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COMPLEMENTARY INFLUENCE FUNCTIONS FOR PREDICTING SUBSIDENCE CAUSED BY MINING*

by H. J. Sutherland and D. E. Munson

Geotechnical Engineering Division
and
Experimental Programs Division
Sandia National Laboratories
Albuquerque, New Mexico 87185

ABSTRACT

Surface subsidence caused by underground mining is described through complementary influence functions. The complementary functions developed here differ from the simple functions previously used in that the surface displacement is the result of the combined contributions of the mined and unmined zones. This eliminates computational difficulties experienced with the simple functions in determining the deflections above the rib side and in the eventual application of influence functions to complex room-and-pillar configurations. Although the analysis framework presented is intended for predicting subsidence over complex mine configurations, use of the complementary functions is illustrated adequately by application to a longwall panel of the Old Ben No. 24 coal mine.

INTRODUCTION

In the first part of this century, mining engineers realized that they needed the capability to predict ground displacements and strains caused by subsurface mine workings. This need to predict surface movements, commonly termed subsidence, is especially critical in Europe because of the extensive surface utilization over economic coal and ore bodies. A similar need is now developing in the United States.

Early considerations of subsidence, before the advent of computer analysis, led to empirical functions that describe mathematically the

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observed surface displacements. Two classes of empirical functions are commonly used: profile functions and influence functions (Brauner, 1973 and Hood et al, 1981). Profile functions are direct fits to empirical data and are typically used for subsidence predictions over long wall panels (Munson and Eichfeld, 1980). These functions cannot describe geometrically complex mining areas such as found in room-and-pillar mining. Influence functions are more applicable to these complex mining areas and have been previously developed from the viewpoint of the excavated material. Each unit element of the ground surface above the mined volume is assigned the same response, and integration of this elemental influence over all the elements in the mined area yields the subsidence prediction. These influence functions are widely used, with considerable success. There is, however, a major problem with this formulation because it significantly overpredicts the subsidence directly over the rib side. Typically the problem is handled by integrating the elements to an imaginary rib location within the mined area rather than to the actual rib location. Solution of the rib side subsidence problem is crucial in developing the capability to predict subsidence over room-and-pillar, as well as over longwall, mines.

In recent analyses, numerical computer methods have been used to predict subsidence. Instead of analyzing the mined material, these techniques analyze the behavior of the coal (ore) and overburden layers remaining after mining. Calculations of considerable detail are possible and have shown how elastic bending, breaking and bulking and void volume are transmitted through the overlying strata to cause surface subsidence ((Munson and Benzley, 1980 and Sutherland and Schuler, 1982). Normally, such large scale analyses are beyond the means of mine operators, and a simpler analysis method is required, especially for room-and-pillar configurations.

In this work, we examine the motivating concepts for influence functions in order to remove the inadequacy of the prediction at the rib side. In the examination, it is apparent that the remaining material in the seam is as influential as that of the open volume left by the excavation. This viewpoint leads to a formulation based on the concept of complementary influence functions.

COMPLEMENTARY INFLUENCE FUNCTIONS

The fundamental concept of complementary influence functions is that the separate influence functions describing the response of mined and unmined zones act together to produce the observed subsidence. Each influence function is defined by the response of a unit element: an "unmined" element for the coal (ore) left by the mining process, and a "mined" element for the open volume created by the removal of the coal (ore). These elements are shown in Figure 1. When these elements are integrated (appropriately summed) over the entire seam (both the mined and the unmined zones) a subsidence prediction results. Thus, the subsidence is the sum of the influence of the mined and unmined response.

