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FACTORIZATION OF THE DRELL-YAN CROSS SECTION*

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Abstract

We state the weak and strong factorization theorems for the Drell-Yan cross section and outline the ingredients involved in their proof. We also discuss the physical picture implied by the factorization results and its phenomenological consequences.

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Introduction

The basic Drell-Yan process for lepton pair production is shown in Fig.1. A quark from one hadron annihilates with an anti-quark from another hadron to produce a virtual photon, which in turn produces a lepton pair. We find it convenient to work in the hadron center of mass frame, with one hadron moving in the plus-z direction and the other hadron moving in the minus-z direction. In terms of the light-cone variables ($p^+ = p^0 + p^z$, $p^- = p^0 - p^z$, $p_\perp$), the upper hadron has momentum $P_1=(0,P,0_\perp)$ and the lower hadron has momentum $P_2 = (P,0,0_\perp)$. The invariant momentum-squared of the virtual photon is

$$Q^2 = x_q x_{\bar{q}} P^2 , \tag{1}$$

where x_q and $x_{\bar{q}}$ are the longitudinal momentum fractions of the quark and antiquark respectively.

In QCD we would like to maintain the simple parton model picture of Fig. 1 as much as possible. Of course the basic Drell-Yan process can be "dressed" with any number of QCD corrections, such as real and virtual gluon radiation, real and virtual pair production, quark-gluon interactions, and gluon-gluon interactions. Thus, the naive parton picture cannot survive. However, we would still like to be able to associate a factor in the cross section with each hadron (the structure function) and a remaining factor with a perturbatively calculable QCD correction to the basic parton model process (the hard process).

The QCD corrections to the cross section can be characterized in terms of regions of momentum space of the gluonic momenta. The regions that are of importance for the discussion of factorization are

(1) Collinear

(a) to + : $\ell^+ \gg \ell^-$, $\ell^+\ell^- \sim \ell_\perp^2$

(b) to − : $\ell^- \gg \ell^+$, $\ell^+\ell^- \sim \ell_\perp^2$

(2) Central ($\ell^+ \sim \ell^- \sim \ell_\perp$)

(a) U.V. : $|\ell_\mu| \gg P$

(b) Hard : $\ell_\mu \sim P$

(c) Soft : $m < |\ell_\mu| \ll P$

(d) Very Soft : $|\ell_\mu| \ll m$

(3) Glauber : $|\ell^+\ell^-| \ll \ell_\perp^2$.

Here m is a typical hadronic mass and we think of P as a large momentum scale (P>>m). The very soft region is the one discussed by Landshoff and Stirling[1)] in connection with the on-shell--off-shell ambiguity in the K-factor. The Glauber region corresponds to the kinematically allowed region of gluon momentum for the case of nearly on-shell to nearly on-shell quark-quark scattering(semi-classical scattering) by gluon exchange.

Let us consider the role played by each of these regions in a diagrammatic calculation of the cross section. The U.V. region is removed by the usual renormalization counter-terms. The contributions from the hard region are perturbatively calculable because of asymptotic freedom. However, the collinear, soft, very soft, and Glauber regions potentially destroy the calculability of radiative corrections, and thus factorization, by driving the loop integrals into regions of small invariant momentum-squared.

A further obstacle to factorization was pointed out by Bodwin, Brodsky, and Lepage[2]. They showed that spectator interactions like the one shown in Fig.2 give a leading twist (leading in 1/P) contribution to the cross section. Since such interactions can involve both the upper and lower hadron, they cannot be associated with either hadron or with the basic process in any obvious way. What is worse, they can become entangled with the QCD corrections to the basic process or with each other.

The Factorization Program

A number of authors have discussed programs for putting the Drell-Yan cross section into a factored form[3]. However, the factorization scheme to which we refer here is the one due to Collins, Soper, and Sterman[4],[5]. Their proposal consists of two parts:

(1) Weak Factorization:

At large Q^2, $d\sigma/dQ^2\, dQ_\perp^2$ is a convolution of a hard process with structure functions $\mathscr{P}_{q/1}$ and $\mathscr{P}_{q/2}$, where the subscripts indicate that these are the structure functions for finding a quark or antiquark in a hadron 1 or 2. The $\mathscr{P}$'s contain all the collinear and all spectator contributions.

