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SEISMIC SAFETY MARGINS RESEARCH PROGRAM
LOAD COMBINATION PROJECT - TASK 3
LOAD COMBINATION METHODOLOGY DEVELOPMENT
INTERIM REPORT I

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FOREWORD

The objective of this report is to provide the NRC with a preliminary overview of a portion of the approach we are developing for Task 3 of the Load Combination Project. More details and examples of results will be provided in future reports.

This report was developed through the efforts of a team consisting of M. K. Ravindra (Sargent & Lundy), C. A. Cornell (Professor of Civil Engineering at the Massachusetts Institute of Technology), J. Collins (J. H. Wiggins Company), R. P. Kennedy (Structural Mechanics Associates) and C. K. Chou, K. Vepa, P. D. Smith, and R. Mensing of Lawrence Livermore Laboratory. In keeping with the spirit of the technical approach used in the SSMRP, the order of the authors' names on this report was selected randomly.

SUMMARY

This is the first interim report giving the results to date on the development of a load combination methodology. After a brief background, the objectives and scope of the load combination methodology task are listed. This is followed by user oriented requirements on the methodology. The proposed methodology is then introduced and simply demonstrated. Examples of similar applications of the reliability based methodology are presented in Section 6 and accompanied by a listing of some of the unique considerations in applying this type of methodology to nuclear design. A fairly detailed exposition of a component reliability and design code optimization scheme is given along with a brief discussion of system and plant reliability considerations.

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1. INTRODUCTION

1.1 Background

Current load combination criteria have been developed on a conservative and somewhat subjective basis. The probability of occurrence and consequences of events have been considered in formulating these criteria, but not as completely as possible. Studies have been conducted to address portions of load combination issues, but a unified approach has never been undertaken. Consequently, the Nuclear Regulatory Commission (NRC) and the nuclear industry have differed on criteria, on a number of occasions. The issues are too complex to develop convincing arguments for adopting or modifying portions without a comprehensive approach which is removed from immediate licensing questions. In some cases, positions have been adopted which result in increased plant costs or other problems at some later date when they are applied to new load combinations or conditions.

The NRC and the industry need a unified approach to load combination. This approach must evolve from a rational procedure to determine what loads <u>need</u> to be combined, how, and at what service level or structural load category. The broad aspects of this approach should be such that both the NRC and the industry agree it would resolve the issues, if adopted and developed. The approach should be systematic so areas of disagreement can be isolated, and focus provided to resolve the disagreement.

The basic problems that need to be addressed are:

1. What loads should be considered as concurrent; what response should be combined?

2. What factors should be applied to load effects or responses?

3. What are appropriate service levels, load categories, and stress limits?

4. How should responses be combined.

1.2 Objective and Scope of the Load Combination Methodology Development Task

The objectives of the load combination methodology development task are to:

1. Develop a methodology for appropriate combination of loads or responses for nuclear power plant design under normal plant operation, transients, accidents, and natural hazards.

2. Establish design criteria, load factors, and component service levels for appropriate combinations of loads or responses to be used in nuclear power plant design or design review.

The scope of the project covers the following:

- Review and evaluate existing industry and NRC requirements for treating loads and load combinations.

- Identify the problems in combining loads.

- Develop a methodology for establishing subsystem and/or reliability based on system or nuclear power plant reliability requirements.

- Develop a methodology for relating probability of component failure to the design criteria for components of a nuclear power plant.

- Demonstrate the load combination methodology for some representative components and subsystems.

- Assist the NRC in revising the Standard Review Plan and in developing regulatory changes.

1.3 Definitions

Listed below are definitions of several terms which are important for understanding the topics discussed in this report.

1. Event: An environmental, accidental or operation condition which is to be considered in design.

2. Load: Effects associated with an event exclusive of the influence of properties of structures, systems and components such as mass, stiffness and damping; the input to a static or dynamic analysis.

3. Response: Effects of loads computed either by a static or dynamic analysis and including the influence of properties of structures, systems and componnets such as stiffness, mass and damping; the output of a static or dynamic analysis

4. Component: The smallest unit whose design is regulated by a by a single specific part of the applicable design code.

5. <u>Limit State:</u> An undesirable condition of a unit. Limit states may be grouped by different levels of implications and, hence, by different degrees of undesirability. Examples of different limit states are: 1) a unit may continue to function but be sufficiently damaged to require replacement, 2) a unit may remain structurally intact but fail to function, and 3) a unit may experience catastrophic failure.

6. <u>Limit State Probability:</u> The probability that the defined undesirable condition exists.

2. STATEMENT OF THE PROBLEM

The ASME code for component design has not traditionally specified which loads should be combined. On the other hand loads are combined in recent ACI and AISC codes. The combinations in these codes were not completely accepted by the NRC, thus, load combinations for structures are specified instead in the Standard Review Plan (SRP). Load combinations for mechanical components have evolved but are not included in the SRP. The differences in philosophy in structural and mechanical design have their counterparts in the resulting load combination criteria for structures and equipment. The following paragraphs give some examples of load combinations, these differences, and identify issues that need to be addressed.

2.1 Pressure Retaining Equipment

The ASME Boiler and Pressure Vessel Code, Section III, Division 1, Nuclear Power Plant Components (ASME Code), governs the design of vessels, pumps, valves, piping, steel containment and component supports. The ASME Code defines six conditions for load combinations; Design, Service Levels A, B, C, and D, and Testing. The philosphy in the ASME Code is to place limits on stress for these conditions for which various unfactored load effects are combined. The four service levels allow combinations of loads of increasing severity and decreasing probability of occurrence to be placed in separate categories with different stress limits. Load combinations are specified by the designer for the six conditions. Thus, the combinations are not included in the code, and they may vary from component to component and plant to plant. The code also allows limit and inelastic analyses for Service Levels C and D in lieu of stress limits based on linear analysis.

The ASME Code differentiates between primary, secondary, and peak stresses. Secondary and peak stresses must be addressed for Design and Testing Conditions, and Service Levels C and D (with exceptions).

The primary stress limits must be met for design conditions and all service levels for metal containment structures. Typical load combinations for Service Levels B and D for a BWR are:

Level B = Normal Pressure + Dead Weight + OBE + Safety Relief Valve Discharge.

Level D = LOCA Pressure + Dead Weight + SSE + LOCA Dynamic Effects.

Combinations for Level B have a relatively-high probability of occurrence and the component should survive them without damage. Since Service Level B assumes a high probability of occurrence, it might be justified to specify that the combination of OBE and SRV with normal loads is a Service Level C combination. However, the code implication is that Level C loads may cause some damage while Level B loads may not. Additionally, loads may take different forms, e.g., the OBE load is vibratory while the SRV load is pulsatory. There is some small probability that responses to both loads will be phased in time so the peak responses will coincide. Further, the responses to dynamic loads may not be as damaging as those for static loads, because of the limited energy content of the dynamic loads. All these factors need to be considered in determining the service level.

For Service Level D, the primary requirement is to safely shutdown the plant. The probability of occurrence of an earthquake-induced large LOCA is believed to be extremely low. The probability of peak responses to LOCA and earthquake loads occurring simultaneously is even lower so the combination of SSE and LOCA may not be required. However, this load combination is prudent provided the resulting design does not increase risk from normal operating conditions.

2.2 Structures

The design of structures is governed by the ACI and AISC codes. Both codes use elastic working stress design (WSD) and load factor design (LFD) criteria. The codes do not distinguish between primary (load controlled) and secondary (displacement controlled) stress, and load and displacement controlled conditions are included in load combinations which are then compared to load controlled limits.

The SRP specifies load combinations for safety-related structures. The ACI and AISC codes do not address service levels so there are two groups of load categories in the SRP.

The first includes normal and severe loads such as dead, live, thermal, wind, and loads due to OBE. Several combinations of responses to these loads are specified and the combinations must meet either a WSD or LFD criteria. Factors used in the LFD criteria vary, depending on the loads. Examples of the two criteria for the same combination of loads are:

WSD Criteria $D + L + E + T_o + R_o \leq 1.35S$

LFD Criteria $.75\,(1.4D + 1.7L + 1.9E + 1.7\,T_o + 1.7\,R_o) \leq U$

where S is a working stress limit and U is the ultimate strength. D, L, E, T_o and R_o are stresses induced by dead, live, OBE, normal thermal loads, and normal pipe reaction, respectively.

The load factors applied in the LFD method are individual "safety factors" on each response. Ideally, they should consider the probability of occurrence of the loads, the probability the loads will be concurrent, and the acceptable level of damage. For example, neither a thermal or dynamic load should be as damaging as a static load.

In the WSD method, all responses are given equal weights and response sums compared to a stress limit. This is comparable to the ASME approach; however, displacement and load-controlled loadings are considered equally damaging.

The second group includes all normal, severe and extreme environmental, and abnormal loadings. Several combinations are specified using the LFD approach. The factors are lower than those for the normal plus severe environmental grouping. This allows some damage, but it should be limited so that a safe shutdown can be achieved.

Development of the load factors for structures has implicitly considered the probability of occurrence, dynamic characteristics of the loadings, source of the load (load or displacement controlled), energy absorption capacity, and service experience of structures. However, no in-depth nuclear industry studies have been conducted to evaluate these factors using reliability methods.

2.3 Rationale for Specifying Load Combinations

Design by analysis can be achieved economically only by a predominant use of linear methods at this time. ASME, ACI and AISC codes use this philosophy; nonlinearities are only considered implicitly in setting acceptance criteria. These design codes are not likely to change significantly unless studies provide technical justification, and then will change only gradually. Therefore, the methodology developed in this task should meet their requirements, insofar as possible. Some considerations in specifying loads, combinations and factors are as follows:

2.3.1 Loads

- What are the sources of and uncertainty in the loads?
- What are the probabilities of occurrence of different load intensities?
- What are the durations of the loads?
- Is the loading function load or displacement controlled?
- What are the dynamic characteristics of the loads?

2.3.2 Load Combinations

- What is the probability of two or more concurrent loads?
- What consequences can be allowed due to concurrent loads? This determines the equipment service level or structural load acceptance criteria.
- What is the probability that the specified consequences (deformation, cracking, etc.) would be exceeded with the specified load combination?
- What is the effect on safety for the specified load combinations (sensitivity of overall risk)?

- What response combination criteria is to be associated with the selected load combination criteria?

2.3.3 Load Factors

- What is the nature of the load, i.e., static, vibratory, pulsatory, load controlled, displacement controlled?

- What are the dynamic characteristics of the structure?

- What factors for combined loads will result in the code intended "safety margins"?

- What effect do the factors have on the probability of failure? (sensitivity of risk)

All these issues must be addressed to develop a rational and uniform basis for combining loads.

2.4 Response Combinations

The method of combination of multiple dynamic responses should account for 1) the probability that the peaks of each response will occur at the same time, 2) the consequence of the peak response exceeding a functional acceptance criteria, and 3) the consequence of peak responses exceeding a strength acceptance criteria.

Traditionally, responses have been combined by using the absolute sum of the peaks. This is always "conservative" and is necessary for static loads. However, when responses due to dynamic loads are combined, many designers have selected the square-root-of-the-sum-of-the-squares (SRSS) method. This is an accepted practice for combining earthquake modal or component responses that recognizes the stochastic nature of earthquakes. However, should the SSE occur, there is a significant probability that the SRSS response would be exceeded. Some probability of exceedance is thus acceptable as a response combination criteria; that is, absolute sums are not necessarily required.

Dynamic loads are energy-limited and may be less damaging than static loads of the same magnitude. They may be combined in a manner less conservative than absolute sum, depending upon their dynamic characteristics and energy absorption capacity. When computed on a linear basis, combined dynamic responses may thus exceed code stress limits without decreasing the intended margins.

In summary, criteria for combining responses to multiple dynamic loads must account for the:

1. Probability that the combined responses will exceed the response combination criteria.

2. Energy content of the dynamic loads.

3. Relative dynamic characteristics of the load and component.

4. Functional requirements.

5. Energy absorption capacity of the component.

Response and load combination issues are interrelated so that neither should be addressed independently in studies on load combination issues.

3. REQUIREMENTS FOR A LOAD COMBINATIONS DESIGN CRITERIA/REVIEW METHODOLOGY

The purpose of this section is to describe some requirements on the design criteria/review methodology developed in this task.

It does not seem practical at this time to require the designer or reviewer to use complex methods beyond what is accepted practice. Thus we have:

Requirement 1: The designer or reviewer should only have to combine peak responses from individual loads.

This requirement would exclude from consideration, for example, combining stress time histories at various time lags.

Various methods of combining the responses could be considered, including statistical ones. Again, it is not practical at this time to add any more complexity to the design process. Thus we have:

Requirement 2: The response combinations in Requirement 1 should be specified in simple deterministic terms.

A large body of thought and experience is reflected in existing design codes. This professional experience should form a basis for the design criteria in this task. Thus we have:

Requirement 3: For mechanical components the ASME code stress limit and service level philosophy should be used.

