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MASTER

STUDY OF MATTER CURRENT EFFECTS IN TWO-BODY INELASTIC COLLISIONS

by

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CHAPTER I

INTRODUCTION

At the present time no comprehensive model of high energy hadronic scattering exists. Most of the existing models have a very restricted range of application. The geometrical model[1] of hadronic scattering proposed by Chou and Yang has had great success in predicting high energy scattering phenomena. For example, the dip in pp elastic scattering[2], and pion and kaon radii[3] were quite accurately predicted by the model. Use of pp differential cross section data also yields an excellent fit of the measured proton form factor in this model[3].

The possible existence of hadronic matter current inside a polarized hadron and an experimental test of this idea were discussed[4] by Chou and Yang in 1973. Subsequently the geometrical model was generalized[5] to include the matter current effect by the same authors in 1976. The proposed experimental test of the matter current idea consists of determining the spin-rotation parameter, R, in polarized elastic meson-proton scatterings. The measurement of R usually requires a second scattering of the recoil proton off a known analyzer such as C^{12}.

The purpose of this work is to offer another method by which the hadronic matter current effect can be detected. Instead of elastic scattering we shall consider the two-body inelastic scattering process

$$\pi^- \ p \rightarrow K^\circ \Lambda$$

in which the final state hyperon is unstable against weak decay. By observing the angular distribution of the decay products the rotation parameter, R, may be inferred. This method eliminates the need for a second scattering of the recoil target, however it must be noted that the cross section for this reaction is very much smaller than that of the elastic scattering.

This proposed experiment constitutes a test of the geometrical model on two counts. First, is the hadronic matter current effect present in two-body inelastic processes, i.e., is the rotation parameter non-zero? Second, if it is non-zero, does the model outlined in this work correctly predict the magnitude of the matter current effect?

Chapters II and III review the formulation of the elastic and inelastic geometrical models in the absence of matter current effects. Chapter IV is the generalization of the elastic geometrical model to include these effects. The extension of the geometrical model to include matter current effects in inelastic scattering, and its application to the two-body inelastic scattering $\pi^- \ p \rightarrow K^\circ \Lambda$ are the subject of Chapter V. A numerical estimate of the rotation parameter, R, for the process $\pi^- p \rightarrow K^o \Lambda$ is carried out in Chapter VI.

CHAPTER II

TWO-BODY PROCESSES IN THE GEOMETRICAL PICTURE - ELASTIC CASE

Experiments[6] performed in the last few years indicate that a geometrical model, which involves the interaction of two extended structures, is capable of describing high energy hadronic collisions quite accurately. It is known experimentally, from electron-proton scattering for example, that the proton has an extended electromagnetic structure. It seems inevitable that it will also have an extended hadronic structure. The relatively small elastic cross section at large scattering angles[7] tends to support this assumption as well, since large momentum transfers (large scattering angles) would tend to break up an extended structure.

The geometrical model considered here involves three basic assumptions which are well founded in the energy region of interest (P_{inc} > 100 GeV/c). They are: (a) the eikonal approximation[8] of the scattering, (b) the exponential form of the transmission coefficient, and (c) the opaqueness in the form of a convolution of hadronic densities.

The large forward peak in the elastic scattering data suggest that the scattering may be of the diffraction type. If this is so, the large forward peak implies that a large number of partial waves must be contributing, and so the quantity kR (k = wave vector of the incident particle, R = approx. range of the interaction) must be large compared to 1. In this case the scattering may be described by the eikonal approximation familiar from wave optics[9]. For P_{inc} = 200 GeV/c and R = 1 fm

$$kR = P_{inc}R/\hbar \simeq 10^3 >> 1 \ . \qquad (2.1)$$

Assuming that the scattering is accurately described in this eikonal approximation, we view the scattering as the passage of two absorptive spheres through one another. Carrying over from optics the result that the fraction of incident intensity absorbed is linearly dependent upon the thickness of material (x) traversed and upon the absorption coefficient (a) we find

$$dI/I = -adx$$

$$\ell nI = -ax$$

$$I = \exp(-ax) \qquad (2.2)$$

We now make the identifications

$I \rightarrow S$	the transmission coefficient
$ax \rightarrow \Omega(b)$	the opaqueness of the collision at two-dimensional impact parameter b

S is thus seen to be a function of impact parameter, b, only.

$$S = \exp(-\Omega(b)) \tag{2.3}$$

Finally, we must approximate the previously mentioned opaqueness. It will be assumed to be in the form of a convolution of the two hadronic densities. In the case of π^- p scattering for example

$$\Omega(b) = K_{\pi-p} D_{\pi-} \otimes D_p \ . \tag{2.4}$$

$D_{\pi-}$ and D_p are the hadronic matter density of pion and proton respectively. The constant $K_{\pi-p}$ is a measure of the strength of the interaction. $K_{\pi-p}$ may be dependent upon the incident projectile energy. $\otimes$ denotes the two-dimensional Fourier convolution. Note that $\Omega(b)$ is unchanged by interchange of colliding particles, and is linear in each hadronic density. This is the product density assumption, about which more will be said later.

The above assumptions are now developed in a more quantitative fashion. As usual we write the differential cross section as

$$d\sigma/dt - \pi|a|^2 \tag{2.5}$$

where the scattering amplitude, a, is given by

$$a = \frac{1}{k^2}\sum_{\ell=0}^{\infty} \tfrac{1}{2}(2\ell+1)P_\ell(\cos\theta)\,[1-S(b)]. \tag{2.6}$$

$$b \sim \frac{\ell+\frac{1}{2}}{k}$$

We may also obtain another form for the scattering amplitude which is valid at high energies by drawing an analogy with wave optics. As a result of the scattering, the incident wave amplitude is reduced to S(b). Since we do not wish to consider the contribution of the incident wave to the scattering amplitude, we subtract out the incident wave amplitude. The total scattering amplitude at momentum transfer (k_x, k_z) is obtained by summing the contributions from each point of impact parameter space.

This is precisely analogous to Fraunhofer diffraction of a wave by an opaque screen[9]. The quantity [S(b)-1] corresponds to the aperture function in optics. Thus we write

$$a(k_x, k_z) = \frac{1}{2\pi} \int [S(b)-1] e^{-i\vec{k}\cdot\vec{b}} d^2b \quad . \tag{2.7}$$

Note that $\vec{b}$ and $\vec{k}$ are two-dimensional vectors in the plane perpendicular to the incident direction. As the scattering amplitude itself is not an observable quantity, but rather its square, we may write the scattering amplitude in the more conventional form

$$a(k_x, k_z) = \frac{1}{2\pi} \int [1-S(b)] e^{-i\vec{k}\cdot\vec{b}} d^2b \quad . \tag{2.8}$$

The scattering amplitude is now expressed as a two-dimensional Fourier transform in momentum transfer space, denoted by

$$a = \langle 1 - S(b) \rangle \quad . \tag{2.9}$$

Since the transmission coefficient has the exponential form given earlier in equation 2.3, it will be convenient to express the scattering amplitude in terms of $\Omega(b) \equiv -\ln S(b)$.

$$\langle\Omega(b)\rangle = -\langle\ln S(b)\rangle \tag{2.10}$$

$$\begin{aligned}\langle a\rangle &= 1-S(b) = 1-\exp(\ln S(b)) \\ &= -\ln S(b) - \frac{1}{2!}[\ln S(b)]^2 - \frac{1}{3!}[\ln S(b)]^3 - \ldots\end{aligned} \tag{2.11}$$

$$\langle a\rangle = \Omega(b) - \frac{1}{2!}\Omega(b)\Omega(b) + \frac{1}{3!}\Omega(b)\Omega(b)\Omega(b) - \ldots \tag{2.12}$$

Using the property of Fourier transform convolutions

$$\langle a\rangle\langle b\rangle = \langle a \otimes b\rangle \tag{2.13}$$

we obtain

$$a = \langle\Omega(b)\rangle - \frac{1}{2!}\langle\Omega(b)\rangle \otimes \langle\Omega(b)\rangle + \frac{1}{3!}\langle\Omega(b)\rangle \otimes \langle\Omega(b)\rangle \otimes \langle\Omega(b)\rangle + \ldots \tag{2.14}$$

The scattering amplitude has now been expressed as an infinite series in the Fourier transform of the collision opaqueness, $\langle\Omega(b)\rangle$.

The problem has now been reduced to a determination of the opaqueness at two-dimensional impact parameter $\vec{b}$. Since we consider the collision as the passage of two extended structures through one another, the situation may be illustrated as in figure 2.1. Point Q within the incoming pion (for example) sees a compressed proton density given by

$$D(b_x', b_z') = \int_{-\infty}^{\infty} \rho_p(b_x', b_y', b_z')\, db_y' \ . \tag{2.15}$$

That is, the proton appears to the pion to be a disc with

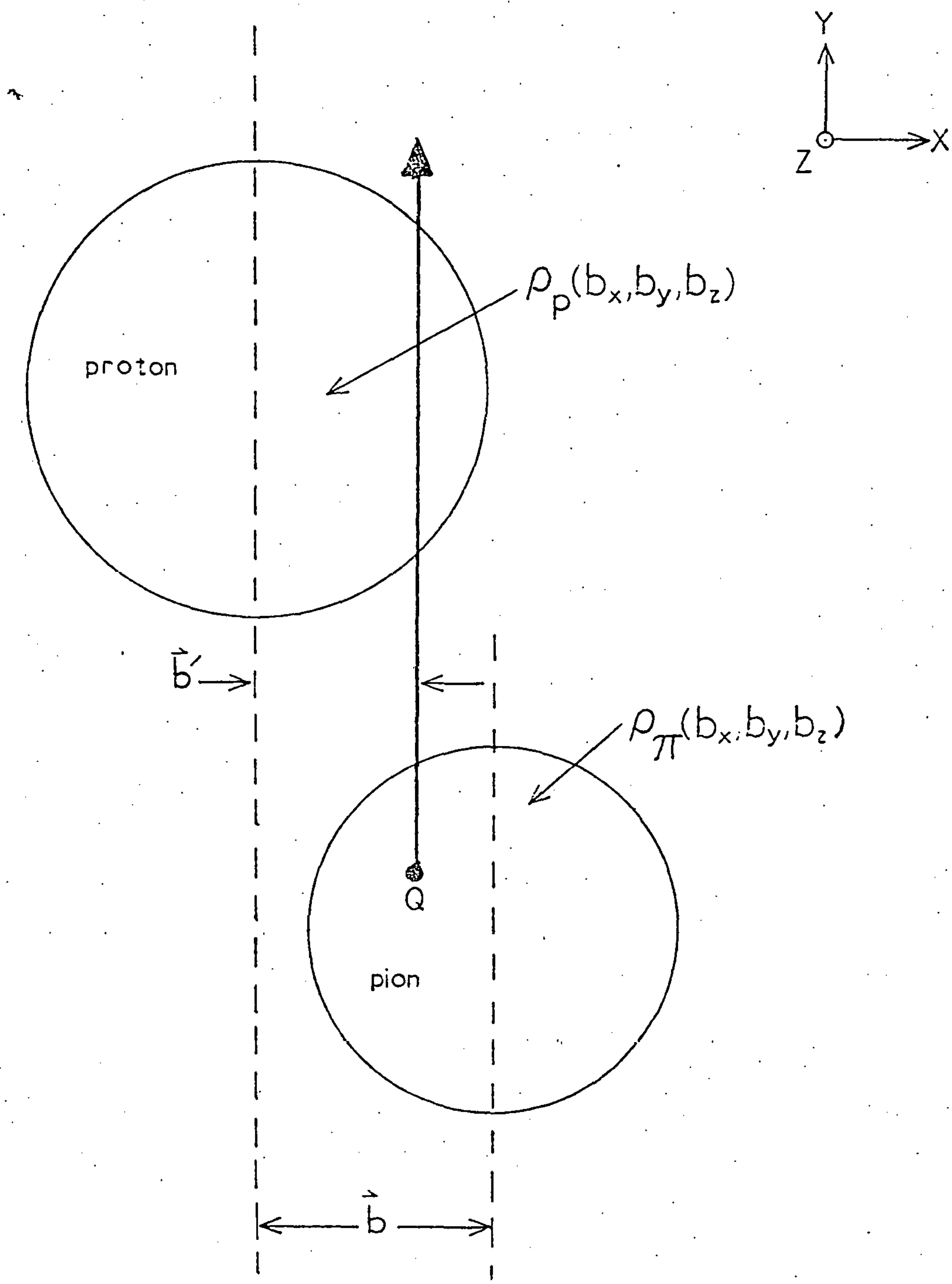


Figure 2.1. Schematic drawing for π^-p collision.

two-dimensional density $D(b'_x,b'_y)$. The total resultant opaqueness is obtained by integrating the product of the two compressed densities over all possible values of $\vec{b}'$ and including a factor which is in some way a measure of the interaction strength between the two particles involved. This is the constant $K_{\pi-p}$ mentioned previously.

$$\Omega_{\pi-p}(b) = K_{\pi-p} \iint D_{\pi-}(\vec{b}-\vec{b}')D_p(b')d^2b'$$
$$= 2\pi K_{\pi-p} D_{\pi-} \otimes D_p \qquad (2.16)$$

Taking the Fourier transform of this we obtain $\langle\Omega(b)\rangle$, the quantity required in our expansion for the scattering amplitude.

$$\langle\Omega(b)\rangle = 2\pi K_{\pi-p}\langle D_{\pi-}\rangle\langle D_p\rangle \qquad (2.17)$$

To connect $\langle D_{\pi-}\rangle$ and $\langle D_p\rangle$ with known quantities, note that

$$\langle D_{\pi-}\rangle = \frac{1}{\sqrt{2\pi}}\langle\rho^{\pi-}_{hadr.}\rangle|K_x,K_z,0^{*}$$

and (2.18)

$$\langle D_p\rangle = \frac{1}{\sqrt{2\pi}}\langle\rho^{p}_{hadr.}\rangle|K_x,K_z,0^{*}$$

$\rho_{hadr.}$ is the density of hadronic matter. $\rho_{hadr.}$ and $\rho_{chg.}$ are assumed proportional, then the Fourier transform of the two-dimensional density may be related to the charge form

$$\langle D_p\rangle = (const.)C^p_E(t) \ .$$
$$-t = k_x^2 + k_z^2 \qquad (2.19)$$

---

*Note that these are 3-dim. Fourier transforms evaluated at $k_y = 0$.

