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MASTER

APPLICATION OF SURVIVAL STATISTICS TO THE IMPULSIVE FRAGMENTATION OF DUCTILE RINGS[1]

D. E. Grady

Sandia National Laboratories[2]

Albuquerque, New Mexico 87185 USA

An analysis of fragmentation due to impulsive stress loading of solid materials is developed which results in analytic expressions for distributions in fragment sizes. The analysis is restricted to a linear (one-dimensional) distribution of material which is loaded uniformly in tension until fracture, and ultimately fragmentation, occurs. Concepts of survival statistics consistent with simple physical laws governing the fracture process are used to account for the spatial and temporal distribution in fracture nucleation sites. Analytic fragment distribution curves for ductile fracture are derived and found to provide a good representation of data obtained from impulsive fragmentation studies on aluminum rings.

I. INTRODUCTION

The fragmentation of a solid body subjected to the intense forces of high velocity impact, explosive loading, or radiation deposition is a destructive event seemingly outside of the scope of predictive calculations. Even when a high degree of symmetry has been incorporated in both the geometry and the application of the load the resulting irregularities in fragment sizes and velocities appear to belie the care taken in performing the experiment.

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DISCLAIMER

It seems clear that intrinsic instabilities in the deformation processes leading to the disruption of the body necessitate the introduction of statistical as well as physical concepts in the analysis of dynamic fragmentation.

A problem in dynamic fragmentation which has received active and continuing attention since the early treatment of Mott (1) is the rapid expansion and fragmentation of metal rings or cylinders due to the application of an impulsive load to the inner circumference. The objective is the prediction of the fragment size distribution resulting from the event. Theoretical treatments of this problem include the work of Mott (1) and Taylor (2) and extensive experimental results have accumulated through numerous efforts (3,4,5,6).

The present study was motivated by the original work of Mott (1). In that work the concept of statistically random fracture activation was coupled with the physical concept of the growth of plastic tensile relief waves and resulted in a method for calculating fragment size distributions. Subsequent studies have primarily focused on more accurate characterization of the fracture event; the essential statistical concepts have not apparently been extensively pursued.

This work will focus on the statistical concepts of the fragmentation process initiated by Mott. A formalism of survival statistics is introduced to treat the randomness of fracture initiation and analytic fragment size distribution expressions are derived for the fragmentation of ductile rings. The results are compared with recent experimental work of Wesenberg and Sagartz (5).

II. STATISTICAL CONCEPTS

The present fragmentation analysis will be restricted to a one-dimensional distribution of material which is loaded uniformly in tension until stress relief occurs by multiple fracturing throughout the body. The objective will be to determine a statistical distribution of fragment sizes (lengths) that depends on the material properties of the body as well as the imposed loading conditions. The geometry is most easily visualized as a ring of material, whose width and breadth are vanishingly small compared to the circumference, which is subjected to a uniform outward impulse directed along the radius vector as shown in Fig. 1. This geometry is an idealization of the problem considered by Mott and will be called the "one-dimensional Mott" problem in the present study.