Requirements 1, 2 and 3 together mean the designer or reviewer will have to check response combination formulas such as (1)

$$Q_1 + Q_2 + \cdots \leq R_{all} \qquad (1a)$$

$$\gamma_1 Q_1 + \gamma_2 Q_2 + \cdots \leq R_{all} \qquad (1b)$$

$$\sqrt{(\gamma_1 Q_1)^2 + (\gamma_2 Q_2)^2} + \gamma_3 Q_3 + \cdots \leq R_{all} \qquad (1c)$$

$$G[\gamma_1 Q_1, \gamma_2 Q_2] + \gamma_3 Q_3 + \cdots \leq R_{all} \qquad (1d)$$

where $Q_1, Q_2, \cdots$ are peak responses to loads 1, 2,$\cdots$, and R_{all} is some allowable stress. Equation (1a) means the peak responses are combined. Equation (1b) introduces the possibility that load factors γ_i *may* be used. Equations (1c) and (1d) illustrate the possibility that the responses may be combined in ways other than *absolute sum*, *e.g.*, *using* the SRSS Rule, Equation (1c), or other *generalized function* G(·), Equation (1d).

These three requirements insure that no significant additional burden will be placed on designers.

Finally, it is important that the methodology developed be realistic. Thus we have:

<u>Requirement 4</u>: The methodology, when used as an evaluation tool, should reflect expected performance and not contain unnecessary or unacceptable conservatism. When used for developing design criteria, the methodology should accommodate the uncertainties in loads, responses and resistances in a realistic way.

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(1) In the sequel, formulas like (1b) will be used for illustrative purposes, but this should not be interpreted to imply the use of formulas like (1a), (1c), or (1d) is excluded.

In summary, the load combination methodology developed in this task is to be such that

1. It does not put additional burdens on designers or reviewers.
2. It follows the general design format of the ASME Code.
3. It realistically reflects expected performance and expected uncertainties.

4. PROPOSED CONCEPT

A concept has been developed which is being proposed as a solution to the problem as described in Section 2 and the requirements as specified in Section 3. This concept incorporates a reliability basis along with the use of load and resistance factors. Aspects of this approach are currently being used with structures and have been incorporated into at least one building code (NBCC-1975). The methodology from the standpoint of the user is outlined in this section.

In the proposed methodology, target reliabilities are provided or established for the systems within the plant based on a specified low risk target for the plant. From these system reliabilities, allocations are made to establish minimum requirements for component reliabilities. These allocations both at the system and component level will reflect design redundancies and complexities but can also reflect the relative costs to increase reliability. A method for allocating reliability to the component level will be developed, or target component reliabilities will be provided for certain generic systems. Further discussion of some aspects of the system problem are included in Section 8.

In this approach, one or more acceptable or target reliabilities will be defined for each component. These would be defined for various kinds of failures or limit states. Examples of limit states for which target reliabilities would be specified are: 1) a component can continue to function but may have been sufficiently damaged so it must be replaced; 2) a component can remain structurally intact but have a load induced failure to function; or 3) a component can experience catastrophic failure.

Once a target limit state probability (complement of target reliability) has been specified for a component, the designer or reviewer will be provided with a table relating load factor values with various probabilities of failure. Implicit in the table will be the instructions of what loads need to be combined (the selection of "which loads" will have been accomplished along with the optimization of the factors as a result of the Load Combinations Program. Of course, the selection of load combinations and optimization of load factors depends on many inputs required for the optimization process. These inlcude component fragilities and frequencies of event such as earthquakes, wind, etc.). A typical table is shown in Table 4-1. It should also be noted that along with the specification of factors will be the special functional form of the combination equation, i.e., Equation 1b, 1c or 1d in Section 3. Stating the factors in tabular form enalbes the methodology to accommodate different target component limit state probabilities for different systems of different plants by simply adjusting the load factors for concurrent loads.

The approach which will be used to develop the 'optimal' design format, load combinations and load factors is discussed in Section 6.

Table 4-1. Example of Format of Results of Load Combination Methodology For "Loss of Function" Limit State

PROBABILITY OF LOSS OF FUNCTION	LOAD FACTORS FOR DESIGN LOADS(*)				
	D	L	E	T_0	R_0
10^{-2}	1.0	0.6	0	0.4	0
10^{-3}	0.9	0.8	0.5	0.6	0.3
10^{-4}	1.0	1.2	1.4	1.0	0.8
10^{-5}	1.5	1.8	1.7	1.8	0
10^{-5}	1.6	2.0	0	2.5	0
10^{-5}	1.4	1.7	1.9	1.7	1.7

*See Section 2-2 for the definitions of the loads used in the table

5. RELATED APPLICATIONS OF PROPOSED METHODOLOGY

The reliability approach we describe in this report has been used since 1964 to develop design code rules, load factors, and load combination formulas for the design of ordinary building structures.

5.1 Historical Background of Comparable Structural Reliability Methods

It was recognized in the early 50's mainly through the pioneering efforts of Freudenthal (1974) that the uncertainties in the design process can only be consistently treated by probabilistic models. Later research in the area of structural reliability indicated that design by reliability analysis is iterative and not suited for routine design. Several decisions may have to be made by the designer for which adequate data may not exist. Such judgmental decisions regarding acceptable failure probability, consequence of failure, and relevance or absence of data are best done by the code. Hence, the need for deterministic formats for probabilistic design of structures became clear (Lind - 1968). The tools that are available for developing such deterministic codes for probabilistic design are code calibration and code optimization (Ravindra and Lind - 1973).

In recent years, design codes for probabilistic design of steel, concrete and wood structures have been developed. Further details can be found in Appendix B.

The point we want to emphasize here is that the concepts of reliability analysis, deterministic format for probabilistic design and code optimization are not new and have been successfully employed in related areas of structural engineering to develop practical design rules.

5.2 Special Considerations in Applying Reliability Methods in Nuclear Design

Design rules developed for ordinary buildings are not directly applicable to the design of nuclear power plants. Several unique aspects of nuclear power plants contribute to this observation. The objective of the following discussion is to highlight the differences between ordinary buildings and nuclear power plants that substantiate the need for the proposed study of load combinations.

1. In the probabilistic code development for ordinary buildings based on second moment methods, safety index (see Appendix B) is used as a relative measure of reliability. This measure may be too crude for nuclear design, hence, it is more appropriate to use the probability of the component reaching some limit state as the central parameter in probabilistic design methodology for nuclear power plants. Also, development of design criteria based on component limit state probability is consistent with the refinement expected in a probabilistic system safety analysis. Therefore, the second moment methods will not be used; instead fully probabilistic methods will be employed.

2. Failure of an ordinary structure does not have the same consequences as the failure of a nuclear power plant. Because of this, the reliability level desired of a nuclear power plant is much higher than the level desired of an ordinary structure. The safety index method used to derive the partial safety factors for the design of ordinary structures is too simplified and therefore will not give consistent results.

3. Code development for the design of ordinary structures has been carried out by calibrating to the reliabilities implied in current designs. This circumvents the need to specify acceptable limit state probabilities and demonstrate by calculation or test that the code provisions achieve the intended target. The philosophy behind the calibration is that the experience with ordinary structures is sufficiently broad and over a long period of time, so that the safety inherent to these structures is acceptable. On the other hand, the experience with nuclear power plants is limited to a few plants and over a short time span. In addition, the existing design rules may be too conservative. Calibration to existing design rules is thus not very meaningful for nuclear power plants. This is because our experience with the results of these design rules (nuclear power plants) is limited, so limited that the absence of failures should not be interpreted as success considering the extreme loads the design should survive, and the general absence of repeated applications of such extreme loads. This means design codes for nuclear power plants have not been validated by the traditional method used in engineering and that calibration is not generally applicable.

4. Because a large number of ordinary structures are designed using a design code, it is possible to monitor the performance of a code over a relatively short period of time. If a code leads to an unacceptably high (or low) probability of failure of structures as observed by the failure rates, partial safety factors in the code can be adjusted to yield the desired levels. Such a validation or adjustment

based on the monitoring of the code performance is not feasible for nuclear design codes. This calls for a detailed study of the load combinations founded on component and system reliability analyses.

5. The design loads acting on ordinary structures have moderately high probabilities of occurrence (e.g., 100 year wind speed and 500 year earthquake). In the context of nuclear plant design, these can be categorized into normal or operating loads. In addition, nuclear power plants are designed to withstand extreme and transient loads with much lower probabilities of occurrence than ordinary structures. Partial safety factors on these extreme loads need to be derived.

6. Nuclear power plants are to be designed to withstand a large number of loads from a variety of sources. Since the ordinary structures are not designed against these loads, partial safety factors on these loads are not available.

7. Major loads in nuclear power plants are dynamic; partial safety factors considering component response to such dynamic loads need to be developed.

8. Nuclear components/systems are expected to perform under extreme environmental (pressure, temperature, radiation and humidity) conditions. The partial safety factors should reflect this requirement.

9. Probabilistic design criteria for ordinary structural

elements exist; corresponding criteria for piping and equipment have not been developed.

10. Nuclear plant design and construction is done to exacting QA/QC requirements far exceeding ordinary design and construction. Any development of design criteria should take this into account.

These unique aspects of nuclear design and the extreme loads considered in this design clearly call for an extensive investigation of the load combination problem specific to nuclear design.

6. DEVELOPMENT OF THE DESIGN CRITERIA BASED ON COMPONENT RELIABILITY

This section provides a general description of a method to develop design criteria for nuclear components subject to multiple loads based on the component reliability approach. The criteria will be expressed as a series of equations in a load and resistance factor design format as given by the general formula:

$$\phi R \geq \sum_{j=1}^{J} \gamma_j Q_j \qquad (2)$$

where R - nominal resistance of the component

ϕ - resistance (performance) factor

Q_j - load effect (peak response) for the jth nominal load

γ_j - load factor for the jth load

The resistance and load factors reflect the uncertainties in the resistance, loads and responses used in the design. This format allows a significant flexibility compared to some others, for example, stress limit formats where all $\gamma_j = 1$.

As discussed in Appendix B, the resistance factor, ϕ , is used to reflect uncertainty in the strength, and is always less than unity. This will depend on the material (steel, concrete, etc.) and limit state under consideration (functionality, collapse, etc.). The Q_j reflects the response due to the jth nominal load under consideration at the point where the formula is being checked by the designer.

For example, Q_j may represent the peak response due to the OBE, and γ_j is the corresponding load factor which reflects the uncertainties associated with responses due to the OBE. Other formulas and γ's may be used for other loads, e.g., the SSE. For a given component, a number of formulas like (2) may have to be checked by the designer.

A number of different formats are possible. For example, it may be possible to adopt a format based on considering only the responses due to two loads at a time, that is, J=2 in (2). Alternative formats of this type will be evaluated both with respect to any technical limitations or advantages and the implications in design. For various reasons, for example, ease of checking or quality assurance, it may be desirable to have a number of equations, but combining the responses from only two loads in each. Other possibilities arise also. For example, there may be good reasons to have each load described by two or more nominal sizes as has been done for the earthquakes.

The method has as its goal the determination of an "optimal" set of design rules for a given target limit state probability, P_{f0}. These rules include the determination of which responses need to be combined (which γ_j in (2) are not zero) and the values of the load factors. The derived resistance and load factors will depend on the target limit state probability, the characteristics of the loads and the responses to be combined, as well as the component.

The proposed methodology involves two steps--a "design" step followed by an evaluation step. For a given design format (i.e., load combination, method of analysis, and calculation procedure for component resistance) these two steps are iterated to derive the "optimal" design rules (i.e., resistance and load factors). Optimization is determined with respect to an appropriate measure of closeness of the evaluated component limit state probability, P_f, based on a given set of design rules, to the target component limit state probability, P_{f0}. The measure of closeness is evaluated over the space (denoted the data space) of all possible design situations (e.g. reactor type, component type, geographic location and magnitudes of different load effects).

The data space is considered to assure that the resistance and load factors derived using this methodology is applicable for a large spectrum of design situations. A schematic description of the methodology is given in Figure 6.1. Notationally,

$\underset{\sim}{\phi}$ = resistance factors

$\underset{\sim}{\gamma}$ = load factors

$\underset{\sim}{\omega}$ = data point in the data space, $\mathcal{D}$

$f(\underset{\sim}{\omega})$ = frequency of occurrence of $\underset{\sim}{\omega} \varepsilon \mathcal{D}$

$W(.,.)$ = measure of closeness, e.g. $W(a,b) = (a-b)^2$

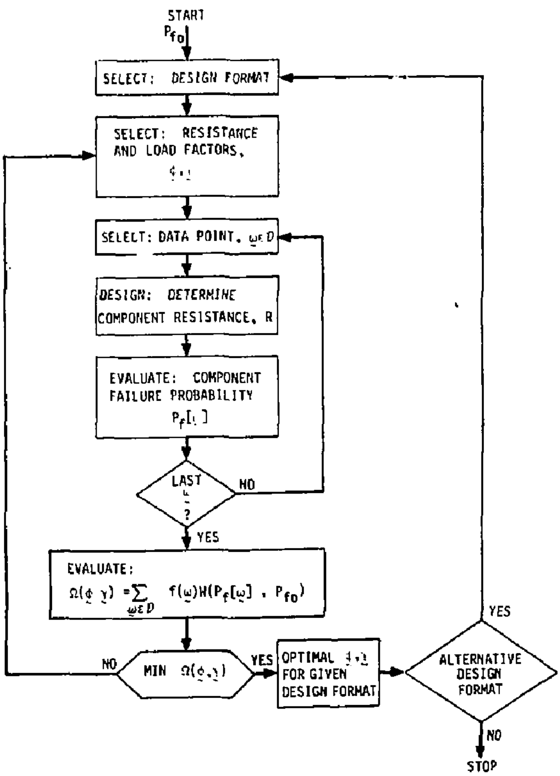

Figure 6-1. Block Diagram of Proposed Method

Given a design format, and an initial set of resistance and load factors, the "design" step involves the determination of the component resistance parameter, R, at each data point $\underset{\sim}{\omega}$ and corresponding set of deterministic influence coefficients, $C(\underset{\sim}{\omega})$, based on the set of design equations

$$\phi R_{\ell} \geq \sum_{j=1}^{J} C_{\ell j}(\underset{\sim}{\omega}) \gamma_j L_j \qquad \ell = 1, 2, \ldots L \qquad (3)$$

where L_j - nominal load intensities due to the jth load

$C_{\ell j}(\omega)$ - influence coefficients which transform the load intensities into load effects

L - number of load combination equations appropriate for the component design

$\gamma_{\ell j}$ - load factor for the jth response in the ℓth load combination equation

The design resistance, R, is

$$R = \max_{\ell} R_{\ell}$$

which is a function of the design format, the resistance and load effects, the data point, $\underset{\sim}{\omega}$, and the corresponding set of influence coefficients, $\underline{C}(\underset{\sim}{\omega})$. It should be emphasized that "design" used in the present context is a much simpler process of proportioning and does not cover planning and detailing.