(2) Strong Factorization:

The Drell-Yan and deep-inelastic structure functions are simply related:

$$\int_{|P_\perp|<Q} d^2P_\perp \; \mathscr{P}_{q/A}(x,P_\perp) = f_{q/A}(x) \; . \tag{2}$$

In the context of nucleus collisions strong factorization appears paradoxical. For deep-inelastic scattering we know that, away from kinematic boundaries, cross sections are approximately proportional to nuclear mass:

$$d\sigma^{A}_{D-I} \approx A^{1} d\sigma^{H}_{D-I} \ . \qquad (3)$$

Then strong factorization implies that, for pion-nucleus collisions, the Drell-Yan cross section is also proportional to A^1:

$$d\sigma^{\pi-A}_{D-Y} \approx A^{1} d\sigma^{\pi-H}_{D-Y} \ . \qquad (4)$$

It seems that nuclear matter is transparent to quarks. That is, a quark on the back face of the nucleus has the same probability as a quark on the front face to annihilate with an antiquark from the pion. Of course we know that this picture must break down for a macroscopic target: pions beams are depleted when they pass through a macroscopic length of material. Thus there must be a constraint on the region of validity of any factorization theorem that depends on the length of the target.

Let us now outline a procedure by which leading twist contributions to the Drell-Yan cross section can be put into the weak factored form[6),7)].

(1) One deforms loop-integration contours into their complex planes in order to eliminate contributions from the Glauber region. This step makes use of the analyticity properties of the Feynman amplitudes and the unitarity cancellation of final state interactions. It is at this stage that a target length condition emerges.

(2) For each set of graphs in nth order, $G^{(n)}$, one constructs a set of collinear subtractions, $S^{(n)}$, such that $G^{(n)}-S^{(n)}$ contains only central contributions. It turns out that $G^{(n)}-S^{(n)}$ contains only interactions involving the active quark and antiquark.

(3) One uses Ward identities to disentangle the $S^{(n)}$ into a topological form that is consistent with factorization.

(4) Soft divergences cancel by a generalized Bloch-Nordsieck mechanism[8)].

(5) Contributions from the very soft region have the net effect of putting the active quark and antiquark on-shell.

The result of this procedure is shown schematically in Fig.3. The cross section is now topologically factored into structure functions (associated with each hadron) and a perturbatively calculable part involving only the active quark and antiquark ($\sigma_{central}$). The net effect of the collinear and spectator interactions is to append an "eikonal" line--indicated by a double line in the figure--on to the hadron wave functions. The eikonal line actually represents a path-ordered exponential of the line integral (along the collision axis) of the hadron's chromodynamic field. It is the effect of the long-range part of the hadronic vector potential, which is a pure gauge. The hadronic wave functions alone are, of course, gauge-dependent, but the eikonal lines combined with the wave functions are a gauge-invariant structure.

Strong factorization is easily proven by making use of the analyticity properties of the Feynman amplitudes and a cancellation between real and virtual emission[9)]. The cancellation requires that one integrate over the lepton pair transverse momentum.

Given these results, we can explain the resolution of the A^1 "paradox." The nucleus is not transparent to nuclear matter. There are spectator interactions in the Drell-Yan process, but to leading twist they have the same effect as the spectator interactions in deep-inelastic scattering. That is, they result in gauge-artifact eikonal lines. Their effect vanishes if one writes the Drell-Yan cross section in terms of the deep-inelastic structure functions. Their effect is manifest for a long target, where the weak factorization procedure breaks down, and for $d\sigma/dQ^2\, dQ_\perp^2$, where there is no strong factorization connection between structure functions.

Implications for Phenomenology

Now let us discuss the phenomenological consequences of the factorization results.

First, there is a target length condition that must be satisfied if factorization is to hold. Namely,

$$Q^2 \gg x_q LMk^2_\perp \propto A^{2/3} \quad . \tag{5}$$

Here, L is the target length, M is the target mass and $k_\perp$ is a typical soft gluon momentum transfer ~200 MeV. For uranium, then, we expect the A^1 behavior that is characteristic of the factorized cross section to change to the $A^{2/3}$ behavior that is characteristic of interactions on the nuclear surface at Q^2~10 GeV^2.