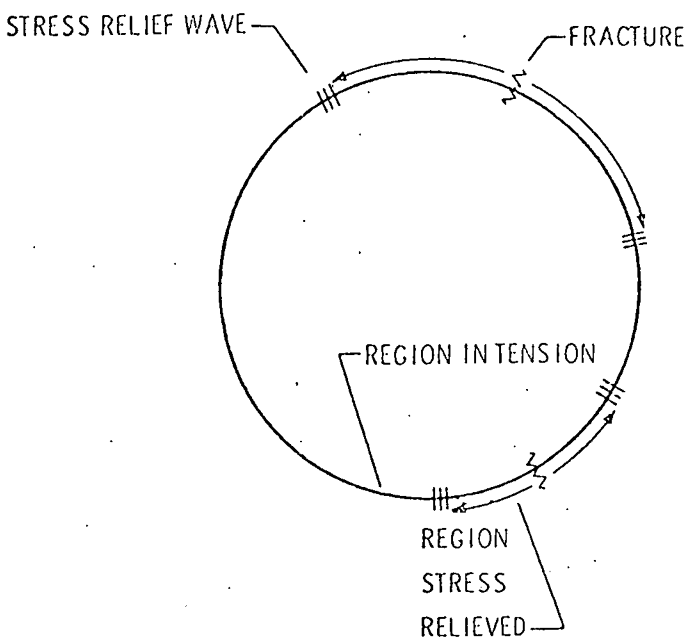

FIGURE 1. The one-dimensional Mott problem. A one-dimensional distribution of material is given an outward radial impulse resulting in circumferential tension and fracture at random sites on the ring. Waves originating at the points of fracture propagate at finite velocities and reduce to zero stress the initial tensile stress in the body. Time-dependent fracture continues only in the regions not yet stress relieved.

To account for the random location at which fractures nucleated in the unrelieved portion of the ring, Mott used a graphical method and arrived at a coarse prediction of the resulting fragment distribution. Recently, Wesenberg and Sagartz (5) compared experimental data with the Mott method. Having the advantage of high-speed computers, they made use of a random number generator routine to locate fractures and, when averaged over 1000 rings, determined a statistical fragmentation curve in reasonable agreement with their data.

In the present analysis concepts from the theory of survival statistics will be applied to the one-dimensional Mott problem. The statistical method will not necessarily provide a predictive capability better than the earlier work. It will, however, provide an alternative point of view, and results in analytic relations which are readily compared with the experimental data and are potentially amenable to generalization to more complex fragmentation problems.

The process of dynamic fracture involves spatially random nucleation and growth of cracks and has similarities to other nucleation and growth phenomena (melting, recrystallization, explosive detonation, etc.). In fracture, as in some of the other phenomena, nucleation and growth of a single crack can be treated in substantial detail. It is the impingement or influence of one crack, or region of growth, on others which compounds the complexity of the total nucleation and growth process.

To treat problems of nucleation and growth, Johnson and Mehl (7) and Avrami (8) introduced the concept of an extended volume fraction, D_x, defined as the volume fraction of the body transformed without regard for nucleation or for growth within previously transformed material. The extended volume fraction can exceed unity.

The actual transformed volume fraction, D, of the body is determined from the ratio in the change of the extended and actual transformed volume fraction. Namely,

$$\frac{dD}{dD_x} = 1 - D, \qquad [1]$$

which integrates to

$$D = 1 - e^{-D_x} \qquad [2]$$

Equation 2 is the essential relation derived in this section and will hereafter be called the JMA relation. The JMA relation is applicable to two-dimensional bodies, or areas, as well as to one-dimensional bodies or lines.

Although the work of Johnson and Mehl (7) and Avrami (8) was focused on phase transformations, their results have more general application. Their results are based strictly on the statistics of survival; in this case, survival at any time of the as yet untransformed volume. The concept is independent of the physics involved and is applicable to any nucleation and growth process which is random in nature. The JMA relation allows initial attention to focus on the physics of the nucleation and growth process at a single site. Equation 2 will then account for the coalescence of multiple transforming regions.

III. PHYSICAL CONCEPTS

The process of fracture nucleation and the growth of stress-relieved regions during tensile loading will depend on both material properties and the conditions of loading. In particular, the nucleation rate is expected to depend on strain, strain rate, or time, coupling the activation process intimately with the loading conditions impressed on the body. In addition the defect structure which provides sites for fracture nucleation will characterize a material property important to the fracture rate. The nucleation process is poorly understood and a simple uniform nucleation rate will be assumed in the present calculation.

The law governing the growth of stress-relieved regions after fracture initiation, or equivalently, the velocity of the stress-release waves shown in Fig. 1, is determined by the response of material still in tension. Following Mott we will assume that response of the material preceding fracture is rigid plastic and governed by a flow stress σ_y. Mott has shown through momentum conservation concepts that the release wave originating at the point of fracture and propagating into the deforming plastic region depends on both time and loading rate. The result is not well known and the derivation will be outlined briefly.

