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CHARGE CONJUGATION AND ITS VIOLATION IN UNIFIED MODELS†

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ABSTRACT

Yang-Mills theories admitting a charge conjugation C, which reflects the representation of left-handed fermions $\underline{f}_L$ onto itself, are reviewed with particular attention to flavor chiral theories, where $\underline{f}_L$ is non-self-conjugate. Simple cases of the fermion mass matrices in SO_{10} and E_6 are studied, and it is observed that the weak isospin $\underline{I}^W$ conserving part of the mass can be classified into its C conserving and C violating pieces. If the left-handed fermions are assigned to families of $\underline{16}$'s of SO_{10} or $\underline{27}$'s of E_6, then the hypothesis of the $\underline{I}^W$ invariant mass violating C maximally, with the C conserving part put to zero, gives a simple explanation of the low-mass "$\overline{\underline{5}}$ + $\underline{10}$" structure of the families.

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This talk begins with a sketch of the solution of the problem of finding the possible charge conjugation operators C, which reflect the left-handed fermions to their left-handed antiparticle states, in Yang-Mills theories where C can be defined. The general analysis can be found in a paper with M. Gell-Mann [1]; besides reviewing those results, this report provides an explicit construction of the C operator for SO_{10} and E_6 theories that unify electromagnetic, weak, and strong interactions. The uniqueness of the construction follows from the general analysis; although it adds little in principle, it does ease the analysis of the fermion mass matrix. For example, the $\underline{27}$ of E_6 has five neutral lepton states, but setting up the mass matrix and identifying the properties of the elements under C is not complicated, and is carried out here explicitly. The main reason for showing these examples is to demonstrate that the mass matrix may have special properties under C. Specifically, it is found that the hypothesis of maximal C violation of the $\underline{I}^W$ (weak isospin) invariant part of the fermion mass matrix can provide large $I^W = 0$ masses in some models to just those states that have not been observed. Of course, confirmation of the hypothesis is not possible until the assignment of the left-handed fermions to the correct $\underline{f}_L$ has been discovered. However, the hypothesis may be a helpful guide in searching for the gauge group G and the representation $\underline{f}_L$ to which the left-handed fermions should be assigned.

Let us consider first the problem of defining C for a "family" of left-handed fermions consisting of the u, $\bar{u}$, d', $\overline{d'}$, e^-, e^+, and ν_e. In the SU_5 model [2] it is not possible to define C, since although u and $\bar{u}$ are both in the same $\underline{10}$, the d' and e^+, also in $\underline{10}$, are not in the same irreducible representation (irrep) as their antiparticles $\overline{d'}$ and e^-, which are in the $\underline{\bar{5}}$. Thus a reflection taking the SU_5 quantum numbers of the u onto those of the $\bar{u}$ in the $\underline{10}$ will not reflect the $\overline{d'}$ and e^+ onto the correct quantum numbers in the $\underline{\bar{5}}$; C cannot be defined.

In the standard SO_{10} model [3] the state of affairs is much neater. Each family is assigned to a 16-dimensional spinor, and the u, d', e^-, and their antiparticles are in the same irrep. In addition the ν_L is matched up by the C operation defined below with a neutral, weak singlet state $(\bar{\nu})_L$ to form the two halves of a Dirac spinor. It is possible to define C in a unified model if a particle and its antiparticle image are always assigned to the same irrep of the gauge group G.

The generators of the unifying gauge group G must include the electric charge operator Q^{em} and the eight color generators of $SU_3{}^c$ of the strong interactions. Charge conjugation must flip the sign of Q^{em}, that is, C must anticommute with Q^{em}. Similarly, C must invert the $SU_3{}^c$ root diagram through the origin, exchanging $I_+{}^c$ for $I_-{}^c$, $U_+{}^c$ for $U_-{}^c$, and $V_+{}^c$ for $V_-{}^c$. Thus the color generators $F_1{}^c$, $F_3{}^c$, $F_4{}^c$, $F_6{}^c$, and $F_8{}^c$ anticommute with C and $F_2{}^c$, $F_5{}^c$, and $F_7{}^c$ commute with C. Note that $F_2{}^c$, $F_5{}^c$, and $F_7{}^c$, which are left invariant by C, form an SO_3 subgroup of $SU_3{}^c$; it is a symmetric subgroup, as is now discussed.