Note that G_E^p is probably the correct form factor with which to associate $<D_p>$, since in the Breit frame it is precisely the charge density[10]. However, since G_E^p and G_M^p are found experimentally to be proportional[11] (scaling law),

$$G_E^p = G_M^p/\mu_p \qquad (\mu_p = \text{proton magnetic moment}) \qquad (2.20)$$

we may alternatively write

$$\begin{aligned} <D_p> &= (\text{const.})\, G_M^p(-t) \ . \\ -t &= k_x^2 + k_z^2 \end{aligned} \qquad (2.21)$$

For the pion there is but one form factor since the pion is a pseudoscalar particle, and we associate this form factor with the Fourier transform of the pion density as

$$<D_{\pi^-}> = (\text{const.})F_{\pi^-}(-t) \ . \qquad (2.22)$$

Finally then, the scattering amplitude at high energies is given by the infinite series

$$a_{\pi^-p} = <\Omega_{\pi^-p}(b)> - \frac{1}{2!}<\Omega_{\pi^-p}(b)> \otimes <\Omega_{\pi^-p}(b)> + \ldots$$

where (2.23)

$$<\Omega_{\pi^-p}(b)> = (\text{const.})\, F_{\pi^-}(-t)\, G_M^p(-t) \ .$$

Thus in the geometrical model the measured cross section may be used to extract form factors, or accepted values for the form factors may be used to calculate expected cross sections.

Note that in the case of electron-proton scattering the form factor G_M^p appears as $[G_M^p]^2$ in the scattering cross section. If hadronic elastic scattering is viewed as the passage of two extended objects through each other, and if we identify the charge distribution with the hadronic matter distribution, then it is to be expected that in proton-proton scattering the leading term will appear as $[G_M^p]^4$ in the cross section[12]. This is in fact borne out in the scattering data, and the cross section is seen to have the dependence

$$d\sigma/dt = (\text{const.})\,[G_M^p(-t)]^4 \tag{2.24}$$

for small t. This corresponds to taking only the first term in our series expansion, the higher order terms being viewed as small corrections due to the shielding of the back of the target by the front. This suggests that the identification of the charge form factor with the hadronic matter form factor is a valid and useful one.

CHAPTER III

TWO-BODY PROCESSES IN THE GEOMETRICAL PICTURE - INELASTIC CASE

We now consider the more general collision A B → C D. In order to generalize the results of the previous chapter to include inelastic two-body collisions, the form of the scattering amplitude must be altered. Recall that the scattering amplitude for the elastic case was given by

$$a_{el} = \frac{1}{k^2} \sum_{\ell=0}^{\infty} \tfrac{1}{2}(2\ell+1) P_\ell(\cos 0)[1-S(b)] \qquad (3.1)$$

or equavalently

$$a_{el} = \frac{1}{2\pi} \iint [1-S(b)] \exp(-i\vec{k}\cdot\vec{b}) d^2b \ . \qquad (3.2)$$

Recall further that the scattering matrix element has the form

$$S_{ab} = \delta_{ab} + f_{ab} \qquad (3.3)$$

where a and b refer to the initial and final states. The origin of the 1 in the elastic scattering amplitude is now clear--it corresponds to the unscattered portion of the incident wave. In the present case of inelastic collisions, the final and initial states are not the same, and the inelastic spin non-flip amplitude is thus

$$a_{in} = \frac{1}{k^2} \sum_{\ell=0}^{\infty} \tfrac{1}{2}(2\ell+1) P_\ell(\cos\theta) S'(b) \qquad (3.4)$$

or analogously, in the wave equation derivation

$$a_{in} = \frac{1}{2\pi}\iint S'(b)\exp(-i\vec{k}\cdot\vec{b})\,d^2b \quad . \tag{3.5}$$

The rest of the simplification of the scattering amplitude in the eikonal approximation follows exactly as in Chapter II, and the differential inelastic cross section is given by

$$d\sigma/dt = \pi|a_{in}|^2$$

where (3.6)

$$a_{in} = \langle S'(b)\rangle \quad .$$

Recall that $\langle\ \rangle$ denotes the two-dimensional Fourier transform.

The factor S'(b) must now be specified. Whereas for the elastic case we had only to consider the attenuation of the incident particle amplitude as it passed through the target, we must now consider a three-step process, which may be pictured as follows[13].

In the first step the incident particle, A, is attenuated as it begins to pass through the target, B. In step two the rearrangement from initial state configuration to final state configuration occurs: A B → C D. In the third step the final state particle C is attenuated as it leaves the rearrangement site. A correct description of the scattering process A B → C D must account for the three phenoma.

Clearly the steps one and three may be treated in a

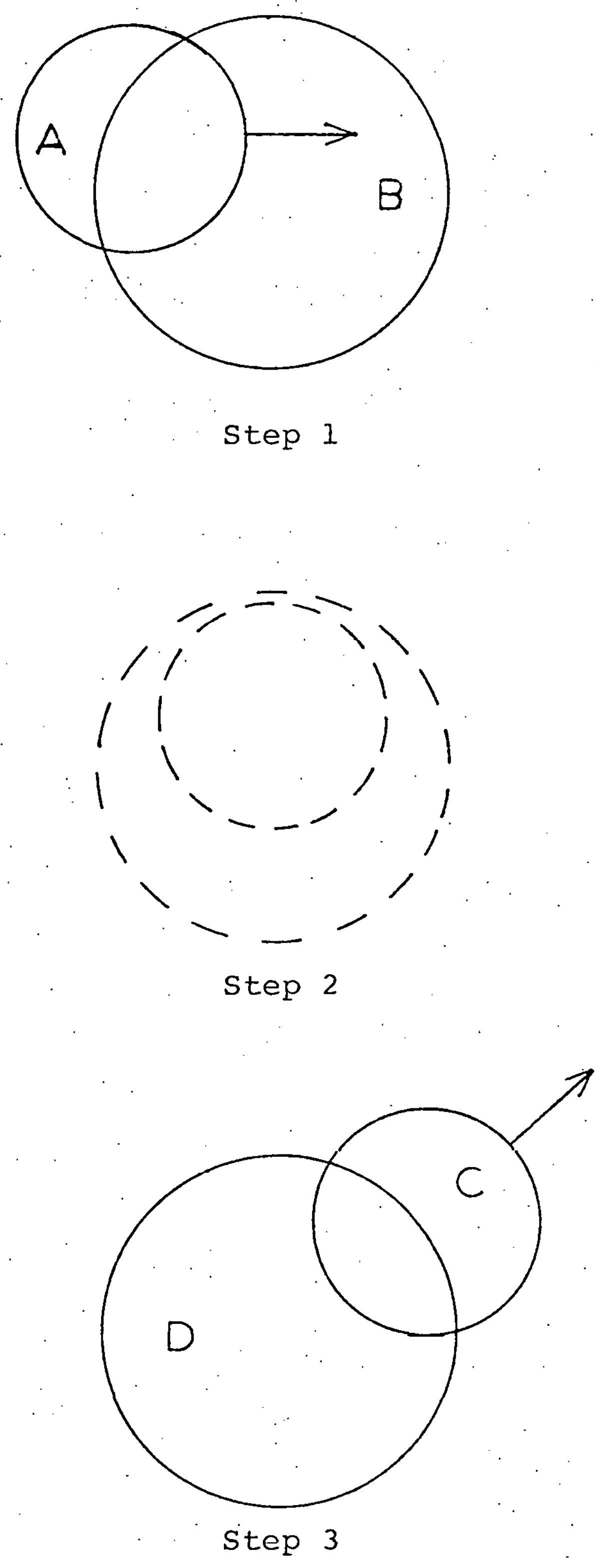


Figure 3.1. Schematic drawing for inelastic exchange reaction.

way analogous to the passage of projectile through target in the elastic case. The attenuation of particle A as it begins to pass through target B is given by

$$\exp[-\Omega^{AB}_{-\infty\to b_y}(\vec{b})] \ . \tag{3.7}$$

Similarly, the attenuation of final state particle C as it leaves final state particle D is given by

$$\exp[-\Omega^{CD}_{b_y\to\infty}(\vec{b})] \ . \tag{3.8}$$

Note that in these expressions $\vec{b},b_y$ is the point at which the rearrangement occurs. $\vec{b}$ is again a two-dimensional vector in the plane perpendicular to the incident direction. Also note that the exponential argument in each case is not the total opaqueness in the sense of Chapter II, but rather only the opaqueness up to (or away from) the rearrangement site.

Step two, the rearrangement process itself, is accounted for by including a factor

$$[\rho^{AB}(\vec{b},b_y)\rho^{CD}(\vec{b},b_y)]^{\frac{1}{2}}$$

where $\rho^{AB}(\vec{b},b_y)$ and $\rho^{CD}(\vec{b},b_y)$ are the densities of the initial and final states respectively. They are defined in the following way.

$$\rho^{AB} = \rho^{A} \otimes \rho^{B} \qquad \text{and} \qquad \rho^{CD} = \rho^{C} \otimes \rho^{D} \tag{3.9}$$

This factor is a measure of the probability that the rearrangement occurs at the point $(\vec{b},b_y)$, and is the geometrical mean of the two densities. We choose this form (geometrical mean) as opposed to the arithmetic mean since the factor representing step two above should be linear in each density, and further, it should reflect the fact that if either the initial or final state density becomes very small (ostensibly zero) the probability of the exchange process occuring should also become small. Notice that the arithmetic mean does not satisfy this latter requirement.

The modified opaquenesses are obtained in the following way. The opaqueness at a point is proportional to the hadronic density at that point, and so the uncompressed opaquenesses are given by

$$\Omega^{AB}(\vec{b},b_y) = K\rho^{AB}(\vec{b},b_y)$$

and (3.10)

$$\Omega^{CD}(\vec{b},b_y) = K'\rho^{CD}(\vec{b},b_y) \quad .$$

K and K' are constants which depend on incident momentum. The compressed opaquenesses which appear in the attenuation factors of steps one and three are thus

$$\Omega^{AB}_{-\infty\to b_y}(\vec{b}) = K \int_{-\infty}^{b_y} \rho^{AB}(\vec{b},b_y')\,db_y' \tag{3.11}$$

$$\Omega^{CD}_{b_y\to\infty}(\vec{b}) = K' \int_{b_y}^{\infty} \rho^{CD}(\vec{b},b_y')\,db_y' \quad . \tag{3.12}$$

The factor S'(b) (amplitude for occurrence of the process

A B → C D) is then

$$S'(b) = C' \int_{-\infty}^{\infty} e^{-\Omega^{AB}_{-\infty\to b_y}(\vec{b})} \; e^{-\Omega^{CD}_{b_y\to\infty}(\vec{b})} \times [\rho^{AB}(\vec{b},b_y)\,\rho^{CD}(\vec{b},b_y)]^{\frac{1}{2}} db_y \; . \tag{3.13}$$

In principle the inelastic spin non-flip scattering amplitude has been found, and the inelastic cross section is given by

$$(d\sigma/dt)_{in} = \pi |a_{in}|^2 \tag{3.14}$$

where

$$a_{in} = \langle S'(b) \rangle \quad . \tag{3.15}$$

See also reference 14. If, however, the initial and final states have approximately the same density distributions the following simplification occurs

$$\begin{aligned} S'(b) &= C' e^{-\Omega^{AB}_{-\infty\to\infty}(\vec{b})} \int_{-\infty}^{\infty} \rho^{AB}(\vec{b},b_y)\, db_y \\ &= C e^{-\Omega^{AB}_{-\infty\to\infty}(\vec{b})} \, \Omega^{AB}_{-\infty\to\infty}(\vec{b}) \quad . \end{aligned} \tag{3.16}$$

C and C' are constants depending on incident momentum. Assuming this condition applies to the case of the exchange reaction $\pi^- p \to K^\circ \Lambda$, the exchange scattering amplitude is given in the geometrical model by

$$a_{\pi-p\to K^\circ\Lambda} = C \langle \Omega^{AB}_{-\infty\to\infty}(\vec{b}) \exp(-\Omega^{AB}_{-\infty\to\infty}(\vec{b})) \rangle \tag{3.17}$$

and the exchange differential cross section is given by

$$(d\sigma/dt)_{\pi^- p \to K^\circ \Lambda} = \pi |a_{\pi^- p \to K^\circ \Lambda}|^2 \quad . \tag{3.18}$$

CHAPTER IV

CONCEPT OF MATTER CURRENT AND ITS EFFECT ON ELASTIC SCATTERING

Up to now we have considered collisions between particles which have an extended, but internally static, structure. We now extend the geometrical model to allow the "stuff" of which the particle is made to have a rotational velocity distribution. It should be noted that this rotation is not necessarily rigid rotation, but merely some type of orbital motion. It seems reasonable to conjecture that this rotation of the hadronic matter is directly related to the intrinsic spin of the particle.

