Fuchs, Robert:

Detecting Structural Breaks in Financial Data: Simulation and Application to Trading Strategies

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Ausrichtung**

Masterthesis

Detecting Structural Breaks in Financial Data:
Simulation and Application to Trading Strategies

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Abstract

In this master thesis, we examine structural breaks in financial data. For that purpose, the Chow test, the t-test, the OLS-CUSUM test, the K&L test and the I&T test will be analyzed. In various simulations, the influence of different GARCH parameters on the test decision will be checked. A structural break in data can either happen in the first moment or in the second moment. Several simulations show that structural breaks in different GARCH parameters can be detected by structural break tests. By using data of the DAX and the Dow Jones, we create various trading strategies and compare them to a buy-and-hold strategy. These strategies are based on structural break tests. The trading strategies will apply the K&L test, the t-test and the OLS-CUSUM test as these tests have the best results in the accomplished simulations.

Contents

1. Introduction

„You need to divorce your mind from the crowd. The herd mentality causes all these IQ's to become paralyzed. I don't think investors are now acting more intelligently, despite the intelligence. Smart doesn't always equal rational. To be a successful investor you must divorce yourself from the fears and greed of the people around you, although it is almost impossible."[17, Warren Buffett]

Warren Buffett is one of the most famous investors in the world. He tries to invest rationally and many traders see in him a great role model. Traders all over the world try to create profit by predicting the future trend of financial data better than the market. Warren Buffett has been able to do this for a long time. But how can one beat the market? One rational approach is to use the assumption that financial data, like stocks, follow a trend where structural breaks with changing trends can occur. These structural breaks can either be in the returns or in the dynamics of the volatility. In theory one can recognize these structural breaks by using structural break tests to get information about the future trend. The next step would be to create a trading strategy which uses this information to make profit. This can either happen by using breaks in the returns, which leads to a signal for buying or selling the stock, or by using breaks in the volatility. In the second case, we have two options to create a trading strategy: Firstly, we can invest directly in the volatility by buying financial products, like options. For example, a straddle strategy, which includes an ATM („ATM" stands for „at the money") put and an ATM call, can be accomplished; secondly, we can invest in the stock by assuming a negative or a positive impact of the break on the volatility of the returns. Figure 1.1 shows a simulated price process of a random walk model with drift, and two structural breaks in the returns. This simulation is simplified and just used to show what structural breaks in returns and in prices look like.

The random walk model with drift assumes that the price changes are independent of each other. Therefore, a stochastic process can be created where the actual price depends on the previous price added by a drift component and a random error.

Structural Breaks in the Returns
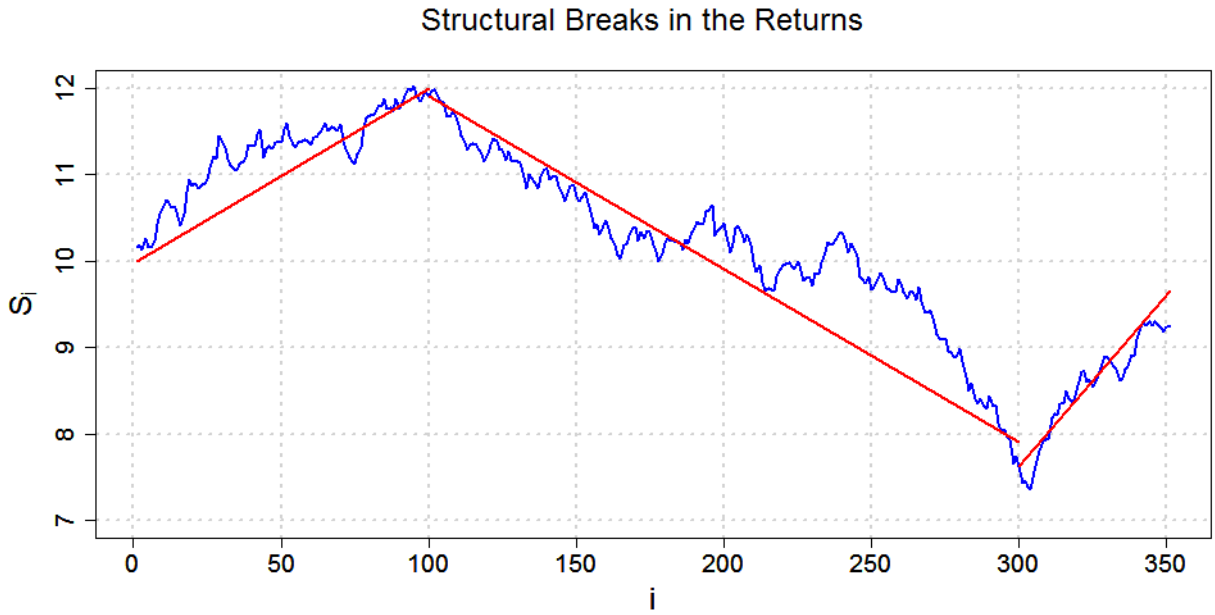
Figure 1.1.: *A random walk with drift model for stock prices S_i with two structural breaks in the drift:*
$S_i = \alpha_t + S_{i-1} + \varepsilon_i$ with $\alpha_1 = 0.02$, $\alpha_2 = -0.02$, $\alpha_3 = 0.04$ and $\varepsilon_i \overset{iid}{\sim} N(0, 0.1)$ for all $i \in \mathbb{R}$. The red lines describe the drift of the process.

This leads to the formula of a random walk model:

$$X_i = \alpha + X_{i-1} + \varepsilon_i, \tag{1.0.1}$$

$$\varepsilon_i \overset{iid}{\sim} (0, \sigma^2), \ \forall i \in \mathbb{R}. \tag{1.0.2}$$

[5, Dupernex, 2007, p. 168]

Figure 1.1 shows breaks in the drift α at day 100 and day 300. This drift component can be seen as returns, while the standard error ε_i determines the volatility.

Henceforth, log returns, which are calculated by

$$r_i = [ln(S_i) - ln(S_{i-1})] \cdot 100\%, \tag{1.0.3}$$

are meant if we talk about returns.

The second kind of structural breaks can be in the volatility of the returns. Therefore, figure 1.2 shows simplified returns with two structural breaks in the volatility by simulating them from a normal distribution. The assumption that returns can be simulated from a normal distribution violates many of the stylized facts [3, Cont, 2001]. For example that the volatility of returns occurs in clusters or that daily returns are not normally distributed. The figure shall just show what breaks in the volatility can look like. As we see, there are two breaks in the simulated returns. Between day 1 and 100 and day 301 and 350 σ^2 of the normal distribution equals 0.4, while between day 101 and 300 σ^2 of the normal distribution equals 0.1. Since the normal distribution has a mean of zero, there is no drift

Structural Breaks in the Volatility
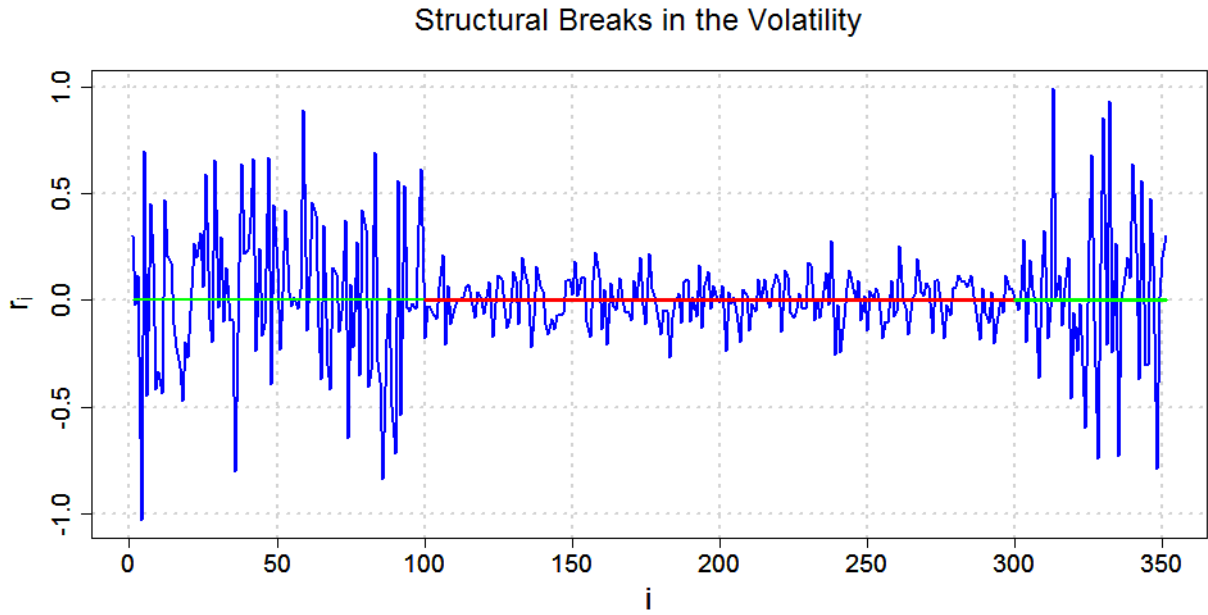
Figure 1.2.: *Simulated normal distributed log returns,*
$r_i = [ln(S_i) - ln(S_{i-1})] \cdot 100\%,$ *with two structural breaks in the volatility:* $r_i \overset{iid}{\sim} N(0, \sigma_t^2)$ *with* $\sigma_1^2 = \sigma_3^2 = 0.4$ *and* $\sigma_2^2 = 0.1$. *The green lines show the time where* $\sigma_1^2 = \sigma_3^2 = 0.4$. *The red line shows the phase where* $\sigma_2^2 = 0.1$.

in the prices, and the returns vary around zero. It would also be possible to change the mean of the standard errors and to create a structural break in the drift.

In the following, in chapter 2 we will discuss some tests for detecting structural breaks. The Chow test [2, Chow, 1960], the two-sample t-test [6, Fahrmeier, Künstler, Pigeot, Tutz 2011, p. 454 – 459] and an OLS-CUSUM test [20, Zeileis, Leisch, Hornik, Kleiber, 2002] will be introduced. Moreover, a CUSUM-based test considered by Kokoszka and Leipus [1, Andreou and Ghysels, 2001], henceforth K&L test, and a CUSUM-based test considered by Inclán and Tiao [1, Andreou and Ghysels, 2001], henceforth I&T test, will be presented. In chapter 3 the GARCH(1,1) model will be introduced and the structural break tests will be examined in various simulations. Firstly, the results of the K&L test and the I&T test from Andreou and Ghysels [1, Andreou and Ghysels, 2001] shall be reproduced. Secondly, the impact of various parameters on the test decision will be analyzed. After describing the used data in chapter 4, we will implement some trading strategies based on structural break tests and apply them to real data in chapter 5. In chapter 6 a short conclusion will be made and an outlook will be given.

2. Tests for Structural Breaks

In this chapter, some tests for detecting structural breaks will be presented. The used tests are the Chow test [2, Chow, 1960], the two-sample t-test [6, Fahrmeier, Künstler, Pigeot, Tutz 2011, p. 454 – 459] and the generalized OLS-based CUSUM test [20, Zeileis, Leisch, Hornik, Kleiber, 2002]. All of them can be applied to r_i, $|r_i|$ and r^2. By using r_i, the test shall recognize a break in the mean of the returns, while by using $|r_i|$ and r^2, the test shall detect a break in the dynamics of the volatility of the returns. Additionally, a CUSUM-based test considered by Kokoszka and Leipus [1, Andreou and Ghysels, 2001] and a CUSUM-based test considered by Inclán and Tiao [1, Andreou and Ghysels, 2001] will be introduced for testing for structural breaks in the volatility of returns. Here, the K&L test will be applied to $|r_i|$ and r^2, while the I&T test will be just applied to r^2.