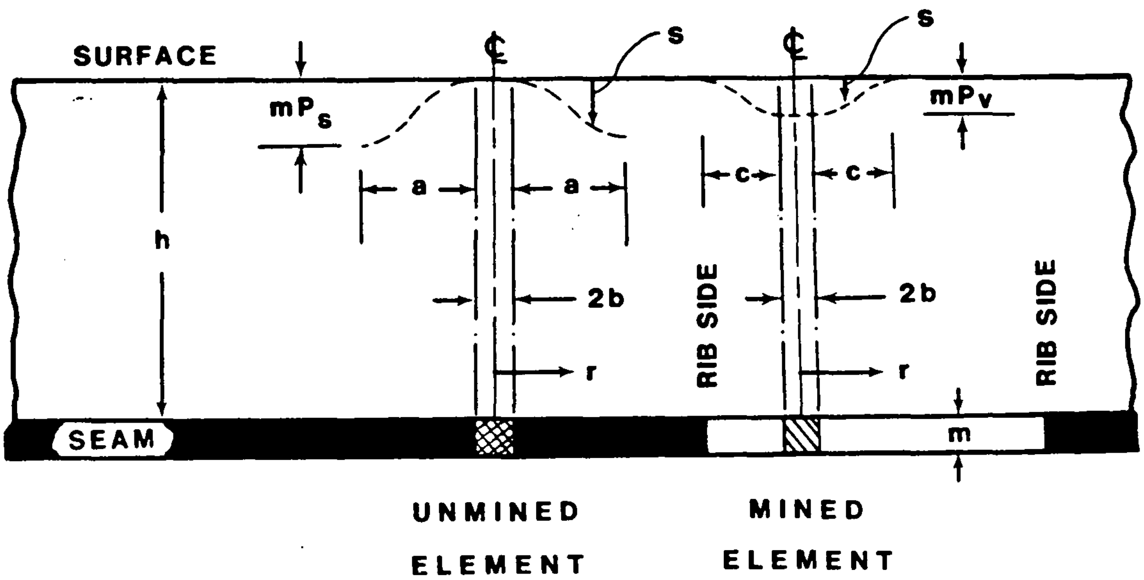

Fig. 1. Complementary Elements

Response of the Unmined Element

The elemental response of the unmined element is based on the elastic response of a plate supported by the element (Timoshenko and Woinowsky-Krieger, 1959) and is given by the function

$$s_s/m = P_s \begin{Bmatrix} \begin{cases} 0 & , 0 \le r \le b \\ \{I\}\{\Delta\}\left\{r_s^2[1 - 2\ln(r_s)]\right\} & , b \le r \le a + b \\ 1 & , a + b \le r \end{cases} \end{Bmatrix} , \qquad (1)$$

where

$$r_s = \frac{r}{a} - \frac{b}{a} ,$$

s_s is the vertical subsidence (positive down) around the unmined element, m is the mined height, P_s is the proportion of the maximum subsidence attributed to the unmined element, r is the radial coordinate from the centerline of the element, b is the half-width of the element, and a is the radial extent of influence outside the element (see Figure 1). $\{I\}$ and $\{\Delta\}$ are functions that express the effects of the moment of inertia of the overlying strata and the crushing of unmined material on the element response, respectively. As this solution suggests, the unmined elements hold the surface above them at its original position, but allow the surface above neighboring mined zones to move down according to the elastic solution.

The elastic solution, however, must be modified to account for possible variation in the thickness of the elastic beam representing the unfractured overlying strata. Physical and numerical models of the response of the overlying strata (Sutherland and Schuler, 1982) have shown that the failure zone above a longwall panel produces a thinning of the remaining elastic beam as one moves away from the rib. This thinning can be accommodated in Equation 1 by using the moment of inertial function $\{I\}$. If $t(r_s)$ is the thickness of the plate as a function of the radius r_s and t_o is the center thickness of the plate, then for $b \leq r \leq a + b$

$$\{I\} = \left[\frac{t_o}{t(r_s)}\right]^3 \quad . \tag{2}$$

The elastic solution must also be modified to incorporate the effect of possible crushing of the unmined element $\{\Delta\}$. Analysis of this effect is the subject of a forthcoming paper and will not be treated further. This crushing effect, while very important for room-and-pillar mines, is less essential to the illustrations involving longwall panels discussed here.