Having computed the design resistance, the component reliability is assessed in the evaluation step. In this step the component limit state probability, P_f, is calculated based on an "ideal" model given the component resistance R from the design step. The method of evaluation, based on the probabilistic data of the loads, the combined responses and the component resistance is described in Appendix A. The output of the evaluation step is the component limit state probability, $P_f(\underset{\sim}{\omega})$.

These two steps are iterated over the appropriate data space, D, and the corresponding set of influence coefficients $\underline{C}(\omega)$, thus resulting in a collection of values of $P_f(\underset{\sim}{\omega})$. These values, in turn, are compared with the target limit state probability, P_{fo}, and the differences are summed over the data space, D. Thus, for a given design format and set of resistance and load factors, $(\underset{\sim}{\phi}, \underset{\sim}{\gamma})$, the objective function $\Omega(\underset{\sim}{\phi},\underset{\sim}{\gamma})$ given by

$$\Omega(\underset{\sim}{\phi}, \underset{\sim}{\gamma}) = \sum_{\omega \varepsilon D} f(\underset{\sim}{\omega})\ W\ [P_f(\underset{\sim}{\omega}),\ P_{fo}] \tag{4}$$

where $W(\cdot)$ is an appropriate distance measure, e.g. $W(a,b)=(a-b)^2$ and $f(\omega)$ is a weight or frequency measure associated with the point $\underset{\sim}{\omega} \varepsilon D$

This general two-step procedure is iterated for different resistance and load factors to determine the "optimal" set of design rules. The term optimal refers to the set of design rules which minimize $\Omega(\underset{\sim}{\phi},\underset{\sim}{\gamma})$ for the given design format. Finally, the proposed method can also be used to compare alternative design formats. For example, alternative formats could be SRSS vs. Absolute Sum, LOCA + SSE combination vs no such combination. One basis of such comparisons could be the initial cost.

The "two-step" code optimization process can be used in a variety of ways. First, one process would leave out the design iteration in the two-step process. For example, the design of a particular nuclear power plant could be accepted as given. The evaluation step could be applied using the "ideal" model. This evaluation process could be used for at least two purposes:

1. Estimate the limit state probability of the various components in the plant. Thus, if this probability varies significantly, some affected components with high probability may be reinforced to improve overall plant safety. This provides a convenient safety checking tool for the regulatory agency.

2. Whenever new loads, load combinations and procedures for combining responses are postulated, their effect on the component limit state probabilities can be estimated. If the effect is unacceptable, design criteria can be modified using the two step process.

Second, the two-step process can be used for a given set of components in various ways:

1. A matrix of component limit state probabilities and optimized code rules (coincident loads, load factors, etc.) could be developed. Such a matrix would no doubt show, as the target limit state probability decreased, the requirement to consider more and more various loads in various combinations in even more complex code rules.
2. A matrix of component limit state probabilities and optimized code rules for the same component in different locations in the plant could be developed.

In summary, the primary advantage of this methodology is that it does not require any new load combinations nor any documented justification for the load combinations. Since the load and resistance factors are adjusted to achieve the target component limit state probability, the questions of the fact that some combinations govern the design, and of the need for some combinations are moot.

7. SYSTEM RELIABILITY MODEL

7.1 Introduction

The safety analysis of a nuclear power plant uses the reliability of the components, subsystems, and systems in the plant to establish the reliability of the plant. In determining expected consequences from failures in the plant, the complements of these reliabilities (the limit state probabilities) are used in fault and event trees and are combined with release categories and release consequences to produce the plant risk. A typical presentation of this risk is in the form of a "Farmer curve", (Farmer - 1967) which plots probability as a function of the number of casulaties.*

This analysis moves from component reliability to plant risk. However, in order to establish load combination requirements in the most general sense, we must establish a required or target component reliability. Thus, the process must be reversed and we must proceed from a Farmer curve or some other statement of plant reliability backward to establish the required component reliability. Moreover, it must be implied that the selected curve represents an acceptable level of risk before component reliabilities can be inferred to be acceptable.

Even if no precise acceptable risk can be specified, a load combination methodology such as we outline here will still lead to the development of more rational and systematic design (load combination) requirements*. It will also provide guidance towards a more effective and balanced

* Interestingly, just such a process is presently being used by the Central Electricity Generating Board to determine load combination requirements for new plants in the United Kingdom.

allocation of resources (steel, concrete, etc.) to the protection of the health and safety of the public, by providing a means to focus on the more important safety issues.

7.2 Establishment of a Risk Reference at the System Level

In the past, the term "acceptable risk" has created controversy because it is largely subjective. Acceptable risk can be presented on an absolute basis such as in a Farmer curve whose level of probability versus consequence is deemed acceptable by some responsible organization or government body. Thus, a system whose safety evaluation produces a curve which never exceeds the acceptable risk curve would be found to have an acceptable risk.

In the absence of a sanctioned risk, system analysts frequently use relative risk to show that the system under evaluation has no greater risk than other systems whose risk is found acceptable either officially or in a de facto sense.

There can also be consideration of incremental risk where one examines the increase in risk due to the addition or modification of a particular system. Incremental risk could be considered in the load combinations problem by determining the additional risk to which the power plant is exposed when certain load combinations are not considered in the design criteria of the plant. This incremental risk however, falls back upon the need for some acceptable risk level. It could be that the incremental risk added to the previous risk still does not exceed a so-called acceptable risk level. Thus, no matter what one calls the risk evaluation, there is a need to seek a reference level upon which to base the resulting requirements for system and component limit state probabilities.

If obtaining consensus on a reference risk level proves difficult, additional studies of the impact of various risk levels may be necessary. For example, studies could be made on the economic impact of various acceptable risk requirements. In this instance, acceptable risk would be used as a parameter in quantitative studies (using current models) to determine cost as a function of the various acceptable risk levels considered.

Another approach to the use of acceptable risk could be based on conditional events. For example, it could be assumed that the SSE or LOCA or other significant events occur with a probability of one. The acceptable risk could be stated in terms of the conditional probability that the system will fail given the event. Conceivably, lower values (more like 1×10^{-4} or 1×10^{-5}) could be considered for individual conditional probabilities considering the fact that the probabilities of the initiating events are generally much less than one.

7.3 Establishment of Individual Safety System Reliability Requirements

As mentioned in the introduction, the process of obtaining the required limit state probability levels for the components is an inverse process from the normal system safety analysis. In this discussion, it has been divided into two steps; component and plant reliabilities. A problem that arises in this inverse analysis is that the specified values, i.e., the acceptable reference or ideal risk levels, are few, while the values to be established are many since they involve all of the limit state probabilities of the components under consideration in the plant. Thus, the problem is one of an underdetermined set of equations which does not establish unique values for the limit state probabilities at either the component or the system level.

What is suggested in this discussion is a staging of the limit state probability requirements and, perhaps, an allocation of limit state probability among systems and components based on specific criteria. These criteria could include:

1. Assigning the same limit state probability requirement (i.e., P_f level) to each safety system.

2. Allowing the owner/engineer to allocate the P_f's among each of the safety systems, giving consideration to cost trade-offs. The allocation would be acceptable as long as, in combination, the safety systems provided the required plant reliability.

3. Establishing classes or categories for the various safety systems and allocating limit state probabilities according to the classes. These classes could be arranged according to the severity of the consequences associated with the failures of those particular systems.

The discussion above reinforces the fact that system safety analyses will be needed in the design or modification phase of the nuclear power plant. It also points out the dependancy of the specification of the system's reliabilities on the acceptable risk level and the need for a general study to evaluate alternative methods for selecting acceptable failure probabilities for safety systems.

7.4 Determination of Target Component Limit State Probability

When a component is one of many within a safety system and the safety system aspires to a single limit state probability, we are once again faced with the problem of underdetermined equations. Therefore, the options available in the development of the load combination methodology are:

1. Requiring the same limit state probability level in all components in the safety system;

2. Establishment of classes of components such that each component in a class has the same limit state probability;

3. Allocation of P_f among the components such that the safety system limit state probability requirement is still satisfied.

If components are treated by classification, for instance, all piping involved in particular systems in the power plant could have the same P_f requirement; this might simplify the P_f selection by the designer. Trade-offs between components would allow the owner/engineer to minimize cost while still achieving the safety goal for the system of which the components are a part. Choosing the same limit state probability for all would probably not be wise because of the relative expense in hardening certain components.

Again, it will be necessary to perform studies evaluating the various alternatives for selecting target P_f for the various components. It should be noted that any time components are grouped by class or system and every component in the class is given the same target P_f, this P_f will have to be lower than that which would be allowed in an optimum allocation. This is because using the classification approach allows flexibility in the use of the components and in the definition of the system. Thus, if a P_f is established for these components by class, no combination of the components should produce a system probability of failure exceeding the specified level of acceptability.

8. VALIDATION

The methods described in this report are largely analytic and consequently the design rule could initially be based totally on models with limited data. The problem then is to establish the validity of the models and the resulting design rules to provide justification to the profession for their adoption.

It appears that conclusive testing to validate the methodology will be very expensive and time consuming, and at this stage it is not possible to structure a definitive test program. Thus, in the meantime, validation will be dependent upon alternatives such as:

1. Extensive peer review of methods and models.

2. Comparison of resulting designs with those from current codes to indicate whether current deficiencies have been removed and whether the new designs show more balance.

3. Calibration wit[illegible]esent standards where possible.

This subject of validation will be developed in more detail in subsequent reports.

APPENDIX A

COMPONENT LIMIT STATE PROBABILITY EVALUATION

As part of the methodology for the derivation of optimal design rules based on a target limit state probability, P_{fo}, the component limit state probability, P_f, must be evaluated. The procedure for evaluating P_f must take into consideration the load combinations considered in the design as well as any loads initiated as a consequence of initial loads. The proposed method of analysis is based on using the expected upcrossing rate $\nu^+(r)$, as a function of stress, r, and the arbitrary point-in-time distribution of stress, as described by the probability density function, $f_L(r)$, for assessing the combined effects of the loading processes.

The proposed approach for calculating P_f is outlined below. Details of the methods used in the analysis follow.

To compute the component limit state probability, the occurrence of different initial load combinations as well as any subsequent additional loads must be taken into consideration. Let q, q=1, ...,Q, index the different load cases, i.e., the occurrence of one or more initial loads e.g. wind plus earthquake, and any subsequent or initiated load, e.g., pipe break. Also, let P(q) denote the probability of occurrence of the qth case. Assuming the different load cases are mutually exclusive, P_f can be written as the sum

$$P_f = \sum_q P(q)\ P(f|q) \qquad \text{(A1)}$$

where P(f|q) denotes the component limit state probability given load case q.

The component limit state is modeled as the limit state being attained at any one of the k "critical" points (or cross sections) in the component. Then,

$$P(f|q) = P[F(q,1) \cup F(q,2) \cup \ldots \cup F(q,K)] \qquad \text{(A2)}$$

where F(q,k) denotes the event that the limit state occurs at point k under load case q. An appropriate estimate of P(f|q), considering the stochastic dependence between the different points in the component, is

$$P(f|q) = \max_{k} P[F(q,k)] \qquad (A3)$$

To determine the limit state probability at point (cross section) k given load case q, let $Z_{k,q}$ denote the stress induced at point k under load case q. Then,

$$P[F(q,k)] = \int_{r=0}^{\infty} P\ [Z_{k,q}>r]\ f_{R_k}(r)dr \qquad (A4)$$

where $f_{R_k}(\cdot)$ is the probability density function of the resistance, R_k, at the point (cross-section) k and $P\ [Z_{k,q}>r]$ is the probability that the stress induced under load case q exceeds r.

