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PROPAGATION OF NORMAL ZONES OF FINITE SIZE IN LARGE, COMPOSITE SUPERCONDUCTORS*

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Abstract

Very large, composite superconductors have been proposed for use in energy storage magnets. A typical conductor, rated at 230 kA, has been discussed by J. Waynert, Y. Eyssa, and X. Huang, namely, a 6-cm aluminum cylinder with superconducting filaments on its outer surface. Owing to its large size and nonuniform distribution of filaments, such a conductor can sustain normal zones of finite size that travel at a uniform velocity along the conductor. This paper presents a simple, analytical model that permits determination of the conditions under which such zones can exist and the size and velocity of such zones. It has been shown that the transport current has a threshold value below which finite normal zones cannot exist and that the propagation velocity corresponding to this threshold current, though not zero, is the smallest possible.

Introduction

Very large, composite superconductors have been proposed for use in superconducting energy storage magnets. A typical conductor is that discussed by Waynert, Eyssa, and Huang [1], namely, a 6.0-cm-diam aluminum (RRR of ~500) cylinder with superconducting filaments on the outer surface. The conductor is rated at a current I of 230 kA. Even if such a conductor is cryostable when the current uniformly fills the

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aluminum, it may nevertheless suffer the propagation of normal zones of finite size. The reason is as follows.

When the superconductor is first normalized, the current spilling into the aluminum is tightly confined to the vicinity of the outer surface. The Joule power is then very high. Thereafter, the current diffuses radially, tending toward a state of uniform current density in the conductor. In this state, the Joule power may be quite low and the conductor may be cryostable. The relaxation time of the current redistribution is $t_d = \mu_0 R^2/\alpha_1^2 \rho$, where μ_0 is the magnetic permittivity of free space ($4\pi \times 10^{-7}$ H/m), R is the radius of the conductor, ρ is the residual resistivity of the aluminum, and α_1 (= 3.832...) is the first zero of the Bessel function of the first kind of order one. During redistribution of the current from the outer surface to uniformity, an excess heat density, $Q = 6.333 \times 10^{-3}\ \mu_0 I^2/R^2$, is produced. If it is large enough, this excess heat density can sustain a normal zone of finite size.

Because the current redistribution energy is released over a short interval of approximate duration t_d following normalization of the filaments, current diffusion is complete far behind the propagating front. If the steady Joule power is low enough, the normal zone may recover far behind the front. At the front, on the other hand, the normal zone advances and normalizes additional lengths of superconductor. Thus, we are led to expect normal zones of finite size that propagate at velocities that cannot be arbitrarily small. These zones should make their first appearance at a threshold determined by the magnitude and time constant of the excess heat density, among other things.

In this paper, I analyze a simple model problem that exhibits the features just described and that can be solved explicitly.

The Model Problem

Figure 1 shows the temperature rise in a finite-size normal zone as a function of distance z along the conductor. The zone is presumed to travel to the right without change of shape with a steady velocity v. (This presumption is without justification; it is made to determine under what conditions a stationary normal zone of finite size could exist, and it begs entirely the question of whether such zones in fact exist.) The origin of the z-coordinate has been chosen so that the locus of the point on the leading edge at which $T = T_c$, the critical temperature, is $z = vt$, where t is the time. Current sharing is ignored, and the conductor is assumed to be unconditionally cryostable when the current is uniformly distributed throughout the aluminum. The heat balance over a length dz of conductor can now be written

$$A\,dz\,S\frac{\partial T}{\partial t} = A\,dz\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) - P\,dz\,h\,(T - T_b) + A\,dz\,[QF\,(t - z/v) + Q_J]\,U\,(t - z/v) = 0 \quad (1)$$