Response of the Mined Element

As a portion of the overlying strata progressively breaks and falls into the mined cavity, vertically nonuniform voids are left throughout the caved overburden. Description of this distribution of void in the panel center has led to the prediction of maximum subsidence (Munson and Benzley, 1980). Near the rib side, both the horizontal and vertical distribution of residual void is non-uniform. The horizontal distribution is simply related to the probability of later migration of a void as it moves to the surface. By assuming a Gaussian probability distribution for this migration process, the integrated effect at the surface has the form of an error function; namely,

$$s_v/m = P_v \left\{ \begin{array}{lr} 1 & , 0 \leq r \leq b \\ \mathrm{erfc}(2r_v) = 1 - \frac{2}{\sqrt{\pi}}\int_0^{2r_v} \exp(-\xi^2)d\xi & , b \leq r \leq c + b \\ 0 & , c + b \leq r \end{array} \right\} , \tag{3}$$

where

$$r_v = r/c - r/b \quad ,$$

and s_v is the vertical subsidence due to the mined element, P_v is the proportion of the maximum subsidence due to the mined element, c is the radial extent of influence outside the element, and ξ is an integration parameter. This probability distribution appears similar to other subsidence analyses (Brauner, 1973); but contrary to the other analyses, we are concerned only with the residual void in the overburden and not the total mined volume. Thus, the mined element response is based on the horizontal distribution of part of the excavated volume (the bulking) in the overlying strata.

APPLICATION TO A LONGWALL PANEL

According to the functions just presented, the parameter set that governs subsidence is a, c, P_s and P_v. In terms of a longwall panel, a and c can be interpreted as the half range of the profile function and the draw angle, and P_s and P_v are related to the centerline displacement and the displacement above the rib side, respectively. Specific values of these parameters, together with simple computational techniques, are applied to the analysis of subsidence over the longwall panel 2N at the Old Ben No. 24 coal mine. This panel is located at a depth h of 189 m and the excavated height m is nominally 2.1 m (Edl and Eichfeld, 1978). In nondimensional form, the parameters a/h, b/h, P_s, c/h and P_v were taken to be 0.48, 0.004, 0.5, 0.48, and 0.08. The resulting displacements and their sum (i.e., the total subsidence) are plotted in Figure 2 for the case where {I} is a constant. The measured subsidence data, also plotted in Figure 2, show an influence of a variable beam thickness. Consequently, a calculation was made that assumed a moment-of-inertia term of the form

$$\{I\} = \left[\frac{t_o}{t(r_s)}\right]^3 = \left[\frac{1}{1 + p\ (1 - r_s)^3}\right]^3 \ . \qquad (4)$$

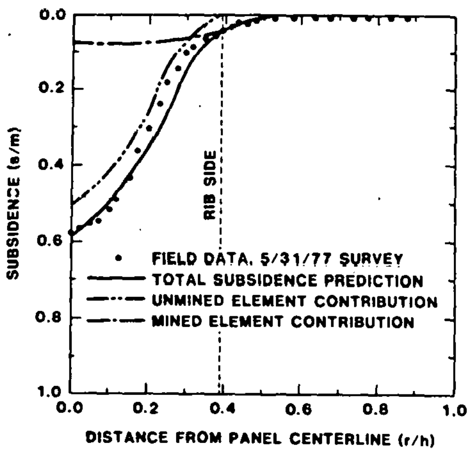

Fig. 2. Subsidence Prediction (Without Variation in in Beam Thickness)

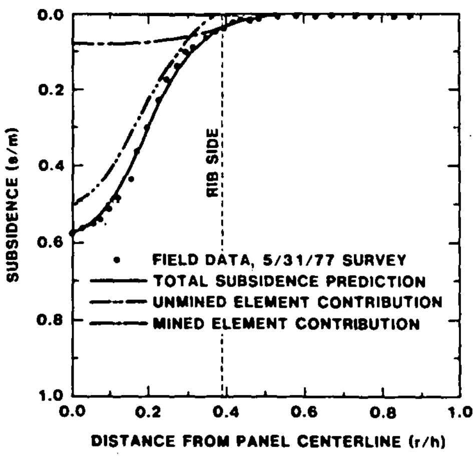

Fig. 3. Subsidence Prediction (With Variation in in Beam Thickness)

This equation implies that the beam forms a "stress arch" above the mined elements, where the ratio of the beam thicknesses at the ends of the span to that at the center of the span is $p + 1$. The result for $p = 0.3$ is shown in Figure 3 and is in much better agreement with the observed profile.

