approximate treatment of collisions shows that collisions have a small effect on the growth rate of the mode until the plasma becomes very collisional ($\lambda_{ii} \leq r_s/2$). Finite Larmor radius theory predicts that the growth rate of the instability normalized to that of MHD depends only on the combination $\bar{s}/e$, where e is the plasma elongation. However, a full Vlasov stability calculation does not appear to show such a scaling.

I. Introduction

Recently we have implemented numerically a kinetic stability analysis of the internal tilt mode (n = 1 ballooning mode) in FRCs.[1,2] Our earlier results[2] can be summarized by saying that when full Vlasov ion kinetic effects are taken into account in the FRC geometry, the growth rate of the mode is reduced from MHD by more than a factor of 30 for existing experimental values of $\bar{s}$ ($0.75 \leq \bar{s} \leq 1.6$). Such a reduction in the growth rate of the instability is consistent with observed characteristic plasma decay times. Our study also indicates that there may be a stability threshold for internal tilting at about $\bar{s}$ of 3. While we believe we have demonstrated the capability to do exact Vlasov stability computations for multidimensional plasma equilibria, detailed application of these results to the experiments is premature, since many improvements on the calculation are needed.

In studies[3,4] of the stability of mirror and EBT systems it has emerged that the stability of certain modes is enchanced or achieved when $\omega^* \geq \gamma_M$, where ω^* is the diamagnetic drift frequency and γ_M is the MHD growth rate of the mode. Since ω^* is determined by the radial dimensions of the plasma and γ_M by the axial dimensions, the stability criterion implies that stability can be achieved for a given plasma radius if the plasma length is increased. Post and Hammer[5] have speculated on whether the same aspect ratio scaling should be observed in our Vlasov stability calculations.

In this paper we report on two extensions of our previous results: we have considered approximately the effect of ion collisions and have done a brief study of the effect of plasma elongation on tilt stability. When collisions are included in an approximate way, we find that for all values of $\bar{s}$ collisions have little effect on the growth rate until the plasma becomes very collisional ($\lambda_{ii} \leq r_s/2$). In our brief survey of the effect of plasma aspect ratio on stability, we find that the Vlasov stability results do not appear to follow the finite Larmor radius (FLR) scaling of normalized growth rate with $\bar{s}/e$. The exact scaling with elongation has not been determined, but we will pursue this study more in the future. We also are extending our calculation by considering a partial expansion of the eigenfunction, rather than a simple trial function approach, but at this time we have not achieved convergence in our computations. Thus the effect on the growth rate of the eigenfunction expansion will have to await more study.

II. Ion Collisions

It is difficult to consider ion-ion collisions exactly using a Vlasov model in an eigenvalue calculation. Instead of attempting a rigorous treatment of collisions, we first attempt an approximate treatment of collisions, see if the effect is big, and then decide whether a better treatment is justified. Our kinetic dispersion functional involves autocorrelation functions of the interaction of the perturbing fields with particles moving along their equilibrium orbits. We reason that the qualitative effect of collisions can be ascertained simply by cutting off the correlation functions at appropriate ion-ion collision times.

The dispersion functional has the form

$$\Delta = -2\delta W + 2\omega^2 K + V(\omega) = 0, \tag{1}$$

where δW is the incompressible MHD δW and K is the MHD inertia term

$$K = \frac{M}{2} \int dq\, n_0 |\xi|^2 \tag{2}$$

$V(\omega)$, which contains all the kinetic contributions, is defined by

$$V(\omega) = -iM^2\omega \int_0^\infty d\tau\, e^{i\omega\tau}[A(-\tau) + \omega B(-\tau) + \omega^2 C(-\tau)], \quad \mathrm{Im}\,\omega > 0 \tag{3}$$

The autocorrelation functions are defined by

$$A(\tau) = \int dz_0\, f_0' e^{in\vartheta[Z(z_0,\tau)]}\, X^*[Z(z_0,0)]X[Z(z_0,\tau)], \tag{4}$$

$$B(\tau) = \int dz_0\, f_0' e^{in\vartheta[Z(z_0,\tau)]}\, \{Y^*[Z(z_0,0)]X[Z(z_0,\tau)] + X^*[Z(z_0,0)]Y[Z(z_0,\tau)]\}, \tag{5}$$

and

$$C(\tau) = \int dz_0\, f_0' e^{in\vartheta[Z(z_0,\tau)]}\, Y^*[Z(z_0,0)]Y[Z(z_0,\tau)] \tag{6}$$

The phase functions X and Y are defined by

$$X = \underline{\underline{vv}} : \nabla\underline{\xi} + \frac{inv_\vartheta}{r}\underline{v}\cdot\underline{\xi} \quad \text{and} \quad Y = -i\underline{v}\cdot\underline{\xi} \tag{7}$$

In Eqs. (3-7), a phase space point (q,p) is denoted by $\underline{z}$, and the solution of the equilibrium equations of motion is $\underline{z} = \underline{Z}(z_0,\tau)$. This means that a particle starting at phase point z_0 will be at position $\underline{z}$ a time τ later.