The Yang-Mills lagrangian must be invariant under C, which implies that C must be an automorphism of the Lie algebra of G that reverses the signs of some generators A of G, while leaving the remaining generators S of G invariant:

$$C(S) = S \quad , \quad C(A) = -A . \tag{1}$$

In order for C to be an automorphism of G, S must define a symmetric subgroup:

$$[S, S] \subseteq S \quad ; \quad [S, A] \subseteq A \quad ; \quad [A, A] \subseteq S. \tag{2}$$

In the cases where the action of C on any irrep of G is to reflect its weight system onto itself, C is an inner automorphism: but if C reflects a complex irrep onto its conjugate, or a self-conjugate irrep onto an inequivalent irrep of the same dimension, then it is an outer automorphism. The mathematical analysis is found in Ref. 1.

Let us examine the simplest example of such a reflection. The CP reflection, which takes $\underline{f}_L$ to $\underline{f}_R$, must invert the root diagram of G through the origin of root space; without it, there can be no gauge invariant kinetic energy term. (Root space is an Euclidean space of dimension equal to the rank of G; the root vectors describe the shift in quantum numbers due to the action of the generators, or currents, on the states of an irrep. A weight vector is a list of rank(G) quantum numbers carried by a Hilbert-space vector in the representation. This language is reviewed in [4].) The inversion of the roots and weights implies that a non-self-conjugate irrep is carried onto its conjugate; for example, CP must reflect a $\underline{3}^c$ weight onto minus itself, which is in the $\bar{\underline{3}}^c$ weight system. Thus, there is no member of the Cartan subalgebra of G in the symmetric subgroup associated with CP. (The Cartan subalgebra is the maximal set of diagonalizable generators of G, of which there are rank(G) in number.) The reflected representation $\underline{f}_R$ must be such that $\underline{f}_R \times \underline{f}_L$ contains the identity and the adjoint, which is the group theoretical restatement of the requirement that the kinetic energy be gauge invariant. The symmetric subgroups that are left invariant by CP are

$$\begin{array}{ll} SU_n \supset SO_n & G_2 \supset SU_2 \times SU_2 \\ SO_{2n+1} \supset SO_{n+1} \times SO_n & F_4 \supset SU_2 \times Sp_6 \\ Sp_{2n} \supset SU_n \times U_1 & E_6 \supset Sp_8 \\ SO_{2n} \supset SO_n \times SO_n & E_7 \supset SU_8 \\ & E_8 \supset SO_{16} \end{array} \tag{3}$$

Notice that in every case, the dimension of the symmetric subgroup is $\frac{1}{2}(\dim(G) - \mathrm{rank}(G))$, which is due to the fact that the symmetric subgroup is generated by $\frac{1}{i\sqrt{2}}(E_\alpha - E_{-\alpha})$, where α is a root and E_α is the corresponding ladder operator (or generator) of G. This is an obvious generalization of the discussion above of $SU_3{}^c$.

In the case of a flavor chiral theory, which is a theory where $\underline{f}_L$ is not self conjugate, the reflection by C of $\underline{f}_L$ onto itself cannot coincide in its group structure with CP, since CP reflects $\underline{f}_L$ onto $\overline{\underline{f}}_L$. Moreover, since $\underline{f}_L \times \underline{f}_L$ does not contain a gauge singlet, any fermion mass violates the gauge symmetry. Only SU_n, SO_{4n+2}, and E_6 have complex representations, so they are the only candidate simple groups that can lead to a flavor chiral theory. The emphasis on flavor chiral theories is, of course, due to the economical way that they incorporate the standard model of the weak interactions.