Consider a linear body of initial length L fixed at the origin, and with initial density ρ and undergoing uniform deformation at a constant strain rate $\dot{\varepsilon}$ in response to a stress σ_y. At time zero the body is released at the origin (simulating fracture) and the resulting stress-release wave propagates in the positive x direction. The trajectory of the wave is determined from momentum conservation. At time zero the momentum of the body is

$$p_0 = \int_0^L v\rho dx = \tfrac{1}{2}\rho L^2\dot{\varepsilon} \quad . \qquad [3]$$

At time t the momentum is

$$p_t = \rho\dot{\varepsilon}x^2 + \int_x^L x\dot{\varepsilon}\rho dx \quad ,$$

or

$$p_t = \tfrac{1}{2}\rho\dot{\varepsilon}x^2 + \tfrac{1}{2}\rho L^2\dot{\varepsilon} \quad . \qquad [4]$$

Equating the change in momentum to the impulse applied,

$$p_t - p_0 = \sigma_y t \quad , \qquad [5]$$

results in a trajectory for the release wave,

$$x = \left(\frac{2\ \sigma_y\ t}{\rho\dot{\varepsilon}}\right)^{\frac{1}{2}} \quad , \qquad [6]$$

and a velocity

$$C = \left(\frac{\sigma_y}{2\rho\ \dot{\varepsilon}t}\right)^{\frac{1}{2}} = \frac{\alpha}{t^{\frac{1}{2}}} \quad . \qquad [7]$$

An exact plastic wave calculation by Lee (9) shows that the release velocity is initially equal to the elastic wave velocity but rapidly approaches Equation 7. The expression in Equation 7 will be assumed for the present analysis.

IV. APPLICATION TO DUCTILE FRACTURE

Before proceeding, one final concept in the fracture process must be introduced. When a fracture initiates at some point on the one-dimensional body, two release waves, one propagating to the left and one propagating to the right, will be created. Each region between the fracture point and the right or left directed release wave will be called a domain of release (DOR). Since the point at which each release wave arrests is statistically independent of the other, it will be necessary, in the bookkeeping process leading to the fragment distribution, to account for each DOR separately.

Consequently, if we assume that the uniform nucleation rate of DOR (twice the fracture nucleation rate) is a constant I_x, then the extended length fraction of stress-relieved material at time t due to fractures nucleated at an earlier time τ is,

$$dD_x = 2I_x\ \alpha(t-\tau)d\tau \quad , \qquad [8]$$

where the release-wave velocity from Equation 9 has been used. Integrating to time t results in a total extended length of

$$D_x = \frac{4}{3}\ I_x\ \alpha t^{3/2} \quad . \qquad [9]$$

To correct for arrest of DOR and nucleation in previously stress-relieved regions, ignored in the calculation of D_x, the JMA relation is applied, resulting in an actual stress-relieved length fraction of,

$$D = 1 - e^{-\frac{4}{3} I_x \alpha t^{3/2}} . \qquad [10]$$

To determine the resulting fragment distribution it will be necessary to account for both nucleation and arrest of DOR. The nucleation rate and number of DOR, ignoring arrest and nucleation in previous stress-relieved regions are I_x and N_x, respectively. The actual nucleation rate and active DOR at time t are then $I = I_x(1-D)$ and $N = N_x(1-D)$. Using Equation 10 results in

$$I = I_x e^{-\frac{4}{3} I_x \alpha t^{3/2}} , \qquad [11]$$

and

$$N = I_x t e^{-\frac{4}{3} I_x \alpha t^{3/2}} . \qquad [12]$$

Since the nucleation rate is given by Equation 11 the arrest rate I_a can be determined from Equation 12 through the relation $dN/dt = I + I_a$. The resulting arrest rate is

$$I_a = -2 I_x \alpha t^{1/2} N . \qquad [13]$$

The form of Equation 13 has an analogy in the kinetic theory of gases where the collision rate is proportional to the number of particles and to some hazard function characterizing the probability of collision.

