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CLASSICAL HADRODYNAMICS APPROACH TO ULTRARELATIVISTIC HEAVY-ION COLLISIONS

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ABSTRACT

We discuss the exact solution of the classical relativistic equations of motion for an action corresponding to nucleons interacting with massive scalar and vector meson fields. This model—the classical analogue of the quantum hadrodynamics of Serot and Walecka—provides a manifestly Lorentz covariant approach to heavy-ion collisions, allows for nonequilibrium phenomena, interactions of correlated nucleon clusters, and particle production, and is valid when interaction times are short. We present an analysis of the nonlocality inherent in the model and discuss effects arising from the finite size of a nucleon.

The conditions found in relativistic heavy-ion collisions occurring at AGS, CERN, and RHIC energies differ greatly from those necessary for the valid application of most previous approximation methods and models used to describe such collisions because the interaction time there is extremely short, and the nucleon mean free path, force range, and internucleon separation are all comparable in size.[1] Additionally, the de Broglie wavelength of projectile nucleons is sufficiently small that quantal coherence effects are negligible and the classical approximation for nucleon trajectories is valid. Classical relativistic hadrodynamics provides a natural means for studying many-body collective dynamics which is valid under the above conditions.[2] It automatically includes spacetime nonlocality, retardation, nonequilibrium phenomena, interactions with correlated clusters of nucleons, and particle production in a single, unified approach, and it avoids the use of a mean field theory, perturbation theory, or non-covariant equations of motion.

In our contribution to the Seventh Winter Workshop on Nuclear Dynamics[2] we presented the equations of motion for point nucleons interacting with massive scalar and vector meson fields and obtained several numerical solutions. We made three physical assumptions: (1) Lorentz invariance, which includes energy and momentum conservation, (2) point nucleons minimally coupled to the meson fields, and (3) the classical approximation.

Here we generalize the equations of motion to account for the finite size of the nucleon, but neglecting its internal dynamics. The simplest covariant generalization that has the correct nonrelativistic and point particle limits is

$$M^* a^\mu = g_s \mathcal{P}^{\mu\nu} \partial_\nu \phi_{\text{ext}} + g_v F^{\mu\nu}_{\text{ext}} v_\nu + f^\mu_s + f^\mu_v \quad , \tag{1}$$

where $\partial\phi_{\text{ext}}$ and F_{ext} are the external scalar and vector fields and

$$f_s^\mu = \frac{g_s{}^2}{12}\mathcal{P}^{\mu\nu}\int_0^\infty d\sigma\left[h'\left(\frac{\sigma}{2}\right) - m_s\int_0^s du\, h'\left(\frac{\sqrt{s^2-u^2}}{2}\right)J_1(m_s u)\right]s_\nu \quad (2)$$

and

$$f_v^\mu = \frac{g_v{}^2}{6}\mathcal{P}^{\mu\nu}\int_0^\infty d\sigma\left[\bar{h}\left(\frac{\sigma}{2}\right) - m_v\frac{s\cdot v}{s\cdot v'}\int_0^s du\,\bar{h}\left(\frac{\sqrt{s^2-u^2}}{2}\right)J_1(m_v u)\right.$$
$$\left. - 2m_v\left(\frac{s\cdot v\, s\cdot v'}{s^2} - v\cdot v'\right)\frac{J_2(m_v s)}{s^2}\right]s_\nu \quad (3)$$

are the corresponding self-forces. The nucleon four-position, four-velocity, and four-acceleration are $q^\mu(\tau)$, $v^\mu = dq^\mu/d\tau$, and $a^\mu = dv^\mu/d\tau$; the four-vector separation is $s^\mu = q^\mu(\tau) - q^\mu(\tau-\sigma)$, with interval $s = (s_\mu s^\mu)^{1/2}$; $v' = v(\tau-\sigma)$. The tensor $\mathcal{P}^{\mu\nu} = g^{\mu\nu} - v^\mu v^\nu$ projects vectors orthogonal to the four-velocity and $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ is the metric. We discussed the values of the various constants of Eqs. (1-3) in Ref. 2. The functions $h'(s)$ and $\bar{h}(s)$, as well as the effective mass M^*, depend on the mass density of the nucleon, $\rho(r)$, through