The symmetric subgroups of SU_{p+q}, where the associated C reflects a complex irrep onto itself, are $SU_p \times SU_q \times U_1$. The number of Cartan subalgebra generators outside the symmetric subgroup is min(p,q); the remaining members of the Cartan subalgebra are invariant under C. Similarly, SO_{p+q} contains the symmetric subgroup $SO_p \times SO_q$, and flavor chiral theories are defined by the constraint that $p + q = 4n + 2$. Then, only if p and q are even does C reflect a complex irrep onto itself. The number of Cartan subalgebra generators that are flipped in sign by C is min(p,q). In addition, SO_{2n} contains $SU_n \times U_1$ as a symmetric subgroup; the integer part of n/2 diagonal generators are changed in sign by C. Finally, in E_6, the C associated with $SU_2 \times SU_6$ changes the signs of four diagonal generators, and that associated with $SO_{10} \times U_1$ changes only two. Thus, the C associated with $SU_2 \times SU_6$ is the only suitable candidate.

In Ref. [1] we carried out in a coordinate independent language the analysis of several models. We carry out the same discussion here using a definite coordinatization of root space. There are practical advantages of each formulation, but they are, of course, physically equivalent.

The discussion of applications begins with the SO_{10} model: after selecting C, we show in detail what it does to the SO_{10} generators. Then, the action of C on the weights in the $\underline{16}$ can be studied, and finally a classification of the neutral lepton mass matrix is possible. We do not study the charged particles in this example, because they have, trivially, just C-conserving, $|\Delta \underline{I}^W| = 1/2$ masses.

There are six symmetric subgroups of SO_{10}: for $SO_5 \times SO_5$, the reflection flips the sign of five diagonal generators and $\underline{16}$ onto $\overline{\underline{16}}$, so the reflection is suitable for CP as it simply reverses the sign of each root and weight; the reflection associated with $SO_4 \times SO_6$ takes $\underline{16}$ onto $\underline{16}$ and flips the signs of four diagonal generators, and turns out to be the only candidate for C; the reflection associated with $SO_3 \times SO_7$ takes $\underline{16}$ onto $\overline{\underline{16}}$; the reflection associated with $SO_2 \times SO_8$ takes $\underline{16}$ onto $\underline{16}$, but flips only two quantum numbers; the reflection for SO_9 takes $\underline{16}$ onto $\overline{\underline{16}}$; and the reflection associated with $SU_5 \times U_1$ flips the sign of only two diagonal generators. This exhausts the list of symmetric subgroups and the action of the associated reflection on complex irreps. We conclude that there is only one candidate for C, and it leaves one quantum number in SO_{10} invariant.

SO_{10} contains color and flavor in a well-known way; for example, we may follow the maximal subgroup chain, $SO_{10} \supset SU_5 \times U_1{}^r$, with $SU_5 \supset SU_2{}^W \times U_1{}^W \times SU_3{}^c$, where Y^W generates the $U_1{}^W$ and Q^r generates the $U_1{}^r$. This embedding can be specified uniquely (up to a Weyl reflection) in terms of the root diagram. If it is required that the highest weight of an SO_{10} irrep is projected onto the highest weights of the SU_5 irreps contained in its branching rule, then the embedding in root space is specified by the matrix [5],

$$P(SO_{10} \supset SU_5) = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}, \tag{4}$$

where this matrix acts on an SO_{10} weight, written in the integer basis of Dynkin, as a column vector, to give the Dynkin labels of the SU_5 weight. The axis defined by the Q^r generator, which is in the Cartan subalgebra, is (-1 1-1 0 1). (The weights and axes in root space are always written here in the Dynkin integer basis, which is dual to the weight written as a linear combination of simple roots. The Dynkin basis is not orthonormal, so the computation of scalar products requires knowledge of the metric tensor, which is essentially the inverse of the Cartan matrix. The reader who wants a more detailed resumé of these points might enjoy looking at Ref. [4].) The $SU_2^W \times SU_3^c$ can be embedded in SU_5 with the projection matrix [5],

$$P(SU_5 \supset SU_2 \times SU_3) = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}. \tag{5}$$