The next objective is to calculate the number of DOR which nucleate at time τ and arrest at time t since we can assign a final length of $\ell = 2\alpha(t - \tau)^{1/2}$ to these DOR. The total number of DOR which arrest at time t within an interval dt is $I_a dt$. Only a fraction of these will have nucleated at time τ. If the number of DOR active at time t and which nucleated at time τ within an interval $\delta\tau$ is $\delta N_\tau(t)$ then the portion arrested at time t is

$$d(\delta N_\tau) = \frac{\delta N_\tau}{N} I_a \, dt . \qquad [14]$$

Substituting from Equation 12 results in

$$d(\delta N_\tau) = -2 I_x \alpha t \delta^{1/2} N_\tau dt , \qquad [15]$$

and integration provides the active DOR at time t which nucleated at time τ,

$$\delta N_\tau(t) = I_x \, \delta\tau e^{-\frac{4}{3} I_x \alpha t^{3/2}} . \qquad [16]$$

Using Equation 16 and $\ell = 2\alpha(t - \tau)^{1/2}$ in Equation 15 results in

$$d(\delta N_\tau) = -\frac{I_x^2}{\alpha}\left(\tau + \frac{\ell^2}{4\alpha^2}\right)^{1/2} \delta\tau e^{-\frac{4}{3} I_x \alpha \left(\tau + \frac{\ell^2}{4\alpha^2}\right)^{3/2}} \ell d\ell . \qquad [17]$$

Integrating over all past times provides the distribution over lengths of the DOR,

$$\frac{dN}{d\ell} = \frac{I_x}{2\alpha^2} \ell e^{-\frac{I_x}{6\alpha^2}\ell^3} . \qquad [18]$$

One further step remains in calculating the fragment distribution. It is readily appreciated that the distribution of DOR is not proportional to the distribution of fragments. Each fragment of specified length results from two DOR whose lengths are arbitrary providing only that the sum of their lengths equals the specified fragment length. Equation 18 can be converted to a probability distribution by normalizing with the total number of DOR (the integral of Equation 18), resulting in,

$$f(\ell) = \beta\left(\frac{I_x}{6\alpha^2}\right)^{2/3} \ell e^{-\frac{I_x}{6\alpha^2}\ell^3} , \qquad [19]$$

where

$$\frac{1}{\beta} = \int_0^\infty \zeta e^{-\zeta^3} d\zeta .$$

The DOR combine in pairs to form fragments and the fraction of fragments of length L within interval dL is,

$$F(L)dL = \int_{L=\ell_1+\ell_2} f(\ell_1)\, f(\ell_2)\, d\ell_1 d\ell_2 \quad , \qquad [20]$$

where the integral is over all ℓ_1, and ℓ_2 which sum to L. The integral can be completed through the transformation

$$L = \ell_1 + \ell_2$$

$$\zeta = \ell_1 - \ell_2$$

$$d\ell_1\, d\ell_2 = \frac{\partial(\ell_1,\ell_2)}{\partial(L,\zeta)}\, dL d\zeta$$

resulting in

$$F(L) = \frac{\beta^2}{4}\left(\frac{I_x}{6\alpha^2}\right)^{4/3} e^{-\frac{I_x}{24\alpha^2}L^3} \int_0^L (L^2 - \zeta^2)\, e^{-\frac{I_x}{8\alpha^2}L\zeta^2}\, d\zeta \qquad [21]$$

IV. COMPARISON WITH EXPERIMENTS ON ALUMINUM RINGS

Wesenberg and Sagartz (5) performed eleven experiments on thin 6061-T6 aluminum cylinders (127 mm diameter by 102 mm length). A capacitive discharge method was used to impart radial impulsive loading at a circumferential strain rate of approximately 10^4/s. Fracture was observed photographically to occur at a strain of about 30%. A total of 125 fragments from 11 equivalent experiments were collected and assigned an effective length by weight.

The fragment distribution function for ductile fracture (Equation 21) scales with respect to a single characteristic length parameter $L^* = (6\alpha^2/I_x)^{1/3}$, where α is defined in Equation 7. A value of $L^* \doteq 22.8$ mm provided the best fit to the data of Wesenberg and Sagartz shown in Fig. 2.

Wesenberg and Sagartz directly applied the method suggested by Mott through the use of a computer program relying on a random number generator routine. The method was applied to 1000 rings before statistical convergence was adequate. Their best fit for the Mott method is also shown in Fig. 2.

