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WAVENUMBERS FOR CURRENTS ON INFINITE- AND FINITE-LENGTH WIRES IN A CHIRAL MEDIUM

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INTRODUCTION

There is increasing interest in determining the electromagnetic properties of material media differing from free space and the effects thereof on the radiation, propagation, and scattering of electromagnetic fields. A material property of special present interest is that of chirality. Chirality manifests itself as a "handedness" wherein a chiral medium does not support propagation of a linearly-polarized plane wave, but which instead decomposes into two circularly-polarized waves that propagate at different speeds. Initial work in this area was devoted to developing various analytical solutions to some basic problems such as the Green's Dyadic for a point current source [Basiri et. al. (1986)]. Attention is now being increasingly devoted to using this early work for a variety of applications such as analyzing antennas in chiral media [Jaggard et. al. (1991)]; scattering from chiral objects [Bhattacharyya (1991); scattering from objects having chiral coatings [Uslenghi (1990)]; and reflection from planar chiral interfaces [Lakhtakia et. al. (1986)].

The focus of the work described here is determining the wavenumbers (= $-\alpha - j\beta$) of the current waves excited on wire antennas located in an infinite chiral medium using two complementary approaches. One is to use an extension of an existing computer model (NEC) [Burke and Poggio (1981)] that permits modeling of arbitrary wire objects located in an infinite chiral medium. The other is to develop a solution for an infinitely long cylindrical antenna also located in an infinite chiral medium. The latter canonical problem is of interest in its own right as well as providing a means for achieving mutual validation with the NEC model.

FINITE-LENGTH WIRES IN A CHIRAL MEDIUM

The thin-wire, electric-field integral equation for a perfectly-conducting wire can be expressed as

$$\mathbf{s}\cdot\mathbf{E}^{inc}(\mathbf{r}) = -j\omega\mu\int_{C(\mathbf{r})} \mathbf{s}\cdot[\mathrm{I}(\mathbf{r}')\cdot\Gamma(\mathbf{r},\mathbf{r}')d\mathbf{r}'];\ \mathbf{r}\in C(\mathbf{r})$$

where a time variation of $e^{j\omega t}$ has been assumed, $\mathbf{r}$ and $\mathbf{r}'$ denote the observation point and source point where the unit tangent vectors are given by $\mathbf{s}$ and $\mathbf{s}'$, respectively, $\mathbf{E}^{inc}(\mathbf{r})$ is the "incident" field exciting the wire, $C(\mathbf{r})$ describes the spatial contour of the wire, and $\Gamma(\mathbf{r},\mathbf{r}')$ is the Green's dyadic for the medium. For an infinite chiral medium, the Green's function can be written [Bassiri et. al. (1986)]

$$\Gamma_{chiral}(\mathbf{r},\mathbf{r}') = a_1\left(\mathbf{u} + \frac{1}{h_1}\mathbf{u}\times\nabla + \frac{1}{h_1^2}\nabla\nabla\right)\frac{e^{-jh_1R}}{4\pi R} + a_2\left(\mathbf{u} - \frac{1}{h_2}\mathbf{u}\times\nabla + \frac{1}{h_2^2}\nabla\nabla\right)\frac{e^{-jh_2R}}{4\pi R}$$

with $$a_1 = \frac{h_1^2 - k^2}{h_1^2 - h_2^2},\ a_2 = -\frac{h_2^2 - k^2}{h_1^2 - h_2^2}$$

and $h_{1,2} = \pm\omega\mu\gamma + \sqrt{(\omega_{\cdot}\gamma)^2 + k^2}$

where **u** is the unit dyad and the constant γ c ·ermines the degree of medium chirality as expressed by the constitutive relationships [Basiri, et. al. (1986)]

$$\mathbf{D} = \epsilon\mathbf{E} + j\gamma\mathbf{B} \text{ and } \cdot = j\gamma\mathbf{E} + \mathbf{B}/\mu.$$

The wavenumber $k = \omega\sqrt{\epsilon\mu}$ is the achiral-me\ a wavenumber and ε and μ are the medium permittivity and permeability respectively. As can be seen, the proportionality quantities a_1 and a_2 depend on the wavenumbers h_1 and h_2 for the two circularly-polarized waves that propagate in the chiral medium.