The distribution of the resistance, $f_{R_k}(\cdot)$, is a function of the component design and will depend on the limit state. It is an input into the evaluation procedure.[1] The probabilities, $P[Z_{k,q}>r]$, are a function of $\nu^+(r)$ and $f_L(r)$. To evaluate these probabilities, potential loads on a component are partitioned into two classes:

a. Initial loads

   Loads which have a potential of initiating additional loads on the component due to failures of other parts of a system. This class of loads includes loads due to earthquake, wind, hurricane, normal operation and operating incidents, etc. Some examples of the time histories of responses to initial loads are given in Figure A-1.

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1 The probability, P(q), is determined from the rates of occurrence of the loads associated with the qth sequence of events which are inputs.

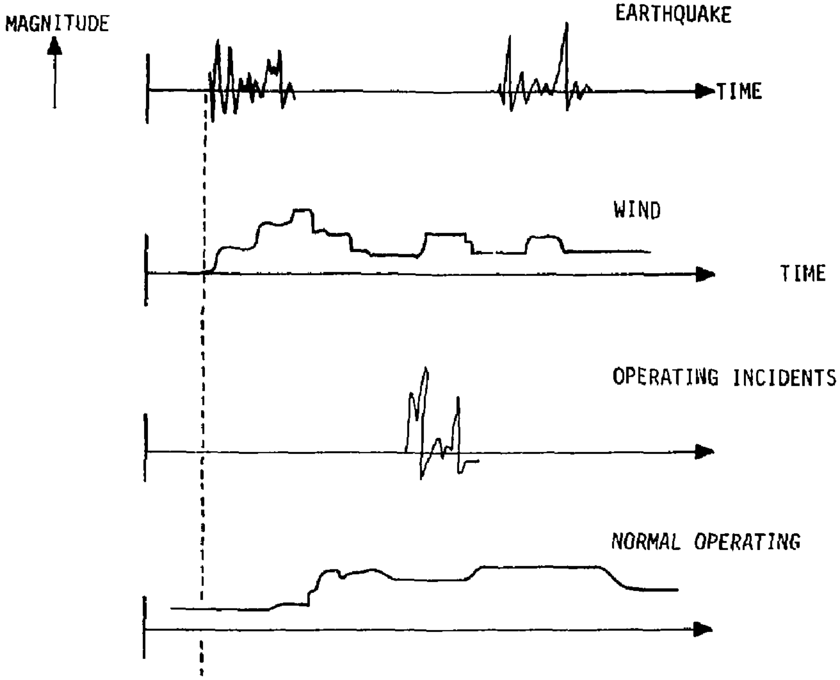


Figure A-1. Examples of Time Histories of Responses to Initial Loads.

b. Initiated Loads

Loads on a component due to the response and/or failure of another part of a system as a consequence of some initial load. Loads in this class arise from pipe breaks, valves failing to close, turbine trip, etc. An example of the relationship between the time history of the response to an initial load, e.g., due to an earthquake, and the response to an initiated load, e.g., a pipe break, is given in Figure A-2.

Responses to initial loads are assumed to be either continuous (e.g., normal operation) or intermittently continuous (e.g., the earthquake, windstorm and operating incident) processes. A stationary continuous process can be described by the expected upcrossing rate $\nu^+(r)$ and the arbitrary point in time probability density function, $f_L(r)$, both a function of stress, r. The expected upcrossing rate, $\nu^+(r)$, is the expected number of crossings, per unit time, of the response process from a stress level less than r to a stress level greater than r. For a response process, L(t), let

$$G_L(r,T) = P[\max_{0 \leq t \leq T} L(t) > r] \tag{A5}$$

Then,

$$G_L(r,T) = P[L(0) > r] + P[L(0) < r] \sum_{j=1}^{\infty} P\,[\text{Exactly } j \text{ upcrossings}) \text{ of } L(t) \text{ at level } r \text{ occur in } (0,T) \mid L(0) < r]$$

But, assuming a stationary process,

$$\begin{aligned} \nu_T^+(r)T &= E[\text{Number of upcrossings of } L(t) \text{ at level } r \text{ in } (0,T)] \\ &= \sum_{j=1}^{\infty} j\, P[\text{ Exactly } j \text{ upcrossings at level } r \text{ occur in } (0,T)] \end{aligned} \tag{A6}$$

Then,

$$\nu_L^+ (r)T \geq G_L(r,T) \tag{A7}$$

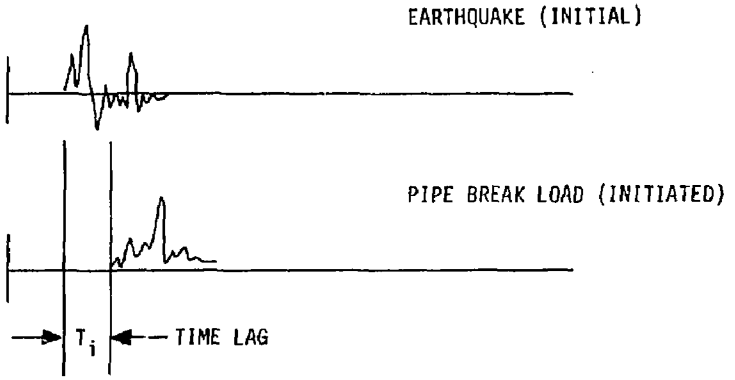


Figure A-2. Relationship Between the Response to an Initial Load and the Response to an Initiated Load.

Thus, $\nu_L^+(r)T$ provides an upper bound and also a close approximation to the probability of the $\max_{0\le t\le T} L(t)$ exceeding r.

An intermittent continuous process, L(t), can be viewed as a product X(t)Y(t) of a continuous process X(t) and a (0,1) rectangular renewal pulse process Y(t) with expected renewal rate, λ, and expected duration μ_D (i.e., the expected length of time Y(t) = 1 given an event occurs--Y(t) changes from 0 to 1). These latter two parameters are associated with the distributions of the random variables D and I illustrated in Figure A-3.

Figure A-3. Intermittent Continuous Process

The variable D denotes the duration of the response (e.g., response to the load due to a windstorm) and I denotes the interarrival (renewal) times between events (e.g., occurrence of a windstorm). The expected value of D is μ_D and the expected interarrival time, μ_I, is $1/\lambda$.

If $\nu_X^+(r)$ is the expected upcrossing rate for the continuous process X(t), the expected upcrossing rate, $\nu_L^+(r)$, for the intermittent continuous process, L(t), is given by

$$\nu_L^+(r) = q\nu_X^+(r) \tag{A8}$$

where q is the ratio of the expected duration of a response, μ_D, to the expected interarrival time, μ_I, or $q = \lambda\mu_D$.

The arbitrary point in time probability density function describes the distribution of stress levels of a response process viewed at an arbitrary point in the process. For an intermittent continuous process, L(t),

$$f_L(r) = p\delta(r) + (1-p)\, f_X(r) \tag{A9}$$

where

$$\delta(r) = \begin{cases} 1 & \text{if } r = 0 \\ 0 & \text{otherwise,} \end{cases}$$

and p is the probability the process is at zero, i.e., an event is not occurring, and $f_X(r)$ is the arbitrary point in time probability density function for the continuous process X(t).

For purposes of the computation of the actual component limit state probability, P_f, the response to each initial load is assumed to be described by four parameters, $\lambda, \nu_L^+(r)$, $f_L(r)$, $f_D(d)$ or μ_D, where $f_D(d)$ is the density function for the random duration D.

Similarly, the responses to initiated loads are assumed to be described by three parameters, $\nu_L^+(r)$, $f_L(r)$, $f_T(t)$ or μ_T, where $f_T(t)$ is the probability density function for the delay between the response to the occurrence of an initial load and the beginning of the response to the initialized load and μ_T is the expected delay.

To illustrate the determination of $\nu^+(r)$ and $f(r)$ for a combination of loads, consider the scenario of a windstorm followed by the occurrence of an earthquake. The concurrent responses are shown in Figure A-4

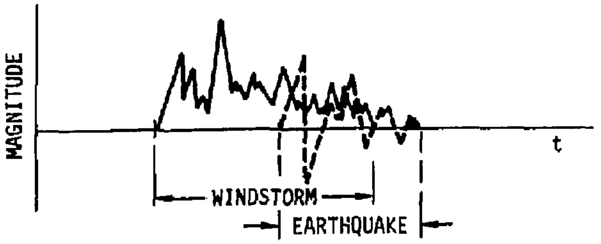


Figure A-4 Concurrent Responses Due to a Windstorm and an Earthquake

Let (ν_W^+, f_W) and (ν_E^+, f_E) be the expected upcrossing rate and arbitrary point in time probability density function for the continuous response processes associated with the windstorm and earthquake respectively. Then, conditional on concurrent responses, the expected upcrossing rate for the responses to the intermittent windstorm and earthquake processes are respectively

$$\hat{\nu}_W^+(r) = q'_W \nu_W^+(r) \tag{A10}$$

and

$$\hat{\nu}_E^+(r) = q'_E \nu_E^+(r) \tag{A11}$$

where

$$q'_W = \mu_W/(\mu_E + \mu_W)$$

$$q'_E = \mu_E/(\mu_E + \mu_W)$$

Then, the upcrossing rate for the combined response is approximated by the convolution of the upcrossing rate of each process with the arbitrary point in time distribution of the other. That is.

$$\hat{\nu}^+_{W;E}(r) = \int_X \hat{\nu}_W^+(r-x)\hat{f}_E(x)dx + \int_X \hat{\nu}_E^+(r-x)\hat{f}_W(x)\,dx \tag{A12}$$

where $\hat{f}_W$ and $\hat{f}_E$ are the appropriately transformed functions as given by Equation (A9). Similarly,

$$\hat{f}_{W+E}(r) = \int_x \hat{f}_W(r-x)\ \hat{f}_E(x)dx \qquad \text{(A13)}$$

In the case of the response to an initial load combined with the response to an initiated load, (ν^+, f), the response to the initiated load will have to be transformed to accommodate the lag time prior to taking the convolution.

The initial step in the determination of $P(q)$ and $P[Z_{k,q}>r]$ is to identify all possible load combinations which lead to potential failure at the kth point. The analyses will involve the construction of a load event tree as illustrated in Figure A-5.

The expected frequency of occurrence of case q is denoted λ_q. For example, for the combination, windstorm + earthquake + pipebreak,

$$\lambda_q = \lambda_W \cdot \mu_{DW}\lambda_E \cdot P(PB|E,W) \qquad \text{(A14)}$$

where

- λ_E, λ_W - expected frequency of an earthquake, windstorm respectively
- μ_{DW} - expected duration of response to a windstorm load
- $P(PB|E,W)$ - conditional probability of a pipe break given an earthquake plus wind.

The probability of occurrence of load case q during the life, L, of the component is:

$$P(q) = \lambda_q \cdot L \qquad \text{(A15)}$$

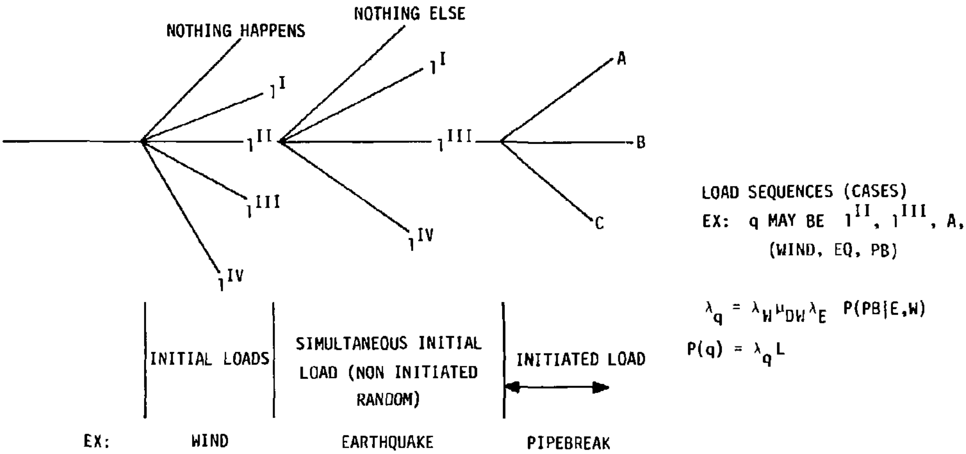


Figure A-5. Load Event Tree for Any Time Increment.

For load case q the expected upcrossing rate $\hat{\nu}_q^+(r)$ and arbitrary point in time probability density function $\hat{f}_q(r)$ are evaluated as described previously. If $Z_{k,q}$ denotes the maximum stress at point k due to load case q, i.e.

$$Z_{k,q} = \max_{0 \leq t \leq T} S_{k,q}(t) \tag{A16}$$

where $S_{k,q}(t)$ denotes the concurrent response process for case q at point k, then

$$P[Z_{k,q} \leq r] \simeq \exp\left[-\nu_q^+(r)L\right] \tag{A17}$$

Thus, based on life, L, an approximation to the probability of the peak stress at point k exceeding r is

$$P[Z_{k,q} > r] \simeq \hat{\nu}_q^+(r)L \tag{A18}$$

Equations (A4) and (A1) are used to evaluate the component limit state probability.

The attractiveness of using the expected upcrossing rate and arbitrary point in time probability density function is that both functions can be estimated from response spectra data associated with the analysis of initial and initiated loads.