where A is the cross-sectional area of the aluminum, S is the heat capacity per unit volume of aluminum, P is the cooled perimeter, h is the heat transfer coefficient, Q (as before) is the excess heat density produced during current redistribution, Q_J is the Joule power density when the current is uniformly distributed throughout the aluminum, and U is Heaviside's step function. The function $F(t)$ prescribes how the excess heat Q at a point is released over the course of time. The origin of time in $T(t)$ is, of course, the instant the superconductor goes normal. Thus F must appear in Eq. (1) as a function of the argument $t - z/v$ since the superconductor at point z goes normal at time $t = z/v$. [Another argument leading to the same conclusion is this: if a traveling-wave solution for $T - T_b$ exists, so that $T - T_b$ is a function of $x = z - vt$ only, then Eq. (1) implies that for this solution F must also be a function of $x = z - vt$.] For points ahead of the normalizing front, the last term on the left-hand side of Eq. (1) is absent.

For a traveling-wave solution, which is what we are seeking, Eq. (1) then becomes

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) + vS\frac{dT}{dx} - \frac{Ph}{A}(T - T_b)$$

$$+ Q\frac{1}{t_*}F\left(-\frac{x}{vt_*}\right) + Q_J = 0 \;\;, \quad x < 0 \quad (2a)$$

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) + vS\frac{dT}{dx} - \frac{Ph}{A}(T - T_b) = 0 \;\;, \quad x > 0 \quad (2b)$$

Here we have added an arbitrary scale factor, t_*, with the dimensions of time to F in order that both F and its argument be dimensionless. Next, we introduce the dimensionless variables

$$\theta = (T - T_b)\,/\,(T_c - T_b) \qquad (3a)$$

$$\xi = x\,(Ph/kA)^{1/2} \qquad (3b)$$

$$\beta = v\,S\,(A/Phk)^{1/2} \qquad (3c)$$

$$\tau = Pht_*/AS \qquad (3d)$$

$$\gamma = QA/Pht_*\,(T_c - T_b) \qquad (3e)$$

$$\alpha = Q_JA/Ph\,(T_c - T_b) \qquad (3f)$$

in terms of which Eqs. (2a) and (2b) become

$$\frac{d^2\theta}{d\xi^2} + \beta\frac{d\theta}{d\xi} - \theta + \gamma\,F\left(-\frac{\xi}{\beta\tau}\right) + \alpha = 0\,, \quad \xi < 0 \qquad (4a)$$

$$\frac{d^2\theta}{d\xi^2} + \beta\frac{d\theta}{d\xi} - \theta = 0\,, \quad \xi > 0 \qquad (4b)$$

Equations (4a) and (4b) must be solved together with the boundary conditions

$$\theta\,(\pm\infty) = 0 \qquad (5a)$$

$$\theta\,(0\pm) = 1 \qquad (5b)$$

and

$$d\theta/d\xi \text{ continuous at } \xi = 0 \qquad (5c)$$

Equations (5) overdetermine the solution of Eqs. (4a) and (4b) and thus convert the problem to a nonlinear eigenvalue problem for β in terms of γ and τ.

Solution of the Model Problem

The solution of Eq. (4b) that we seek is

$$\theta = \exp(\lambda_- \xi) \quad , \quad \xi > 0 \tag{6a}$$

where λ_- is the negative root of the quadratic equation

$$\lambda^2 + \beta\lambda - 1 = 0 \tag{6b}$$

The solution of Eq. (4a) that we seek is of the form

$$\theta = A\exp(\lambda_+ \xi) + \text{particular solution of Eq. (4a)} \ , \quad \xi < 0 \tag{6c}$$

where A is a constant yet to be determined. A particular solution may be determined by the method of variation of parameters by using $\exp(\lambda_\pm \xi)$ as the two solutions of the homogeneous equation. A straightforward calculation gives

$$\frac{1}{\lambda_+ - \lambda_-}\left\{\exp(\lambda_-\xi)\int_{-\infty}^{\xi}\exp(-\lambda_-\xi)\left[\gamma F\left(-\frac{\xi}{\beta\tau}\right)+\alpha\right]d\xi \right.$$
$$\left. -\exp(\lambda_+\xi)\int_{-\infty}^{\xi}\exp(-\lambda_+\xi)\left[\gamma F\left(-\frac{\xi}{\beta\tau}\right)+\alpha\right]d\xi\right\} \tag{7}$$

as the particular solution when $\xi < 0$. The solutions (6a), (6c), and (7) already satisfy the boundary conditions (5a).