Since $A(\tau)$, $B(\tau)$, and $C(\tau)$ contain the characteristic powers of velocity v^4, v^3, and v^2, respectively, we have put smooth cutoffs in the autocorrelation functions at ion-ion collision times calculated from effective thermal velocities obtained from Maxwellian distributions weighted by the corresponding powers of velocity. If we define a collisionality parameter $c = r_s/\lambda_{ii}$, then it can be shown that $c \sim (sB)(x_s T^{3/2})$. Thus c and $\hat{s}$ can be varied independently by an appropriate sequence of equilibrium states. For a given value of $\hat{s}$ we choose c to be in the set {0, 0.98, 4.9}. The curves of γ/γ_M versus $\hat{s}$ for each c are shown in Fig. 1. As can be seen from Fig. 1, for $c = 0.98$, or $\lambda_{ii} = r_s$, collisions have almost no effect. This is because the FLR like autocorrelation function $A(\tau)$ (this term dominates in (3)) decays in a time short compared to a typical collision time τ_i. Only when the plasma is so collisional that $c = 4.9$, do we start to approach the MHD limit. Based on

Fig. 1 we could estimate that collisional effects start to become important when $c = r_s/\lambda_{ii} \geqslant 2$.

III. Elongation Study

We now show that in the finite Larmor radius (FLR) limit our dispersion functional shows the elongation scaling suggested by Post and Hammer, then we test to see if the scaling is true in general. The elongation parameter e is defined to be $e = (2r_s / \ell_s)^{-1}$, where ℓ_s is the total length of the FRC. The MHD growth rate is approximately $\gamma_M \approx v_i/\ell_s$, so δW can be scaled relative to K by

$$\delta W \sim -\gamma_M^2 K. \tag{8}$$

In the FLR limit we retain only the term $A(-\tau)$ in Eq. (3). We can approximate

$$\int d\tau \sim \tau_c \text{ and } A(\tau) \sim -\left(\frac{1}{T}\right)\int dg\ n_0\ v_i^4/r_s^2|\xi|^2, \tag{9}$$

where τ_c is the correlation time, the length of time for which the particle's equilibrium motion contributes coherently to the normal mode. Then Eq. (3) becomes

$$V(\omega) \sim M^2\omega\ \tau_c\ \frac{(-1)}{T}\int dg\ n_0|\xi|^2\ v_i^4/r_s^2 \tag{10}$$

(The factor of i in $V(\omega)$ was dropped, since we know that V becomes real in the FLR limit.) Divide the dispersion functional (1) by 2K and use $T = Mv_i^2/2$ to get

$$0 = \gamma_M^2 + \omega^2 - 2\omega(\tau_c v_i/r_s)(v_i/r_s). \tag{11}$$

In an FLR theory it is appropriate to choose τ_c to be Ω_{ci}^{-1}. Define the small parameters $\delta = e^{-1}$ and $\epsilon = \rho_i/r_s$. Then (11) becomes

$$0 = \gamma_M^2 + \omega^2 - 2\omega\ \epsilon\ \gamma_M/\delta \tag{12}$$

The requirement for stabilization from (12) is

$$\delta^{-1} = e > \epsilon^{-1} \tag{13}$$

Thus the elongation scaling alluded to by Post and Hammer appears in the FLR limit of our dispersion functional. The question is to what extent is this scaling valid for the full kinetic calculation.

The scaling of the growth rate with elongation as predicted by Eq. (13) was tested computationally using the sequence of three equilibria[6] shown in Fig. 2 (note the different horizontal scales). For each $\bar{s}$ in the set {0.75, 1.0, 2.0, 7.0, and 15.0}, we calculated the normalized growth rate for the three values of elongation. If γ/γ_M were a function of the single variable s/e, then our data should lie on a smooth curve. In Fig. 3 we see that the normalized growth rate is not a function of single variable s/e (for instance, the data is not even approximately monotonic). However, we point out that the short elongation case ($e = 5.1$) is not in the asymptotic MHD regime, since γ_M is not scaling linearly with e. That is, γ_M is not given accurately by v_i/ℓ_s, but there is additional r_s dependence. Figure 3 suggests that there may be a scaling of normalized growth rate with s/e for small values of that parameter. However, this is the highly kinetic regime where the FLR assumptions are not justified, so any scaling with $\bar{s}/e$ is probably fortuitous (also the error bars on the data points is larger for very small growth rates).