The basis for the detection of this matter current within the hadron is the experimentally observed increase of the total scattering cross section[15] with increasing incident energy in the region 200-1500 GeV. See figure 4.1.

The basic principle is the following. Recalling the concept of a hadronic matter density $\rho(b_x, b_y, b_z)$ from Chapter II, we now consider, in addition, a hadronic matter current density $\vec{j}(b_x, b_y, b_z)$. Consider the effect of such a matter current density on the scattering of two hadrons, bearing in mind the above-mentioned increase of cross section with energy. Due to the non-zero matter current

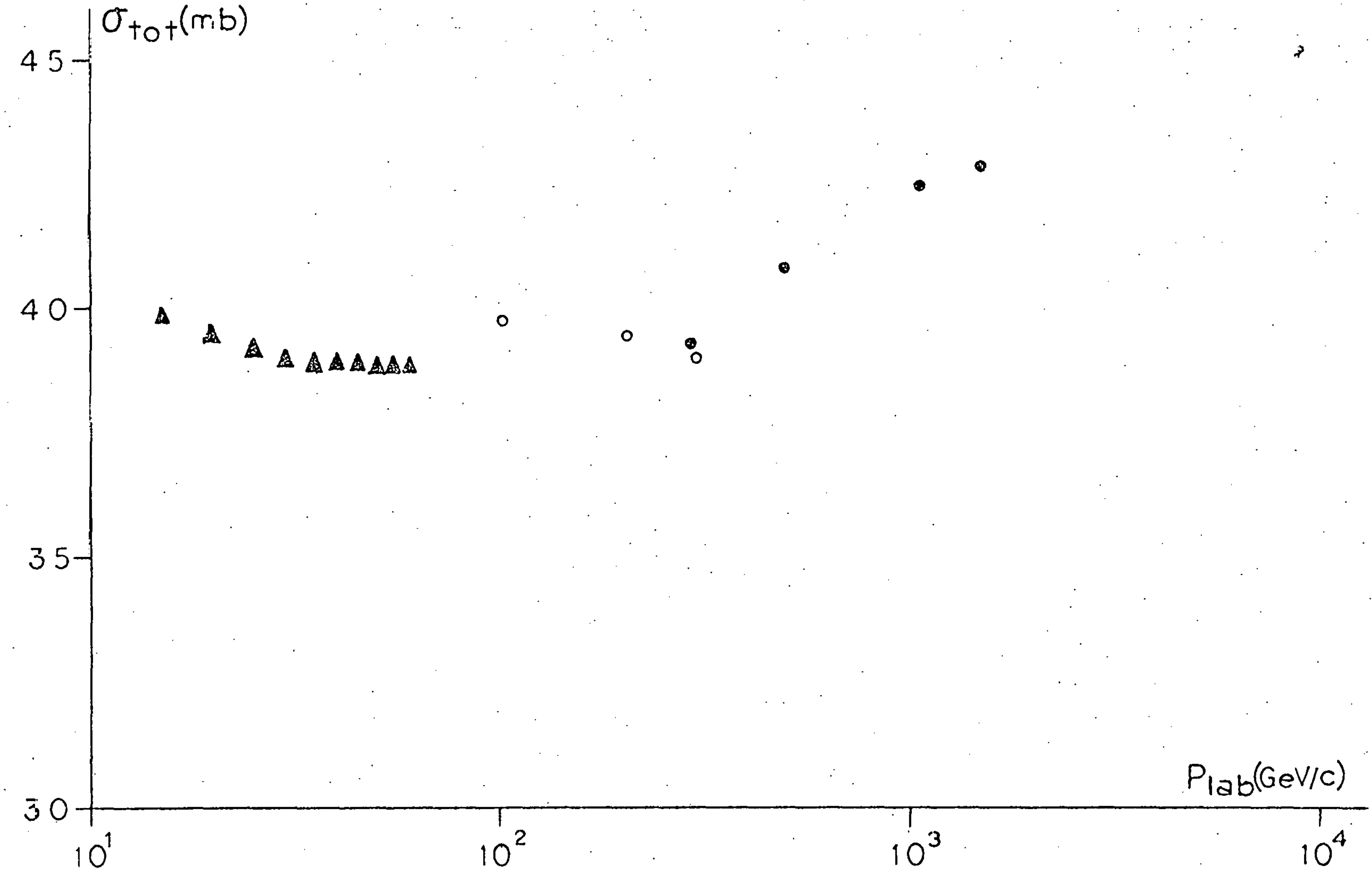


Figure 4.1. The increase of total pp cross section with incident energy (from ref. 15).

density (rotation of hadronic "stuff") one half of the target will appear more opaque to the incident projectile than the other half. This is precisely due to the increase of opaqueness with increasing relative colliding particle velocity. Thus, for a proton polarized along the $\hat{z}$-axis, the left half appears more opaque to a pion incident in the $\hat{y}$ direction than does the right half. See figure 4.2.

It has been assumed in the above conjecture that the only spin effect in elastic scattering at the high energies we are considering is this opaqueness difference. It will be shown below that the result of the opaqueness difference of the two halves of the target is a rotation of the polarization vector of the target about an axis perpendicular to the scattering plane, i.e., a non-zero rotation parameter, R(t).[16] It will also be shown in this chapter that a measurement of the differential cross section along with a measurement of the rotation parameter are sufficient to determine the value of the matter current density distribution $\vec{j}(b_x,b_y,b_z)$.

Note that the only requirement for observing the above rotation effect is a gradient of cross section with energy. In principle, a decrease of cross section with incident energy would do as well. This particular means of observing the effect is probably less advantageous since at the lower energies where cross section decreases with increasing energy the spin dependence may have a much more complicated form that that assumed above. This will render

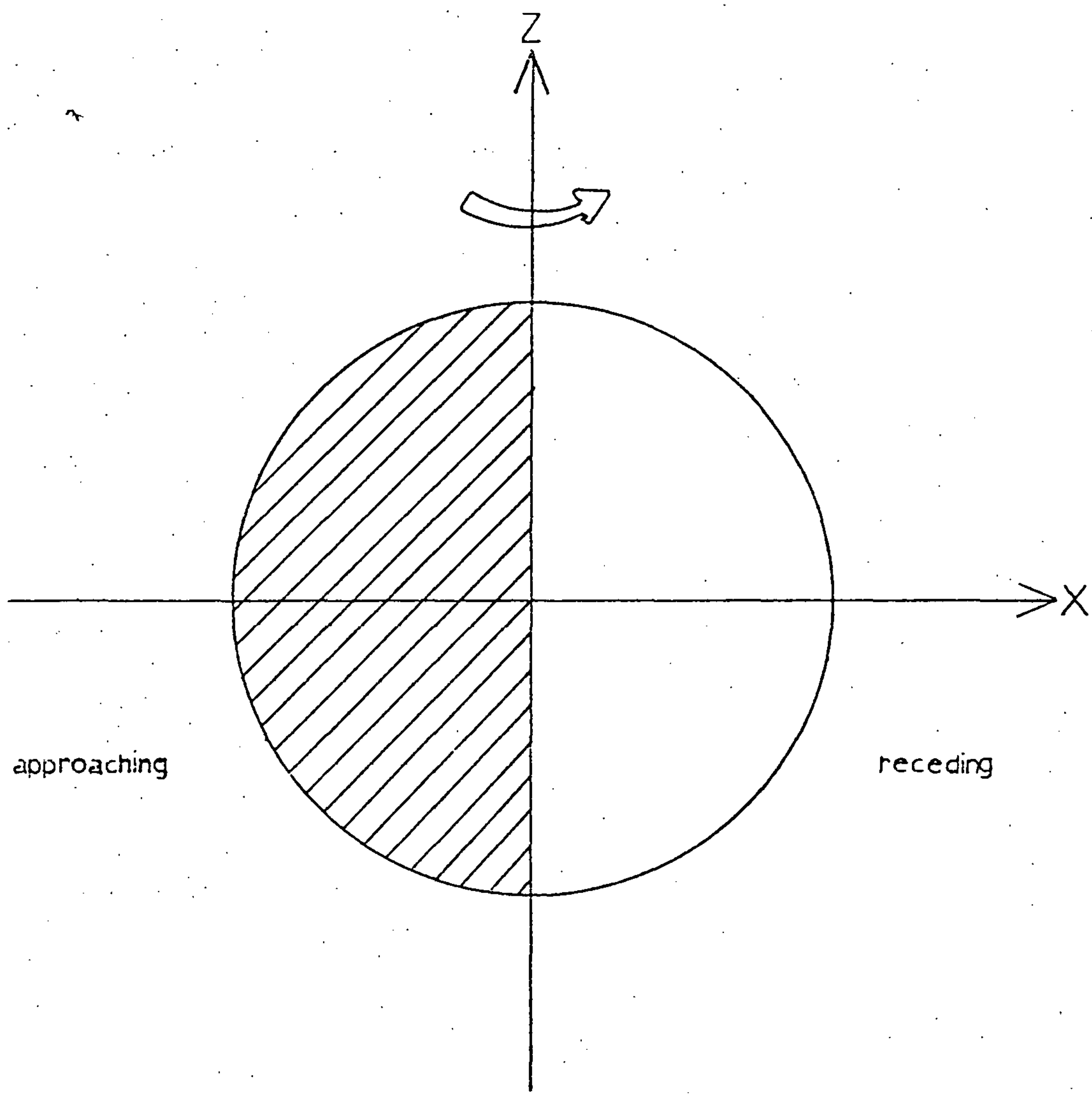


Figure 4.2. Illustration of the opaqueness of a spinning proton.

interpretation of results very difficult.

We now examine the modifications which must be made to the geometrical model of Chapter II to accommodate matter current effect. From the introductory discussion it should be clear that only the opaqueness of the interaction will be affected by the introduction of the matter current density. We recapitulate the main results of Chapter II.

$$\left(\frac{d\sigma}{dt}\right)_{elastic} = \pi |a_{elastic}|^2 \qquad (4.1)$$

$$a_{elastic} = \frac{1}{2\pi}\iint [1-S(\vec{b})]e^{-i\vec{b}\cdot\vec{k}}d^2b \equiv \langle 1-S(\vec{b})\rangle \qquad (4.2)$$

where

$$S(\vec{b}) = \exp[-\Omega(\vec{b})] \qquad (4.3)$$

and

$\Omega(\vec{b})$ = the opaqueness at two-dimensional impact parameter $\vec{b}$.

It is this opaqueness which must be modified.

To explicitly exhibit the concepts involved in the model, consider the following elastic scattering of particle A (S = 0) by particle B (S = ½).

a) target B is infinitely heavy

b) incident momentum of A; (0,k,0,ik)

c) outgoing momentum of A; $(q_x,k,0,ik)$

d) target B polarized with $J_z = m = \pm\frac{1}{2}$.

Note that the limits $q_x \ll k$ and $M_A \ll k$ have been taken.

Since there is no momentum transfer along the $\hat{y}$ direction, we will be interested in the compressed hadronic matter density and the compressed hadronic matter current density at two-dimensional impact parameter $\vec{b} = (b_x, b_z)$. Recall that these densities are given by

$$\rho(b_x, b_z) = \int_{-\infty}^{\infty} \rho(b_x, b_y, b_z)\, db_y$$
$$\vec{j}(b_x, b_z) = \int_{-\infty}^{\infty} \vec{j}(b_x, b_y, b_z)\, db_y \quad . \tag{4.4}$$

Assume now that the opaqueness which the target presents to a <u>point projectile</u> is given by

$$\Omega_{point}(\vec{b}, b_y) = C(P_{eff})^{\alpha} \rho^{B}(b_x, b_y, b_z) \quad . \tag{4.5}$$

P_{eff} is the momentum of the projectile in the rest frame of the point (b_x, b_y, b_z). α is a parameter characteristic of the particular collision under consideration. It is determined by fitting the total cross section for the collision with $\sigma_{tot} \propto (P_{inc})^{\alpha}$. To express P_{eff} in terms of the incident particle momentum, P_{inc}, note that

$$P_{eff} = \frac{m(V_{inc} - V_y)}{\sqrt{1 - V_{inc}^2}\ \sqrt{1 - V_y^2}} \tag{4.6}$$

so that neglecting terms of order V_y^2 and higher

$$(P_{eff})^{\alpha} = \left(\frac{mV_{inc}}{\sqrt{1 - V_{inc}^2}} \right)^{\alpha} [1 - \alpha V_y / V_{inc}]$$
$$\simeq (P_{inc})^{\alpha} [1 - \alpha V_y] \tag{4.7}$$

where V_y is the velocity of rotation of point (b_x, b_y, b_z) in the rotating target. Using this result in the expression for Ω_{point} yields

$$\Omega_{point}(\vec{b},b_y) \propto P_{inc}{}^{\alpha}[1-\alpha V_y]\rho^B(\vec{b},b_y). \tag{4.8}$$

Compressing the densities along b_y, we obtain

$$\Omega_{point}(\vec{b}) = 2\pi K[\rho^B(\vec{b})-\alpha j_y(\vec{b})] \tag{4.9}$$

where K is a function of the incident momentum.