Now that color and flavor are embedded explicitly into SO_{10}, we can identify the physical significance of each of the 45 SO_{10} roots. It is easy to find that the nonzero color roots are (0 1 0 0 0), (1 0 0-1 1) and (-1 1 0 1-1), and their negatives, and the electric charge axis, properly normalized is $\frac{1}{3}$(-2-2 3-1 1). The action of C on the generators is to flip the signs of these roots and axis. The remaining equations can be gotten from the generators, but it is slightly simpler to study the weights in the 10. The procedure is to write out the weights of the 10, compute their flavor and color content according to (4) and (5), and then require that the action of C on the weights do what it must to color and electric charge. It follows that the action of C on the SO_{10} weights is

$$C(SO_{10}) = \begin{pmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} . \tag{6}$$

Thus C leaves invariant the axis with Dynkin labels (0 0 1-1-1), which corresponds to the diagonal generator $3Y^W - 4Q^r - 10I_3^W$; C inverts the SU_3^c roots, electric charge, and $2Q^r + Y^W$, where the Y^W axis is $\frac{1}{3}$(-4-1 6-5-1).

We now study the action of C on the weights in the <u>16</u>. The u quark weights, (0 0 0 0 1), (-1 0 0 1 0), and (0-1 0 0 1) are reflected to the $\bar{u}$ weights, (0 0-1 1 0), (1 0-1 0 1), and (0 1-1 1 0), respectively; the d quark weights (0 1 0-1 0), (-1 1 0 0-1), and (0 0 0-1 0) are reflected to the $\bar{d}$ weights, (0-1 1 0-1), (1-1 1-1), and (0 0 1 0-1), respectively; and the e^- (1 0 0 0-1) is reflected to the e^+ weight (-1 0 1-1 0). Finally, the ν_L with weight (1-1 0 1 0) is reflected to (-1 1-1 0 1), which is the SU_5 singlet and is called the $(\bar{\nu})_L$.

The weights of the neutral lepton mass matrix is the sums of the weights of the corresponding states. Thus, the ν_L mass matrix element $\langle\nu_L|M|\nu_L\rangle$ has weight (2-2 0 2 0) with $|\Delta \underline{I}^W| = 1$; certainly we expect it to be less than about 1 eV. It is reflected by C onto $\langle(\bar{\nu})_L|M|(\bar{\nu})_L\rangle$, which has weight (-2 2-2 0 2) and is a weak isospin singlet. The off-diagonal element $\langle\nu_L|M|\bar{\nu}_L\rangle$ and its transpose have weight (0 0-1 1 1), $|\Delta \underline{I}^W| = 1/2$, and are invariant under C. The mass matrix can be written in the useful form

$$\begin{array}{cc} & \begin{array}{cc} \nu_L & \bar{\nu}_L \\ (1\text{-}1\ 0\ 1\ 0) & (\text{-}1\ 1\text{-}1\ 0\ 1) \end{array} \\ \begin{array}{c} \nu_L \\ (1\text{-}1\ 0\ 1\ 0) \\ \bar{\nu}_L \\ (\text{-}1\ 1\text{-}1\ 0\ 1) \end{array} & \begin{pmatrix} \begin{array}{c}(2\text{-}2\ 0\ 2\ 0) \\ |\Delta \underline{I}^W| = 1\end{array} & \left[\begin{array}{c}(0\ 0\text{-}1\ 1\ 1) \\ |\Delta \underline{I}^W| = 1/2\end{array}\right] \\ \left[\begin{array}{c}(0\ 0\text{-}1\ 1\ 1) \\ |\Delta \underline{I}^W| = 1/2\end{array}\right] & \begin{array}{c}(\text{-}2\ 2\text{-}2\ 0\ 2) \\ |\Delta \underline{I}^W| = 0\end{array} \end{pmatrix} \end{array} \quad (7)$$

where the [...] signify that the mass matrix element is reflected onto itself by C.