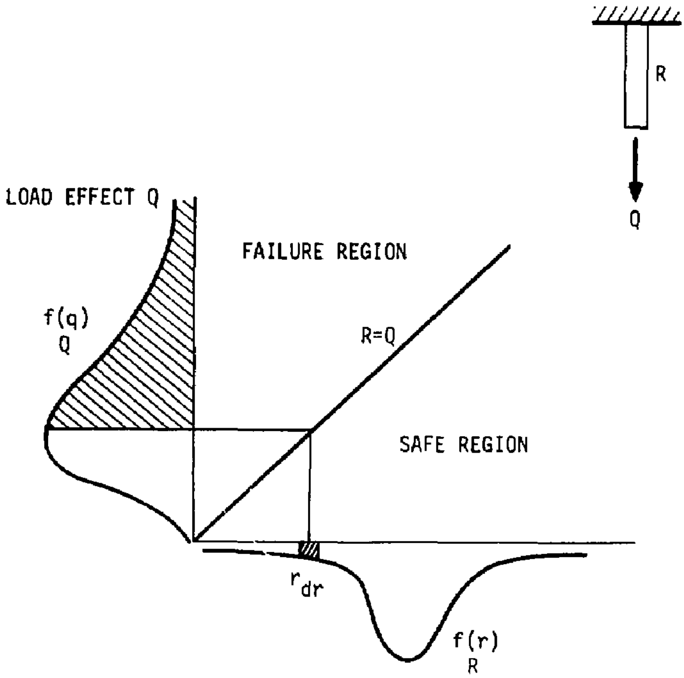
APPENDIX B

DETAILED EXPOSITION OF RELATED APPLICATIONS

This appendix describes recent research and code development activities pertaining to the probabilistic analysis and design of structures. The discussion herein highlights the following topics: need for probabilistic design, second moment design formats, calibration, Load and Resistance Factor Design (LRFD), load combination studies, levels of probabilistic design codes and code optimization.

B.1 Need for Probabilistic Design

The sophistication achieved in analyzing structures using computers has not been matched by the procedures of selecting the design loads and allowable stresses or load factors. The level of structural safety is only implied in these selection procedures. Freudenthal (1947) showed that the use of safety factors may not lead to consistent levels of safety in all design situations. Uncertainties present in the design and construction processes because of random loads, variability in material strengths and the imperfect modeling of structures for analysis and design should be considered in the selection of safety factors. Freudenthal (1947) argued that since the probability of failure is the only rational measure of structural safety, any criteria used in design should result in consistent failure probabilities for different design situations. The probability of failure, P_f, is the probability that the load effect, Q, on a structure exceeds the resistance, R. Both the load effect and the resistance (measured in the same units) are random variables; probability density functions for Q and R are denoted $f_Q(q)$ and $f_R(r)$ respectively. Referring to Fig. B-1, failure occurs when Q is larger than R; the region $R < Q$ is the failure region. Thus, the probability of failure is evaluated as:



$$P_f = P[R<Q] = \int_0^{\infty} [1-F_Q(r)]f_R(r)dr$$

Figure B-1. Fundamental Case

$$P_f = P[R < Q] = \int_0^{\infty} \int_r^{\infty} f_Q(q)dq \; f_R(r) \, dr$$

$$= \int_0^{\infty} [1 - F_Q(r)] \; f_R(r)dr \quad \text{(B1)}$$

where $F_Q(q)$ is the cumulative distribution function of Q. The above equations assume that R and Q are stochastically independent. The probability of failure is highly sensitive to the distribution assumptions. Figure B-2 shows the failure probability as a function of the central safety factor ($\theta = m_R/m_Q$) for several families of probability distributions (Rosenblueth and Esteva - 1971). The above equation for failure probability is applicable for a structural member under a single applied load. A structural system generally consists of many members and is subjected to a number of loads. Models for system reliability analyses that are traditionally used are series systems ("weakest link" model), parallel systems, combined systems and conditional systems (Benjamin, 1969). Evaluation of the probability of failure of a structural system should take into account the spatial and temporal correlations between the applied loads and between member resistances. Since the resulting multiple integrals are difficult to evaluate and the information on correlations is generally lacking, bounds on the system failure probability have been developed (Cornell, 1967; Moses and Kinser, 1967; Vanmarke, 1972).

In order to design a structure using structural reliability analysis, the acceptable probability of failure of the system should be specified. This is a controversial topic in the building design. The individual designer is not permitted to select the failure probability; the code writers currently make this choice only in an implicit manner. Even the specification of target or acceptable

Curve	Load Distribution	V_L	Strength Distribution	V_R	Symbol
1	Normal	0.10	Normal	0.10	———
2	Normal	0.30	Normal	0.10	———
3	Lognormal	0.10	Lognormal	0.10	- - - -
4	Lognormal	0.10	Lognormal	0.30	- - - -
5	Lognormal	0.30	Lognormal	0.10	- - - -
6	Lognormal	0.30	Lognormal	0.30	- - - -
7	Normal truncated	0.10	Lognormal	0.10	— · —
8	Normal truncated	0.10	Lognormal	0.30	— · —
9	Normal truncated	0.30	Lognormal	0.10	— · —
10	Normal truncated	0.30	Lognormal	0.30	— · —
11	Extreme type I	0.10	Lognormal	0.10	—— ——
12	Extreme type I	0.10	Lognormal	0.30	—— ——
13	Extreme type I	0.30	Lognormal	0.10	—— ——
14	Extreme type I	0.30	Lognormal	0.30	—— ——

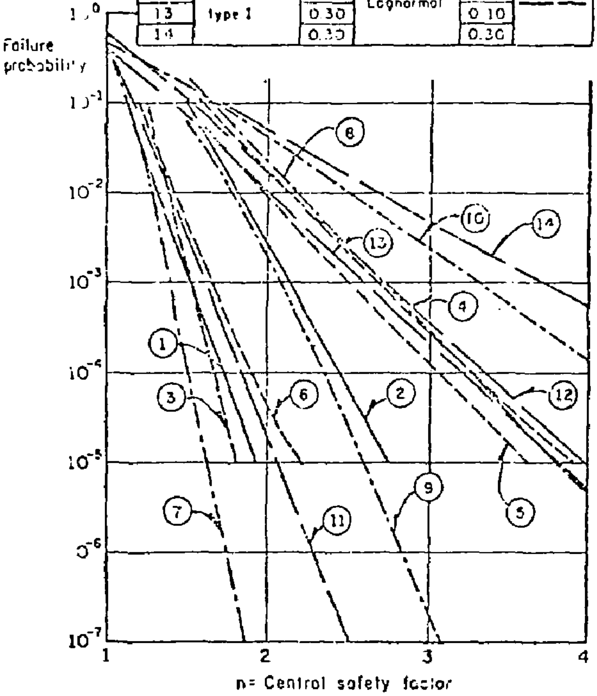


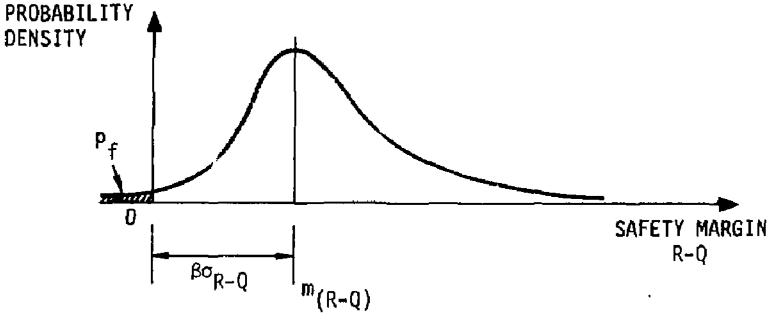
Figure B-2. Failure Probabilities for Several Families of Distributions failure probability for the structural system will not help in obtaining a unique design, because many designs/structural configurations could have the same system failure probability. Since the primary objective of structural engineering is to produce designs, as opposed to analysis of the structural reliability, a need existed for probabilistic structural design criteria that incorporates the essential features of probabilistic safety analysis and is practical for routine design office use. Therefore, research since 1966 has focused on the development of probabilistic design codes.

B.2 Second Moment Design Formats

The first practical probabilistic design code format was proposed by Cornell (1969). The essential features of this format are that the random variables of interest to structural safety are modeled by the first two statistical moments, viz, mean and variance. The mean of a random variable indicates the central tendency of the variable and the variance reflects the dispersion about the mean. In this format, structural safety is characterized by a relative measure of reliability known as "safety index", β. The safety index is defined as the ratio of the mean to the standard deviation of safety margin; safety margin is the value of the resistance of the member minus the load effect acting on it. Referring to Figure B-3, safety index is expressed as the distance between the mean of safety margin and the point of failure (i.e., when the safety margin equals zero) in terms of the standard deviation of safety margin. Thus,

$$\beta = \text{Safety Index} = \frac{m_{R-Q}}{\sigma_{R-Q}} \qquad \text{(B2)}$$

If both R and Q have Gaussian distributions the probability of failure of the member can be calculated as $P_f = \Phi(-\beta)$



$$\beta = \text{SAFETY INDEX} = \frac{m_{R-Q}}{\sigma_{R-Q}}$$

Figure B-3. Cornell Model

where Φ is the standard normal integral. If no distribution assumptions are made, we can only say that a higher value of β indicates higher reliability. Two designs are considered consistent if they have the same reliability. Such consistency can be judged approximately by evaluating the safety indices of the designs.

The safety index concept can be extended to design situations where a large number of variables such as loads, material properties, geometrical dimensions and factors representing the modeling uncertainties are to be considered. The failure criterion can be written as

$$g(X_1, X_2 \ldots, X_n) = Z = 0 \tag{B3}$$

The random variables X_i are described by their mean, m_X, and standard deviation, σ_X. A value of Z less than zero corresponds to failure. Linearization of Eq. B3 using a Taylor series expansion gives the approximation.

$$g(x_1^*, x_2^*, \ldots, x_n^*) + \sum_{i=1}^{n} \frac{\partial g}{\partial x_i}(x_i - x_i^*) = Z = 0 \tag{B4}$$

The point $(x_1^*, x_2^*, \ldots x_n^*)$ about which the expansion is carried out, is suggested by Cornell (1969) to be the mean $(m_{X_1}, m_{X_2}, \ldots m_{X_n})$. The mean and standard deviation of Z are estimated as

$$m_Z \approx g(m_{x_1}, m_{x_2}, \ldots, m_{x_n}) \tag{B5}$$

and

$$\sigma_Z \approx \left[\sum_{i=1}^{n} \left(\frac{\partial g}{\partial x_i}\right)^2 \sigma_{x_i}^2\right]^{1/2} \tag{B6}$$

The derivatives in Eq. B6 are evaluated at $(m_{X_1}, \ldots, m_{X_n})$. A design is considered to have a reliability represented by the safety index β if

$$m_Z - \beta\sigma_Z \geq 0 \tag{B7}$$

For routine design purposes, it is necessary to specify a set of partial safety factors (Ravindra, Heaney and Lind - 1969). This can be done by writing

$$\sigma_Z = \sum_{i=1}^{n} \alpha_i \frac{\partial g}{\partial x_i} \sigma_{X_i} \tag{B8}$$

where

$$\alpha_i = \frac{\partial g}{\partial x_i} \sigma_{x_i} \left[\sum_{j=1}^{n} \left(\frac{\partial g}{\partial x_j} \sigma_{x_j}\right)^2 \right]^{-1/2} \tag{B9}$$

If g(.) is linear in X_i, Eq. B7 can be rewritten as:

$$\sum_{i=1}^{n} \frac{\partial g}{\partial x_i} m_{xi} - \beta \sum_{i=1}^{n} \alpha_i \frac{\partial g}{\partial x_i} \sigma_{x_i} \geq 0 \tag{B10}$$

or,

$$\sum_{i=1}^{n} \frac{\partial g}{\partial x_i} (m_{x_i} - \beta\, \alpha_i\, \sigma x_i) \geq 0 \tag{B11}$$

The derivatives are -1 or +1 in this linear case. The partial safety factors to be applied to m_{X_i} are $\gamma_i = 1 - \beta\alpha_i V_{X_i}$ where V_{X_i} is the coefficient of variation of the random variable X_i.

For the design problem with two variables, e.g., load effect Q and resistance R, the design criterion becomes

$$m_R(1 - \alpha_R\, \beta V_R) \geq m_Q(1 + \alpha_Q\, \beta V_R) \tag{B12}$$

The second moment design format achieves most of the stated objectives of probabilistic design: simplicity, consistency and rational (but approximate) description of the uncertainties.

However, the probability information contained in the safety index β is poor. The linear approximation of the g-function at the mean of the design variables may be too inaccurate since most design situations exhibit severe nonlinearities. Also, this design format (Eqs. B5 and B6) fails to be invariant with the mechanical formulation of the problem; the safety index β depends on this formulation and on the number of variables used in the analysis. Recently proposed second moment formats (Ditlevsen, 1973; Hasofer and Lind, 1974; Veneziano, 1974; Paloheimo, 1973; Lind, 1974; Rackwitz, 1976) overcome these problems. The procedure developed by Rackwitz (1976) follows:

Let $g(X_1, X_2, ----, X_n)$ be the mechanical formulation function of the reliability problem under study, where $X_1, X_2, ---, X_n$ are the basic variables. Failure occurs if and only if $g < 0$. The safety of the member can be assessed, called "safety checking", by measuring the random distance from the mean to any point in the sample space of the structural variables on the surface representing the failure criterion. This point is denoted $\{x_1^*, x_2^*, \ldots, x_n^*\}$. The safety index is defined as $\beta = m_{g_0}/\sigma_{g_0}$ where g_0 is the linear approximation to $g(.) = 0$

$$g_0 = g(x_1^*, x_2^*, \ldots, x_n^*) + \sum_{i=1}^{n} \frac{\partial g}{\partial x_i} (x_i - x_i^*) \doteq 0 \qquad \text{(B13)}$$

in which all the derivatives are evaluated at the point $(x_1^*, --- x_n^*)$.