The structure of projectile A may now be taken into account. Since the projectile is spinless it has only a compressed matter density (j = 0) denoted by

$$\rho^A(\vec{b}) = \int_{-\infty}^{\infty}\rho^A(b_x,b_y,b_z)\,db_y \quad . \tag{4.10}$$

The total resultant opaqueness is obtained by taking the folding integral of Ω_{point} and ρ^A.

$$\Omega(\vec{b}) = \rho^A(\vec{b}) \otimes \Omega_{point}(\vec{b}) \tag{4.11}$$

This yields the differential elastic cross section.

$$(d\sigma/dt)_{elastic} = \pi|<1-e^{-\Omega(\vec{b})}>|^2$$
$$\Omega(\vec{b}) = \rho^A(\vec{b}) \otimes \Omega_{point}(\vec{b}) \tag{4.12}$$

We now specialize the collision under consideration even further to exhibit the properties of the model when target B has spin ½, an example of such a collision being $\pi^- p \to \pi^- p$.

Since the target, B, has a definite parity, the density must be even under the parity operation.

$$\rho(\vec{r}) = \rho(-\vec{r}) \tag{4.13}$$

where $\vec{r}$ is a three-dimensional vector, and $\vec{b}$ is the two-dimensional projection of $\vec{r}$ on the x-z plane. The y-component of $\vec{r}$ is called b_y. Using the fact that ρ is the product of ψ and ψ^* (each with angular momentum ½), ρ is seen to consist of a scalar part plus a vector part. If ρ is to be even under the parity operation, ρ must be spherically symmetric, and so the compressed density $\rho(\vec{b})$ is cylindrically symmetric. The above argument may be modified to yield the same result for the projectile, A.

As intimated earlier in this chapter, the matter current density is responsible for rotating the target polarization, and so we now allow $\vec{j}$ to become a 2×2 matrix. This allows the scattering to introduce components of final state polarization which are <u>not</u> along the initial polarization direction. Since $\vec{j}$ is a 2×2 matrix, each component of $\vec{j}$ may be written in terms of the complete set $(\sigma_x, \sigma_y, \sigma_z, I)$. Imposing the further condition that $\vec{j}$ behave as a vector under the parity operation, we find the general form of $\vec{j}$ to be

$$\vec{j} = f\vec{r}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - g\vec{r} \times \vec{\sigma} \tag{4.14}$$

provided that f and g are functions of r. The hadronic current is certainly conserved. This places a restriction on $\vec{j}$ that

$$\int_V \vec{\nabla}\cdot\vec{j}\,d^3x = 0 \quad \text{or} \quad \int_S \vec{j}\cdot d\vec{a} = 0 \tag{4.15}$$

Note that for $f \lesseqgtr 0$, the term $f\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\vec{r}\,d\vec{a}$ is always $\lesseqgtr 0$. Since $\vec{\nabla}\cdot(g\vec{r}\times\vec{\sigma}) \equiv 0$, $f = 0$.

$$\vec{j} = -g\vec{r}\times\vec{\sigma} \tag{4.16}$$

or in component form

$$\begin{aligned} j_x &= -g(b_y\sigma_z - b_z\sigma_y) \\ j_y &= -g(b_z\sigma_x - b_x\sigma_z) \\ j_z &= -g(b_x\sigma_y - b_y\sigma_x) \quad . \end{aligned} \tag{4.17}$$

For the kinematics specified above, namely initial polarization perpendicular to the scattering plane, the polarization direction cannot change as a result of the scattering. This can be seen by invoking reflection and rotation invariance for example. We need only consider the diagonal elements of $\vec{j}$: $m = \pm\frac{1}{2} \to m = \pm\frac{1}{2}$. Then it is found

$$\begin{aligned} j_x &= \mp g b_y \\ j_y &= \pm g b_x \qquad (\pm \text{ for } m_i = \pm\tfrac{1}{2}) \\ j_z &= 0 \end{aligned} \tag{4.18}$$

The compressed current densities are thus

$$\begin{aligned} j_y &= \pm g_1 b_x \\ j_x &= j_z = 0 \end{aligned} \tag{4.19}$$

where

$$g_1(b) = \int_{-\infty}^{\infty} g(\vec{b},b_y)\,db_y \quad . \tag{4.20}$$

The Fourier transforms of the pertinent quantities are

$$\langle\rho\rangle = \frac{1}{2\pi}\int \rho e^{-i\vec{q}\cdot\vec{b}} d^2b \tag{4.21}$$

$$\langle j_x\rangle = \langle j_z\rangle = 0 \tag{4.22}$$

$$\begin{aligned}\langle j_y\rangle &= \frac{1}{2\pi}\int(\pm g_1 b_x)e^{-i\vec{q}\cdot\vec{b}} d^2b \\ &= \pm i \frac{\partial}{\partial q_x}[\frac{1}{2\pi}\int g_1 e^{-i\vec{q}\cdot\vec{b}}]d^2b \\ &= \mp i q_x \cdot (\text{cyl. symmetric fcn.})\end{aligned} \tag{4.23}$$

where $\vec{q}$ and $\vec{b}$ are two-dimensional vectors. Expressing these Fourier transforms in terms of form factors, we define

$$\begin{aligned}\langle\rho\rangle &= \frac{1}{2\pi}G_1^{had}(t) \\ & \qquad\qquad (-t = q_x{}^2 + q_z{}^2) \\ \langle j_y\rangle &= \mp\frac{iq_x}{2\pi}\, G_2^{had}(t) \quad .\end{aligned} \tag{4.24}$$

$G_1^{had}(t)$ and $G_2^{had}(t)$ are the hadronic density and current form factors respectively for the proton. They are analogous to the electromagnetic form factors. Note that as with electromagnetic form factors, spinless particles possess only one form factor.

For the kinematics considered here, the scattering amplitude is given by

$$a(q_x,q_z)\,|_{q_z=0} = \langle 1-e^{-\Omega(\vec{b})}\rangle|_{q_z=0} \tag{4.25}$$

with

$$\Omega(\vec{b}) = \rho^A(\vec{b}) \otimes 2\pi K(\rho_{inc},\alpha)[\rho^B(\vec{b})-\alpha j_y(\vec{b})]$$

$$\equiv \Omega_0(\vec{b})+\Omega_1(\vec{b}) \tag{4.26}$$

where

$$\Omega_0(\vec{b}) = 2\pi K\rho^A(\vec{b}) \otimes \rho^B(\vec{b})$$

$$\Omega_1(\vec{b}) = -2\pi K\alpha\rho^A(\vec{b}) \otimes j_y(\vec{b}) \quad . \tag{4.27}$$

K is a constant which depends on the incident momentum.

Expanding a in a Taylor's series

$$a(q_x,q_z) = \langle 1-e^{-\Omega_0}\rangle+\langle\Omega_1 e^{-\Omega_0}\rangle+ \ldots \tag{4.28}$$

$$\equiv a_0+a_1$$

where

$$a_0 = \langle 1-e^{-\Omega_0}\rangle$$

$$a_1 = \langle\Omega_1 e^{-\Omega_0}\rangle \tag{4.29}$$

and

$$\langle\Omega_0\rangle = KG_A^{had}G_{1B}^{had}$$

$$(\pm \text{ for } m_i=\pm\tfrac{1}{2}) \tag{4.30}$$

$$\langle\Omega_1\rangle = \pm i\alpha Kq_x G_A^{had}G_{2B}^{had}$$

The phase of the scattering amplitude is given by

$$e^{i\phi} = \frac{a}{|a|} = \frac{a_0+a_1}{[a_0^2-a_1^2]^{\frac{1}{2}}}$$

$$e^{i\phi} \sim \frac{a_0+a_1}{a_0} + \theta(a_1^2) \quad . \tag{4.31}$$

Since $a_1 = \langle\Omega_1 e^{-\Omega_0}\rangle = i\, q_x \cdot$(cyl. symmetric fcn. of q_x, q_z), the measurement of a_1 at $q_z=0$ for all values of $q_x \geq 0$ allows the determination of a_1 for all q_x and q_z. It should also be noted that a_1 is purely imaginary.

The left-right asymmetry produced by the scattering is

$$\varepsilon = \frac{d\sigma/dt)_R - d\sigma/dt)_L}{d\sigma/dt)_R + d\sigma/dt)_L} \tag{4.32}$$

where

$$d\sigma/dt)_R = |a_0+a_1|^2 = a_0^2-a_1^2$$

$$d\sigma/dt)_L = |a_0-a_1|^2 = a_0^2-a_1^2 \quad .$$

Since a_1 is purely imaginary we find that

$$\varepsilon = 0 \quad . \tag{4.33}$$

This means that the polarization parameter for the scattering is zero; P=0.

The phase angle of the scattering amplitude is related to the rotation parameters A and R in the following way.

$$\begin{aligned} R &= -\sin 2\phi \\ A &= \cos 2\phi \end{aligned} \tag{4.34}$$

This can be seen by using the fact[15] that the parameters A, P, and R are related to the scattering amplitudes a_0 and a_1 by

$$A = \frac{|a_0|^2-|ia_1|^2}{|a_0|^2+|ia_1|^2}$$

$$P = \frac{2\mathrm{Im}[ia_0{}^*a_1]}{|a_0|^2+|ia_1|^2}$$

$$R = \frac{-2\mathrm{Re}[ia_0{}^*a_1]}{|a_0|^2+|ia_1|^2} \quad .$$

Let us examine the meaning of this rotation parameter, R. Writing the scattering amplitude a as

$$\begin{aligned} a &= a_0+a_1 \qquad e^{i\phi} = \frac{a_0+a_1}{|a_0+a_1|} \qquad (m=+\tfrac{1}{2}) \\ a &= a_0-a_1 \qquad e^{-i\phi} = \frac{a_0-a_1}{|a_0+a_1|} \qquad (m=-\tfrac{1}{2}) \end{aligned} \tag{4.35}$$

and using the fact that the target initially polarized in the $\hat{z}$ direction cannot have its polarization direction altered by a scattering in the $\hat{x}$-$\hat{y}$ plane, we find the initial states transformed as follows

$$\begin{aligned} \begin{pmatrix} 1 \\ 0 \end{pmatrix} &\to \begin{pmatrix} a_0+a_1 \\ 0 \end{pmatrix} \\ \begin{pmatrix} 0 \\ 1 \end{pmatrix} &\to \begin{pmatrix} 0 \\ a_0-a_1 \end{pmatrix} \quad . \end{aligned} \tag{4.36}$$

Consider now a target polarized initially in the $\hat{x}$ direction.

$$\chi_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{4.37}$$

It will be transformed into

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix} a_0+a_1 \\ a_0-a_1 \end{pmatrix} = \frac{|a_0+a_1|}{\sqrt{2}} \begin{pmatrix} e^{i\phi} \\ e^{-i\phi} \end{pmatrix} \simeq \frac{|a_0|}{\sqrt{2}} \begin{pmatrix} e^{i\phi} \\ e^{-i\phi} \end{pmatrix} \tag{4.38}$$

as a result of the scattering. This final state has polarization given by $(\cos 2\phi, -\sin 2\phi, 0)$. Thus the polarization vector is rotated through an angle 2ϕ about the normal to the scattering plane. This result is general, and can be shown to hold for any initial polarization. It is now clear that a measurement of the rotation parameter is equivalent to the knowledge of the angle through which the polarization vector of the target is rotated.

We are now in a position to determine the compressed current density from experimentally measurable quantities. We write the cross section

$$d\sigma/dt = \pi |a|^2 \tag{4.39}$$

$$|a|^2 = |a_0 + a_1|^2 = |a_0|^2 + |a_1|^2 + 2\mathrm{Re}(a_0 a_1^*) \ . \tag{4.40}$$

Since a_0 is purely real while a_1 is purely imaginary the last term in the above equation is identically zero. It is thus seen that to first order in Ω_1 (i.e., to first order in rotational velocity effects)

$$\frac{d\sigma}{dt} = \pi |a_0|^2 = \pi |\langle 1 - e^{-\Omega_0} \rangle|^2 \ . \tag{4.41}$$

This result does <u>not</u> depend upon rotation effects; it is the same result found in Chapter II where no rotation effects were considered. a_0 can be determined from the differential cross section data in the usual way. ϕ is determined from a measurement of the rotation parameter, $R(t)$. This may be obtained by a second scattering of the final state of the target. In the next chapter we will examine an alternative way of determining the rotation parameter.