The $|\Delta \underline{I}^W| = 1/2$ mass has the same weight as the u quark, and is expected to have a value of a few MeV. In order for the small eigenvalue of (7) to be a few eV or less, the $|\Delta \underline{I}^W| = 0$ term must be huge, and if we ignore the $|\Delta \underline{I}^W| = 1$ term, the mass matrix has the form [6],

$$\begin{pmatrix} 0 & m \\ m & M \end{pmatrix} , \quad (8)$$

which has small eigenvalue m^2/M, approximately. Note that (8) can be restated as: the weak isospin conserving mass violates C maximally, while the $|\Delta \underline{I}^W| = 1/2$ mass conserves C.

The second example is less trivial: the unifying group is E_6 and a single family is assigned to a $\underline{27}$ [7]. The $\underline{27}$ has two charge -1/3 quarks and their antiparticles, so there is an opportunity to study the C properties of the quark masses in this example.

The symmetric subgroups of E_6 are Sp_8, $SU_2 \times SU_6$, $SO_{10} \times U_1$, and F_4. Of these, the reflection associated with Sp_8 and F_4 reflect $\underline{27}$ to $\overline{\underline{27}}$; CP is associated with Sp_8. We have already argued that C must be associated with $SU_2 \times SU_6$, because the reflection associated with $SO_{10} \times U_1$ flips the signs of only two diagonal generators. Thus C leaves invariant two of the six quantum numbers in E_6.

The embedding of color and flavor in E_6 can be desc- ed by the subgroup chain $E_6 \supset SO_{10} \times U_1{}^t \supset SU_5 \times U_1{}^r \times U_1{}^t \supset SU_2 \times U_1{}^w \times SU_3{}^c \times U_1{}^r \times U_1{}^t$, with the projection of the E_6 to SO_{10} weights given by [5],

$$P(E_6 \supset SO_{10}) = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} ; \tag{9}$$

the remaining projections are given by (4) and (5).

The C reflection is constructed in the same fashion as (6) for SO_{10}. It is

$$C(E_6) = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & -1 & -1 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} . \tag{10}$$

It inverts color roots and reverses the signs of electric charge and $2Q^r + Y^w$, while leaving invariant $3Y^w - 4Q^r - 10I_3{}^w$ and Q^t.

The weight diagram for the 27 is derived from the highest weight (1 0 0 0 0 0) in the usual way [4]. Three of the neutral lepton weights are eigenvectors of $C(E_6)$ with eigenvalues +1; (-1 0 1-1 0 0), (0 1-1 0 1 0), and (1-1 0 1-1 0). The other two neutral weights (0 0 0 1 0-1) and (1 0-1 0 0 1) are transformed into one another by C [8]. The remaining weights carry electric charge and transform under C as expected.

The charge 2/3 u quark has weights (1 0 0 0 0 0), (1-1 0 0 1 0), and (1 0 0 0 0-1), which are reflected by C in (10) to the $\bar{u}$ weights, (0 0-1 1 0 0), (0 1-1 1-1 0), and (0 0-1 1 0 1), respectively. The u quark mass carries weight (1 0-1 1 0 0), which is a C conserving, $|\Delta \underline{I}^W| = 1/2$ mass.