The application of complementary functions to three-dimensional panel geometry of Old Ben is illustrated in Figure 4. Superimposed on a quadrant of a longwall panel are the predicted subsidence contours for the cases of the influence of the unmined element alone, the mined element alone, and the total subsidence of the complementary functions. As with the subsidence profiles, the contours are in quantitative agreement with field measurements (Edl and Eichfeld, 1978).

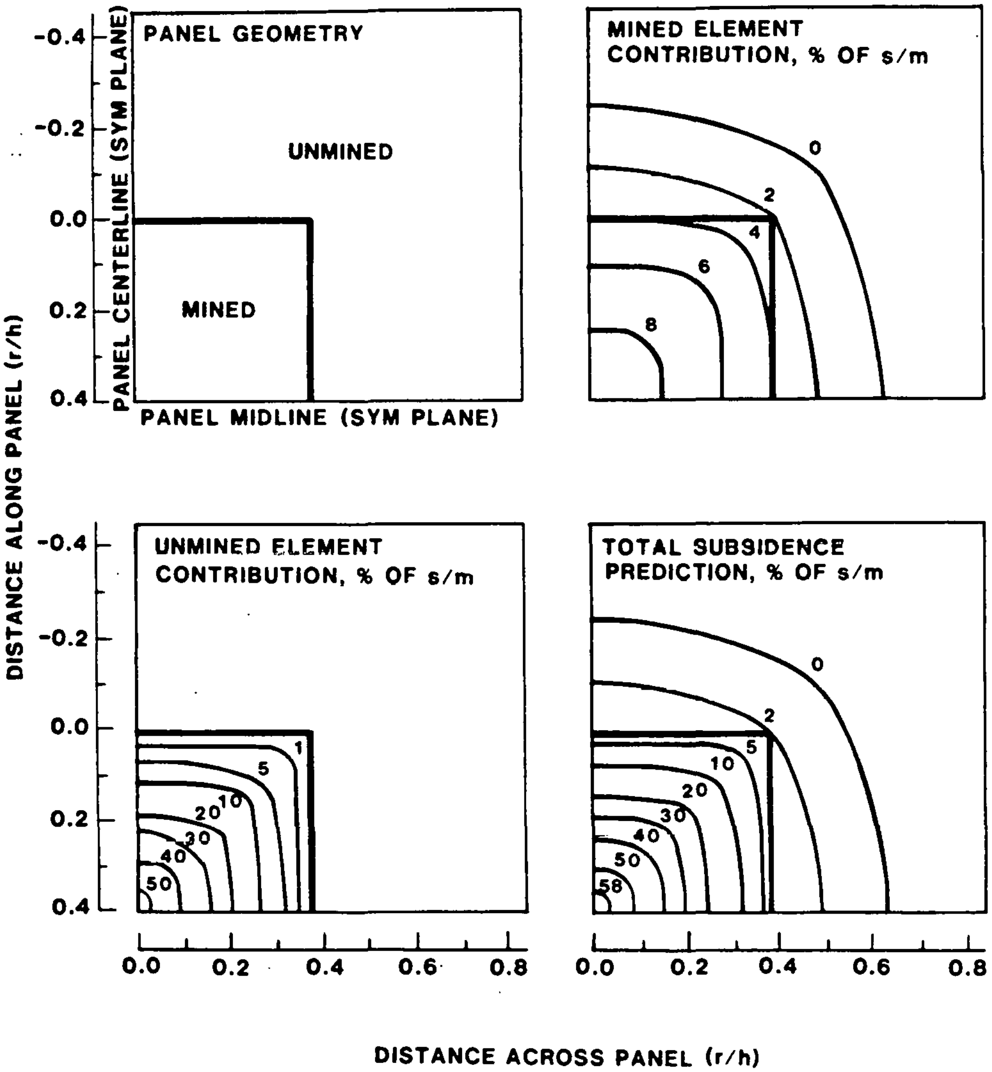

Fig. 4. Subsidence Contours Over a Longwall Panel

CONCLUSIONS

A new approach is developed for the use of influence functions in the prediction of mine subsidence. In this approach, complementary influence functions representing the response of both mined and unmined elements are integrated over the area and summed. Both elements strongly affect the subsidence. Development of complementary influence functions is essential in advancing the state-of-the-art in subsidence analysis of complicated room-and-pillar mines. Limited comparisons between field data and predictions are very encouraging.

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