$$M^* = M - \frac{g_s{}^2}{6}\left[\frac{G_0(m_s)}{m_s} - 2G_1(m_s)\right] - \frac{g_v{}^2}{6}\left[5\frac{G_0(m_v)}{m_v} + 2G_1(m_v)\right]$$
$$+ g_s\phi_{ext} + m_s g_s{}^2\left[\int_0^\infty d\sigma\frac{J_1(m_s s)}{s} - \frac{1}{2}\right] \quad , \quad (4)$$

$$G_n(m) = \int_0^\infty ds\, h(s)s^n e^{-2ms} \quad , \quad (5)$$

$$h(s) = 32\pi^2\int_0^\infty ds'\left(s'^2 - s^2\right)\rho(s+s')\rho(|s-s'|) \quad , \quad (6)$$

$h'(s) = dh/ds$, and $\bar{h}(s) = h'(s) - 6m_v{}^2\int_0^s ds' h(s')$.

The nucleon mass density is assumed to have a root-mean-square radius of $R_{rms} = 0.862 \pm 0.012$ fm and is very well approximated by the exponential $\rho(r) \propto \exp(\sqrt{12}\, r/R_{rms})$.[3] For the point particle limit, $R_{rms} \to 0$, we recover the equations of Ref. 2.

The finite size of the nucleon greatly affects the nature of the nonlocality present in the equations of motion; in fact, for large enough R_{rms}, there is no preacceleration and the runaway solutions of the point particle case are absent. We demonstrate this by considering the characteristic equation obtained by assuming an exponential growth of the acceleration, $a^i(\tau) \sim C^i\exp(\kappa\tau)$, namely

$$M^*\kappa^2 + \frac{g_s{}^2}{3}\left[D_s G_0(D_s) - m_s G_0(m_s)\right] + \frac{g_v{}^2}{3}\left[\frac{3\kappa^2 - D_v{}^2}{D_v}G_0(D_v) + m_v G_0(m_v)\right] = 0, \quad (7)$$

where $D_{v,s} = \sqrt{m_{v,s}{}^2 + \kappa^2}$. The inverse of κ gives the range in proper time of the preacceleration; as long as there exist solutions to Eq. (7) for which the real part of κ is positive, runaway solutions will exist. Figure 1 shows that no exponentially

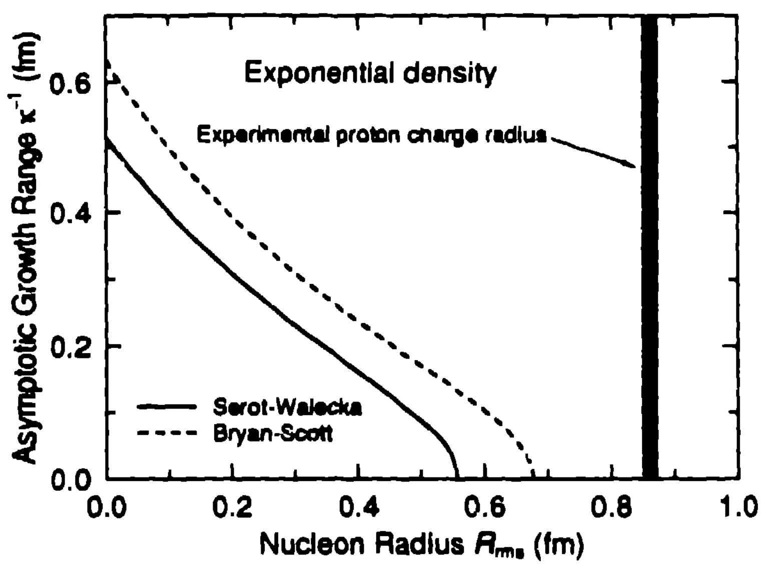

Fig. 1. Asymptotic growth range as a function of root-mean-square radius for an exponential mass density using the Serot-Walecka and Bryan-Scott values for the coupling constants;[2] the shaded bar indicates the experimentally determined proton charge radius.[3]

growing solutions are present for the experimental value of the proton radius; a similar situation occurs in the case of classical nonrelativistic electrodynamics,[4] which is a special case of our theory.

We are currently attempting the exact numerical solution of Eq. (1) at AGS energies. Preliminary results indicate that interaction starts much more sharply than in the point particle case of Ref. 2—confirming our analysis of the characteristic equation—and that peak accelerations are much higher. It remains to be seen whether or not these accelerations are consistent with our neglect of the internal dynamics of the nucleon.

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