The mass matrix of the charge -1/3 quarks and the charged lepton mass matrix are similar; they have precisely the same weight structure. The charge -1/3 quarks in the SU_5 $\underline{10}$ of the SO_{10} $\underline{16}$, to be denoted $\underline{10}(\underline{16})$, have the weights, (0 0 0 0-1 1), (0-1 0 0 0 1), and (0 0 0 0-1 0); the C partners (0-1 1 0 0-1), (0 0 1 0-1-1), and (0-1 1 0 0 0), respectively, are in $\underline{\bar{5}}(\underline{16})$. The other charge -1/3 quark is in $\underline{5}(\underline{10})$, with weights (-1 1 0 0 0 0), (-1 0 0 0 1 0), and (-1 1 0 0 0-1), with C partners (0 0 0-1 1 0), (0 1 0-1 0 0), and (0 0 0-1 1 1), respectively, in $\underline{\bar{5}}(\underline{10})$. Let us write out the mass matrix for one color state, (1 0) for quarks, (-1 0) for antiquarks as

$$
\begin{array}{c|cccc}
 & \begin{matrix} D\ \underline{5}(\underline{10}) \\ (-1\ 1\ 0\ 0\ 0\ 0) \end{matrix} & \begin{matrix} d\ \underline{10}(\underline{16}) \\ (0\ 0\ 0\ 0\text{-}1\ 1) \end{matrix} & \begin{matrix} \bar{d}\ \underline{\bar{5}}(\underline{10}) \\ (0\ 0\ 0\text{-}1\ 1\ 0) \end{matrix} & \begin{matrix} \bar{D}\ \underline{\bar{5}}(\underline{16}) \\ (0\text{-}1\ 1\ 0\ 0\text{-}1) \end{matrix} \\
\hline
\begin{matrix} D\ \underline{5}(\underline{10}) \\ (-1\ 1\ 0\ 0\ 0\ 0) \end{matrix} & 0 & 0 & \begin{bmatrix} (-1\ 1\ 0\text{-}1\ 1\ 0) \\ |\Delta \underline{I}^W| = 0 \end{bmatrix} & \begin{matrix} (-1\ 0\ 1\ 0\ 0\text{-}1) \\ |\Delta \underline{I}^W| = 0 \end{matrix} \\
\begin{matrix} d\ \underline{10}(\underline{16}) \\ (0\ 0\ 0\ 0\text{-}1\ 1) \end{matrix} & 0 & 0 & \begin{matrix} (0\ 0\ 0\text{-}1\ 0\ 1) \\ |\Delta \underline{I}^W| = 1/2 \end{matrix} & \begin{bmatrix} (0\text{-}1\ 1\ 0\text{-}1\ 0) \\ |\Delta \underline{I}^W| = 1/2 \end{bmatrix} \\
\begin{matrix} \bar{d}\ \underline{\bar{5}}(\underline{10}) \\ (0\ 0\ 0\text{-}1\ 1\ 0) \end{matrix} & [(-1\ 1\ 0\text{-}1\ 1\ 0)] & (0\ 0\ 0\text{-}1\ 0\ 1) & 0 & 0 \\
\begin{matrix} \bar{D}\ \underline{\bar{5}}(\underline{16}) \\ (0\text{-}1\ 1\ 0\ 0\text{-}1) \end{matrix} & (-1\ 0\ 1\ 0\ 0\text{-}1) & [(0\text{-}1\ 1\ 0\text{-}1\ 0)] & 0 & 0
\end{array}
\qquad (11)
$$

[The charged lepton mass matrix has the same weight structure if the weight of D is replaced by the charge 1 lepton weight, (0 0 1-1 1-1), the weight of d by (1-1 1-1 0 0) of charge 1, the weight of $\bar{d}$ by (-1 1-1 0 0 1) of charge -1, and the weight of $\bar{D}$ by (-1 0 0 1-1 0) of charge -1.] There are two candidate assignments for the weak isospin conserving mass: either the (-1 0 1 0 0-1) mass is nonzero, the d state is left massless (before the weak breaking), and C is maximally violated; or the (-1 1 0-1 1 0) mass is nonzero, the D is massless, and the mass is C conserving. For the purposes of studying the charged particle masses, these situations appear interchangeable, although the $\underline{\bar{5}}$ + $\underline{10}$ left massless in the limit of no weak breaking differs in the two cases. Recall that $\underline{27} = \underline{16} + \underline{10} + \underline{1}$ of SO_{10}. In the first case (d massless), the $\underline{\bar{5}}$ belongs to the $SO_{10}\underline{10}$; in the second case (D massless), the $\underline{\bar{5}}$ comes from the $SO_{10}\underline{16}$. The same considerations also apply to the two charged leptons in the $\underline{27}$.