Then,

$$m_{g_0} = g(x_1^*, x_2^*, \ldots, x_n^*) + \sum_{i=1}^{n} \frac{\partial g}{\partial x_i} (m_{x_i} - x_i^*) = 0 \qquad \text{(B14)}$$

$$\sigma_{g_0} = [\sum_{i=1}^{n} (\frac{\partial g}{\partial x_i} \sigma_{x_i})^2]^{1/2} = \sum \alpha_i \frac{\partial g}{\partial x_i} \sigma_{x_i} \qquad \text{(B15)}$$

$$\alpha_i = \frac{\partial g}{\partial x_i} \sigma_{x_i} \left\{ \sum (\frac{\partial g}{\partial x_j} \sigma_{x_j})^2 \right\}^{1/2} \qquad \text{(B16)}$$

By the definition of the safety index,

$$m_{g_0} - \beta\sigma_{g_0} = 0 \tag{B17}$$

Substituting Eqs. (B15), (B16) in Eq. (B17), we obtain

$$g(x_1^*, x_2^*, \ldots, x_n^*) + \sum_{i=1}^{n} \frac{g}{x_i}(m_{x_i} - x_i - \alpha_i \beta\sigma_{x_i}) = 0 \tag{B18}$$

The first term is zero; the second term becomes zero if

$$x_i^* = m_{x_i} - \alpha_i \beta\sigma_i \tag{B19}$$

An iterative procedure is used to obtain the design point $\{x_1^*, x_2^*, \ldots, x_n^*\}$.

$$g(x_1^*, x_2^*, \ldots, x_n^*) = g(\ldots, m_{x_i} - \alpha_i\beta\,\sigma_{x_i}, \ldots) = 0 \tag{B20}$$

is solved together with the system of equations

$$\alpha_i = \frac{\partial g}{\partial x_i}\Big]_{\underline{X}^*} \sigma_{X_i} \left[\sum\left(\frac{\partial g}{\partial x_i}\Big]_{\underline{X}^*} \sigma_{X_i}\right)^2\right]^{-1/2} \tag{B21}$$

The components of $\underline{X}^*$ are given by

$$x_i^* = m_{x_i} - \alpha_i\beta\,\sigma_{x_i} = m_{x_i}(1 - \alpha_i\,\beta V_{x_i}) \tag{B22}$$

The following examples illustrate the use of the above procedure in evaluating the safety index implied in current design and in obtaining the partial safety factors for a specified safety index.

EXAMPLE 1

Evaluate the safety index implied in a steel beam designed according to AISC Specifications. Tributary area = A_T = 400 ft^2, code dead load (D_c) = 50 psf and code live load (L_c) = 50 psf.

$$L_{rc} = 50(1-\rho) \qquad \rho = 0.0008 \times 400 = 0.32$$

$$= 0.23\left(1 + \frac{D_c}{L_c}\right) = 0.23\left(1 + \frac{50}{50}\right) = 0.46$$

$$L_{rc} = 50(1-0.32) = 50(0.68) = 34 \text{ psf}$$

$$Z^* = \frac{1.7\ell c_D D_c + \ell c_L L_{rc}}{F_y^*} = \frac{1.7\ \ell c(D_c + L_{rc})}{F_y^*} = \frac{1.7\ell c\ (50+34)}{36}$$

$$= 3.97\ell c$$

Variable	Mean	COV
F_y	$1.10F_y^*$	0.10
Z	Z*	0.05
P	1.02	0.06
E	1.00	0.05
A	1.00	0.04
D	D_c	0.04
B	1.00	0.20
L	$14.9 + \frac{763}{\sqrt{A_I}}$	$\dfrac{\sqrt{11.3 + \frac{15,000}{A_I}}}{14.9 + \frac{763}{A_I}}$

For A_I = 800 ft^2 $\quad m_L$ = 41.88 psf $\quad V_L$ = 0.13

Basic Variables F_y, D, and L

$$g = F_y ZP - c(D + L)$$

$$F_y\ (1.0)(1.02)\ \frac{Z^*}{c} - D - L = 0$$

$$\frac{\partial g}{\partial F_y} = 1.02\ \frac{Z^*}{c} = 1.02 \times 3.97 = 4.05$$

$$\frac{\partial g}{\partial D} = -1 \qquad \frac{\partial g}{\partial L} = -1$$

$$\sigma_{F_y} = 36 \times 1.10 \sqrt{0.10^2 + 0.05^2 + 0.06^2} = 5.02 \text{ Ksi}$$

$$\alpha_{F_y} = \frac{\frac{\partial g}{\partial F_y}\ \sigma_{F_y}}{\sqrt{\left(\frac{\partial g}{\partial F_y}\ \sigma_{F_y}\right)^2 + \left(\frac{\partial g}{\partial D}\ \sigma_D\right)^2 + \left(\frac{\partial g}{\partial L}\ \sigma_L\right)^2}}$$

$$= \left\{(4.05)(5.02)\right\} \div \left\{(4.05 \times 5.02)^2 + 50^2(.05^2 + .04^2 + .04^2) + 41.88^2\ (.05^2 + .20^2 + .13^2)\right\}^{1/2}$$

$$= \frac{20.33}{23.06} = 0.887$$

$$\alpha_D = \frac{(-1)\ 50 \sqrt{0.05^2 + 0.04^2 + 0.04^2}}{23.06} = -0.1637$$

$$\alpha_L = \frac{(-1)\ 41.88 \sqrt{0.05^2 + 0.20^2 + 0.13^2}}{23.06} = -0.434$$

$$[1.10\, F_y^* - 0.887\, \beta(5.02)]\, 1.02\, \frac{Z^*}{c} - (50 + 0.1637\beta \times 0.0755 \times 50)$$

$$- (41.88 + 0.434\beta \times 0.244 \times 41.88) = 0$$

$Z^*/c = 3.97$

Solving for β, $\underline{\underline{\beta = 2.91}}$

Example 2

Design a steel beam carrying a mean dead load of 50 psf and having an influence area of 800 ft^2 for a safety index $\beta = 3.5$.

$$g = F_y ZP - c(D + L)$$

$$F_y(1.00)(1.02)\frac{Z^*}{c} - (D + L) = 0$$

$$\frac{\partial g}{\partial F_y} = 1.02\frac{Z^*}{c}\ ; \quad \frac{\partial g}{\partial D} = -1\ ; \quad \frac{\partial g}{\partial L} = -1.$$

	F_y	D	L	Z^*/c
m_i	39.6	50	41.88	
σ_i	5.02	3.77	10.20	
α^1	0	0	0	2.27
α^2	0.7302	-0.2369	-0.6408	4.40
α^3	0.895	-0.153	-0.414	4.47
α^4	0.903	-0.149	-0.403	4.47

Trial 1

$$\frac{\partial g}{\partial F_y} = 1.02 \times 2.27 = 2.3154; \quad \frac{\partial g}{\partial D} = -1 \; ; \quad \frac{\partial g}{\partial L} = -1$$

$$\alpha_{F_y} = \frac{2.3154 \times 5.02}{\sqrt{[(2.3154)(5.02)]^2 + (3.77 \times 1)^2 + (10.2 \times 1)^2}} = \frac{2.3154 \times 5.02}{15.9171}$$

$$= 0.7302$$

$$\alpha_D = -\frac{3.77}{15.9171} = -0.2369$$

$$\alpha_L = -\frac{10.20}{15.9171} = -0.6408$$

$$F_y{}^* = 39.6 - 0.7302 \times 3.5 \times 5.02 = 26.77$$

$$D^* = 50 + 0.2379 \times 3.5 \times 3.77 = 53.13$$

$$L^* = 41.88 + 0.6408 \times 3.5 \times 10.20 = 64.76$$

$$26.77 \times 1.02 \times Z^*/c - 53.13 - 64.76 = 0$$

$$Z^*/c = 4.318$$

Trial 2

$$\frac{\partial g}{\partial F_y} = 1.02 \times 4.318 = 4.40$$

$$\sqrt{(4.40 \times 5.02)^2 = 3.77^2 + 10.2^2} = 24.64$$

$$\alpha_{F_y} = 0.896 \; ; \quad \alpha_D = -0.153 \; ; \quad \alpha_L = -0.414$$

$F_y^* = 39.6 - 0.896 \times 3.5 \times 5.02 = 23.86$

$D^* = 50 + 0.153 \times 3.5 \times 3.77 = 52.02$

$L^* = 41.88 + 0.414 \times 3.5 \times 10.20 = 56.66$

$(23.86 \times 1.02 \times Z^*/c) - 52.02 - 56.66 = 0$

$Z^*/c = 4.47$

<u>Trial 3</u>

$\frac{\partial g}{\partial F_y} = 1.02 \times 4.47 = 4.56$

$\sqrt{(4.56 \times 5.02)^2 + 3.77^2 + 10.2^2} = 25.34$

$\alpha_{Fy} = 0.903$ $\quad F_y^* = 39.6 - 0.903 \times 3.5 \times 5.02 = 23.73$

$\alpha_D = -0.149$ $\quad D^* = 50 + 0.149 \times 3.5 \times 3.77 = 51.97$

$\alpha_L = -0.403$ $\quad L^* = 41.88 + 0.403 \times 3.5 \times 10.20 = 56.3$

$23.73 \times 1.02\ Z^*/c - 51.97 - 56.30 = 0$

$Z^*/c = 4.47$

$\phi = \frac{23.73}{39.60} = 0.60$

$\nu_D = 1.04$

$\nu_L = 1.34$

B.3 Calibration

The safety index β is the central parameter in the second moment probabilistic formats. In order to develop a consistent set of design criteria (i.e., load and resistance factors), the value of β must be specified. It can be a value agreed upon by the profession to give the desired level of reliability, or it can be obtained by selecting the value of β such that the same degree of reliability is attained in the new criterion as in the existing design method for a number of standard design situations, e.g., simple beams, centrally loaded columns, tension members, high strength bolts and fillet welds. The latter procedure is called "calibration." It has the advantage of utilizing past experience and is based on the precept that the reliabilities inherent in current criteria for "standard" design situations are acceptable.

The concept of code calibration has been used in the past in developing the load factor design criteria for bridges by AASHO. A 40 ft. span bridge was taken as the "standard" design situation; the load factors were derived such that the new designs match the designs by the allowable stress method for this "standard" span. Code calibration as a mathematical problem of adjusting the parameters of a code to achieve a stated objective was first formally proposed by Lind (1968). This concept has been applied extensively in most probabilistic code development work.

B.4 Load and Resistance Factor Design (LRFD)

In this section we will describe the development of load and resistance factor design criteria for steel (hot and cold formed) using a second moment probabilistic format. The objective is to illustrate the code calibration procedure and to demonstrate that practical design rules can be derived using probabilistic formats. The major phases of the design criteria development are also discussed.

The load and resistance factor design criterion is expressed by the following general formula (Galambos, Ravindra, et al, 1978):

$$\phi R_n \geq \sum_{k=1}^{j} \gamma_k \, m_{Q_k} \tag{B23}$$

The left side of the formula relates to the resistance (capacity) of the structure while the right side characterizes the loading acting on it.

The resistance side of the design criterion consists of the product ϕR_n, in which R_n is the "nominal resistance: and ϕ is the "resistance factor". The nominal resistance is the resistance computed according to a formula in a structural code and it is based on nominal material and cross-sectional properties. The resistance factor ϕ, which is always less than unity, together with R_n reflects the uncertainties associated with R. The factor ϕ is dimensionless and R_n is a generalized force: bending moment, axial force, or shear force associated with a limit state of strength or serviceability.

The loading side of the design criterion (Eq. B23) is the sum of products γm_Q, in which m_Q is the "mean load effect", and γ is the corresponding "load factor". Here γ is dimensionless and m_Q is a generalized force (i.e., bending moment, axial force or shear force) computed for mean loads for which the structure is to be designed. The γ-factors reflect potential overloads and the uncertainties inherent in the calculation of the load effects. The summation sign in Eq. B23 denotes the combination of load effects from different load sources. For example, if only dead load and live load effects are considered

$$\sum_{k=1}^{j} \gamma_k m_{Q_k} = \gamma_D m_{Q_D} + \gamma_L m_{Q_L} \tag{B24}$$

in which m_{Q_D} and m_{Q_L} are the mean dead and live load effects, respectively; γ_D and γ_L are the corresponding load factors.