If Q^2 satisfies the target length condition, then the Drell-Yan and deep-inelastic structure functions are related by Eq.(2). That is, the normalization of the Drell-Yan cross section is completely fixed by the deep-inelastic results. This means that the K-factor is not an arbitrary normalization factor but, rather, a specific consequence of higher-order QCD corrections to the parton model. Eq.(2) also implies that if the EMC effect[10)] is present in deep-inelastic scattering, it should appear in lepton pair-production as well.

Finally, the presence of spectator interactions implies a smearing of the lepton pair transverse momentum distribution that increases with nuclear size. If we attempt to estimate this effect by assuming the spectator interactions to be semi-classical, then we arrive at the result[2)]

$$\langle Q_\perp^2 \rangle \approx a + bA^{1/3} \,, \tag{6}$$

where $b \sim (.1 \text{ Gev})^2$. The Chicago-Princeton pion-nucleus Drell-Yan data[11)] for $Q_\perp^2$ as a function of target nucleus mass are shown in Fig.4. As can be seen, the data show no indication of the predicted growth of $Q_\perp^2$ with nuclear size. However, they do not quite rule it out. Furthermore, we know from the EMC effect that there is a softening of the quark longitudinal momentum distribution with increasing nuclear size. Lorentz invariance implies a corresponding softening of the transverse momentum distribution. Bodwin, Brodsky, and Lepage[12)] have estimated the size of this effect and find it to be −10% to −20% for Fe--that is roughly equal in magnitude and opposite in sign to the spectator interaction smearing. If the softening effect saturates at an A of about 27, then one might find that the nuclear dependence of $Q_\perp^2$ is something like the dashed curve in Fig.5. Since the total effect is small at large A, it would be more difficult to observe than a simple linear dependence on $A^{1/3}$.

Acknowledgements

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Figure Captions

Fig. 1 The basic Drell-Yan process.

Fig. 2 An example of a spectator interaction.

Fig. 3 The Weak Factored Form for the Drell-Yan Cross Section.

Fig. 4 $Q_{\perp}^2$ Dependence on Nuclear Mass. The solid and broken lines are of the form given in Eqn.(3). The dashed line is indicative of the effect of nuclear softening as explained in the text.