2.1. Chow Test

The Chow test is a structural break test which was introduced by Gregory Chow [2, Chow, 1960]. A short explanation is also given by Lee [11, Lee, 2008]. The Chow test is constructed in a way that assumes a linear model, given in formula 2.1.1, of the examined data and also assumes that the breakpoint is known. The next step is to divide the dataset in a model with z observations preceding the break, given in formula 2.1.2, and a model with m observations following the break, given in formula 2.1.3.

$$Y = X\beta + u, \text{ with } u \overset{iid}{\sim} (0, \sigma^2), \tag{2.1.1}$$

$$Y_z = X_z\beta_z + u_z, \text{ with } u_z \overset{iid}{\sim} (0, \sigma_z^2,) \tag{2.1.2}$$

$$Y_m = X_m\beta_m + u_m, \text{ with } u_m \overset{iid}{\sim} (0, \sigma_m^2). \tag{2.1.3}$$

After fitting the full model and both submodels by using the method of least squares, given in formula 2.1.4,

$$\hat{\beta} = (X'X)^{-1}X'y, \tag{2.1.4}$$

the residuals u, u_z and u_m can be computed.

The test statistic is then given by

$$F = \frac{(u'u - u'_z u_z - u'_m u_m)/p)}{(u'_z u_z + u'_m u_m)/(z + m - 2p)} \sim F(p, z + m - 2p) \qquad (2.1.5)$$

where p is the number of regression parameters.

Rejection area of H_0:

$$H_0 : \beta_z = \beta_m, H_1 : \beta_z \neq \beta_m :$$
$$F > F(p, z + m - 2p).$$

If we look at the test statistic in formula 2.1.5, we can see that the numerator becomes zero if the squared residuals of the full model have the same value as the sum of the squared residuals of the submodels. This happens when the submodels have the same parameters as the full model and it indicates that there is no structural break.

In the following simulations, the random errors in the linear models are assumed to be normally distributed.

2.2. t-Test

The two-sample t-test, a parametric test, compares the mean of two independent populations. It has some conditions which are violated by the volatility of returns. The stylized facts state that r_i^2 and $|r_i|$ have a slow decay of autocorrelation [3, Cont, 2001]. Thus, the conditions 2.2.1 and 2.2.2 are violated by using quadratic or absolute returns. Nevertheless, the two-sample t-test for quadratic and absolute returns shall be checked in various simulations. A short explanation of the two-sample t-test can be found in [6, Fahrmeier, Künstler, Pigeot, Tutz, 2011, pp. 454 – 459].

Conditions:

$$X_1, ..., X_z \text{ independent iterations of X,} \qquad (2.2.1)$$
$$Y_1, ..., Y_m \text{ independent iterations of Y,} \qquad (2.2.2)$$
$$X_1, ..., X_z, Y_1, ..., Y_m \text{ independent of each other,} \qquad (2.2.3)$$
$$X \sim N(\mu_X, \sigma_X^2), Y \sim N(\mu_Y, \sigma_Y^2), \qquad (2.2.4)$$
$$\text{or X, Y arbitrarily distributed for } z, m \geq 30, \qquad (2.2.5)$$
$$\sigma_X^2 = \sigma_Y^2 \text{ unknown.} \qquad (2.2.6)$$

Test statistic:

$$T = \frac{\bar{X} - \bar{Y} - \delta_0}{\sqrt{\left(\frac{1}{z} + \frac{1}{m}\right) \frac{(z-1)S_X^2 + (m-1)S_Y^2}{z+m-2}}} \sim t(z+m-2), \tag{2.2.7}$$

where

$$S_X^2 = \frac{1}{z-1} \sum_{i=1}^{z} (X_i - \bar{X})^2, \tag{2.2.8}$$

$$S_Y^2 = \frac{1}{m-1} \sum_{i=1}^{m} (Y_i - \bar{Y})^2 \tag{2.2.9}$$

and

$$\bar{X} = \frac{1}{z} \sum_{i=1}^{z} X_i, \tag{2.2.10}$$

$$\bar{Y} = \frac{1}{m} \sum_{i=1}^{m} Y_i. \tag{2.2.11}$$

Rejection area of H_0:

(a) $H_0 : \mu_X - \mu_Y = \delta_0,\ H_1 : \mu_X - \mu_Y \neq \delta_0 :$
$\quad |T| > t_{1-\alpha/2}(z+m-2),$
(b) $H_0 : \mu_X - \mu_Y \geq \delta_0,\ H_1 : \mu_X - \mu_Y < \delta_0 :$
$\quad T < -t_{1-\alpha}((z+m-2),$
(c) $H_0 : \mu_X - \mu_Y \leq \delta_0,\ H_1 : \mu_X - \mu_Y > \delta_0 :$
$\quad T > t_{1-\alpha}(z+m-2).$

Since we test for structural breaks, the t-test will be applied with $\delta_0 = 0$. While the Chow test and the t-test assume that the breakpoint is known, the OLS-CUSUM test, the K&L test and the I&T test check where a structural break is most likely.

2.3. OLS-CUSUM Test

The next test that will be introduced is the OLS-CUSUM test, which is explained in [20, Zeileis, Leisch, Hornik, Kleiber, 2002]. Lets assume again a linear model with formula 2.3.1.

$$Y_i = X_i \beta_i + u_i, \text{ with } i = 1, \cdots, n \text{ and } u_i \overset{iid}{\sim} (0, \sigma^2). \tag{2.3.1}$$

The variance of the residuals can be computed by formula 2.3.2 and 2.3.3 where p is the number of regression parameters and $\hat{\beta}_i^{(n)}$ is the estimator by using the method of least squares, given in formula 2.1.4.

$$\hat{u}_i = y_i - X_i \hat{\beta}_i^{(n)}, \tag{2.3.2}$$

$$\hat{\sigma}^2 = \frac{1}{n-p} \sum_{i=1}^{n} \hat{u}_i^2. \tag{2.3.3}$$

The OLS-CUSUM test then has the following formula for the process:

$$W_n^0(t) = \frac{1}{\hat{\sigma}\sqrt{n}} \sum_{i=1}^{|nt|} \hat{u}_i, \text{ with } 0 \le t \le 1. \tag{2.3.4}$$

The standard Brownian Bridge $W^0(t) = W(t) - tW(1)$ where $W(t)$ is a Wiener process [6, Hull, 2009, p. 282 – 286] then is the limiting process for $W_n^0(t)$. [20, Zeileis, Leisch, Hornik, Kleiber, 2002]

Thus, the boundaries can be calculated by a Kolmogorov-Smirnov type asymptotic distribution because the supremum of a Brownian bridge converges against the Kolmogorov-Smirnov distribution. [1, Andreou and Ghysels, 2001]

If $W^0(t)$ violates this boundary up or down, the null hypothesis that there is no structural break must be rejected. For better programming performance, an approximation for the Kolmogorov-Smirnov distribution will be used in the analysis. In formula 2.3.5 the approximation for $n > 35$ is given, while in formula 2.3.6 an adjusted approximation is shown. [7, Hedderich and Sachs, 2015, pp. 462-463]

The adjusted formula will be used because the test statistic of the OLS-CUSUM test 2.3.4, the K&L test 2.4.1 and the I&T test 2.5.1 are adjusted, too.

$$Q_{1-\alpha} = \frac{\sqrt{-0.5ln(\frac{\alpha}{2})}}{\sqrt{n}}, \tag{2.3.5}$$

$$Q_{1-\alpha} = \sqrt{-0.5ln(\frac{\alpha}{2})}. \tag{2.3.6}$$

As we can see in figure 2.1, the boundaries for small α are almost equal to the values of the Kolmogorov-Smirnov distribution but for a value of $\alpha \ge 0.5$ both lines diverge. Since we will use $\alpha \le 0.2$ in the following chapters, the approximation can be applied. For calculating p-values, the approximation cannot be used as the inverse of the adjusted approximation can have values between [0,2].
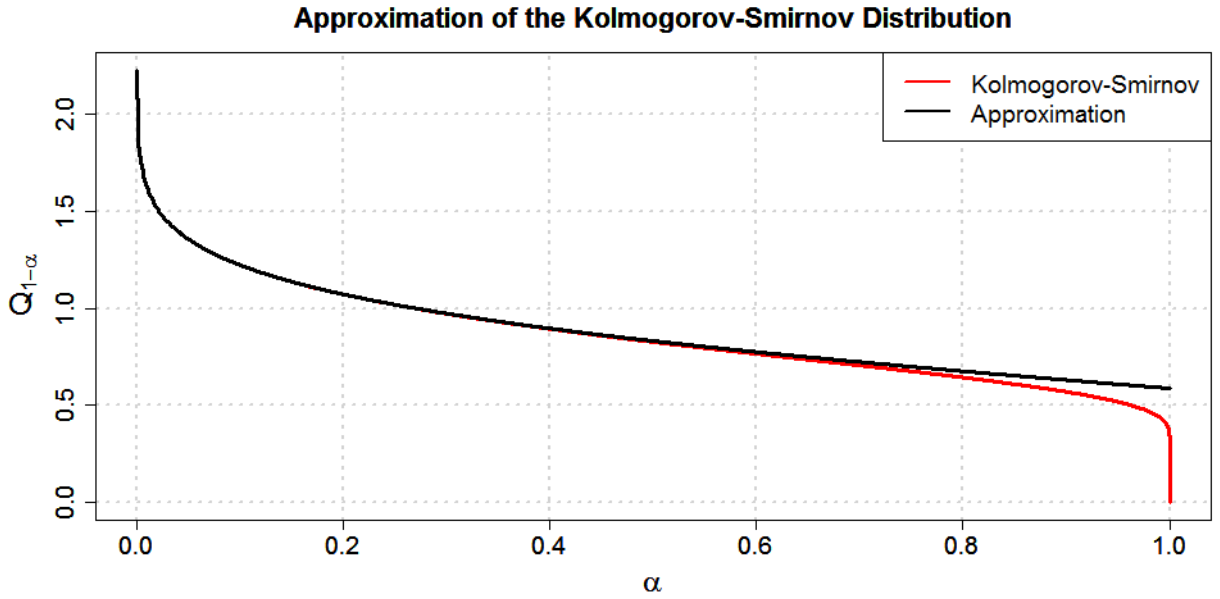
Figure 2.1.: *Quantiles of the Kolmogorv-Smirnov distribution adjusted by $\sqrt{n}$ and the quantiles of the adjusted approximation $Q_{1-\alpha} = \sqrt{-0.5ln(\frac{\alpha}{2})}$. The range of α is from 0.0001 to 1.*

2.4. K&L Test

A similar structural break test to the OLS-CUSUM test is the K&L test. While the previous tests will be used with r_i, $|r_i|$ and r_i^2 in the simulations, the K&L test will be used with $|r_i|$ and r_i^2 and the I&T test will be used wit r_i^2. The K&L test is described in [1, Andreou and Ghysels, 2001] and is a CUSUM based test. It is not necessary to know the point of the structural break. The considered process of the test is given by

$$U_N(k) = \left(\frac{1}{\sqrt{N}} \sum_{j=1}^{k} X_j - \frac{k}{N\sqrt{N}} \sum_{j=1}^{N} X_j \right) \tag{2.4.1}$$

where $0 < k < N$ is true and N is the number of observations.