In order to decide which assignment is more attractive, we turn to a study of the neutral lepton mass matrix, which can be written as a matrix of weights where the labels on the rows and columns should, by now, be obvious (just divide the diagonal entries by 2):

$$\begin{pmatrix}
\text{(0 0 0 2 0-2)} & \text{[(1 0-1 1 0 0)]} & \text{(1-1 0 2-1-1)} & \underline{\text{(-1 0 1 0 0-1)}} & \text{(0 1-1 1 1-1)} \\
\text{[(1 0-1 1 0 0)]} & \underline{\text{(2 0-2 0 0 2)}} & \underline{\text{(2-1-1 1-1 1)}} & \text{(0 0 0-1 0 1)} & \text{(1 1-2 0 1 1)} \\
\text{(1-1 0 2-1 1)} \leftrightarrow & \underline{\text{(2-1-1 1-1 1)}} & \underline{\text{[(2-2 0 2-2 0)]}} & \text{[(0-1 1 0-1 0)]} & \text{[(1 0-1 1 0 0)]} \\
\underline{\text{(-1 0 1 0 0-1)}} \leftrightarrow & \text{(0 0 0-1 0 1)} & \text{[(0-1 1 0-1 0)]} & \text{[(-2 0 2-2 0 0)]} & \underline{\text{[(-1 1 0-1 1 0)]}} \\
\text{(0 1-1 1 1-1)} \leftrightarrow & \text{(1 1-2 0 1 1)} & \text{[(1 0-1 1 0 0)]} & \underline{\text{[(-1 1 0-1 1 0)]}} & \text{[(0 2-2 0 2 0)]}
\end{pmatrix}, \tag{12}$$

where the I_3^w value of the mass matrix element is one-half the sum of the first five Dynkin labels. Let us first assume that the weak isospin conserving part of (12) is maximally C violating, so that only the entries with weights (2 0-2 0 0 2), (2-1-1 1-1 1), and (-1 0 1 0 0-1) are nonzero. For a general choice of parameters, (12) has four nonzero eigenvalues and one zero eigenvalue; the massless fermion has weight (0 1-1 0 1 0), which is in $\overline{\underline{5}}(\underline{10})$. Thus, with maximal C violation, the massless fermions at the weak isospin conserving level are classified by $\overline{\underline{5}}$ + $\underline{10}$.

In the case of C conservation, the elements with weights (2-2 0 2-2 0) and (-1 1 0-1 1 0) are nonzero, and the neutrals in the SO_{10} $\underline{1}$ + $\underline{10}$ get masses. Both neutral states in the $\underline{16}$ remain massless, at least until some C violation is introduced at the SO_{10} level. Thus, the C conservation hypothesis leaves a $\underline{1}$ + $\overline{\underline{5}}$ + $\underline{10}$ of SU_5 to get masses from other sources, such as the weak interactions. If the four component ν mass comes from the weak interactions, then its mass is of order the u mass, not in accord with experience.

Stated in a slightly different way, all the C conserving weak isosinglet masses leave SO_{10} invariant, so the fermions occur in SO_{10} irreps, $\underline{16}$'s in this case, but the C violating masses leave just SU_5 invariant, while violating SO_{10}, and the low mass fermions in the $\underline{27}$ occur in a $\overline{\underline{5}}(\underline{10})$ + $\underline{10}(\underline{16})$ pattern.

In summary, we find that the hypothesis of maximal C violation of the weak isospin invariant masses leads to a satisfactory fermion spectrum in several flavor chiral models. Of course, this selection rule must be tested on the "correct" representation before it can be confirmed or rejected. However, the general structure of the mass matrices is such that the hypothesis may provide a helpful guideline in searching for satisfactory theories.

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