Knowing the quantities ϕ and a_0, the following steps lead to the compressed current density.

$$a_1 = |a_0|e^{i\phi} - a_0$$

$$\langle a_1 \rangle = e^{-\Omega_0}\Omega_1$$

$$\Omega_1 = e^{\Omega_0}\langle a_1 \rangle$$

$$\rho^A \otimes j_y = -\langle a_1 \rangle e^{\Omega_0}/2\pi\alpha K \tag{4.42}$$

This integral equation for j_y may be formally solved since $\rho^A(\vec{b})$ is presumed to be known.

$$j_y = \left\langle \frac{\langle -\langle a_1 \rangle e^{\Omega_0}/2\pi\alpha K \rangle}{\langle \rho^A \rangle} \right\rangle \tag{4.43}$$

We also have the relation

$$j_y = \pm b_x g_1 \tag{4.44}$$

so that

$$g_1 = \pm j_y / b_x \tag{4.45}$$

where j_y is given by equation (4.43). Recall that g_1 is further related to g by

$$g_1(b) = \int_{-\infty}^{\infty} g(b, b_y)\, db_y \tag{4.46}$$

This integral equation for g may be solved as follows. Let

$$b \equiv (b_x^2+b_z^2)^{\frac{1}{2}} \qquad \text{and} \qquad r \equiv (b_x^2+b_y^2+b_z^2)^{\frac{1}{2}} \tag{4.47}$$

Then the integral equation has the form

$$g_1(b) = 2\int_b^\infty rg(r)(r^2-b^2)^{-\frac{1}{2}}dr \tag{4.48}$$

This equation is referred to in the literature as Abel's equation. It has the solution

$$g(r) = \frac{-1}{\pi r}\frac{d}{dr}\int_r^\infty \frac{bg_1(b)}{(b^2-r^2)^{\frac{1}{2}}}\,db \tag{4.49}$$

Finally then the matter current density is given by

$$\begin{aligned} j_x &= \mp b_y g(r) \\ j_y &= \pm b_x g(r) \\ j_z &= 0 \end{aligned} \tag{4.50}$$

It has been shown that a measurement of the differential cross section and the rotation parameter are sufficient to determine the matter current density of a polarized target.

CHAPTER V

APPLICATION OF MATTER CURRENT CONCEPT TO INELASTIC SCATTERING

As a final specialization of the geometrical model of hadronic collisions we consider the effect of the matter current on an inelastic scattering - in particular the exchange reaction $\pi^- p \rightarrow K^\circ \Lambda$

$$\text{weak} \; \llcorner\!\!\longrightarrow \pi^- p$$

As will be shown below, detection of the final state kaon and the decay proton will be sufficient to determine the matter current of the initial state proton. This method does not require the use of a second scattering to determine the rotation parameter as would an elastic scattering. The fact that the strong production process conserves parity while the weak decay process does not is responsible for this simplification. Bear in mind that at high energies the only manifestation of the target spin is through the non-zero matter current. In this treatment we consider only the spin non-flip and spin flip amplitudes due to the matter current to be non-zero. All spin effects other than the matter current effect are assumed to be unimportant at the energies we consider here. In general there may be other contributions to these amplitudes, but inclusion tends to obscure the essentials of this method.

The results of the previous chapter can clearly be specialized to the case under consideration by using the inelastic scattering amplitude results found in Chapter III. As this is the scattering reaction which will be used to illustrate the numerical estimates to be made in the next chapter, we give details of the method explicitly.

Consider the kinematics of the scattering to be those of the previous chapter.

a) target B infinitely heavy

b) incident momentum of A: $(0,k,0,ik)$

c) outgoing momentum of C: $(q_x,k,0,ik)$

d) target polarized with $J_z = m = \pm\frac{1}{2}$

Note that the limits $q_x \ll k$ and $M_A \ll k$ have been taken as before. As there is no momentum transfer in the $\hat{y}$-direction, the compressed density and current will again be of interest.

The differential cross section is given as usual by

$$d\sigma/dt = \pi|a|^2 \tag{5.1}$$

where

$$a = \langle S'(b)\rangle \tag{5.2}$$

The amplitude for occurrence of the process A B → C D is

$$S'(b) = C'\int_{-\infty}^{\infty} e^{-\Omega^{AB}_{-\infty\to b_y}(\vec{b})}\, e^{-\Omega^{CD}_{b_y\to\infty}(\vec{b})}\, [\rho^{AB}(\vec{b},b_y)\,\rho^{CD}(\vec{b},b_y)]^{\frac{1}{2}}db. \tag{5.3}$$

C' is a constant which depends on incident momentum. We assume that the density distributions of the initial and final states are approximately equal.

$$\rho^{AB}(\vec{b},b_y) \simeq \rho^{CD}(\vec{b},b_y) \tag{5.4}$$

Therefore

$$S'(b) = Ce^{-\Omega^{AB}_{-\infty\to\infty}(\vec{b})}\,\Omega^{AB}_{-\infty\to\infty}(\vec{b}) \tag{5.5}$$

C is a constant which depends on incident momentum. Note that it is the assumption of identical density distributions which allows S(b) to be expressed in this simple form.

The extension to the case of non-zero matter current is in the specification of the opaqueness including matter current effects. We again assume the effect of rotation upon the opaqueness which the target presents to a <u>point</u> projectile is approximated well by

$$\Omega^{AB}_{point}(\vec{b},b_y) = A'P_{eff}^{\alpha}\rho^{B}(\vec{b},b_y) \tag{5.6}$$

where P_{eff} is the momentum of the projectile in the rest frame of the point (b_x,b_y,b_z) of the target.

$$\Omega^{AB}_{point}(\vec{b},b_y) \doteq A'P_{inc}^{\alpha}[1-\alpha v_y]\rho^{B}(\vec{b},b_y) \tag{5.7}$$

Compressing this opaquaness yields

$$\Omega^{AB}_{point}(\vec{b}) = 2\pi A[\rho^{B}(\vec{b})-\alpha j_y(\vec{b})] \tag{5.8}$$

A and A' are constants which depend upon the incident momentum. To include the effect of the extended structure of the spinless ($\vec{j} = 0$) projectile we take the folding integral of Ω^{AB}_{point} with $\rho^A(b)$.

$$\Omega^{AB}_{-\infty\to\infty}(\vec{b}) = \Omega^{AB}_{point}(\vec{b}) \otimes \rho^A(\vec{b})$$

where (5.9)

$$\rho^A(\vec{b}) = \int_{-\infty}^{\infty} \rho^A(\vec{b},b_y)\,db_y.$$

Equations 5.1 - 5.9 specify the two-body inelastic exchange differential cross section in this geometrical model.

We now wish to show that in the case of a spin ½ target, measurements of the differential cross section and the rotation parameter are again sufficient to specify the matter current distribution. This is merely the extension of the results of the previous chapter to inelastic scattering.

Using the fact that the target and projectile both have definite parities, it is seen that the particle densities are even under the parity operation. Now, the density, in the case of the spin ½ target, is the product of two spin ½ wave functions thus yielding the density a scalar part plus a vector part. If the density is to be even under parity the density must be scalar and therefore spherically symmetric. The compressed densities are thus cylindrically symmetric. The most general form for the matter current matrix, remembering that it must transform as a vector under the parity operation, is

$$\vec{j} = f\vec{r}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - g\vec{r}\times\vec{\sigma} \qquad (5.10)$$

where $\vec{r}$, $\vec{\sigma}$, and $\vec{j}$ are three-dimensional vectors and f and g are functions of r. As shown in the previous chapter f must be zero if the hadronic current is to be conserved.

$$\vec{j} = -g\vec{r}x\vec{\sigma} \tag{5.11}$$

Since the exchange process is a hadronic process, rotation and reflection invariance prevent the introduction of any polarization components in the scattering plane as a result of the scattering. We may therefore concentrate on the diagonal elements ($m = \pm\frac{1}{2} \rightarrow m = \pm\frac{1}{2}$) of $\vec{j}$.

$$\begin{aligned} j_x &= \mp g b_y \\ j_y &= \pm g b_x \\ j_z &= 0 \end{aligned} \tag{5.12}$$

j_x, j_y, j_z are the components of the three-dimensional vector $\vec{j}$. The compressed currents are

$$\begin{aligned} j_x &= 0 \\ j_y &= \pm g_1 b_x \\ j_z &= 0 \end{aligned} \tag{5.13}$$

where

$$g_1(b) = \int_{-\infty}^{\infty} g(\vec{b}, b_y)\, db_y \tag{5.14}$$

and the two-dimensional vector $\vec{b}$ is the projection of the three-dimensional vector $\vec{r}$ onto the x-z-plane. The y-component of $\vec{r}$ is called b_y.

As the Fourier transforms of the density and current will be required below, we record them now.

$$\langle\rho\rangle = \frac{1}{2\pi} G_1^{had}(t)$$

$$\langle j_y\rangle = \mp \frac{iq_x}{2\pi} G_2^{had}(t) \qquad (-t=q_x{}^2+q_z{}^2) \tag{5.15}$$

$$\langle j_x\rangle = \langle j_z\rangle = 0$$

The hadronic form factors are defined as

$$G_1^{had}(t) = \int d^2b\rho(b)e^{-i\vec{q}\cdot\vec{b}}$$

$$G_2^{had}(t) = 2\pi(\text{cyl. symm. function}). \tag{5.16}$$

This is the same cylindrically symmetric function defined in equation (4.23). Note that both of these form factors are cylindrically symmetric, i.e., functions of t only.

The scattering amplitude may now be written as

$$a = K\langle e^{-\Omega^{AB}}\Omega^{AB}\rangle$$

$$\Omega^{AB} = 2\pi A[\rho^A(\vec{b}) \otimes \rho^B(\vec{b})-\alpha\rho^A(\vec{b}) \otimes j_y(\vec{b})] \tag{5.17}$$

$$\equiv \Omega_0^{AB} + \Omega_1^{AB}$$

where we define

$$\Omega_0^{AB} = 2\pi A\rho^A(\vec{b}) \otimes \rho^B(\vec{b})$$

$$\Omega_1^{AB} = -2\pi A\rho^A(\vec{b}) \otimes j_y(\vec{b})\alpha. \tag{5.18}$$

A and K are constants which depend on the incident momentum.

If we write the scattering amplitude in terms of Ω_0 and Ω_1 and expand the exponential

$$a(q_x,q_z) = K\langle e^{-(\Omega_0+\Omega_1)}(\Omega_0+\Omega_1)\rangle \tag{5.19}$$

$$a(q_x,q_z) \simeq K\langle \Omega_0 e^{-\Omega_0}+\Omega_1 e^{-\Omega_0}-\Omega_0\Omega_1 e^{-\Omega_0}\rangle$$

$$= K[\langle \Omega_0 e^{-\Omega_0}\rangle+\langle(1-\Omega_0)\Omega_1 e^{-\Omega_0}\rangle] \tag{5.20}$$

$$= a_0+a_1 \quad .$$

The definitions

$$a_0 = K\langle \Omega_0 e^{-\Omega_0}\rangle$$
$$a_1 = K\langle \Omega_1 (1-\Omega_0) e^{-\Omega_0}\rangle \tag{5.21}$$

have been made. Note that a_0 is purely real while a_1 is purely imaginary, as in the elastic case of the previous chapter.

The Fourier transforms of Ω_0 and Ω_1 may now be written

$$\langle \Omega_0^{AB}\rangle = \frac{1}{2\pi} AG_{1A}^{had}(t)G_{1B}^{had}(t)$$
$$\langle \Omega_1^{AB}\rangle = \frac{\pm iq_x\alpha A}{2\pi} G_{1A}^{had}(t)G_{2B}^{had}(t) \quad . \tag{5.22}$$

and so, as in the elastic case, a measurement of a_1 for $q_z = 0$ and all values of $q_x \geq 0$ will determine a_1 for all values of q_x and q_z.

The left-right asymmetry of the production process is zero; $\varepsilon = 0$. Therefore the polarization parameter for the production process is zero; $P = 0$. The phase of the scattering amplitude is

$$e^{i\phi} = \frac{a}{|a|} = \frac{a_0+a_1}{|a_0+a_1|} \simeq \frac{a_0+a_1}{|a_0|} + \theta(a_1^2). \qquad (5.24)$$

As shown in Chapter IV this angle ϕ is precisely the angle involved in the rotation parameters A and R.

$$A = \cos 2\phi$$
$$R = -\sin 2\phi \qquad (5.25)$$

2ϕ is the angle about the normal to the scattering plane through which the polarization vector is rotated.

We now use the weak decay properties of the Λ produced in the exchange scattering $\pi^- p \to K^\circ \Lambda$ to determine the rotation parameter, or more precisely the angle 2ϕ. The Λ decays via the following channels.

$$\Lambda \xrightarrow{\text{weak}} \begin{cases} \pi^- p & (64.2\%) \\ \pi^\circ n & (35.8\%) \end{cases} \qquad (5.26)$$

We concentrate on the first of these decay modes.[17] Merely for the purposes of examining the decay of the Λ, let us choose a coordinate system in which the Λ is initially polarized along the z-direction. In the rest system of the $\Lambda, j_z = s_z = \pm\frac{1}{2}$. <u>If</u> parity were conserved in the decay, and the Λ had even (odd) parity with respect to the proton, then the final $\pi^- p$ state would have to be in an $\ell=1$ ($\ell=0$)

state of relative orbital angular momentum. However, if parity is not conserved in the decay of the Λ, then both the $\ell = 1$ and the $\ell=0$ state occur with amplitudes a_p and a_s.