To find the point $\hat{k}$ where the probability of a breakpoint is the highest, we take the maximum of the process in formula 2.4.1, and get:

$$\hat{k} = min\{k : |U_N(k)| = \max_{0 \leq j \leq N} |U_N(j)|\}. \tag{2.4.2}$$

Under the null hypothesis, no structural break exists, the process in formula 2.4.1 follows a Brownian bridge with $\sigma^2 = \Sigma_{j=-\infty}^{\infty} Cov(X_j, X_0)$:

$$U_N(k) \rightarrow_{D[0,1]} \sigma B(k). \tag{2.4.3}$$

Thus, with rearranging the formula, we get

$$sup\{|U_N(k)|\}/\hat{\sigma} \rightarrow_{D[0,1]} sup\{B(k) : k \in [0,1]\} \qquad (2.4.4)$$

which follows a Kolmogorov-Smirnov type asymptotic distribution. [1, Andreou and Ghysels, 2001]

H_0 can be rejected if the test statistic 2.4.4 is greater than the boundary.

The critical value is calculated in the same way as for the OLS-CUSUM test. As estimator for σ^2 an approach of Newey and West [12, Newey and West, 1994] will be applied:

$$\hat{\sigma}^2 = \hat{\Omega}_0 + \sum_{j=1}^{m} (1 - \frac{j}{m+1})(\hat{\Omega}_j + \hat{\Omega}_j'), \qquad (2.4.5)$$

$$\hat{\Omega}_j = \hat{Cov}(X_t, X_{t-j}), \qquad (2.4.6)$$

$$m = N^{\frac{1}{3}}. \qquad (2.4.7)$$

2.5. I&T Test

The I&T test is constructed to find a single and unknown structural break. It is described in [1, Andreou and Ghysels, 2001] and is similar to the OLS-CUSUM test and the K&L test. Usually, this test is applied to i.i.d. data but in the following simulations it is applied to r_i^2, which are not i.i.d. distributed. This could lead to problems in the specificity or the power of the test decision. The test statistic is given by

$$IT = \sqrt{N/2} \max_k |D_k|, \qquad (2.5.1)$$

$$D_k = \frac{\sum_{j=1}^{k} X_j}{\sum_{j=1}^{N} X_j} - \frac{k}{N}. \qquad (2.5.2)$$

Again, the supremum of the test statistic in 2.5.1 converges against the Kolmogorov-Smirnov distribution and the boundaries can be computed in the same way as for the OLS-CUSUM test and the K&L test.

3. Simulation Studies

To check which test gives us the best results for detecting structural breaks in r_i, r_i^2 and $|r_i|$, we will simulate returns by using a GARCH(1,1) model and apply the various tests to these returns. The simulations have implemented structural breaks in various GARCH parameters. All simulations were made in a Monte-Carlo design with 2000 iterations and a burn-in of 100.

3.1. GARCH(1,1)

The GARCH model („GARCH" stands for „generalized autoregressive conditional heteroscedasticity") is a suitable model for simulating returns and volatility. It is described in [10, Kreiß and Neuhaus, 2006, pp. 298-323]. Since we just use a GARCH(1,1) model in the following simulations, in this section only the GARCH(1,1) model will be explained. If $i \varepsilon \mathbb{Z}$, r_i the returns from formula 1.0.3, σ_i the volatility and $u_i \overset{iid}{\sim}(0,1)$ is true, the GARCH(1,1) model can be represented in the following formula:

$$r_i = \mu + \sigma_i \cdot u_i, \tag{3.1.1}$$

$$\sigma_i^2 = \omega + \alpha r_{i-1}^2 + \beta \sigma_{i-1}^2 \tag{3.1.2}$$

where ω, α_1 and β_1 are nonnegative and real parameters and $\omega, \alpha_1, \beta_1 \neq 0$ is true. μ indicates the mean of the returns and can be seen as a drift component.

Since we want to simulate returns and volatility with possible structural breaks, the model is divided in two submodels. The model before the structural break happens is described in formulas 3.1.3, 3.1.4 and 3.1.5, the model after the structural break happens is described in formulas 3.1.6, 3.1.7 and 3.1.8. The breakpoint is determined as k with $k = n_1 n$ where n is the number of observations and $n_1 \in [0,1]$ is the relative breakpoint.

Model preceding the structural break:

$$r_i = \mu_1 + \sigma_i \cdot u_i, \text{ with } u_i \overset{iid}{\sim} N(0, \varepsilon_1), \tag{3.1.3}$$

$$\sigma_i^2 = \omega_1 + \alpha_1 r_{i-1}^2 + \beta_1 \sigma_{i-1}^2, \tag{3.1.4}$$

$$1 \leq i \leq k. \tag{3.1.5}$$

Model following the structural break:

$$r_i = \mu_2 + \sigma_i \cdot u_i, \text{ with } u_i \overset{iid}{\sim} N(0, \varepsilon_2), \tag{3.1.6}$$

$$\sigma_i^2 = \omega_2 + \alpha_2 r_{i-1}^2 + \beta_2 \sigma_{i-1}^2, \tag{3.1.7}$$

$$k + 1 \leq i \leq n. \tag{3.1.8}$$

3.2. ACF in Simulated GARCH(1,1) Model

Before we start with the comparison of the various tests, a brief look at the ACF („ACF"
stands for „autocorrelation function") will be given. [14, Ruppert, 2010, p. 206]
By estimating the sample autocovariance function

$$\hat{\gamma}(h) = \frac{1}{n} \sum_{j=1}^{n-h} (Y_{j+h} - \bar{Y})(Y_j - \bar{Y}), \tag{3.2.1}$$

with $\bar{Y}$ calculated through formula 2.2.11, we get the sample ACF which is defined as

$$\hat{\rho}(h) = \frac{\hat{\gamma}(h)}{\hat{\gamma}(0)}. \tag{3.2.2}$$

Figure 3.1 shows the ACF for simulated returns r_i and the corresponding conditional
volatilities σ_i. The GARCH(1,1) model will be simulated with $\alpha = 0.89$ and $\beta = 0.1$. As
we can see, there is no autocorrelation in the simulated returns. In contrast to the returns,
the simulated conditional volatilities have an autocorrelation with fast decay. In figure
3.2 the ACF for the simulated returns and for the corresponding conditional volatilities
with $\alpha = 0.1$ and $\beta = 0.89$ are given. As in the previous figure, we could not detect any
autocorrelation in the simulated returns. The ACF of the simulated conditional volatilities
shows a slow decay of autocorrelation. To summarize, the simulated returns do not have
any autocorrelation and the GARCH parameter α and β have a positive impact on the
ACF of the volatilities.

3.3. K&L Test and I&T Test

After describing the construction of the simulations, the K&L test and the I&T test will
be examined. To this end, the parameters of the simulation are adapted to the parameters
of the simulation from Andreou and Ghysels. [1, Andreou and Ghysels, 2001, pp. 31-32]
Since a different estimator for $\hat{\sigma}$ are used in this simulation, the results for the K&L test
can differ from the results of [1, Andreou and Ghysels, 2001, pp. 31-32]. Moreover,
the values from the following simulations for the I&T test differ from the values of [1,
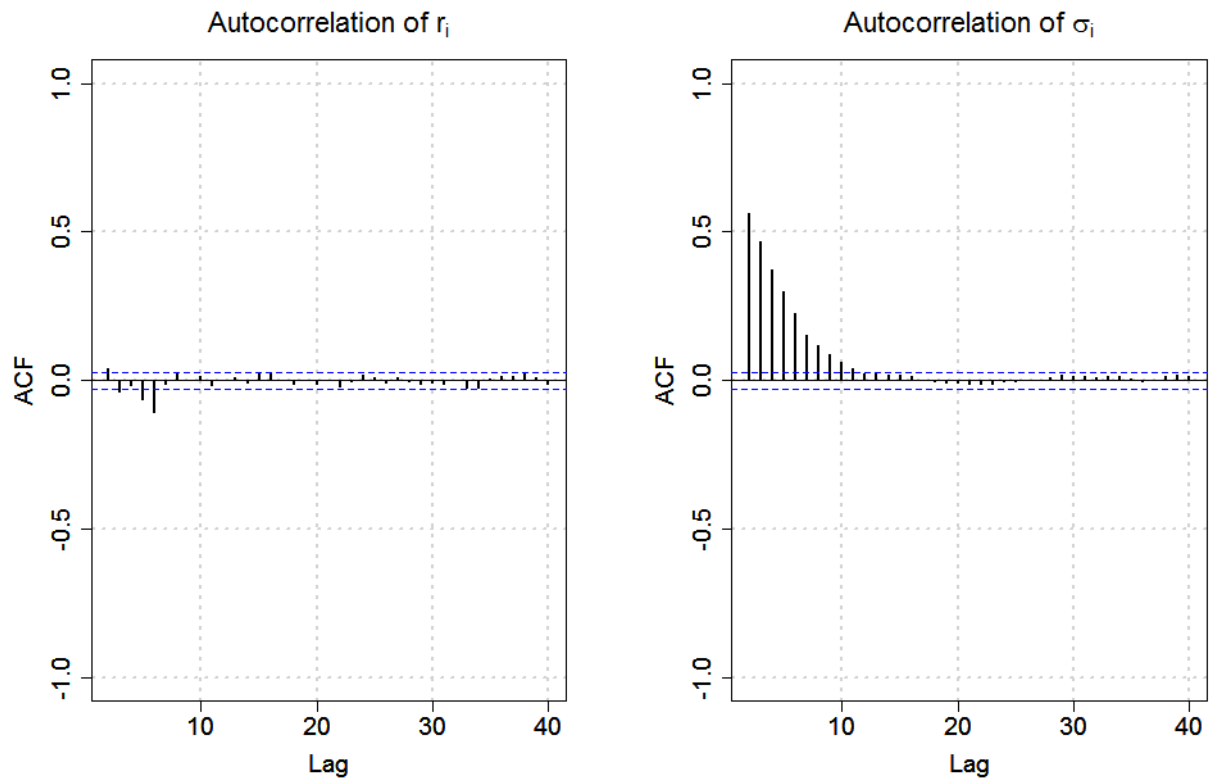
Figure 3.1.: *Autocorrelation of a GARCH(1,1) Model with $\alpha = 0.89$ and $\beta = 0.1$. On the left is the ACF of the simulated r_i, on the right is the ACF of the simulated σ_i.*
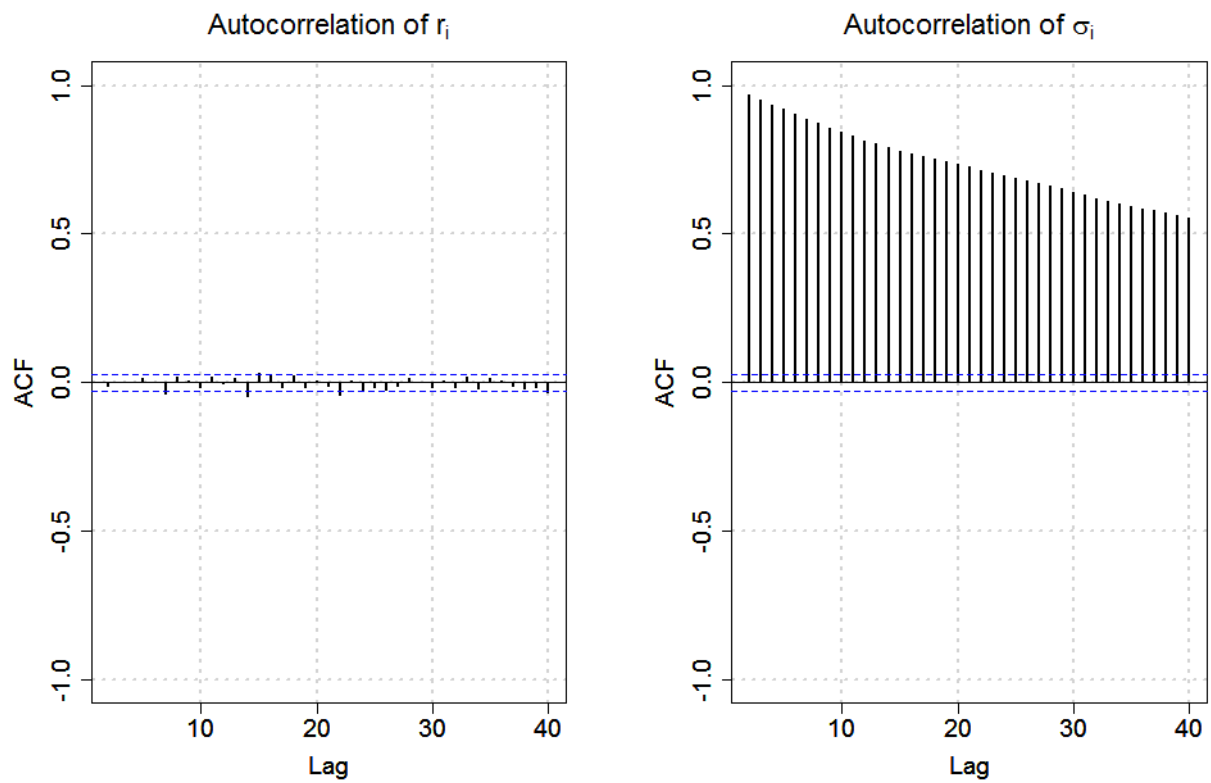
Figure 3.2.: *]*