At $\xi = 0$, the boundary conditions (5b) and (5c) imply

$$1 = A - \frac{1}{\lambda_+ - \lambda_-}\int_0^{\infty}\left[\gamma F\left(\frac{\xi}{\beta\tau}\right)+\alpha\right] \times \left[\exp(\lambda_+\xi) - \exp(\lambda_-\xi)\right] d\xi \tag{8a}$$

$$\lambda_- = \lambda_+ A - \frac{1}{\lambda_+ - \lambda_-}\int_0^{\infty}\left[\gamma F\left(\frac{\xi}{\beta\tau}\right)+\alpha\right] \times \left[\lambda_+\exp(\lambda_+\xi) - \lambda_-\exp(\lambda_-\xi)\right] d\xi \tag{8b}$$

We eliminate A between these equations by multiplying Eq. (8a) by λ_+ and subtracting. Then

$$\lambda_+ - \lambda_- = \int_0^{\infty}\exp(\lambda_-\xi)\left[\gamma F\left(\frac{\xi}{\beta\tau}\right)+\alpha\right] d\xi \tag{9a}$$

or

$$\gamma = \frac{\sqrt{\beta^2+4} - 2\alpha/\left(\beta+\sqrt{\beta^2+4}\right)}{\beta\tau\overline{F}\left[\frac{1}{2}\beta\tau\left(\beta+\sqrt{\beta^2+4}\right)\right]} \tag{9b}$$

where $\overline{F}$ is the Laplace transform of the dimensionless function F. Equation (9*b*) determines the dimensionless propagation velocity β in terms of the dimensionless current redistribution time τ and the dimensionless excess energy density γ.

Analysis of Eq. (9*b*)

The function F is normalized so that $\int_0^\infty F(t)\,dt = 1$, which means that $\overline{F}(0) = 1$. Thus, as $\beta \to 0$, $\gamma \sim (2-\alpha)/\beta\tau \to \infty$. Since $F(t)$ must be integrable at the origin (even if it is singular there), it can be shown* that $\lim_{p\to\infty} \overline{F}(p) = 0$. Therefore, as $\beta \to \infty$, $\gamma \sim 1/\tau\overline{F}(\beta^2\tau) \to \infty$. Thus, γ must have a minimum when considered as a function of β for fixed τ. For γ below this minimum, there can be no propagating normal zones of finite size.

For the sake of definiteness, suppose we take $t_* = t_d/2$, half the current redistribution time, and take $F(t) = e^{-t}$. This assumption means that we consider only the longest-lasting mode in the redistribution of the current and implies a distortion of the early stages of that process. Then from Eq. (9*a*) we find easily that

$$\gamma = \left(\frac{1}{\beta\tau} + \frac{1}{2}\,\beta + \frac{1}{2}\sqrt{\beta^2+4}\right)\left(\sqrt{\beta^2+4} - \frac{2\alpha}{\beta+\sqrt{\beta^2+4}}\right) \tag{10}$$

---

* $\overline{F}(p) = \int_0^\infty e^{-pt}\,F(t)\,dt = e^{-pt}\int_0^t F(t)\,dt\,\Big|_0^\infty$

$$+\,p\int_0^\infty e^{-pt}\left[\int_0^t F(t)\,dt\right]dt \;=\; p\int_0^\infty e^{-pt}\left[\int_0^t F(t)\,dt\right]dt\,.$$

Now, as $p\to\infty$, the region of t that makes a significant contribution to the last integral is of the order of $1/p$. Therefore, $\overline{F}(p)$ is proportional to $\int_0^{1/p} F(t)\,dt$ and so $\to 0$ as $p \to \infty$.