In the LRFD, one expression of the type given in Eq. B24 is derived for each set of load combinations that need to be considered. The nominal resistance always relates to a specific "limit state". Two classes of limit states are pertinent to structural design: the "maximum strength: (or "ultimate") limit state and the "serviceability" limit state. Violation of a strength limit state implies "failure" in the sense that a clearly defined limit of structural usefulness has been exceeded, but this does not necessarily involve actual collapse. In the case of a structural system with "compact" beams this means that a plastic mechanism has formed. Other strength limit states for steel structures are: frame instability, lateral torsional or local instability, incremental collapse, etc. Serviceability limit states include excessive deflection, excessive vibration, and premature yield or slip.

The objective of one LRFD project was to develop the design criteria shown in Eq. B25 for different structural steel elements under a number of load combinations in a consistent way, taking into account the inherent uncertainties of the resistances and load effects. The following second moment probabilistic format (Rosenblueth and

Esteva, 1971) was used to determine the values of ϕ and γ. In this format, the safety index β is defined as

$$\beta = \frac{\ln\left(\frac{m_R}{m_Q}\right)}{\sqrt{V_R^2 + V_Q^2}} \qquad \text{(B25)}$$

The resistance R of a structural element is expressed as:

$$R = R_n MFP \qquad \text{(B26)}$$

in which R_n = the nominal code specified resistance and R, M, F, and P are random variables. The dimensions of R_n are limit state moments, axial forces or shears, and M, F and P are thus nondimensional. The product form of Eq. B26 was chosen for illustration because many relationships in steel design are of this form. It was assumed that the random variables M, F and P are uncorrelated. The coefficient of variation of the resistance, V_R, is written approximately as:

$$V_R \approx \sqrt{V_M^2 + V_F^2 + V_P^2} \qquad \text{(B27)}$$

in which V_M, V_F, and V_P are the coefficients of variation of M, F and P respectively.

The random variable M represents the variation in material strength or stiffness. The statistical parameters, m_M and V_M may be obtained by routine tests. The random variable F characterizes the uncertainties in "fabrication". The term "fabrication" includes the variations in geometrical properties introduced by rolling, fabrication tolerances, initial distortions, welding tolerances, erection variations, and the like. The variations are the differences between the ideal designed member and the member in the structure after erection.

The random variable P, called the "professional" factor, reflects the uncertainties of the assumptions used in determining the resistance from "design" models. These uncertainties could be the result of using approximations for theoretically exact formulas, or of the assumptions such as perfect elasticity, perfect plasticity, homogeneity, or "beam" theory instead of the "theory of elasticity". Comparisons between "design" predictions and test results, or between "design" predictions and approximate or exact theoretical formulas could be used to estimate the values of m_p and V_p.

The load effect, Q, refers to the strength limit state that is examined. It is assumed here that only dead and live load effects are present. Other load combinations are studied later.

The load effect Q for combined dead and live gravity loading is modeled as:

$$Q = E(c_D AD + c_L BL) \qquad \text{(B28)}$$

where D and L are random variables representing dead and live load intensities, respectively; c_D and c_L are deterministic influence coefficients that transform the load intensities to load effects (e.g., moment, shear and axial force); A and B are random variables reflecting the uncertainties in the transformation of loads into load effects; and E is a random variable representing the uncertainties in structural analysis. The corresponding mean values are m_D, m_L, m_A, m_B and m_E and the coefficients of variation are V_D, V_L, V_A, V_B and V_E respectively.

In this load combination, the live load of interest is the maximum live load that occurs in the lifetime of the structure. The random variables D and L include the uncertainties in idealizing the loads which vary randomly in time and space. The random variables A and B account for the uncertainties in the transformation of idealized design loads into load effects such as moments, shears, and axial forces. Their variation characterizes the differences between actual and computed internal forces in the structure.

The variable E includes the uncertainties in modeling a three-dimensional real structure of complex geometry and behavior into a set of members and connection of fixed geometry and stipulated behavior. It also accounts for the uncertainties induced by approximate or simplified structural analysis in lieu of complicated refined theories (e.g., assumption of inflexion points, spring-mass systems in vibration analysis).

The mean and the coefficient of variation of Q are derived as:

$$m_Q \approx C_D\, m_A\, m_D + C_L\, m_B\, m_L \tag{B29}$$

and

$$V_Q \approx \sqrt{V_E{}^2 + \frac{c_D^2 m_A^2 m_D^2 (V_A{}^2 + V_D^2) + c_L^2 m_B^2 m_L^2 (V_B^2 + V_L^2)}{(c_D m_A m_D + c_L m_B m_L)^2}} \tag{B30}$$

The mean value of E is assumed to be unity in Eqs. 28 and 29.

Using the expression for R and Q and applying the linear approximation (Eq. 12), the LRFD criterion is derived as:

$$\exp(-\alpha\beta V_R) m_R \geq \exp(\alpha\beta V_E)\,[(1 + \alpha\beta\sqrt{V_A{}^2 + V_D{}^2})\, c_D m_D$$

$$+ (1 + \alpha\beta\sqrt{V_B{}^2 + V_L{}^2})\, c_L m_L)] \tag{B31}$$

in which $\alpha = 0.55$ gives a good approximation between Eqs. B31 and B25 determined by an error minimization process. Therefore, the resistance and load factors are given by

$$\phi = \exp(-\alpha\beta V_R)\frac{m_R}{R_n} \qquad \text{(B32)}$$

$$\gamma_E = \exp(\alpha\beta V_E) \qquad \text{(B33)}$$

$$\gamma_D = 1 + \alpha\beta\sqrt{V_A{}^2 + V_D{}^2} \qquad \text{(B34)}$$

$$\gamma_L = 1 + \alpha\beta\sqrt{V_B{}^2 + V_L{}^2} \qquad \text{(B35)}$$

These factors can be evaluated for any given value of the safety index β. The specification of β is done through "calibration" to the current AISC specifications and is illustrated herein for simple beams.

A simply supported beam under uniformly distributed dead and live loads, adequately braced and "compact" will require a plastic modulus Z given by

$$Z = \frac{1.7\,[c_D D_c + c_L L_c(1-\rho)]}{F_y} \qquad \text{(B36)}$$

in which F_y = the specified minimum yield stress of the grade of steel used, c_D and c_L are the influencing coefficients equal to $s\ell 2/8$ (where s = the beam spacing and ℓ - the span). The code specified dead load is D_c, L_c is the uniformly distributed code live load intensity specified ANSI A58.1 - 1972 for the type of occupancy and ρ is the live load reduction factor given by

$$\begin{aligned} \rho &= 0 \quad \text{for } A_T \leq 150 \text{ sft} \\ &= 0.0008\,A_T \text{ for } 150 \quad A_T \leq 750 \text{ sft} \\ &= 0.60 \text{ for } A_T > 750 \text{ sft} \end{aligned} \qquad \text{(B37)}$$

$$\text{or} \quad \rho = 0.23\,(1 + D_c/L_c)$$

whichever is smaller. The tributary area is A_T.

A value of β can be obtained by selecting a "standard" design situation and requiring that the LRFD criterion produce the same design (i.e., the same plastic section modulus) as the AISC specification. Instead of limiting the calibration to one preselected data point, a complete spectrum of design situations, characterized by different tributary areas and dead loads was studied in the LRFD project in order to select a representative value of β.

The safety index β was computed from Eq. B25 for the plastic modulus Z which was required by the AISC specification. The mean resistance m_R is equal to

$$m_R = m_Z \, m_{F_{sy}} \, m_{\left(\frac{\text{test}}{\text{prediction}}\right)} \qquad \text{(B38)}$$

The values of the mean and the coefficient of variation of the variables identified earlier are:

$$m_z = z \qquad V_F = 0.05$$

$$m_{F_{sy}} = 1.05\, F_y \qquad V_M = 0.$$

$$m_{\left(\frac{\text{Test}}{\text{Prediction}}\right)} = 1.02 \qquad V_P = 0.06$$

$$m_E = 1.0; \quad m_A = 1.0; \quad m_D = D_c; \quad m_B = 1.0$$

$$V_E = 0.05; \quad V_A = 0.04; \quad V_D = 0.04; \quad V_B = 0.20$$

$$m_L = 14.9 + \frac{763}{\sqrt{2A_T}}; \qquad V_L = \frac{\sqrt{11.3 + 7500/A_T}}{14.9 + 763/\sqrt{2A_T}}$$

The variation of β for different values of D_c and A_T is shown in Fig. B-4. It was concluded from this study that $\beta = 3.0$ is a representative value for the safety index implied in the design of simply supported beams using Part 2 of the 1969 AISC specification. Calibrations were also performed for centrally loaded columns and for high strength bolts and fillet welds. The conclusion of the calibration studies was that $\beta = 3.0$ and $\beta = 4.5$ were to be selected for members and for connectors respectively as the basis for developing load factors and resistance factors.

With the numerical values of the mean and the coefficients of variation chosen previously and the safety index $\beta = 3.0$, the LRFD criterion for the plastic design of simply supported steel beams becomes:

$$0.86\, ZF_y \geq 1.1\, [c_D m_D + c_L m_L] \tag{B39}$$

The resistance factors for beams, columns, beam-columns, plate guiders, composite beams and connectors have been derived. (Galambo and Ravindra et al - 1978)

Load Factors have been derived for the load combinations formulated using Turkstra's rule.

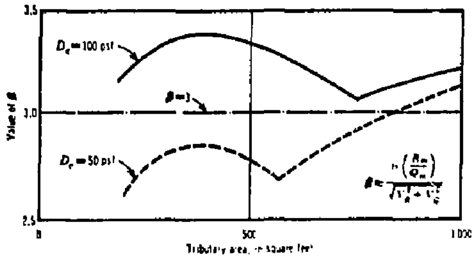


Figure B-4. Variations of Safety Index for Simple Steel Beams Implied in Part 2, AISC Specification

The load combinations are:

Dead Load + Lifetime Max. Live Load

$$\phi R_n \gtrsim 1.1\ [1.1 c_D m_D + 1.4\ c_L m_L] \tag{B40}$$

Dead Load + Sustained Live + Lifetime Max. Wind Load

$$\phi R_n \gtrsim 1.1\ [1.1\ c_D m_D + 2.0\ c_L m_{LS} + 1.6\ c_W m_W] \tag{B41}$$

Lifetime Max. Wind Load + Dead Load

$$\phi R_n \gtrsim 1.1\ [1.6\ c_W m_W + 0.9\ c_D m_D] \tag{B42}$$

Dead Load + Sustained Live Load + Lifetime Max. Snow Load

$$\phi R_n \gtrsim 1.0\ [1.1\ c_D m_D + 2.0\ c_L m_{LS} + 1.7\ c_s m_s\ ] \tag{B43}$$

The major phases of the Load and Resistance Factor Design Project were:

1. Collection of data on load and resistance variables.
2. Calibration studies to derive safety index values.
3. Development of load and resistance factors.
4. Design office studies and criteria development.

The LRFD project has shown that practical design rules can be developed using probabilistic formats. Similar studies have been performed for cold formed steel (Yu and Galambos, 1979), and for concrete (Ellingwood, 1979).

B.5 Load Combination Studies

Most current building codes recommend a load reduction factor when two or more time-varying loads are to be combined. The reasoning behind this reduction factor (or increase in allowable stress) is that the probability of two or more maximum load intensities occurring simultaneously is small. The value of the reduction factor (or "one-third" increase in the allowable stress) is largely based on judgment. Any such factor should be based on the probability of the design combined loading being exceeded in the life of the structure. This probability can be assessed by considering the random occurrences, duration and intensities of different loads. A complete treatment of loads as random processes is conceptually appealing but is computationally extremely difficult.

Recognizing this aspect of computational difficulties and the need for practical design rules, Turkstra (1972) proposed a procedure for finding the maximum combined load effect over the lifetime of the structure. He postulated that when one of the loads in the combination is at its lifetime maximum, the other loads will be at their arbitrary point-in-time values. Using this postulate, the lifetime maximum wind load would for example be combined with the sustained live load. By checking a number of such load combinations, one can ascertain that the maximum combined load effect from all loads is included in the design.

Recent work in the load combination area has focussed on assessing the probability of the maximum combined load effect, S_m. Based on the load coincidence, Wen (1977) has derived the following expression for the cumulative probability distribution of S_m.

$$F_{S_m}(s,T) \simeq \exp\left[-\left(\sum_{i=1}^{N} \nu_i P_i + \sum_{i \neq j}^{N}\sum^{N} \nu_{ij} P_{ij} + \sum_{i \neq}\sum_{j \neq}\sum_{k} \nu_{ijk} P_{ijk}\right) T\right] \tag{B44}$$

in which ν_i = mean occurrence rate of only load $X_i(t)$, ν_{ij}, ν_{ijk} = *mean occurrence rates of coincidence of only* $X_i(t)$ and $X_j(t)$, and only $X_i(t)$, $X_j(t)$ and $X_k(t)$ respectively. P_i, P_{ij}, P_{ijk} are conditional probabilities of S_m exceeding s given the occurrence of load $X_i(t)$ alone, the coincidence of only loads $X_i(t)$ and $X_j(t)$ and the coincidence of $X_i(t)$, $X_j(t)$ and $X_k(t)$, respectively. ν_i, ν_{ij} and ν_{ijk} are given in terms of λ_i and λ_{di}, the mean occurrence rate and mean duration of load $X_i(t)$.