Autocorrelation of a GARCH(1,1) Model with $\alpha = 0.1$ and $\beta = 0.89$. On the left is the ACF of the simulated r_i, on the right is the ACF of the simulated σ_i.

r_i^2	$Q_{0.90}$	H_0: $\alpha = 0.10$	$Q_{0.95}$	H_0: $\alpha = 0.05$	$Q_{0.99}$	H_0: $\alpha = 0.01$
n=250	1.476	0.229	1.650	0.149	1.986	0.056
n=500	1.495	0.264	1.693	0.174	2.039	0.065
n=750	1.486	0.242	1.662	0.152	2.002	0.056
n=1000	1.494	0.262	1.666	0.162	1.982	0.059
n=1250	1.500	0.278	1.731	0.177	2.089	0.066
n=1500	1.531	0.260	1.709	0.175	2.067	0.071
n=2000	1.528	0.269	1.703	0.170	1.972	0.070
n=3000	1.503	0.283	1.701	0.184	1.985	0.064
n=3500	1.527	0.292	1.696	0.190	2.053	0.062

Table 3.1.: *Quantiles and the relative frequencies of H_0 by using the I&T test with r_i^2 under H_0:* $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.5$ *and* $\varepsilon_1 = \varepsilon_2 = 1$.

Andreou and Ghysels, 2001, pp. 31-32]. No reason could be found for this.

First of all, the returns are simulated and the K&L test with r_i^2 and $|r_i|$ will be applied, while the I&T test just will be applied with r_i^2. After repeating this step 2000 times, the quantiles and the relative frequencies of H_0 with given α will be calculated. Both simulations in this section are made under the null hypothesis that no structural break exists. The first simulation with following parameters

$$\mu_1 = \mu_2 = 0,$$
$$\omega_1 = \omega_2 = 0.4,$$
$$\alpha_1 = \alpha_2 = 0.1,$$
$$\beta_1 = \beta_2 = 0.5,$$
$$\varepsilon_1 = \varepsilon_2 = 1$$

will be accomplished with changing n.

In table 3.1 the results are given for the I&T test. As we can see, the changes in the quantiles are marginally for various n. For $\alpha = 0.1$, $\alpha = 0.05$ and $\alpha = 0.01$ the critical values are equal to 1.224, 1.358 and 1.628. The respective quantiles are all higher than the critical values. Furthermore, the rejection rate of H_0 is higher than the α level. For $Q_{0.90}$, $Q_{0.95}$ and $Q_{0.99}$ we get values between $[0.229, 0.292]$, $[0.149, 0.190]$ and $[0.056, 0.071]$ which is above the respective α level. An optimal test under H_0 would result in a rejection rate of H_0 equal to the α level, which is not the case in this simulation. If we look at the K&L test with r_i^2 in table 3.2 and with $|r_i|$ in table 3.3, we get another impression. The quantiles for different n vary around the boundaries which are the same as for the I&T test and the rejection rates are similar to the α level. It can be said that in this simulation under H_0 the K&L test works well and keeps the α level for r_i^2 and for $|r_i|$.

r_i^2	$Q_{0.90}$	H_0: $\alpha = 0.10$	$Q_{0.95}$	H_0: $\alpha = 0.05$	$Q_{0.99}$	H_0: $\alpha = 0.01$
n=250	1.214	0.095	1.335	0.043	1.562	0.005
n=500	1.240	0.109	1.375	0.056	1.615	0.008
n=750	1.207	0.089	1.349	0.048	1.603	0.007
n=1000	1.226	0.102	1.354	0.048	1.624	0.010
n=1250	1.221	0.098	1.375	0.057	1.633	0.011
n=1500	1.237	0.105	1.361	0.051	1.652	0.011
n=2000	1.229	0.104	1.361	0.051	1.616	0.010
n=3000	1.234	0.106	1.374	0.058	1.598	0.008
n=3500	1.238	0.107	1.358	0.050	1.617	0.010

Table 3.2.: *Quantiles and the relative frequencies of H_0 by using the K&L test with r_i^2 under H_0:*
$\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.5$ *and* $\varepsilon_1 = \varepsilon_2 = 1$.

$|r_i|$	$Q_{0.90}$	H_0: $\alpha = 0.10$	$Q_{0.95}$	H_0: $\alpha = 0.05$	$Q_{0.99}$	H_0: $\alpha = 0.01$
n=250	1.233	0.105	1.354	0.049	1.559	0.002
n=500	1.253	0.117	1.390	0.060	1.611	0.009
n=750	1.223	0.099	1.347	0.044	1.632	0.010
n=1000	1.234	0.105	1.367	0.051	1.625	0.010
n=1250	1.234	0.105	1.390	0.058	1.663	0.012
n=1500	1.230	0.103	1.387	0.054	1.662	0.012
n=2000	1.239	0.110	1.367	0.052	1.631	0.010
n=3000	1.237	0.108	1.375	0.056	1.624	0.010
n=3500	1.235	0.105	1.360	0.051	1.609	0.008

Table 3.3.: *Quantiles and the relative frequencies of H_0 by using the K&L test with $|r_i|$ under H_0:*
$\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.5$ *and* $\varepsilon_1 = \varepsilon_2 = 1$.

In another simulation under H_0 we adjust the parameters to

$$\mu_1 = \mu_2 = 0,$$
$$\omega_1 = \omega_2 = 0.2,$$
$$\alpha_1 = \alpha_2 = 0.1,$$
$$\beta_1 = \beta_2 = 0.7,$$
$$\varepsilon_1 = \varepsilon_2 = 1$$

where n changes again. The construction of this simulation is the same as in the previous simulation. Firstly, we want to take a look at the results of the I&T test in table 3.4 which differs from the previous simulation. The values of the quantiles as well as the rejection rates are higher. Due to the fact that only $\omega_1 = \omega_2$ and $\beta_1 = \beta_2$ have changed, an intuitive explanation can be given by higher autocorrelation effects. The autocorrelation effect for σ in the GARCH model increases with rising α or β. Thus, the changed β can explain this decline of the results. Nevertheless, the impact of various parameters on the I&T test will be examined in the following simulations. The same effect of higher autocorrelation can be seen for the K&L test. In table 3.5 and table 3.6 the results for the K&L test with

r_i^2	$Q_{0.90}$	H_0: $\alpha = 0.10$	$Q_{0.95}$	H_0: $\alpha = 0.05$	$Q_{0.99}$	H_0: $\alpha = 0.01$
n=250	1.675	0.347	1.908	0.237	2.360	0.117
n=500	1.770	0.418	1.988	0.307	2.416	0.151
n=750	1.809	0.450	2.011	0.330	2.473	0.164
n=1000	1.785	0.451	1.988	0.326	2.489	0.159
n=1250	1.784	0.463	2.001	0.330	2.361	0.162
n=1500	1.804	0.476	2.029	0.354	2.452	0.181
n=2000	1.787	0.488	1.973	0.362	2.416	0.177
n=3000	1.851	0.492	2.042	0.365	2.369	0.202
n=3500	1.843	0.530	2.043	0.396	2.397	0.201

Table 3.4.: *Quantiles and the relative frequencies of H_0 by using the I&T test with r_i^2 under H_0: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.2$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.7$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i^2	$Q_{0.90}$	H_0: $\alpha = 0.10$	$Q_{0.95}$	H_0: $\alpha = 0.05$	$Q_{0.99}$	H_0: $\alpha = 0.01$
n=250	1.298	0.136	1.419	0.074	1.646	0.012
n=500	1.310	0.155	1.450	0.082	1.698	0.018
n=750	1.325	0.162	1.467	0.086	1.715	0.019
n=1000	1.298	0.139	1.421	0.071	1.659	0.011
n=1250	1.297	0.142	1.422	0.074	1.660	0.011
n=1500	1.314	0.154	1.446	0.078	1.746	0.020
n=2000	1.284	0.133	1.407	0.066	1.687	0.013
n=3000	1.312	0.152	1.438	0.080	1.671	0.016
n=3500	1.318	0.145	1.434	0.082	1.703	0.018

Table 3.5.: *Quantiles and the relative frequencies of H_0 by using the K&L test with r_i^2 under H_0: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.2$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.7$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i^2 and $|r_i|$ are given. As we can see, the rejection rates are higher than the α level and the quantiles are higher than the boundaries. For the squared returns $Q_{0.90}$, $Q_{0.95}$ and $Q_{0.99}$, we have values between $[0.133, 0.162]$, $[0.071, 0.086]$ and $[0.011, 0.020]$ which is above the respective α level. For absolute returns we have similar values. To conclude, under H_0 with increasing autocorrelation effect the test cannot keep the given α level. A more accurate examination will be given in the next section.