A short calculation shows that, for fixed τ, $\beta(\gamma_{\min})$ is determined by

$$\frac{1/\tau - \beta^2/2 - \beta^3/2\sqrt{\beta^2+4}}{1/\tau + \beta^2/2 + \beta\sqrt{\beta^2+4}/2} = \frac{\beta + 2\alpha/\left(\beta + \sqrt{\beta^2+4}\right)}{\sqrt{\beta^2+4} - 2\alpha/\left(\beta+\sqrt{\beta^2+4}\right)} \frac{\beta}{\sqrt{\beta^2+4}} \quad (11)$$

Shown in Fig. 2 are curves of $\beta(\gamma_{\min})$ and $\gamma_{\min}$ plotted versus τ for $\alpha = 0$ and $\alpha = 1$. These curves were created by assuming a value of $\beta(\gamma_{\min})$, calculating the corresponding value of τ from Eq. (11), and then calculating $\gamma_{\min}$ from Eq. (10).

When $\gamma > \gamma_{\min}$, two values of β are possible. The arguments given so far do not determine whether one of these values is physically realizable or whether both are. The following heuristic argument implies that the larger value corresponds to a stable normal zone whereas the smaller value corresponds to an unstable normal zone.

Suppose we consider a conductor described by the power and time parameters γ and τ and choose as our initial condition the steady solution for a value $\gamma' > \gamma$ (point P' in Fig. 3). Now the available power γ is less than the power γ' required for the initial zone to sustain itself as a steady traveling wave. Since the advance of the normal front is caused by conduction of heat from warm to cold conductor, if too little Joule power is being created, the normal zone will slow down. Thus we expect the initial condition P' to approach the steady state Q'. A similar argument holds for initial points P' in the vicinity of Q' for which $\gamma' < \gamma$. Thus the steady state Q' is stable. A similar argument indicates that the steady state Q'' is unstable.

Discussion

Normal zones of finite size can propagate only if the conductor recovers far behind the advancing front. We have taken as the condition for this that the conductor is unconditionally stable when the current is uniformly distributed in the matrix. This means that $\alpha \leq 1$, which for a round conductor becomes

$$\rho I^2 \leq 2\pi^2 f h (T_c - T_b) R^3 \tag{12}$$

where f is the fraction of the total perimeter $2\pi R$ that is cooled. The excess Joule heat density Q is given by

$$Q = g\mu_0 I^2 / R^2 \tag{13}$$

where g is a geometric factor that depends on the initial current distribution. If we combine Eqs. (3e), (12), and (13), we find that

$$\gamma \leq 2\pi^2 g \alpha_1^2 \tag{14}$$

in order to fulfill the criterion of Stekly stability far behind the front.

When the initial current distribution is confined to the outer boundary, $g = 6.333 \times 10^{-3}$, and the right-hand side of Eq. (14) is then 1.836. According to Fig. 2, however, $\gamma_{min} \geq 2 - \alpha$ for all β. Thus the condition of Stekly stability far behind the front allows propagating normal zones of finite size only when $\alpha \geq 0.164$ if the initial current distribution is confined to the outer boundary.

When $1 - \alpha \ll 1$, $\gamma = 1.836$ intersects the curve of γ_{min} at $\tau = 7.935$. So, when $1 - \alpha \ll 1$, propagating normal zones are possible for $\tau > 7.935$ whereas for $\tau < 7.935$ they are not.

According to the figures assembled below, the foregoing analysis shows that the conductor of Waynert et al. cannot support propagating normal zones of finite size at 230 kA.

$I = 230$ kA
$T_b = 1.8$ K
$T_c = 8.0$ K (2.5 T)
$R = 3.0$ cm
$P = 2\pi R/3 = 6.28$ cm
(one-third surface exposure assumed)
$A = \pi R^2 = 28.3$ cm^2