For $\ell=0, j_z=\pm\frac{1}{2}$ the wave function is

$$Y_0^0(\cos\theta)\chi_{\pm\frac{1}{2}} \quad .$$

For $\ell=1, j_z=\pm\frac{1}{2}$ the wave function is

$$\begin{aligned} &\langle 1\pm1\tfrac{1}{2}\mp\tfrac{1}{2}|\tfrac{1}{2}\pm\tfrac{1}{2}\rangle Y_1^{\pm1}(\cos\theta)\chi_{\pm\frac{1}{2}} \\ &+\langle 10\tfrac{1}{2}\pm\tfrac{1}{2}|\tfrac{1}{2}\pm\tfrac{1}{2}\rangle Y_1^{0}(\cos\theta)\chi_{\pm\frac{1}{2}} \quad . \end{aligned} \tag{5.27}$$

The final state wave function assuming a weak (parity non-conserving) decay is thus

$$\psi_{\pm\frac{1}{2}} = \frac{1}{\sqrt{4\pi}}\{a_s\chi_{\pm\frac{1}{2}} - a_p\sin\theta e^{\pm i\phi}\chi_{\mp\frac{1}{2}} \mp a_p\cos\theta\chi_{\pm\frac{1}{2}}\} \tag{5.28}$$

where $\pm$ refers to the initial Λ polarization. The amplitude in the $\hat{z}$-direction is

$$\frac{1}{\sqrt{4\pi}}(a_s \mp a_p)\chi_{\pm\frac{1}{2}} \quad .$$

Then if the Λ is initially polarized along some direction making angles of $\theta_i\phi_i$ with the $\hat{z}$-direction

$$\chi_{\text{initial}} = \begin{pmatrix} \cos\theta_i/2 \quad e^{i\phi_i/2} \\ \\ \sin\theta_i/2 \quad e^{-i\phi_i/2} \end{pmatrix} \tag{5.29}$$

the final state amplitude in the $\hat{z}$-direction is given by

$$\frac{1}{\sqrt{4\pi}}[(a_s-a_p)\cos\frac{\theta_i}{2}e^{i\phi_i/2}\chi_{+\frac{1}{2}}+(a_s+a_p)\sin\theta_i/2e^{-i\phi_i/2}\chi_{-\frac{1}{2}}] \ .$$

The differential cross section at an angle θ_i to the Λ polarization direction is

$$d\sigma/d\Omega|_{\theta_i} = \frac{1}{4\pi}[|a_s|^2+|a_p|^2-2\mathrm{Re}(a_s{}^*a_p)\cos\theta_i] \quad (5.30)$$

$$d\sigma/d\Omega|_{\theta_i} \propto 1+A\cos\theta_i \quad (5.31)$$

where the asymmetry parameter A is defined to be

$$A = -2\mathrm{Re}(a_s{}^*a_p)/[|a_s|^2+|a_p|^2] \quad . \quad (5.32)$$

Experimentally[17] it is found that this parameter A= 0.642, and so decay protons are emitted preferentially in the direction of the initial Λ polarization. The method for determining the angle 2ϕ is now clear. Given the initial polarization direction of the target proton, a measurement of the differential cross section of the decay protons specifies the polarization direction of the Λ before decay. 2ϕ is then the angle between these two polarization directions.

The matter current may now be found as follows. The differential cross section of the final state kaons is given by

$$d\sigma/dt = \pi|a|^2 = \pi[|a_0|^2+|a_1|^2]$$
$$d\sigma/dt \simeq \pi|a_0|^2+\theta(a_1{}^2) \quad . \quad (5.33)$$

Thus a measurement of this cross section yields a_0. This result is independent of rotation effects. The phase angle is determined as described in the preceding paragraph by a measurement of the differential cross section of decay protons.

$$
\begin{aligned}
a_1 &= |a_0|e^{i\phi}-a_0 \\
\langle a_1\rangle &= K\Omega_1(1-\Omega_0)e^{-\Omega_0} \\
K\Omega_1 &= +\langle a_1\rangle e^{\Omega_0}/(1-\Omega_0) \\
K\rho^A(b)\otimes j_y(\vec{b}) &= -\langle a_1\rangle e^{\Omega_0}[2\pi A\alpha(1-\Omega_0)]^{-1}
\end{aligned}
\tag{5.34}
$$

Since the density $\rho^A(b)$ is known, we solve the integral equation for $j_y(\vec{b})$. K and A are constants which depend on incident momentum.

$$
j_y(\vec{b}) = \left\langle \frac{\dfrac{-\langle a_1\rangle e^{\Omega 0}}{2\pi\alpha A(1-\Omega_0)K}}{\langle\rho^A(b)\rangle} \right\rangle \tag{5.35}
$$

Now $$j_y(\vec{b}) = \pm g_1 b_x$$

so that $$g_1 = \pm j_y(\vec{b})/b_x$$

where $j_y(\vec{b})$ is given in equation (5.35). g_1 and g are related by

$$
g_1(b) = \int_{-\infty}^{\infty} g(\vec{b},b_y)\,db_y \tag{5.36}
$$

As in the previous chapter we let

$$
b = (b_x{}^2+b_z{}^2)^{\frac{1}{2}} \quad \text{and} \quad r = (b_x{}^2+b_z{}^2+b_y{}^2)^{\frac{1}{2}}
$$

Then

$$g(r) = \frac{-1}{\pi r} \frac{d}{dr} \int_r^\infty \frac{b g_1(b)}{(b^2 - r^2)^{\frac{1}{2}}} db \tag{5.37}$$

and the matter current is given by

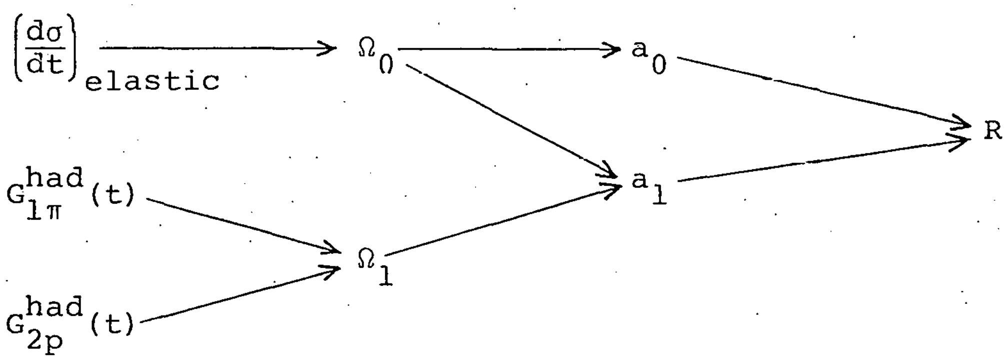
$$\begin{aligned} j_x &= \mp g(r) b_y \\ j_y &= \pm g(r) b_x \\ j_z &= 0 \end{aligned} \tag{5.38}$$

The hadronic matter current may be determined by considering the exchange scattering $\pi^- p \rightarrow K^\circ \Lambda$ and measuring the differential cross sections of the final state kaon and the decay proton.

CHAPTER VI

NUMERICAL ESTIMATES

In this chapter we attempt a quantitative estimate of the rotation parameter, R, using the geometrical model applied to the inelastic exchange process considered in the previous chapter. The general calculational scheme is shown below.



As input we take π^-p elastic differential cross section data at 200 GeV, and the form factors of the pion and proton. The results of this calculation are the real and imaginary parts of the exchange scattering amplitude, as well as the rotation parameter, R, for the process $\pi^-p \to K^\circ\Lambda$.

Starting with the π^-p elastic differential cross section data at 200 GeV, we proceed to calculate Ω_0. A fit to this data has been given by C. W. Akerlof <u>et al</u>., in reference 18, for $|t|<1.15$ $(\mathrm{GeV/c})^2$.

$$\left(\frac{d\sigma}{dt}\right)_{el} = (30.12\times10^3)\exp(9.26t+2.0t\ )\,[\mu b(\mathrm{GeV/c})^{-2}]$$

$$a_{el} = (4.962)\exp(4.63t+1.0t^2)\,[(\mathrm{GeV/c})^{-2}] \tag{6.1}$$

For large values of $-t$ there is no available data and so it is assumed that the cross section falls off exponentially, i.e.,

$$a_{el} = Ae^{bt} \quad . \tag{6.2}$$

Joining these two forms for the scattering amplitude smoothly at $-t=1.15$ $(\mathrm{GeV/c})^2$ yields

$$a_{el} = (1.322)\exp(2.33t)\,[(\mathrm{GeV/c})^{-2}] \tag{6.3}$$

for $|t|\geq1.15$ $(\mathrm{GeV/c})^2$. The complete form of a_{el} for use in our calculations is thus

$$\begin{aligned} a_{el} &= (4.962)\exp(4.63t+1.0t^2)\,(\mathrm{GeV/c})^{-2} \\ &\qquad\qquad \text{for } 0\leq-t\leq1.15\ (\mathrm{GeV/c})^2 \\ &= (1.322)\exp(2.33t)\ (\mathrm{GeV/c})^{-2} \\ &\qquad\qquad \text{for } -t\geq1.15\ (\mathrm{CcV/c})^2\ . \end{aligned} \tag{6.4}$$

Recall from equation 4.29 that the opaqueness, Ω_0, and elastic scattering amplitude, a_{el}, are related by

$$a_{el} = \langle 1-e^{-\Omega_0}\rangle \tag{6.5}$$

where < > denotes the two-dimensional Fourier transform. Ω_0 is thus obtainable by inverting equation (6.5).

$$1-e^{-\Omega_0} = \int_0^\infty J_0(b\sqrt{-t})\, a_{el} \sqrt{-t}\; d\sqrt{-t} \qquad (6.6)$$

The two-dimensional Fourier transform has been reduced to a Fourier-Bessel transform by invoking the cylindrical symmetry of a_{el}. A plot of Ω_0 vs. b is given in Figure 6.1.

The real part of the inelastic scattering amplitude may now be calculated using equation 5.21).

$$a_0 = K\langle\Omega_0 e^{-\Omega_0}\rangle \qquad (6.7)$$

Since Ω_0 is a function of b only, we may again use the Fourier-Bessel form of the two-dimensional Fourier transform. Thus

$$a_0 = K \int_0^\infty J_0(b\sqrt{-t})\,\Omega_0 e^{-\Omega_0} b\,db \quad . \qquad (6.8)$$

As can be seen in Fig. 6.1, Ω_0 is appreciable only for $b \le 15\ (\text{GeV/c})^{-1}$, and so the infinite upper limit is replaced for computational purposes by 15 $(\text{GeV/c})^{-1}$. This corresponds to approximately 3 fm.

The constant K can be determined by the relation

$$\left.\frac{d\sigma}{dt}\right)_{exchange}\Bigg|_{t=0} = \pi K^2 \left[\langle\Omega_0 e^{-\Omega_0}\rangle\big|_{t=0}\right]^2 \qquad (6.9)$$

The value of $\left.\frac{d\sigma}{dt}\right)_{exch}$ for t=0 at 200 GeV is not available, but may be estimated by extrapolating the existing data.[19] In fact, $\left.\frac{d\sigma}{dt}\right)_{exch}$ for t=0 has been observed to fall off with

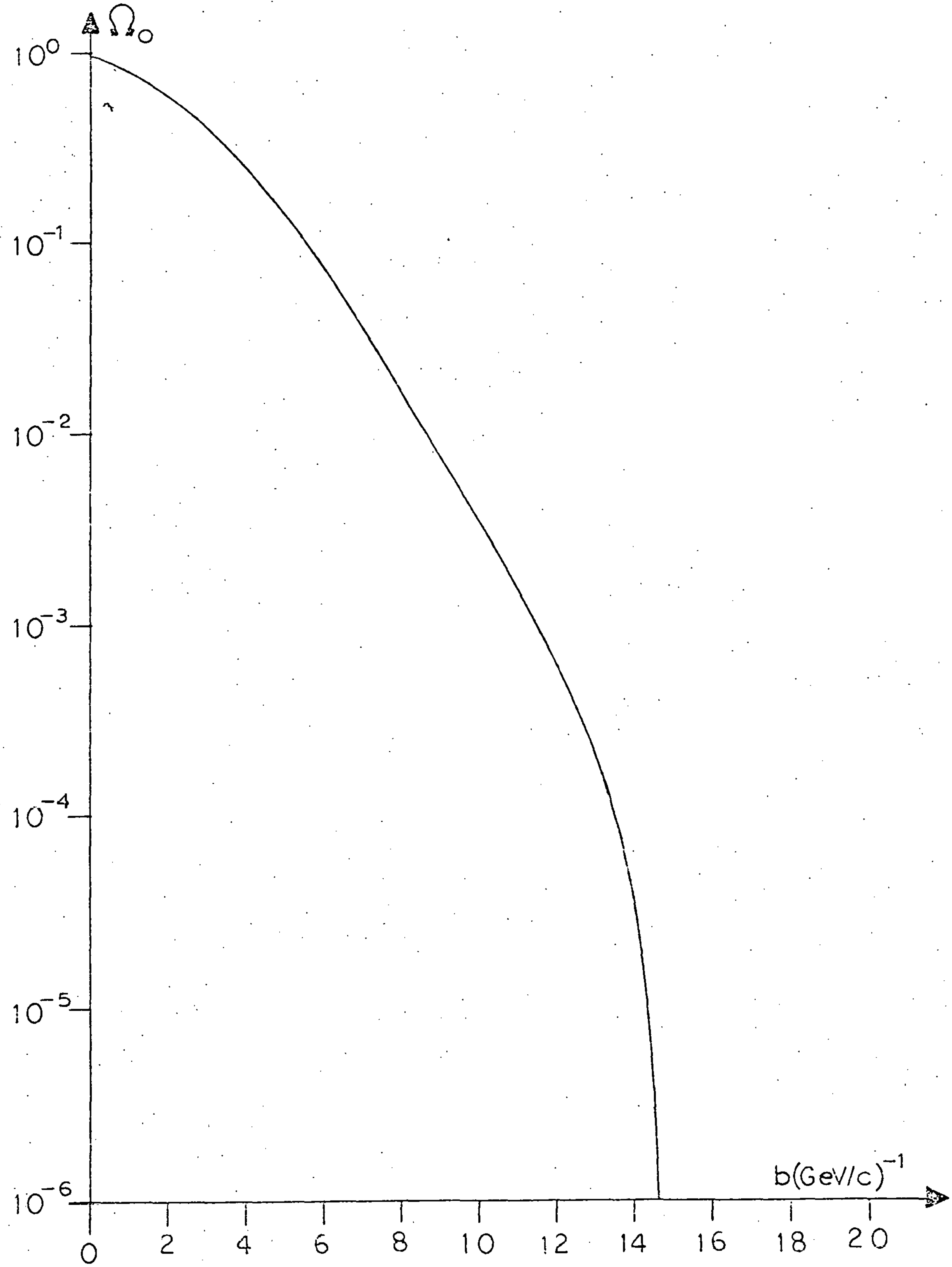


Figure 6.1. The opaqueness versus impact parameter (Ω_0 vs. b).