3.4. Comparison of the Tests

In this section, the influence of changing GARCH parameters will be examined. The construction of the simulations is equal to the previous simulations. After simulating the log returns, the Chow test, the t-test, the OLS-CUSUM test, the K&L test and the I&T test will be applied to the data. Therefore, we compare the tests for the first and the second moments. If a test is applied with r_i, then it should check for a break in the first moment, the mean of the log returns. If a test is applied with $|r_i|$ or r_i^2, then it should check for a break in the second moment, the variance or rather the volatility of the log returns. In

$\lvert r_i \rvert$	$Q_{0.90}$	H_0: $\alpha = 0.10$	$Q_{0.95}$	H_0: $\alpha = 0.05$	$Q_{0.99}$	H_0: $\alpha = 0.01$
n=250	1.315	0.144	1.439	0.080	1.650	0.012
n=500	1.333	0.159	1.470	0.087	1.722	0.020
n=750	1.345	0.161	1.484	0.096	1.717	0.020
n=1000	1.310	0.145	1.428	0.076	1.660	0.014
n=1250	1.305	0.150	1.424	0.072	1.680	0.016
n=1500	1.302	0.152	1.459	0.075	1.713	0.021
n=2000	1.295	0.135	1.406	0.065	1.662	0.014
n=3000	1.315	0.154	1.459	0.080	1.702	0.015
n=3500	1.318	0.158	1.446	0.084	1.694	0.015

Table 3.6.: *Quantiles and the relative frequencies of H_0 by using the K&L test with $\lvert r_i \rvert$ under H_0: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.2$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.7$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

this section, the results are shown only graphically for an $\alpha = 0.05$. All simulation results of this section are tabled in the appendix. Additionally, the appendix contains the same simulations with $\alpha = 0.10$.

3.4.1. Influence of n

Firstly, we want to compare the influence of the sample size n under H_0. Therefore, all five tests will be accomplished for

$$\mu_1 = \mu_2 = 0,$$
$$\omega_1 = \omega_2 = 0.4,$$
$$\alpha_1 = \alpha_2 = 0.1,$$
$$\beta_1 = \beta_2 = 0.5,$$
$$\varepsilon_1 = \varepsilon_2 = 1$$

where n changes. In figure 3.3 and figure 3.4 the results of the simulations for the first moment and the second moment are displayed. The x axis represents the number of observations n. Since the simulation has no structural break, we do not need to set a breaking point n_1. The y axis represents the relative frequencies of H_0. In figure 3.3 the results for testing for structural breaks in the first moment are shown. This means that only r_i was applied. In figure 3.4 the results for testing for structural breaks in the second moment are shown. Thus, r_i^2 and $\lvert r_i \rvert$ were applied. The structure of the graphics will be the same in the remaining chapter. If we look at the results for the first moment, we can see that for all tests the results are similar and vary around a value of 0.95. This implies that all tests keep the α level. Only for a small size of observations, the CUSUM test has a higher rejection rate of H_0. It can be said that n has no influence on the test results of the Chow test, the t-test and the CUSUM test in this simulation.

If we take a look at the second moments in figure 3.4, a different impression can be
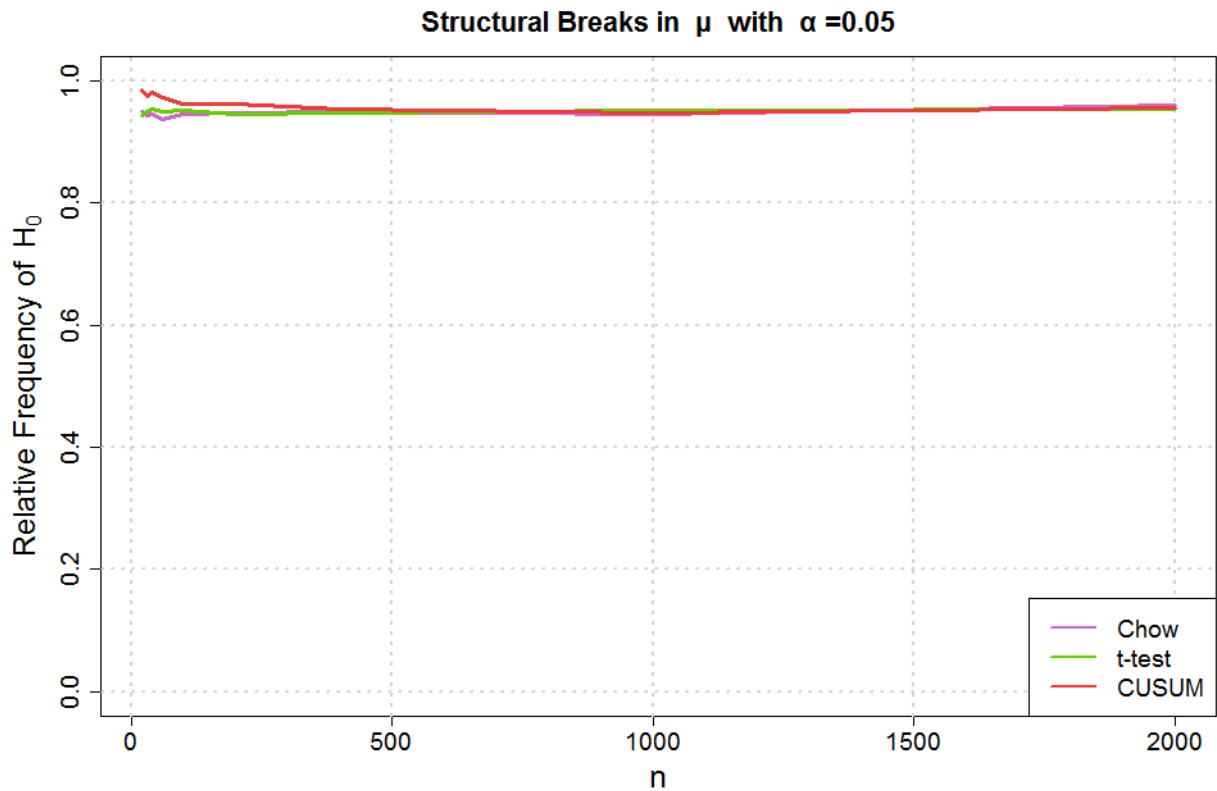
Figure 3.3.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.5$, and $\varepsilon_1 = \varepsilon_2 = 1$.*

gained. For all tests, the rate of H_0 declines for increasing n. The results for squared returns and absolute returns are similar. While the K&L test has a value near to 0.95, for a high enough sample size, the t-test and the Chow test have values around 0.90. The OLS-CUSUM test and the I&T test decline to a value of 0.80. It can be determined that there is just a marginal influence of n for the K&L test, the Chow test and the t-test if n is high enough. The K&L test gives the best results for this simulation.
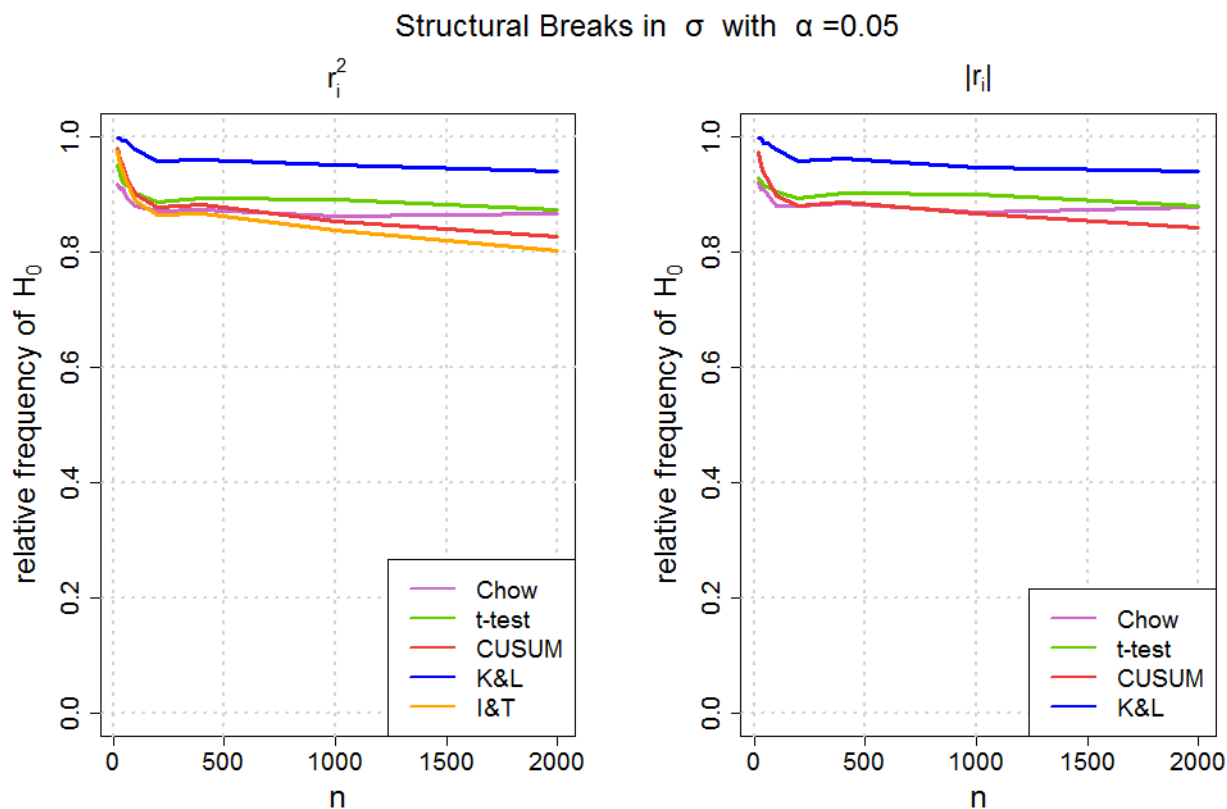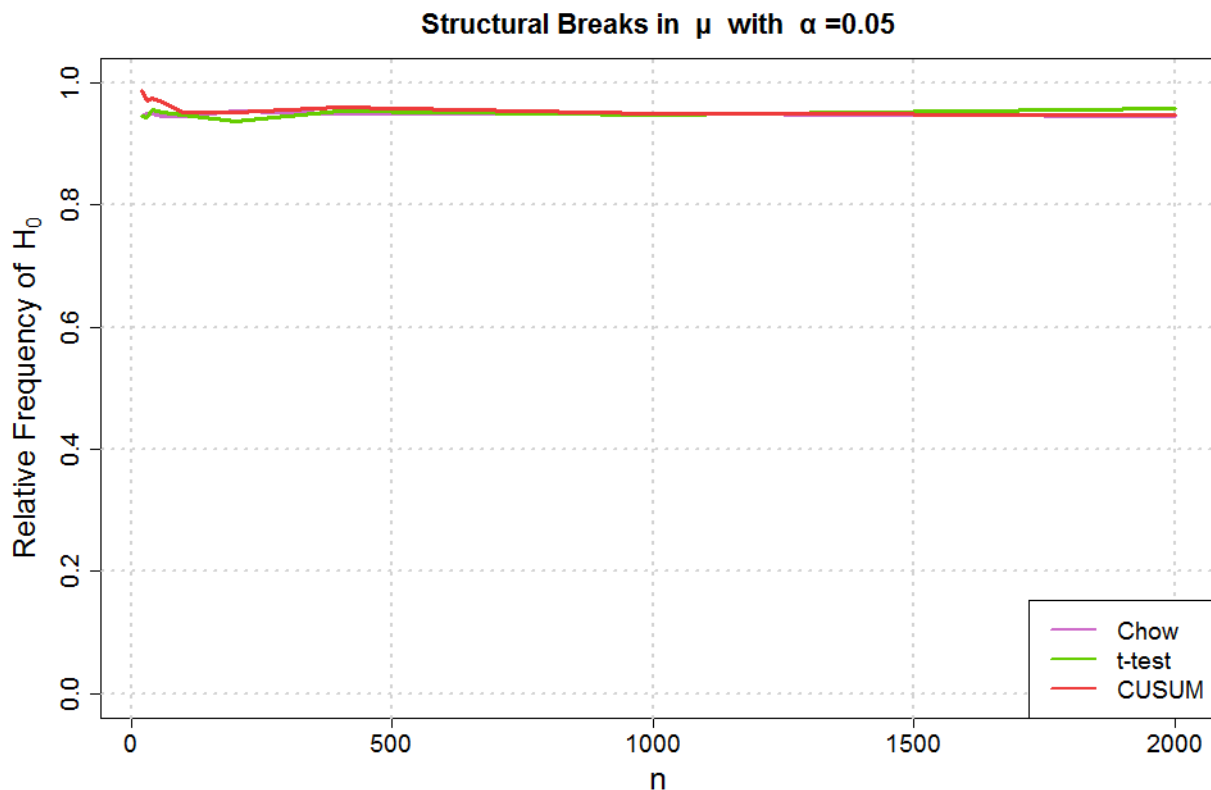
Figure 3.4.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.5$, and $\varepsilon_1 = \varepsilon_2 = 1$.*