$$
\begin{aligned}
\rho &= 2.8\ \mu\Omega\cdot\text{cm}/500 = 5.6\times10^{-9}\ \Omega\cdot\text{cm} \\
&\quad \text{(assumed to include magnetoresistance)} \\
k &= 7.88\ \text{W}\cdot\text{cm}^{-1}\cdot\text{K}^{-1}\ \text{(Wiedemann-Franz law)} \\
t_d &= 1.38\ \text{s} \\
Q &= 0.468\ \text{J/cm}^3 \\
Q_J &= 0.370\ \text{W/cm}^3 \\
S &= 1.13\ \text{mJ}\cdot\text{cm}^{-3}\cdot\text{K}^{-1} \\
S(T_c - T_b) &= 7.00\ \text{mJ/cm}^3 \\
h &= 0.5\ \text{W}\cdot\text{cm}^{-2}\cdot\text{K}^{-1}\ \text{(Kapitza limited)} \\
\alpha &= 0.538 \\
\tau &= 67.8 \\
\gamma &= 0.986 < \gamma_{\min} \\
\beta(\gamma_{\min}) &= 0.137 \\
\gamma_{\min} &= 1.77
\end{aligned}
$$

Further calculations show that propagating normal zones become possible for currents greater than 281 kA. Unconditional stability far behind the propagating front fails at 314 kA.

The conductor of Waynert et al. is rather large, and to use it as an object of experimental study to verify the phenomena under discussion would be cumbersome, to say the least. How small a conductor can we conveniently use as an object of study? Now, if we take $t_* = t_d/2$, as we have been doing, then for a cylindrical conductor $\tau = \mu_0 hR/\alpha_1^2 S\rho$. Now, if τ takes the value at which $\gamma_{\min}$ equals the limit of Eq. (14), then

$$R = \alpha_1^2 \tau S\rho/\mu_0 h \tag{15}$$

For radii exceeding this radius, propagation of normal zones is possible. Table 1 shows these radii for several values of α. The first set of three entries (above the dashed line) refers to a cylindrical conductor with the superconductor confined to the outer boundary. The second set of three entries (below the dashed line) refers to a cylindrical conductor in which the superconductor is confined to a core of radius $0.3R$ (Al/SC = 10.1). For such a conductor the geometric factor $g = 0.0190$. It appears, then, that wires in the millimeter range of radii should allow an experimental search for propagating normal zones of finite size.

Discussion (concluded)

If a conductor is unstable against propagation of a finite normal zone, the propagation velocity is given by $v = (\beta/S) \times (Phk/A)^{1/2}$. For the conductor of Waynert et al., $v/\beta = 8.27$ m/s. At 300 kA, for which this conductor allows propagation of finite normal zones, $\beta = 0.441$, and therefore $v = 3.65$ m/s. Since a typical energy storage magnet has of the order of 1000 km of conductor, a finite normal zone could take 3.17 days to traverse the entire winding! While traversing the winding, the normal zone would release a steady power *in excess of* $vQA = 8.22$ kW that would have to be removed by the refrigerator. It would seem, therefore, that the conductors of energy storage magnets must be designed to operate in the stable regime that does not permit the propagation of normal zones of finite size.

Reference

[1] J. Waynert, Y. Eyssa, and X. Huang, "The Transient Stability of Large Scale Superconductors Cooled in Superfluid Helium," *Adv. Cryog. Eng.* **33**:187–194 (1988).

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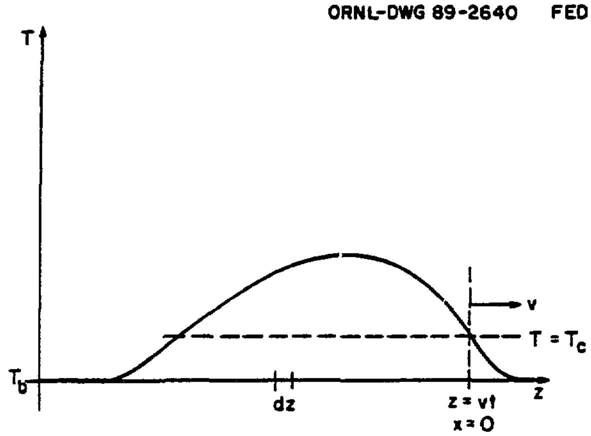