Larrabee and Cornell (1979) have proposed an approximation to the probability of S_m exceeding s in T as:

$$1 - F_{S_m}(s,t) \leq \nu_S^+(s)T \tag{B45}$$

where $\nu_S^+(t)$ is the mean upcrossing rate of the sum of Poisson renewal pulse processes, for the case of sum of two processes,

$$\nu_S^+(s) = \int f_{x_2}(x)\, \nu_{x_1}^+(s-x)\, dx + \int f_{x_1}(x)\, \nu_{x_2}^+(S-x) dx \tag{B46}$$

in which X_1, X_2 are arbitrary point-in-time values and $\nu_{X_1}^+$, $\nu_{X_2}^+$ are upcrossing rates of processes $X_1(t)$ and $X_2(t)$.

Approximate methods for the estimation of the first two moments of $S_m(t)$ have been suggested by Wen (1977) and Der Kiureghian (1978).

B.6 Levels of Probabilistic Design Codes

Many researchers (Rosenblueth, 1972; Rackwitz, 1976) have advocated the adoption of probabilistic design procedures at a graduated pace. They have identified four levels of probabilistic design:

Level I: A design method in which appropriate levels of reliability are provided on a structural element basis by the specification of a number of partial safety factors and load combinations, e.g., LRFD.

Level II: A design method requiring safety checks at selected points on the failure boundary. Reliability levels are defined by safety indices. An example of this design method is the iterative procedure suggested by Rackwitz (see Section B.2)

Level III: A design method wherein the structural elements or systems are designed to specified failure probabilities.

Level IV: A design method wherein the structural system is designed to minimize the total expected cost; the total expected cost is defined as the initial cost plus the product of cost of failure and the probability of failure summed over all failure modes.

While the levels II, III and IV are useful for research, they are not practical for routine design. Level I method with the specified set of partial safety factors is most appropriate for this purpose. The code writers may use the results of studies with the higher level (II, III and IV) design methods in specifying the partial safety factors for level I method.

B.7 Code Optimization

The need for deterministic codes to perform the probabilistic design of structures has been emphasized by Lind (1968). Specification of the partial safety factors in a deterministic code format can be done whatever be the underlying probabil stic model (Lind - 1976) Fig. B-5 is a schematic diagram that shows how the code writers can process the results of the second moment analysis, full structural system reliability analysis an of the expected cost optimization to obtain practical load factor type equations. The tools needed for this transformation are calibration and code optimization (Ravindra and Lind - 1973). The partial safety factors--code parameters--can be derived by matching the safety indices or reliabilities implied by the level I design code to the desired value of β or probability of failur or by minimizing the total expected costs over the set of all "future" designs. In the set-theoretical formulation of design codes this problem of code parameter selection is reduced to one of nonlinear mathematical programming.

This concept has been employed by Lind and his associates (Siu, Parimi and Lind - 1975, Nowak and Lind - 1979) for developing the limit states design criteria for the National Building Code of Canada. The limit states design criterion is expressed as:

$$\phi R_n \geq \gamma_D D + \omega\psi \left\{ \gamma_L L + \gamma_W W \text{ (or } \gamma_E E) + \gamma_T T \right\} \qquad \text{(B47)}$$

ϕ = resistance factor for each material and limit state

R_n = nominal resistance of the structural element

ω = importance factor to account for the use and occupancy of the structure; for normal occupancy and use = 1.0

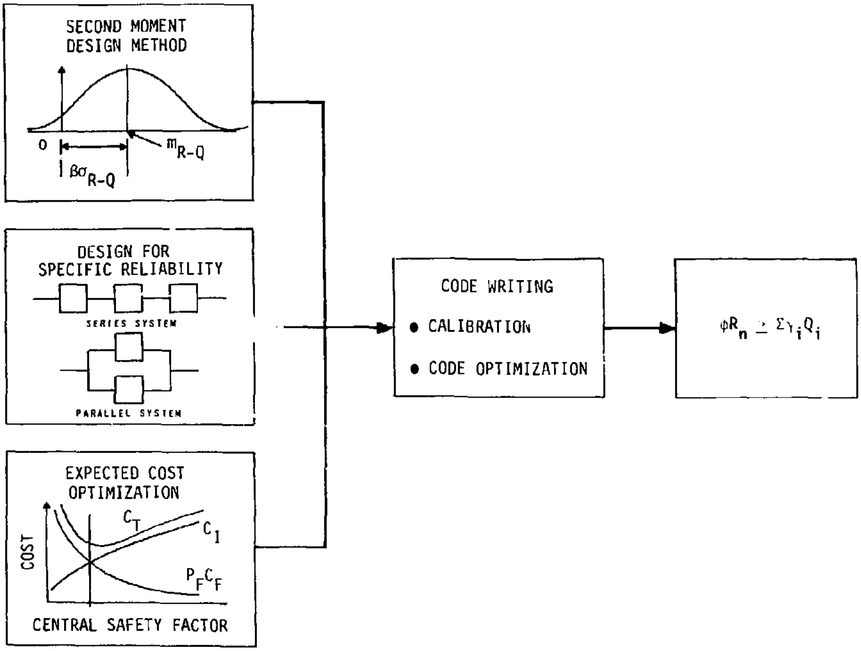


Figure B-5. Deterministic Format for Probabilistic Design

γ_D, γ_L, γ_E, γ_W, γ_T - load factors on the nominal load D, L, E, W and T.

X = combination factor to account for the reduced probability of simultaneous occurrence of maximum loads. $X = 1.00$ for one load in the brackets; $X = 0.70$ for two loads and $X = 0.60$ for three loads within the brackets.

ϕ γ_D, γ_L, γ_W, γ_E, γ_T, ω and X are selected using calibration to current design codes for different materials.

(1) For each limit state for the structural material (e.g., cold formed steel under yielding), the safety index values implied in the code for all design situations (known as points in data space) are calculated. A design situation is characterized by a set of particular values for the ratios of dead to live loads and wind to dead loads. The value of safety index β is determined by using Eq. B25. A weighted average value of safety box index, β_{avg}, is found for the structural material for the limit state under consideration

$$\beta_{avg} = \sum f_i \beta_i \tag{B48}$$

where f_i is the weighing factor based on the frequency of occurrence of the specific design situation.

(2) From an analysis of the β_{avg} values for different materials under different limit states, select representative "target" β values for different limit states as constants for all structural members, e.g., flexural and tension, $\beta = 4.00$; compression, $\beta = 4.75$; and shear, $\beta = 4.25$.

The evaluation of ϕ, $\gamma_D, \gamma_L, \gamma_W, \ldots$ is carried out by using the code optimization procedure. For a selected set of $\gamma_D, \gamma_L, \gamma_W, \ldots$ and ϕ and for a given material under a particular limit state, find the implied value of the safety index, denoted b, for the new code at a given data point. An objective function in terms of the residuals is formulated as

$$\Omega = \sum_{\text{matls}} \sum_{\substack{\text{limit}\\ \text{state}}} \sum_{\substack{\text{data}\\ \text{space}}} (\beta - b)^2 f \qquad \text{(B49)}$$

The minimization of Ω results in optimal values of $\gamma_D, \gamma_L, \gamma_W$, and ϕ.

(4) The above procedure has been applied to derive the optimal values of fourteen code parameters including three load factors (γ_D, γ_L, and γ_W) and eleven resistance factors for the following materials and limit states:

Hot-rolled steel:	Yielding
	Compression
Reinforced concrete:	Flexure
	Compression
	Shear
Wood:	Flexure and tension
	Compression parallel to grain
	Compression perpendicular to grain
	Shear
	Buckling
Cold-formed steel:	Yielding

With these code parameters, the design criteria become:

$$\text{and} \quad \begin{array}{l} \phi R > 1.25D + 1.50L \\ \phi R > 1.25D + 1.70(1.50L + 1.50W) \end{array} \qquad \text{(B50)}$$

Resistance Factors: (a) Cold formed steel - yielding (ϕ_1 = 0.90); (b) hot-rolled steel - yielding (ϕ_2 = 0.85) and compression (ϕ_3 = 0.74); (c) reinforced concrete - flexure (ϕ_4 = 0.83); compression (ϕ_5 = 0.68), and shear (ϕ_6 = 0.64); and (d) wood - flexure (ϕ_7 = 0.92), compression parallel to grain (ϕ_8 = 0.76); compression perpendicular to grain (ϕ_9 = 0.64), shear (ϕ_{10} = 0.90), and buckling (ϕ_{11} = 0.70).

REFERENCES

Cornell, C. A., (1967), "Bounds on the Reliability of Structural Systems," Journal of the Structural Division, ASCE, Vol. 93, No. ST 1. February.

Cornell, C. A., (1969), "A Probability Based Structural Code", Journal of the American Concrete Institute, Vol. 66, No. 12, December, pp. 974-985.

Der Kiureghian, A., (1978), "Second Moment Combination of Stochastic Loads", Journal of the Structural Division, ASCE, Vol. 104, No. ST 10, October.

Ditlevsen, O., (1973), "Structural Reliability and the Invariance Problem", Report No. 22, Solid Mechanics Division, University of Waterloo, Waterloo, Canada.

Ellingwood, B., (1979), "Reliability Based Criteria for Reinforced Concrete Design", Journal of the Structural Division, Vol. 105, No. ST 4, April, pp. 713-725.

Farmer, F.R., "Reactor Safety and Siting: A Proposed Risk Criterion," Nuclear Safety, Vol. 8, 1967, Nov.-Dec.

Fruedenthal, A. M., (1947), "The Safety of Structures", Trans. ASCE, Vol. 112.

Galambos, T. V., Ravindra, M. K., et al (1978), "Load and Resistance Factor Design for Steel", A series of eight papers in Journal of the Structural Division ASCE, Vol. 104, No. ST 9, September.

Hasofer, A. M. and Lind, N. C. (1974), "An Exact and Invariant First Order Reliability Format", Journal of the Engineering Mechanics Division, ASCE, Vol. 100, EM 1, pp 111-121.

Larrabee, R. D., and Cornell, C. A., (1978), "Upcrossing Rate Solution for Load Combinations", Journal of the Structural Division, ASCE, Vol. 105, No. ST 1, January, pp 125-132.

Lind, N. C. (1968), "Deterministic Formats for the Probablistic Design of Structures", in An Introduction to Structural Optimization, Solid Mechanics Study No. 1, University of Waterloo, Waterloo, Ontario, Canada, pp 121-142.

Line, N. C. (1974), "A Formulation of Probablistic Design", Solid Mechanics Division Paper No. 128, University of Waterloo, Ontario, Canada.

Lind, N. C. (1976), "Application to Design of Level I Codes", CEB Bulletin No. 112, Munich, West Germany, July, pp 73-89.

Moses, F. and Kinser, D. E. (1967), "Analysis of Structural Reliability", Journal of the Structural Division, ASCE, Vol. 93, October.

Nowak, A. S. and Lind, N. C., (1979) "Practical Code Calibration Procedures", Canadian Journal of Civil Engineering, Vol. 6, pp 112-119.

Palheimo, E., and Hannus, M., (1974), "Structural Design Based on Weighted Fractiles", Journal of the Structural Division, ASCE, Vol. 100, No. ST 7, pp 1367-1378.

Rackwitz, R., (1976), "Practical Probabilistic Approach to Design", in First Order Reliability Concepts for Design Codes, Comite Europeen du Beton, Bulletin D'Information, No. 112, Munich, West Germany, July.

Ravindra, M. K., Heaney, A. C., and Lind, N. C., (1969), "Probabilistic Evaluation of Safety Factors", Final Report, Symposium on Concepts of Safety of Structures and Methods of Design, International Association for Bridge and Structural Engineering, London, September 1969, pp 35-46.

Ravindra, M. K. and Lind, N. C., (1973), "Theory of Structural Code Optimization", Journal of the Structural Division, ASCE, Vol. 99, No. ST 7, July, pp 1541-1553.

Rosenblueth, E., and Esteva, L., (1971), "Reliability Basis for Some Mexican Codes", in Probabilistic Design of Reinforced Concrete Buildings, ACI Special Publication, SP-31, 1971, pp 1-42.

Rosenblueth, E., (1972), "Code Specification of Safety and Serviceability", State of Art Report No. 2., Tech. Committee No. 10, Structural Safety and Probabilistic Methods, ASCE IABSE Symposium on Planning and Design of Tall Buildings, Lehigh University, PA, August.

Siu, W. C., Parimi, S. R. and Lind, N. C., (1975), "Practical Approach to Code Calibration", Journal of the Structural Division, ASCE, Vol. 101, No. ST 7, July, pp 1469-1480.

Turkstra, C. J., (1972), "Theory of Structural Design Decisions", Solid Mechanics Study No. 2, University of Waterloo, Canada, 1972.

Vanmarcke, E. H., (1972), "Matrix Formulation of Reliability Analysis and Reliability-Based Design", International Journal of Computers and Structures, Vol. 3, No. 6, pp 757-770.

Veneziano, D. (1974), "Safe Regions and Second Moment Reliability Vectors", Research Report R74-33, Dept. of Civil Engineering, M.I.T.

Wen, Y. K., "Statistical Combination of Extreme Loads", Journal of the Structural Division, ASCE, Vol. 103, No. ST 5, May 1977.

Yu, W. W., and Galambos, T. V., (1979), "Load and Resistance Factor Design of Cold-Formed Steel Structures", Research Project sponsored by AISI, University of Missouri, Rolla.