In a second simulation under H_0, the influence of n for a higher autocorrelation effect should be checked. For this reason, $\beta_1 = \beta_2$ will be increased and the new parameters are given by

$$\mu_1 = \mu_2 = 0,$$
$$\omega_1 = \omega_2 = 0.4,$$
$$\alpha_1 = \alpha_2 = 0.1,$$
$$\beta_1 = \beta_2 = 0.85,$$
$$\varepsilon_1 = \varepsilon_2 = 1.$$

In figure 3.5 and figure 3.6 the results of the simulation for the first and the second moments are displayed. The results for r_i are similar to the previous simulation. All three tests vary around 0.95 and keep the α level. A completely different picture as in the previous simulation gives the second moment. Squared returns and absolute returns are almost identical. All tests have a decline in H_0 with increasing n but between $1000 < n < 1500$ it rises again for the K&L test and the t-test. Nevertheless, no test can keep the α level. Although the K&L test and the t-test are better than the remaining tests, the results under H_0 get worse with increasing autocorrelation effect. The effect of higher autocorrelation will be examined in further simulations.
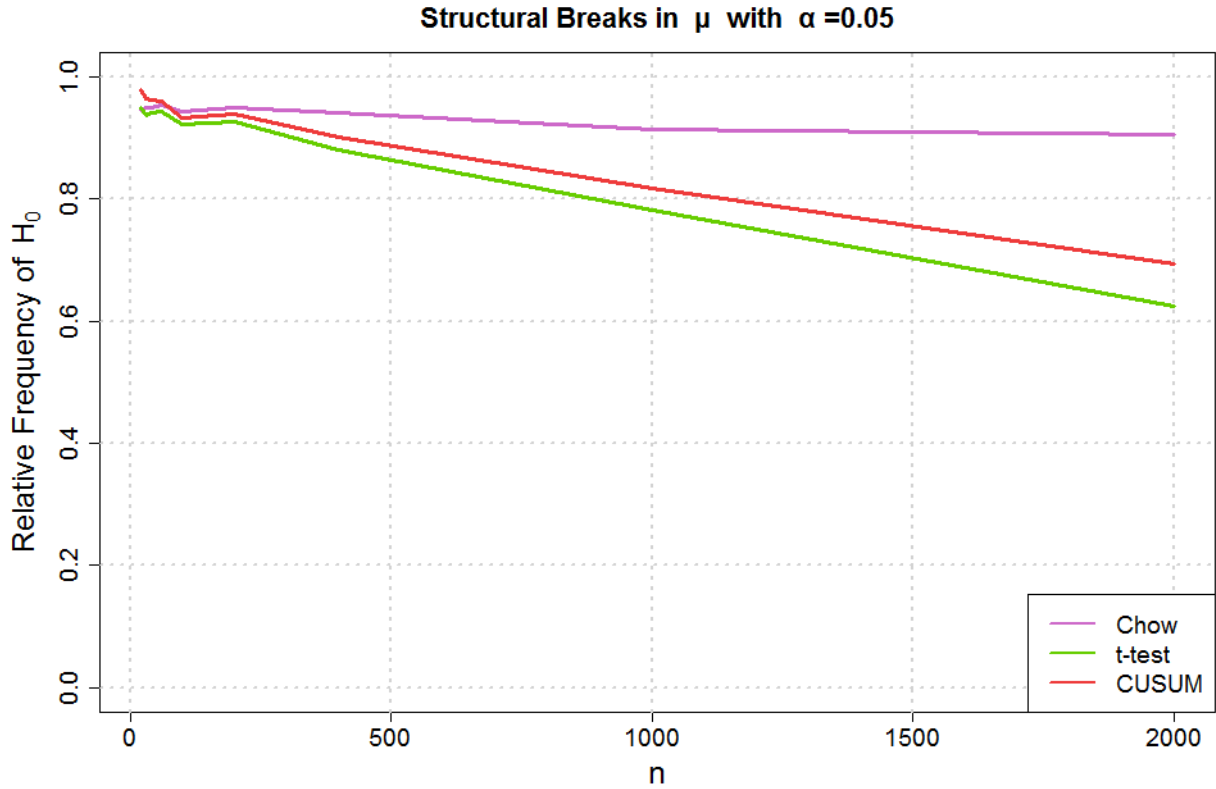
Figure 3.5.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.85$, and $\varepsilon_1 = \varepsilon_2 = 1$.*
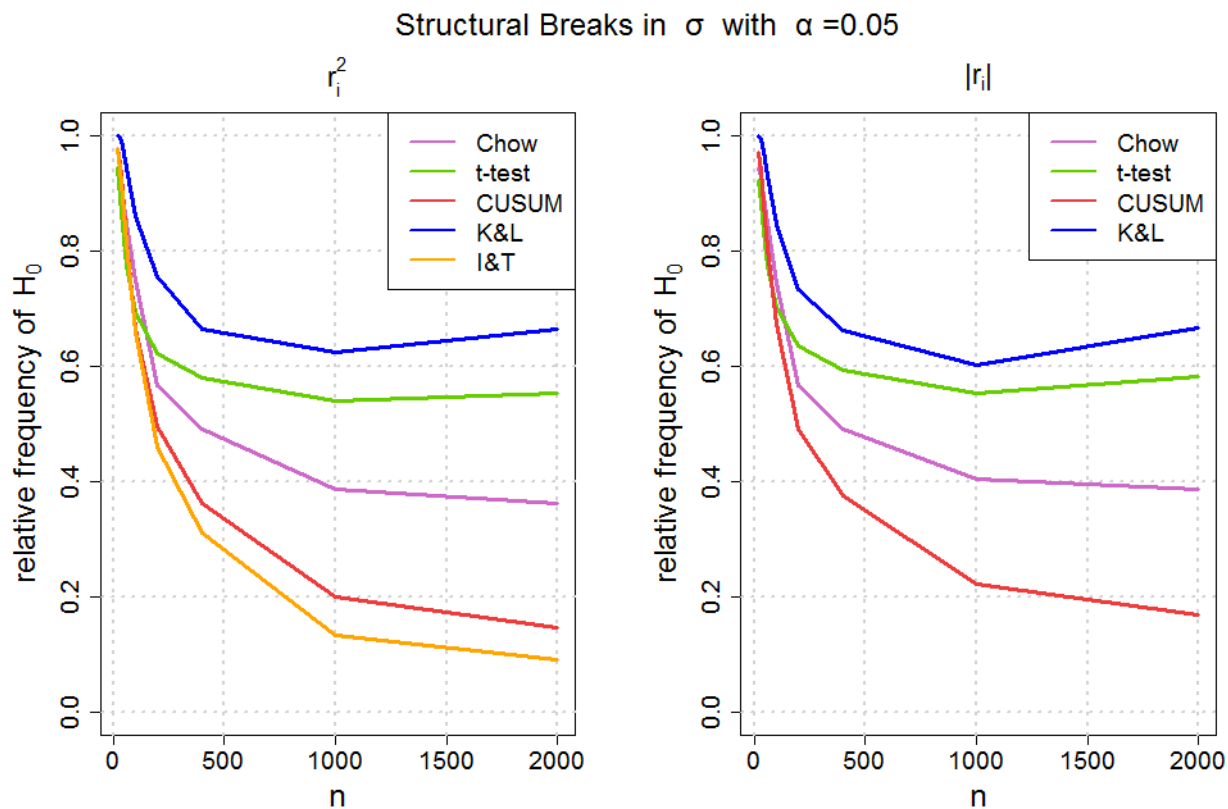
Figure 3.6.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.85$, and $\varepsilon_1 = \varepsilon_2 = 1$.*
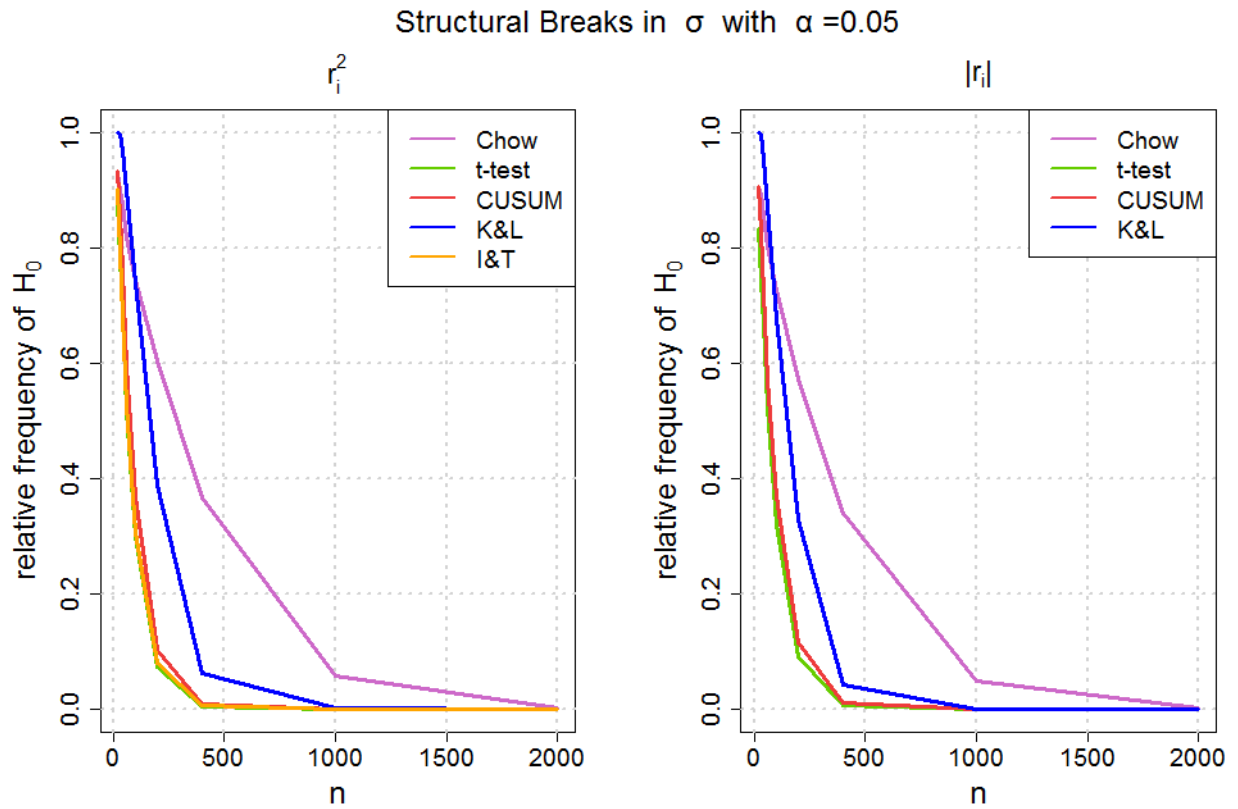
Figure 3.7.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