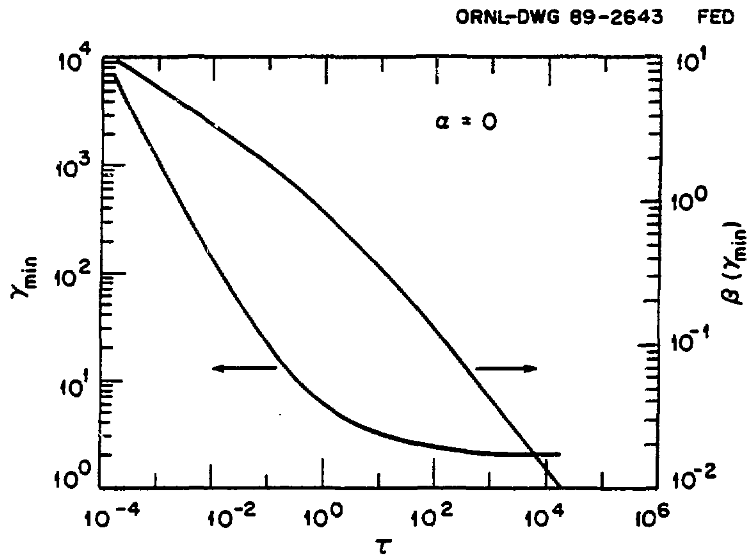
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α = 0
γ_{min}
$\beta(\gamma_{min})$
τ
10^4
10^3
10^2
10^1
10^0
10^1
10^0
10^{-1}
10^{-2}
10^{-4}
10^{-2}
10^0
10^2
10^4
10^6

F.2 (top)

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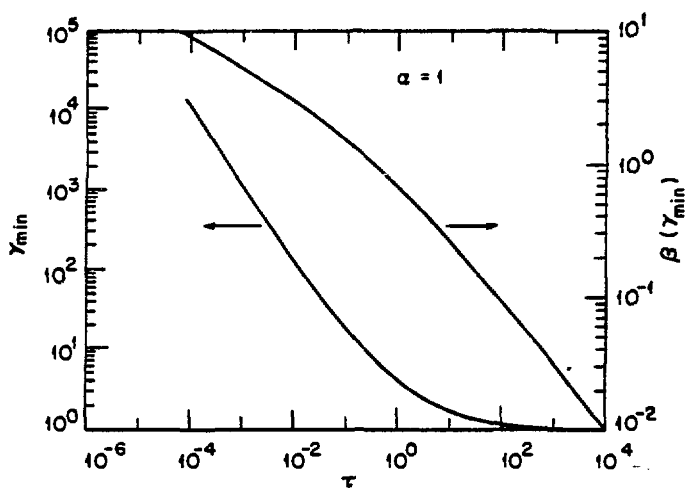

F.2 (bottom)

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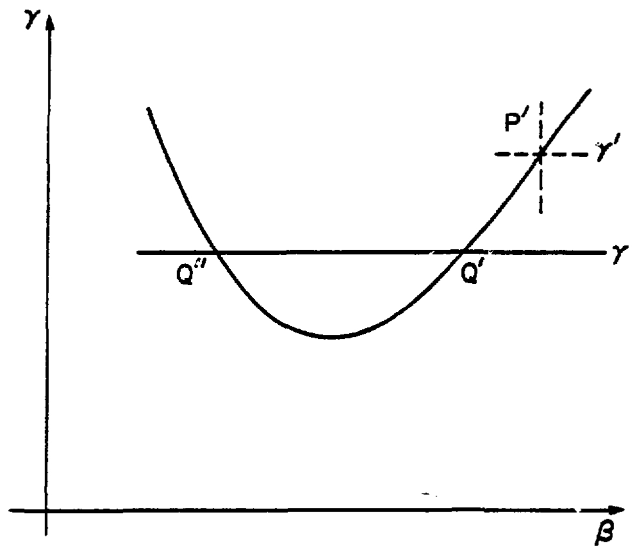

F. 3