incident momentum according to a power law. See Figure 6.2.

$$\log \frac{d\sigma}{dt}\bigg)_{exch} \bigg|_{t=0} = A\log(P_{beam}) + B \qquad (6.10)$$

where the constants A and B are determined by a fit to the data yielding

$$A = -1.184$$

$$B = 3.21 \ .$$

$\frac{d\sigma}{dt}\Big)_{exch}$ for t=0 has units of $\mu b(GeV/c)^{-2}$ and P_{beam} has units of (GeV/c). An evaluation of equation (6.10) at P_{beam}=200 (GeV/c) yields

$$\frac{d\sigma}{dt}\bigg)_{exch} \bigg|_{t=0} = 3.06 \ \mu b(GeV/c)^{-2}$$

or (6.11)

$$a_{exch} \bigg|_{t=0} = 5\times10^{-2} \ (GeV/c)^{-2}$$

K is thus determined from equation 6.9 to be

$$K = .0121 \ . \qquad (6.12)$$

The evaluation of a_0 according to equation 6.8 is now carried out using Simpson's rule approximation of the integral. The results are shown in Figure 6.3, and tabulated in Table I.

We now wish to calculate a_1, the imaginary part of the exchange scattering amplitude, and so must first estimate Ω_1, the rotation-dependent part of the opaqueness by equation 5.18.

$$\Omega_1 = -2\pi A\alpha\rho^{\pi^-}(b) \otimes j_y(\vec{b}) \qquad (6.13)$$

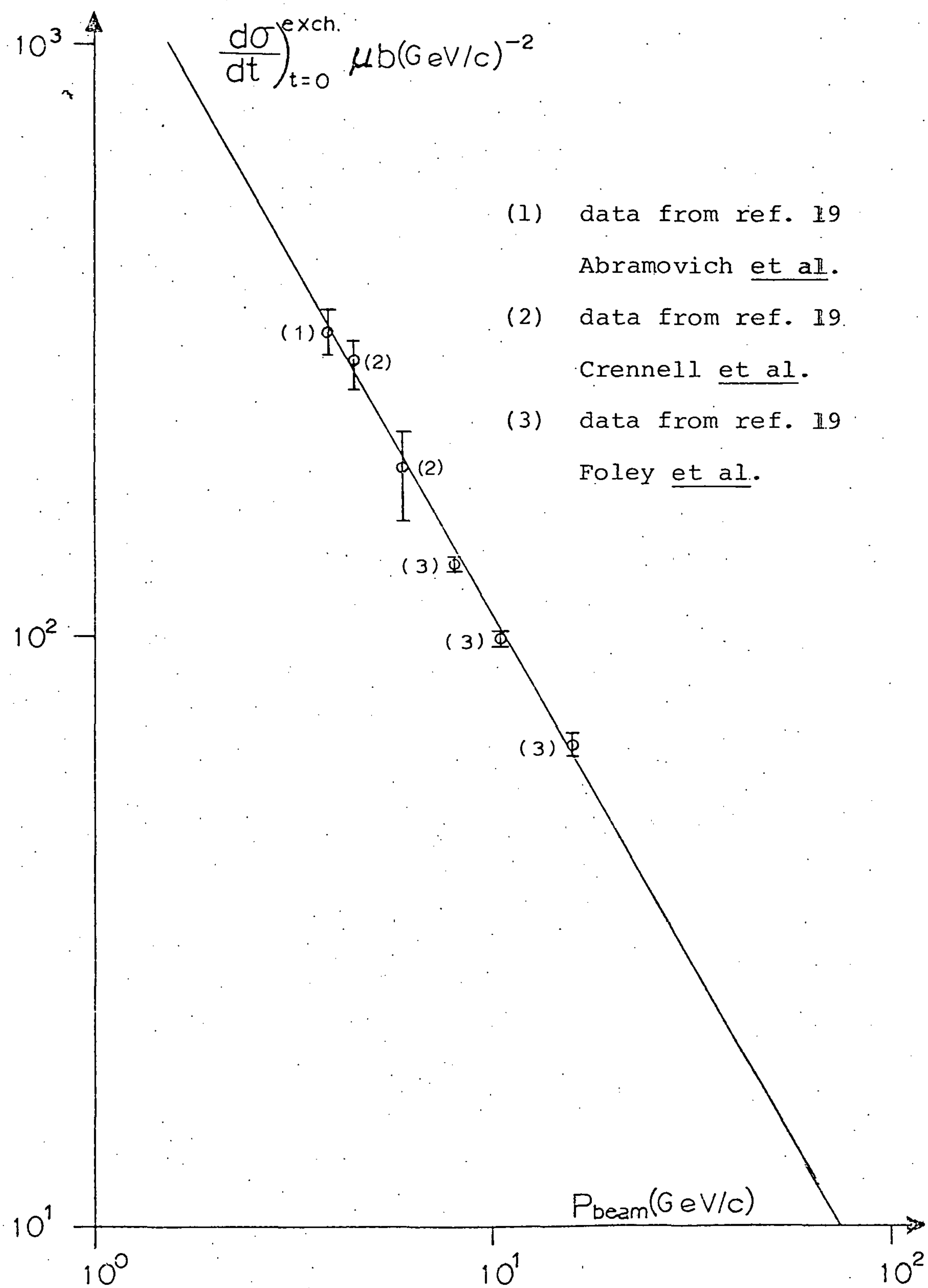


Figure 6.2. Differential cross section for the exchange reaction $\pi^- p \to K^\circ \Lambda$.

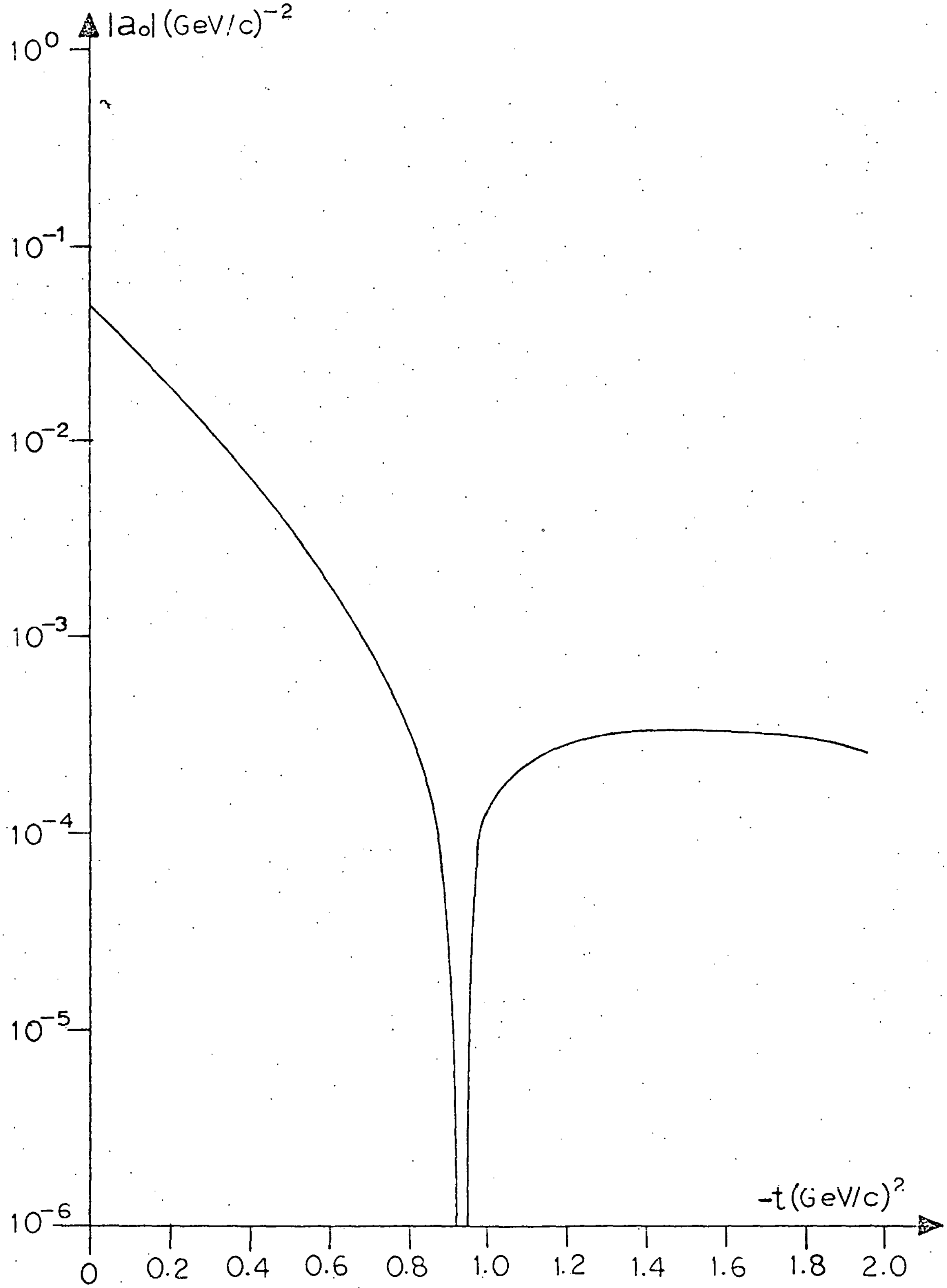


Figure 6.3. Real part of scattering amplitude versus momentum transfer (a_0 vs. $-t$).

If this equation is Fourier transformed the result is

$$\langle\Omega_1\rangle = \pm\frac{iq_x}{2\pi}\,\alpha A G_{1\pi}^{had}(t)\,G_{2p}^{had}(t)\,. \tag{6.14}$$

All factors on the right side of this equation are known. α is a constant which at 200 GeV has the value

$$\alpha = 0.03(\mathrm{GeV/c})^{-1}. \tag{6.15}$$

$G_{1\pi}^{had}$, the hadronic form factor for the pion, is

$$G_1^{had}(t) = \frac{1}{1+|t|/.602} \quad (-t \text{ in } (\mathrm{GeV/c})^2). \tag{6.16}$$

G_{2p}^{had}, the hadronic form factor for the proton, is assumed to have the dipole form

$$G_{2p}^{had}(t) = \left(\frac{1}{1+|t|/.71}\right)^2 \quad (-t \text{ in } (\mathrm{GeV/c})^2). \tag{6.17}$$

The constant A is evaluated using equation 5.22.

$$\langle\Omega_0\rangle = \frac{1}{2\pi}AG_{1\pi}^{had}G_{2p}^{had}\,. \tag{6.18}$$

At t=0 both form factors are equal to one, thus

$$\begin{aligned} A &= 2\pi\langle\Omega_0\rangle_{t=0} \\ &= \left[2\pi\int_0^\infty J_0(b\sqrt{-t})\,\Omega_0\,b\,db\right]_{t=0} \qquad (6.19) \\ &= 37.4\ (\mathrm{GeV/c})^{-2}. \end{aligned}$$

Taking the inverse Fourier transform of equation 6.14 we have

$$\hat{\Omega}_1 = \frac{\pm\alpha A}{2\pi} \frac{\partial}{\partial b_x} \int_{-\infty}^{\infty}\int \frac{1}{2\pi} G_1^{had}(t)\, G_{2p}^{had}(t)\, e^{i(q_x b_x + q_z b_z)}\, dq_x dq_z \tag{6.20}$$

$$= \frac{\pm\alpha A}{2\pi} \frac{\partial}{\partial b_x} G(b)$$

where the two-dimensional integral is defined as G(b). G(b) can be evaluated numerically, and to facilitate later calculations G(b) is fit with a sum of Gaussians.

$$G(b) = A_1 e^{-c_1 b^2} + A_2 e^{-c_2 b^2} + A_3 e^{-c_3 b^2} - A_4 e^{-c_4 b^2}\ (\mathrm{GeV/c})\,. \tag{6.21}$$

$A_1 = 0.014$	$C_1 = 0.0257$
$A_2 = 0.098$	$C_2 = 0.0761$
$A_3 = 0.0657$	$C_3 = 0.295$
$A_4 = 0.01$	

b^2 has units of $(\mathrm{GeV/c})^{-2}$.