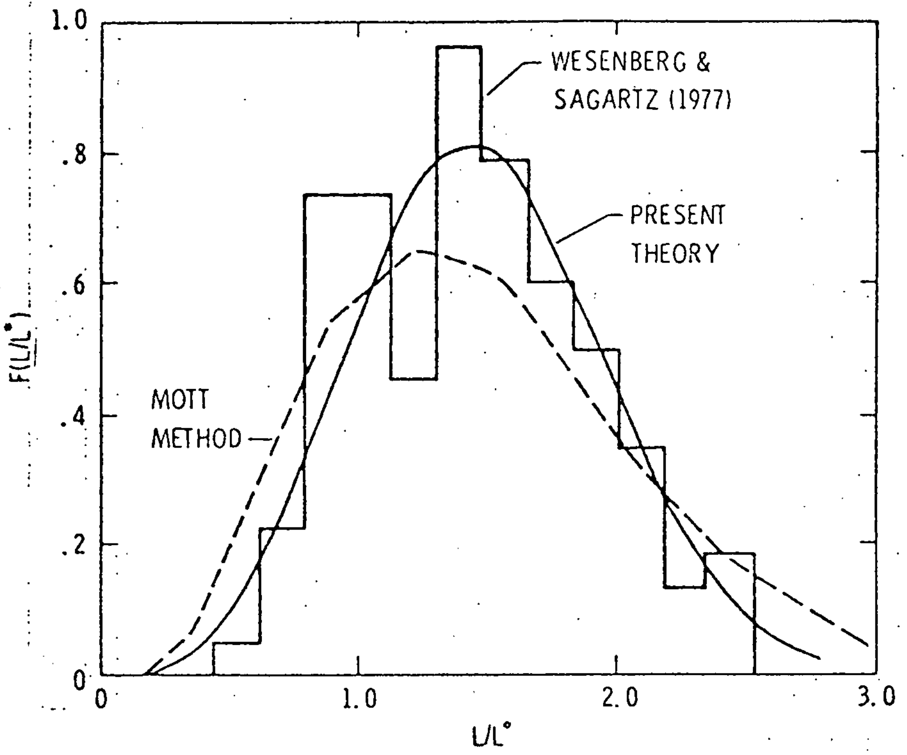

Figure 2. Fragment distribution curves for 6061-T6 aluminum. The data of Wesenberg and Sagartz (5) was obtained by fragmentation of impulse loaded rings. The solid curve represents a best fit of the data for the present theory with a characteristic length of $L^* = 22.8$ mm. The dashed curve is a best fit of the data by Wesenberg and Sagartz using the method suggested by Mott (1).

It is not clear why the distribution curve from the present theory and that predicted by the Mott method are different. The nucleation law assumed by Wesenberg and Sagartz differed from the constant rate of nucleation assumed in the present work, and was determined by two parameters, although in fitting their data, the authors found that only one parameter sensitively influenced the curve. The differences in nucleation laws might account for the difference in distribution curves. Alternatively, the Mott method applied by Wesenberg and Sagartz assumed the nucleation of fracture was incremented on the strain history rather than allowing for the statistical occurrence of fracture in time. This might also account for the differences.

V. SUMMARY AND DISCUSSION

The current study has addressed the problem of predicting fragment size distributions resulting from the tensile fracture of impulsive loaded bodies. The theory has been restricted to one-dimensional bodies subjected to uniform loading. A rigorous treatment of the statistics of dynamic fragmentation has been attempted using concepts of survival statistics and incorporated here through the relation derived by Johnson and Mehl (7) and Avrami (8). Physical concepts are introduced through the assumption of a uniform nucleation rate of fracture and the propagation of stress-relief waves governed by the tensile response of an ideally ductile material.

An analytic distribution curve was derived for ductile fracture and compared with the fragmentation data of Wesenberg and Sagartz (5) on aluminum rings. The analytic expression was found to provide a good representation of the data although further experimental work in this area is strongly needed.

The present analysis and resulting fragment size distribution expressions are fairly complex even for the very simple geometry and loading conditions considered. Direct application of the method to more complicated fragmentation events would probably be difficult. Perhaps the greatest value of the present type of analysis will be the insight that it provides on the statistical nature of impulse fragmentation; hopefully a source of fresh ideas for computational models currently under development (10,11,12).

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