In a last simulation for changing n, we will create a structural break in $n_1 = 0.5$ with

$$\mu_1 = 0, \ \mu_2 = 0.1,$$
$$\omega_1 = 0.4, \omega_2 = 0.5,$$
$$\alpha_1 = 0.1, \alpha_2 = 0.2,$$
$$\beta_1 = 0.5, \beta_2 = 0.6,$$
$$\varepsilon_1 = \varepsilon_2 = 1.$$

We have a break in both the mean and the dynamics of the volatility. Firstly, the mean will be more closely examined in figure 3.7. Despite there is a structural break in the simulation, the Chow test cannot recognize this. The frequency of H_0 varies around [0.91,0.95]. Furthermore, the t-test and the OLS-CUSUM test can not reliably detect the break. Although their power increases with a raising number of observations, the power gets a minimum for $n = 2000$ with a value of 0.62 and 0.69 for the t-test and the OLS-CUSUM test. If we want to apply these tests for real data to create a trading strategy, it is necessary that the used test recognizes the structural break as soon as possible. In figure 3.8 the power for the second moments are displayed. As we can see, the power increases rapidly. While the t-test and the OLS-CUSUM test are nearly equal, the Chow test delivers the worst result. The K&L test begins from a higher value than the other tests but needs longer for
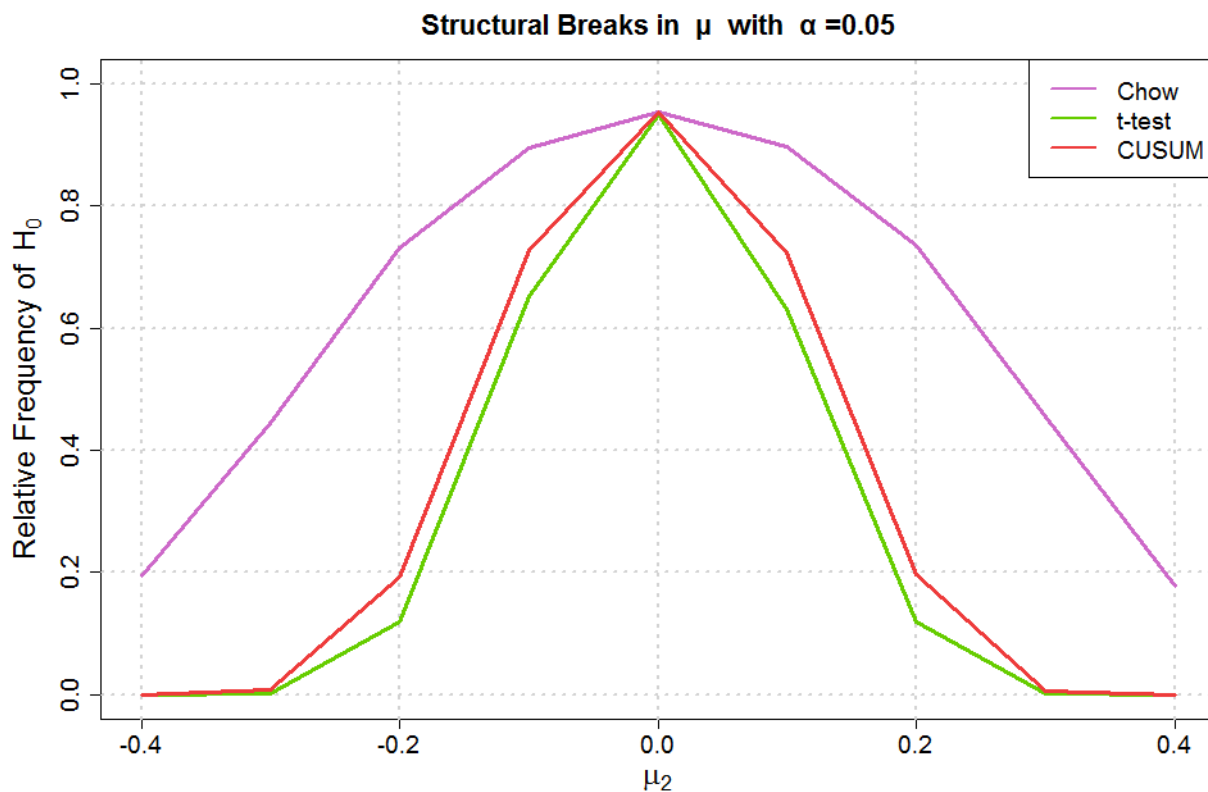
Figure 3.8.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

decreasing to zero. We can summarize for the three simulations that with increasing n the power of the tests gets better for the second moments. The specificity, the test decision keeps H_0 if H_0 is true, gets worse with increasing n and increasing autocorrelation effect. The specificity for the first moment neither depend on the autocorrelation effect nor on n and the power for the first moments gets better with increasing n.

3.4.2. Structural Breaks in μ

Next, we will examine the influence of a changing μ on the test decisions. μ represents the mean of the returns in the GARCH model in formula 3.1.1. To this end, a simulation with changing μ will be carried out with following parameters:

$$\mu_1 = 0,$$
$$\omega_1 = \omega_2 = 0.4,$$
$$\alpha_1 = \alpha_2 = 0.1,$$
$$\beta_1 = \beta_2 = 0.5,$$
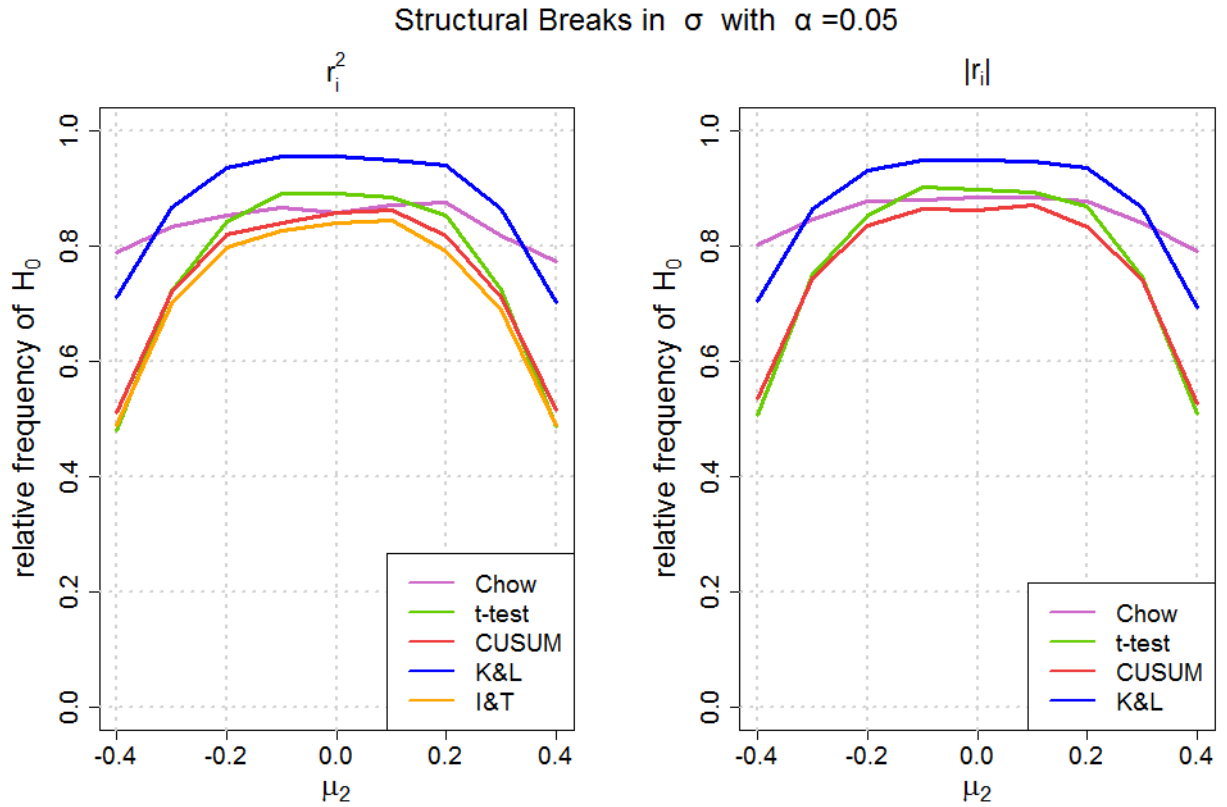$$n = 1000, \; n_1 = 0.5,$$
$$\varepsilon_1 = \varepsilon_2 = 1.$$

Figure 3.9.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

In figure 3.9 the results are given for the structural break tests in μ. If $\mu_2 = 0$, the test is accomplished under H_0 where the specificity of all three tests is nearly 0.95 and keeps the α level. Under H_1, the rejection rate of H_0 proceeds symmetrical. The further the distance between μ_1 and μ_2, the better the power of the different tests. The best results are delivered by the t-test, while the Chow test has the worst results. The changes in μ also have an impact on the power of testing for structural breaks in the volatility which is displayed in 3.10 and proceeds similar for squared returns and absolute returns. Under H_0, the K&L test has the best result and keeps the α level, while the other tests reject H_0 too often. The bigger the distance between μ_1 and μ_2, the higher the rejection rate of H_0. Nevertheless, an influence on the K&L test can only be observed for a distance greater 0.2. Thus, a structural break in μ has an impact on the test results for the first moment as well as on the test results of the second moment.