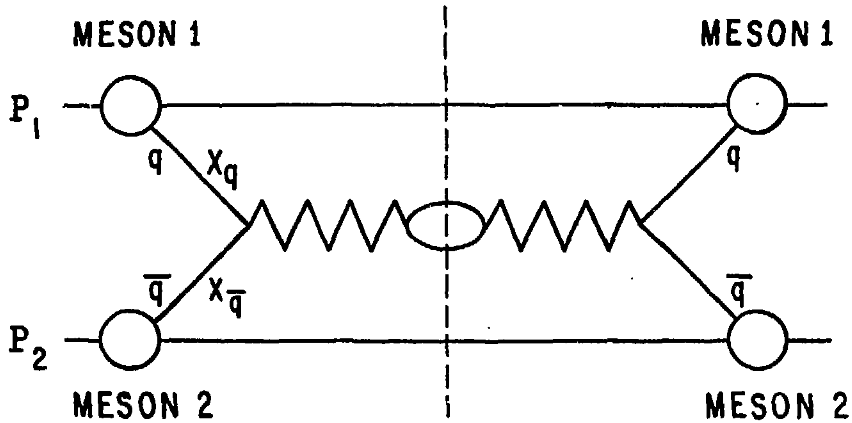


Fig. 1

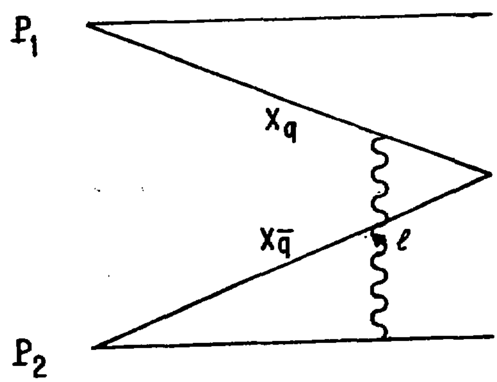


Fig. 2

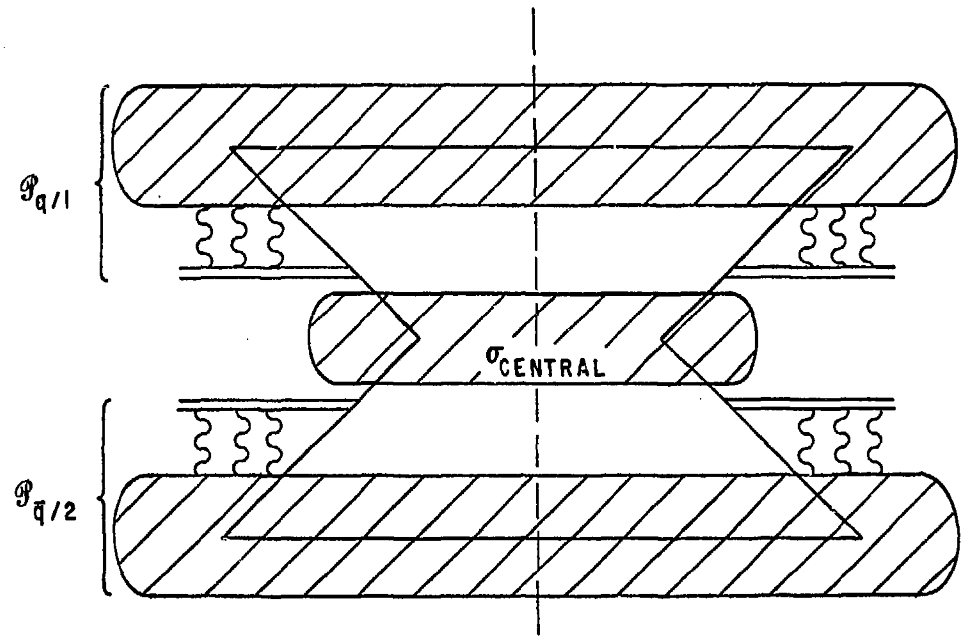
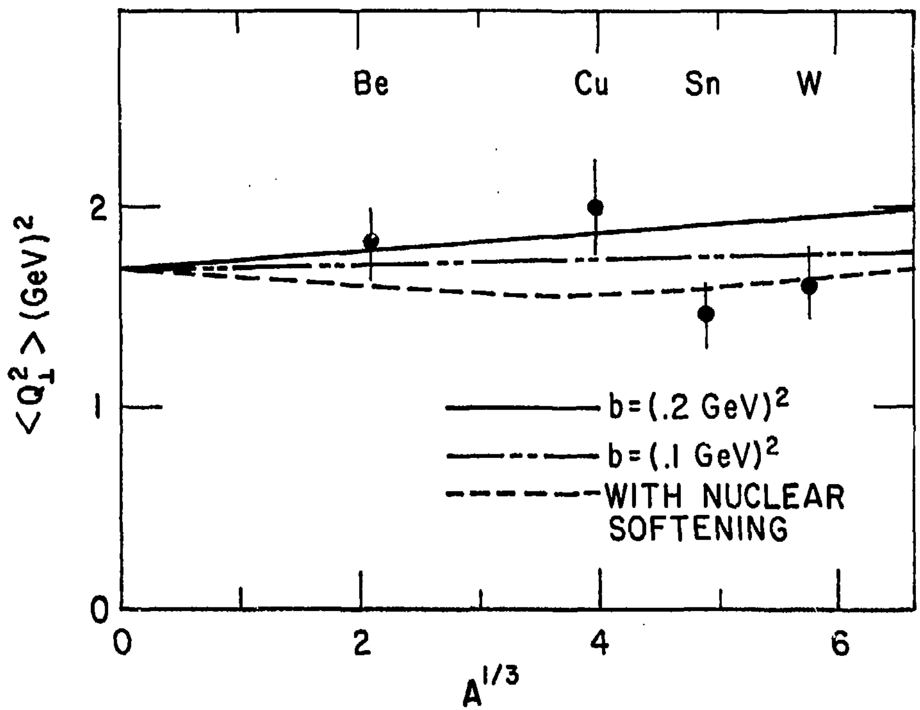


Fig. 3



Be
Cu
Sn
W
$\langle Q_\perp^2 \rangle$ (GeV)^2
2
1
0
0
2
4
6
$A^{1/3}$
b=(.2 GeV)^2
b=(.1 GeV)^2
WITH NUCLEAR SOFTENING

Fig. 4