There may be a caveat with our present elongation study, and the verification and resolution of this difficulty must await future calculations. The total displacement vector for the FRC geometry can be written as

$$\xi(\psi,\chi,\vartheta) = e^{in\vartheta}[\hat{r}\xi_r(\psi,\chi) + \hat{\vartheta}\xi_\vartheta(\psi,\chi) + \hat{z}\xi_z(\psi,\chi)],$$

where ψ labels a flux surface and χ labels the position along a flux surface. ξ_ϑ is determined from incompressibility, and if the FRC has a large enough elongation, ξ_r can be neglected. We have seen from earlier work[7] that if the FRC has nearly elliptical flux surfaces (such as in Figs. 2a and 2b) then it is a good approximation to assume that each flux surface moves rigidly.

$$\xi_z(\psi,\chi) = \xi_z(\psi). \tag{14}$$

Approximation (14) is being made in all our current stability computations. However, if the FRC is more racetrack (perhaps like in Fig. 2c) then the displacement is concentrated more towards the tips of the flux surfaces, and large underestimates of the growth rate could result if assumption (14) is made.[7] Thus, all our results for the Fig. 2c equilibrium should be viewed as preliminary and subject to further verification when our eigenfunction expansion becomes fully three dimensional (i. e., includes χ dependence).

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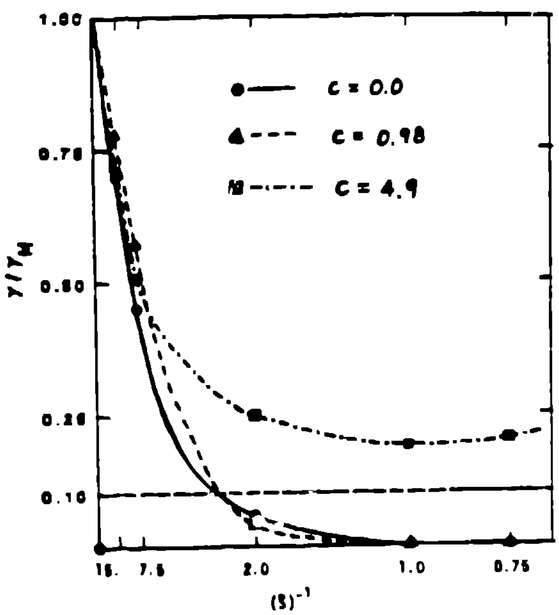

Figure 1. Normalized kinetic growth rate versus $\bar{s}$ for various collisionalities $c \equiv r_s/\lambda_{ii}$. (c = 0.0 is the collisionless case.)

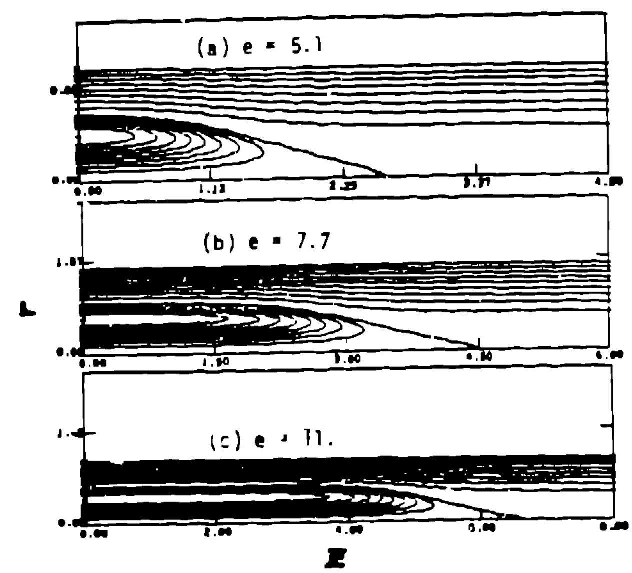

Figure 2. Three Spencer-Hewett equilibria with increasing elongations $e \equiv \ell_s/(2r_s)$.

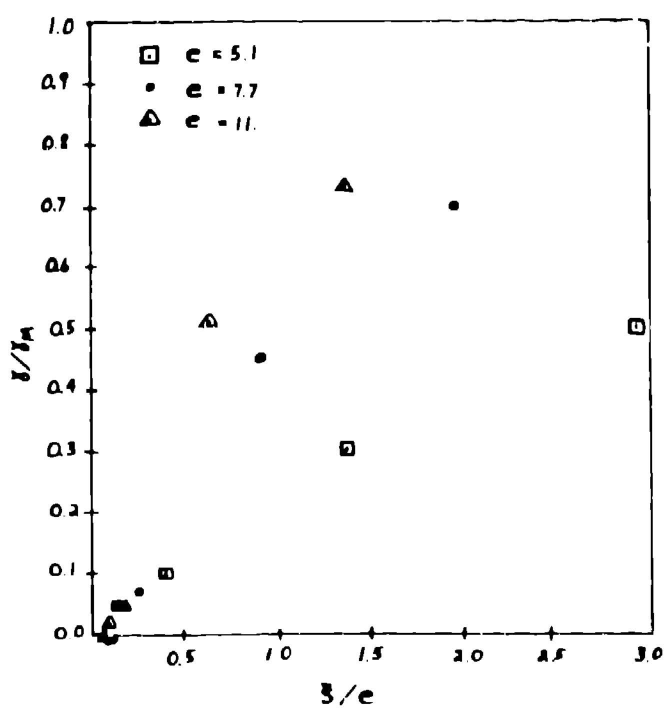

Figure 3. Normalized growth rate versus $\bar{s}/e$ for three different elongation FRCs. If FLR theory applied, all the data would lie on a smooth curve.