The NEC computer model has been modified to include the chiral Green's function to permit the modeling of arbitrary wire objects excited as antennas and scatterers in a chiral medium for which some initial results have been reported elsewhere [Bhattacharyya, Burke and Miller (1992a), (1992b)]. Extending NEC to the problem of a chiral medium somewhat complicates computing the electric fields of a general wire object because it is no longer possible to obtain the fields of the sine and cosine terms of the three-term current basis used in NEC in closed form because of the two different propagation constants. For example, the z-component of the field has the form

$$E_z(\rho,z) = \frac{-j\eta I_0}{4\pi k_1}\left\{\begin{pmatrix}\sin k_s z' \\ \cos k_s z'\end{pmatrix}\left[(z-z')^2\frac{1+jk_1R}{R^2} - k_s\begin{pmatrix}\cos k_s z' \\ -\sin k_s z'\end{pmatrix}\right]\frac{e^{-jk_1R}}{R}\right]_{-\delta}^{\delta}$$

$$+ (k_1^2 - k_s^2)\int_{-\delta}^{\delta}\begin{pmatrix}\sin k_s z' \\ \cos k_s z'\end{pmatrix}\frac{e^{-jk_1R}}{R}dz'\Bigg\}$$

for the sine and cosine terms respectively and a current "segment" 2δ long located on the z-axis of a cylindrical coordinate system. When the medium propagation constant k_1 equals the basis propagation constant k_s, the terms requiring numerical integration cancel, and the field forms revert to those employed for an infinite, achiral, medium. Good results are obtained from NEC using $k_s = \sqrt{h_1 h_2}$. Evaluating the field of the constant term of the current basis, which has no propagation constant for the current, is essentially unaffected by the medium chirality except for the fact that a field components is produced for h_1 and h_2. The NEC estimates for the propagation constant are based on solving for the current on a wire six or so wavelengths long excited a quarter wavelength from one end and having a resistive load at the other end to reduce the current standing-wave ratio. The complex wavenumber was then obtained from a least-squares fit to the log of magnitude and the phase of the current using five match points over each segment.

INFINITE ANTENNA IN A CHIRAL MEDIUM

One starting point for modeling an infinite cylindrical antenna of radius a located on the z axis of a cylindrical coordinate system is to write an integral equation for the current as

$$j\omega\mu\int_{-\infty}^{\infty} I(z')G_{chiral}(a,|z-z'|)dz' = E_z^{inc}(z)$$

where for a chiral medium $G_{chiral}(a,|z-z'|)$ is here the zz component of the chiral Green's dyadic. The solution to this equation can be expressed as a Fourier integral of the current, i.e., [Miller (1967)]

$$I(z) = C\int_{-\infty}^{\infty} e^{j\beta z} i(\beta)d\beta$$

where $i(\beta)$ is determined from a boundary-value solution obtained by expressing the exciting and induced fields in spectral form and C is a normalizing constant. Because of the convolutional form of the integral equation, $i(\beta)$ is available in closed form, being given by

$$i(\beta) = \frac{-j}{\eta} S(\beta) \frac{\left[\frac{a_1 \lambda_1}{h_1} \frac{\partial}{\partial(\lambda_1 a)} H_0^{(2)}(\lambda_1 a) + \frac{a_2 \lambda_2}{h_2} \frac{\partial}{\partial(\lambda_2 a)} H_0^{(2)}(\lambda_2 a) \right]}{\left[\frac{a_1 \lambda_1^2}{h_1^2} H_0^{(2)}(\lambda_1 a) + \frac{a_2 \lambda_2^2}{h_2^2} H_0^{(2)}(\lambda_2 a) \right]}$$

where $S(\beta)$ is the spatial spectrum of the incident field, H_0 is a Hankel function of the second kind, and $\lambda_i = \sqrt{h_i^2 - \beta^2}$.

Evaluation of the infinite-antenna current can be accomplished using numerical quadrature or analytical approximation of the dominant contributions to the integral. For our purposes, we do not need the explicit z-dependent current, but can instead determine the propagation constant from the poles of the current spectrum to obtain the results which follow. We note that the denominator of $i(\beta)$ involves both chiral-medium waves which suggests that the mode associated with the denominator zero might be described as a "hybrid" wave.

NUMERICAL RESULTS

Wavenumber results for the current as obtained from the infinite-antenna model and from NEC are shown in Fig. 1. The attenuation ($-\alpha$) and propagation ($-j\beta$) components of the wavenumber are compared respectively as a function of the chirality constant for two different values of wire radius. Good agreement is obtained between the NEC and infinite-antenna values over a chirality range of 10^{-4} to about 0.05. For the values of chirality shown here, the chiral effect is seen to be greater with larger wire radii. The excellent agreement obtained confirms both the NEC and infinite-wire solutions.

The magnitude of the NEC current on the wire for various wire radii and a chirality constant $\gamma = 0.01$ for a 30-wavelength (free space) wire modeled using 500 segments is shown in Fig. 2. It can be seen for the case investigated, that as wire radius is increased, there is a transition from a more rapidly-attenuated current near the source to a less-attenuated current further down the wire. The near-source current is a "fast-wave" mode where the propagation constant is less than in the ambient medium which makes a transition to an even "faster" mode which exhibits smaller attenuation, as shown in Fig. 3. The attenuation and propagation constants (normalized to k_0) of the first region as obtained from NEC, -0.12?3 -j0.9446 match those computed from the pole of the current spectrum, -0.1280 -j0.9444, quite well. In the faster-wave region, the NEC propagation constant, -j0.4134, is close to the value of h_2 which is -j0.4142.