Now define the function H(b) by

$$H(b) \equiv (1-\Omega_0)\, e^{-\Omega_0}\,. \tag{6.22}$$

This function is also fit by a sum of Gaussians

$$H(b) = 1 - A_5 e^{-c_5 b^2} + A_6 e^{-c_6 b^2}\,. \tag{6.23}$$

$A_5 = 0.99$	$C_5 = 0.056$
$A_6 = 0.014$	$C_6 = 0.566$

b^2 again has units of $(\mathrm{GeV/c})^{-2}$.

The imaginary part of the scattering amplitude can now be calculated.

$$a_1 = K\langle \Omega_1 (1-\Omega_0) e^{-\Omega_0} \rangle$$

$$= \frac{\pm K\alpha A}{(2\pi)^2} \iint_{-\infty}^{\infty} \left(\frac{\partial}{\partial b_x} G(b) \right) H(b) e^{-iq_x b_x} db_x db_z \ . \tag{6.24}$$

This integration can be done analytically with the results shown in Figure 6.4 and tabulated in Table I.

Finally the spin-rotation angle 2ϕ (i.e., the angle through which the polarization of the target is rotated), and the rotation parameter, R, may be calculated.

$$2\phi = 2\tan^{-1}\left(\frac{-ia_1}{a_0}\right) \tag{6.25}$$

$$R = -\sin 2\phi \tag{6.26}$$

The rotation parameter is shown in Figure 6.5 as well as in Table I.

It has been shown in this chapter that by use of $\pi^- p$ elastic cross section data, an estimate of the rotation parameter in the exchange scattering $\pi^- p \to K^\circ \Lambda$ can be made. It should be noted that the order of magnitude of the rotation parameter calculated in this way is in agreement with that of reference 5.

An alternative test of the matter current idea can be made by performing an experiment in which pions are scattered off a polarized proton target whose initial polarization direction is known. Detection of the angular distribution of hyperon decay products as a function of momentum transfer gives an experimental value of the rotation parameter to be compared with the result of this chapter. The range of momentum transfer over which such an experiment might be feasible is bounded by two criteria: (1) $-t$ must be large enough so that the rotation angle 2ϕ is measurable and (2) $-t$ must be small enough so that the cross section for the exchange process is not too small. Recall that at 200 GeV the cross section at $t = 0$ is only 3 $\mu b(GeV/c)^{-2}$. The cross section at higher t values is approximately given by

$$\left.\frac{d\sigma}{dt}\right)_{exch} = \frac{d\sigma}{dt}\Big|_{exch}^{t=0} e^{bt}$$

where $b \approx 8.5\ (GeV/c)^{-2}$. Therefore at $-t = .5\ (GeV/c)^2$ the cross section has already dropped by a factor of approximately 100 from its $t = 0$ value. For $-t \gtrsim .1\ (GeV/c)^2$ the rotation angles predicted in this chapter are readily measurable. The optimum experimental range should thus be roughly $0 \lesssim -t \lesssim .5\ (GeV/c)^2$.

In summary, it has been shown that the existence of an hadronic matter current in this model is detectable in the form of the rotation parameter. Two possible methods for its measurement are the second scattering and the weak decay of the Λ. The advantage of the second scattering method is the relatively large cross section. It suffers

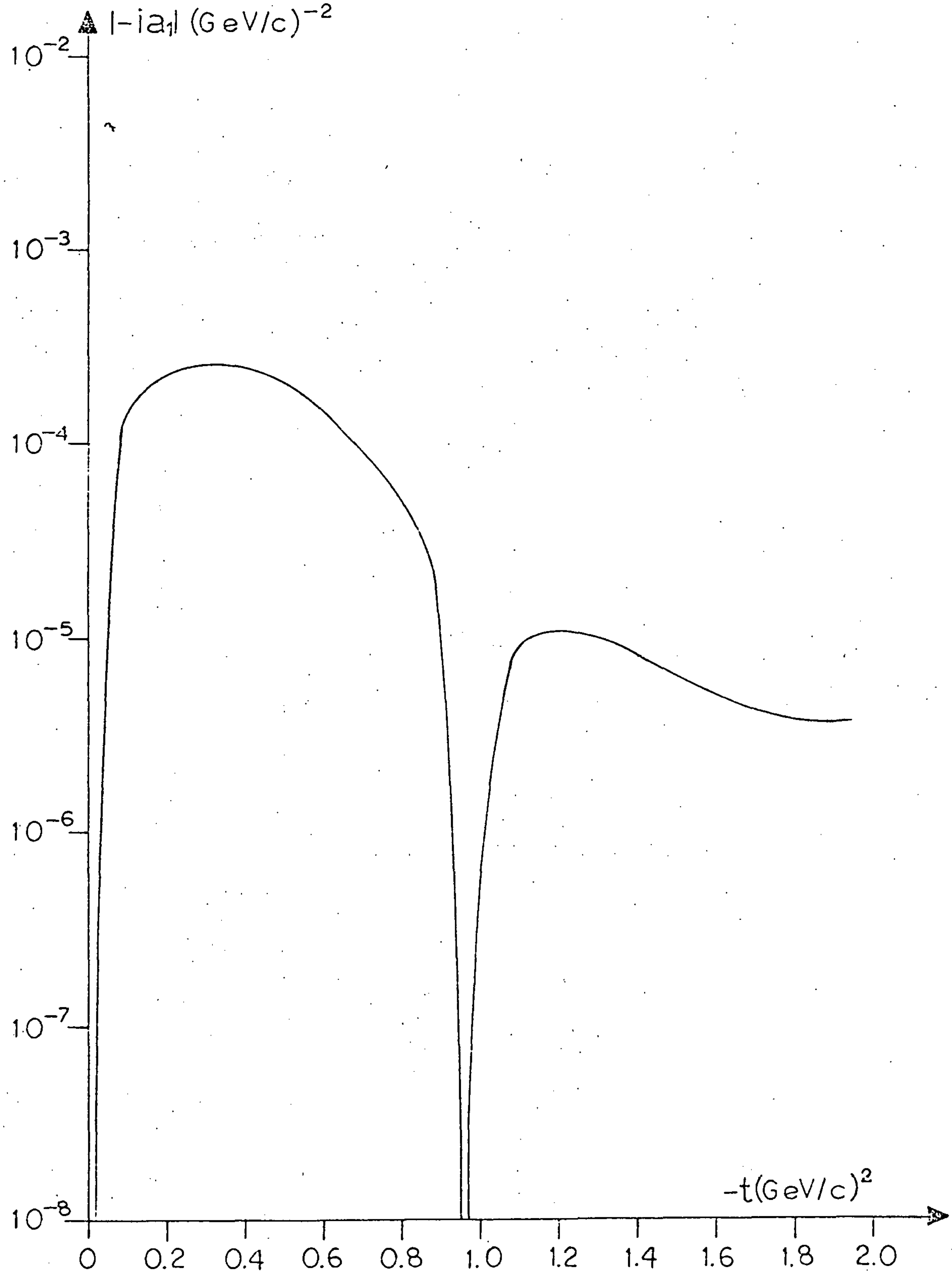


Figure 6.4. Imaginary part of scattering amplitude versus momentum transfer (a_1 vs. $-t$).

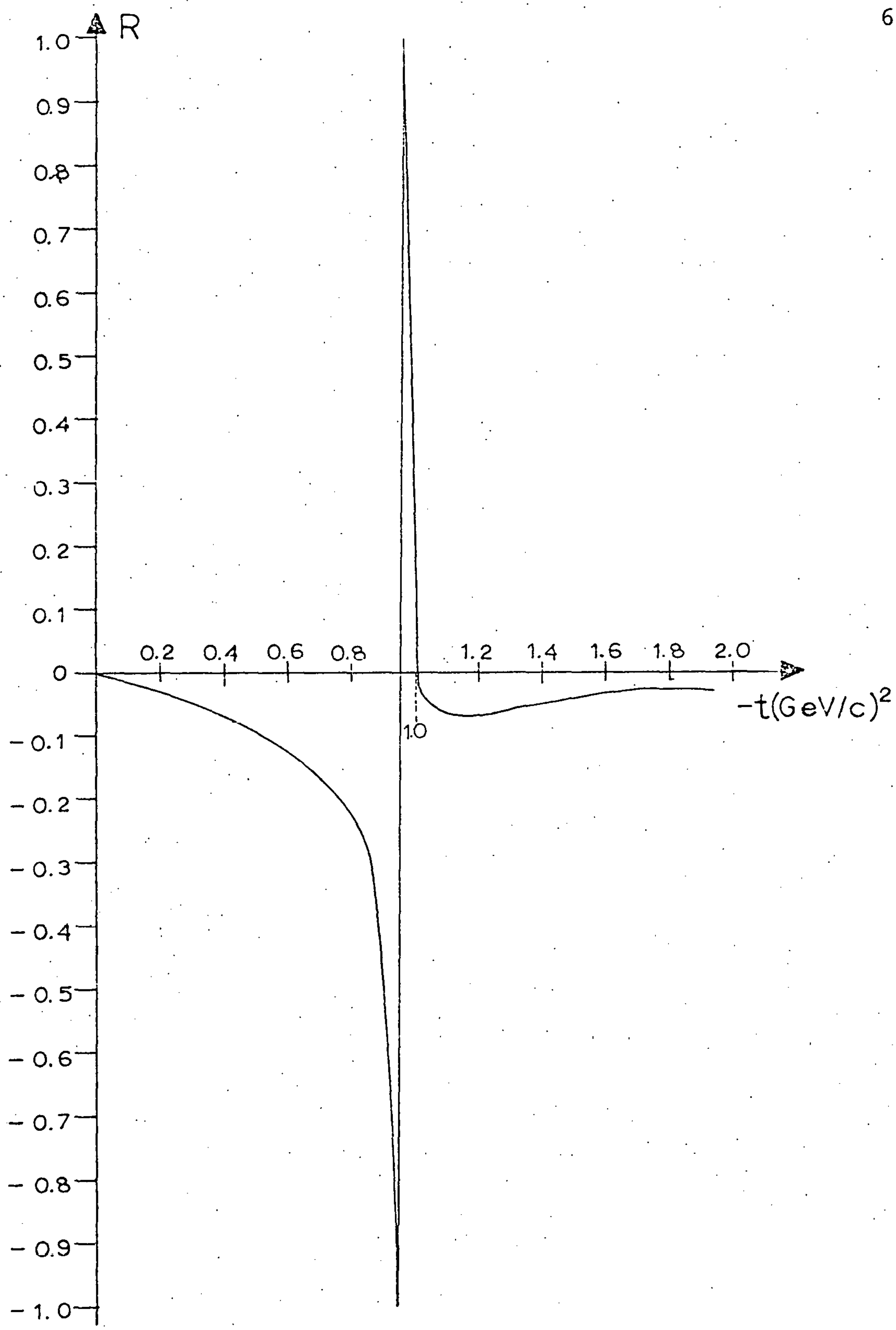


Figure 6.5. Rotation parameter versus momentum transfer in $\pi^- p \to K^0 \Lambda$ collision at 200 GeV (R vs. −t).

TABLE I

Scattering amplitude, rotation angle, and rotation parameter for $\pi^- p \rightarrow K^\circ \Lambda$ collision

$-t$	$-ia_1$	a_0	2ϕ(degrees)	$R=-\sin 2\phi$
0.0	0.0	5×10^{-2}	0.0	0.0
0.1	1.44×10^{-4}	2.99×10^{-2}	0.55	-0.01
0.2	2.41×10^{-4}	1.79×10^{-2}	1.54	-0.03
0.3	2.73×10^{-4}	1.07×10^{-2}	2.93	-0.05
0.4	2.51×10^{-4}	6.32×10^{-3}	4.55	-0.08
0.5	2.01×10^{-4}	3.65×10^{-3}	6.30	-0.11
0.6	1.42×10^{-4}	2.01×10^{-3}	8.09	-0.14
0.7	8.83×10^{-5}	1.01×10^{-3}	9.96	-0.17
0.8	4.58×10^{-5}	4.15×10^{-4}	12.59	-0.22
0.9	1.64×10^{-5}	6.26×10^{-5}	29.23	-0.49
1.0	-8.70×10^{-7}	-1.39×10^{-4}	0.72	-0.01
1.1	-8.68×10^{-6}	-2.49×10^{-4}	3.99	-0.07
1.2	-1.07×10^{-5}	-3.06×10^{-4}	3.98	-0.07
1.3	-9.76×10^{-6}	-3.32×10^{-4}	3.37	-0.06
1.4	-7.94×10^{-6}	-3.40×10^{-4}	2.67	-0.05
1.5	-6.27×10^{-6}	-3.39×10^{-4}	2.12	-0.04
1.6	-5.10×10^{-6}	-3.31×10^{-4}	1.77	-0.03
1.7	-4.39×10^{-6}	-3.19×10^{-4}	1.58	-0.03
1.8	-3.97×10^{-6}	-3.05×10^{-4}	1.49	-0.03
1.9	-3.66×10^{-6}	-2.88×10^{-4}	1.45	-0.03

from the difficulty of performing the second scattering. The weak decay method is experimentally more easily performed, however the cross section for $\pi^- p \to K^\circ \Lambda$ is very small. However, the weak decay method is probably preferably since the small cross section can, to some extent, be compensated by increased beam intensity.

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