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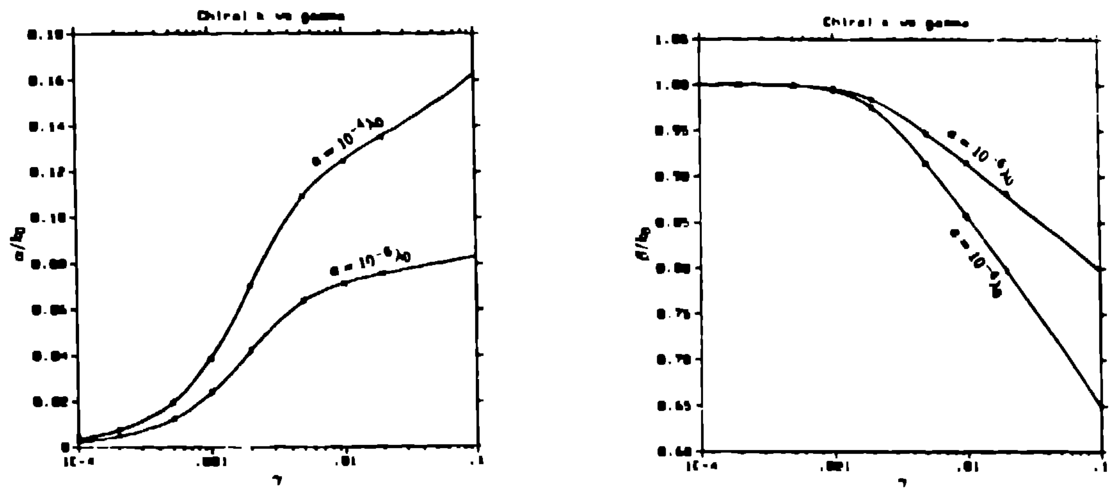

Fig. 1. Comparison of the attenuation (α) and phase (β) constants normalized to k_0 for a wire antenna in a chiral medium as a function of chiral parameter γ as obtained from NEC (the x's) and an infinite-antenna model (the solid lines).

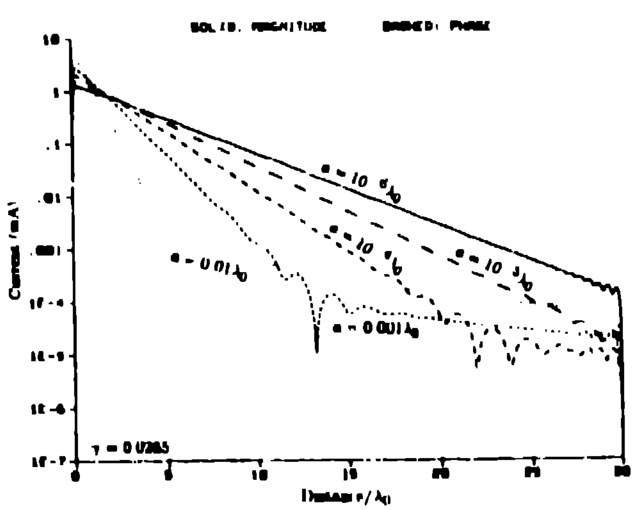

Fig. 2. Current magnitude as a function of distance for 30-wavelength wires of various radii as obtained from NEC for a chiral parameter $\gamma = 0.0265$.

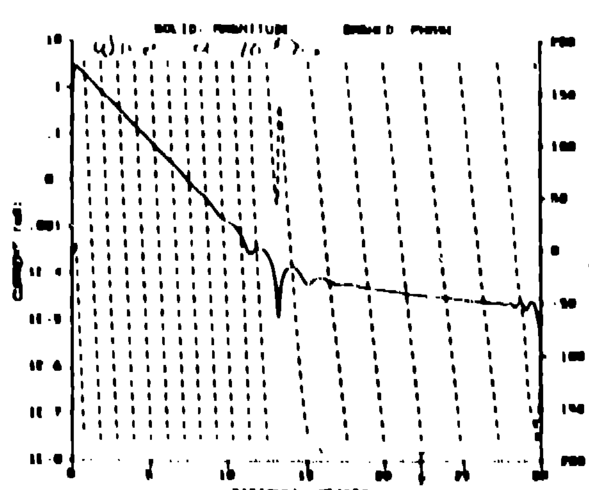

Fig. 3. Current magnitude and phase for a wire of radius 0.001 λ_0 for a chiral parameter $\gamma = 0.0265$.