3.4.3. Structural Breaks in σ

After having checked the influence of n and μ on the test results, we will look at the impact of changing volatility dynamics on the rejection rates of H_0. Remember that the formulas of the GARCH model are given in 3.1.3, 3.1.4 and 3.1.5 for the model preceding
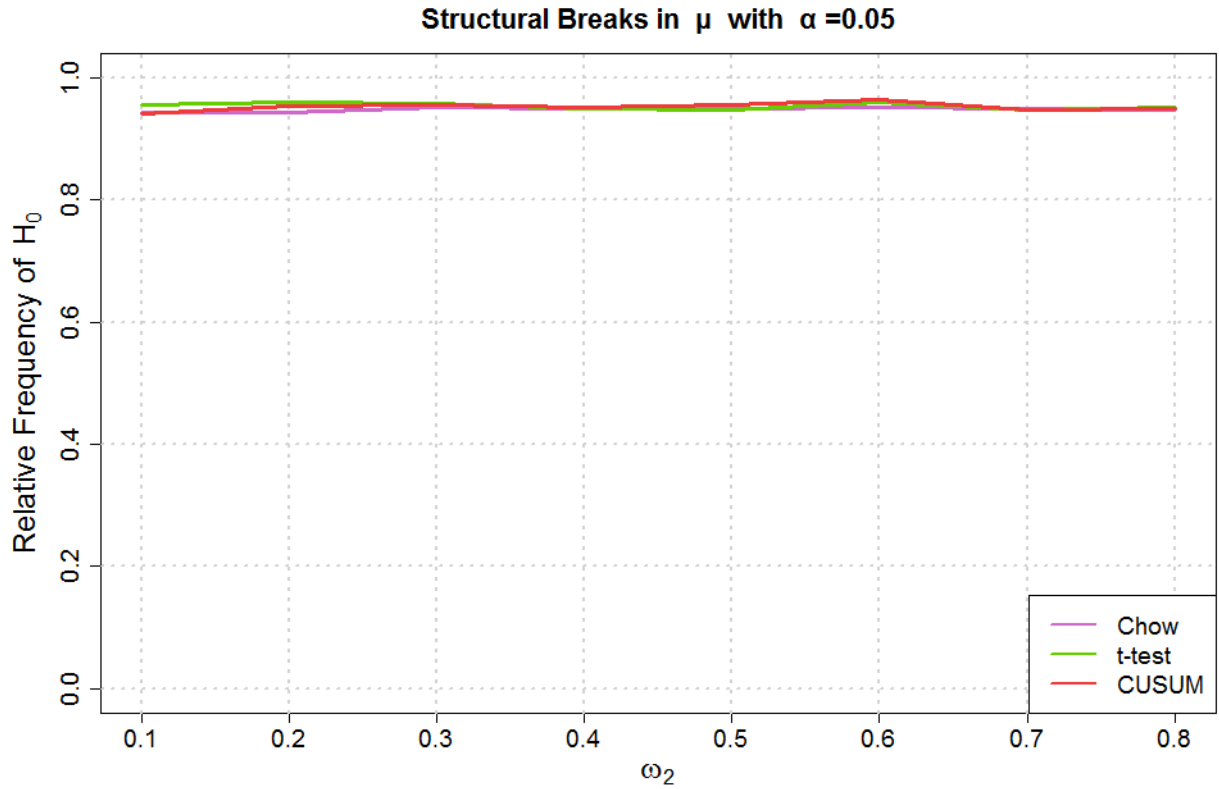
Figure 3.10.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

the structural break, and are given in 3.1.6, 3.1.7 and 3.1.8 for the model following the structural break. Thus, the dynamics of the volatility can change in four different ways. First of all, we want to examine the impact of a break in $\omega_1 \neq \omega_2$. The second and third option for a changing volatility is a break in $\alpha_1 \neq \alpha_2$ respectively $\beta_1 \neq \beta_2$. The last possibility is a break in the random error $\varepsilon_1 \neq \varepsilon_2$.

For a start, consider the first option where a simulation will be accomplished with

$$\mu_1 = \mu_2 = 0,$$
$$\omega_1 = 0.4,$$
$$\alpha_1 = \alpha_2 = 0.1,$$
$$\beta_1 = \beta_2 = 0.5,$$
$$n = 1000, \ n_1 = 0.5,$$
$$\varepsilon_1 = \varepsilon_2 = 1$$

and with changing ω_2. We can find the results in figure 3.11 for the first moment and in figure 3.12 for the second moment. A changed ω_2 has no impact on the rejection rate of all three tests by testing for structural breaks in μ. They keep the α level irrespectively of ω_2. In the second moment we have H_0 if $\omega_2 = 0.4$. At this point, the K&L test has the best specificity with a given $\alpha = 0.05$, followed by the t-test. With increasing distance

Figure 3.11.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*
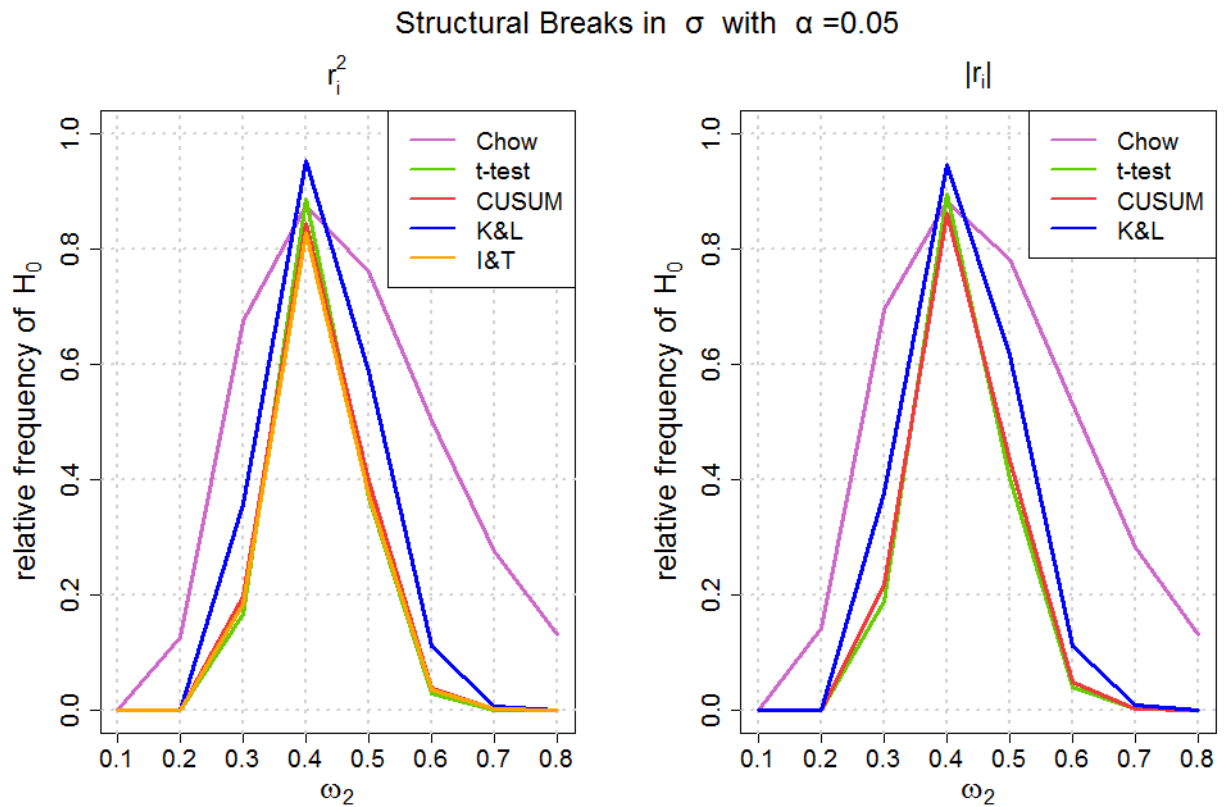
Figure 3.12.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

between ω_1 and ω_2 the power of all tests becomes better but the Chow test has the worst results. Moreover, it is interesting that the power does not proceed symmetrically. With decreasing ω_2, the rejection rate decreases steeper as with increasing ω_2. All tests can detect a structural break with falling ω faster than a break with rising ω. Two tests can be highlighted. Although the K&L test has a worse power than the Chow test, the t-test and the OLS-CUSUM test, it has a better specificity. The second test is the t-test whose specificity is better and whose power is marginal better than for the Chow test and the OLS-CUSUM test.

Two further simulations shall now check the influence of a break in $\alpha_1 \neq \alpha_2$. To this end, the first simulation with
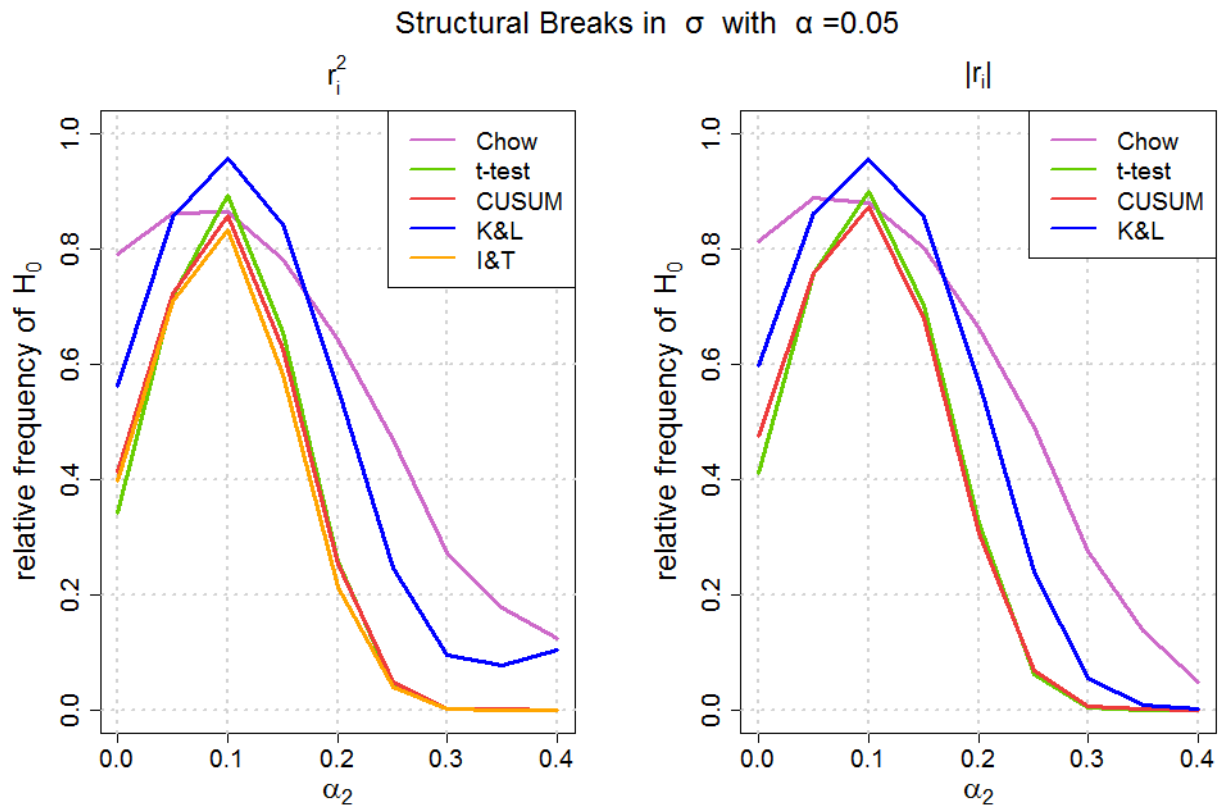
$$\mu_1 = \mu_2 = 0,$$
$$\omega_1 = \omega_2 = 0.4,$$
$$\alpha_1 = 0.1,$$
$$\beta_1 = \beta_2 = 0.5,$$
$$n = 1000,\ n_1 = 0.5,$$
$$\varepsilon_1 = \varepsilon_2 = 1$$

and with changing α_2 will be accomplished. We can find the results in figure 3.13 for the first moment and in figure 3.14 for the second moment. As in the previous simulation, we can see that a change in the volatility, in this case by changing α_2, has no impact on the test results for the first moment. The Chow test, the t-test and the OLS-CUSUM test are keeping the α level. The rejection rate of H_0 varies around a value of 0.05 for all three tests.

For the second moment we can recognize an impact on the test results. Under H_0, $\alpha_2 = 0.1$, again the K&L test delivers the best outcome, followed by the t-test. The Chow test has the worst outcome under H_1 and the power of the t-test, the OLS-CUSUM test and the I&T test proceeding similarly. An interesting fact is the comparison of the K&L test for squared returns and absolute returns. While the rate of H_0 for absolute returns falls to zero if $\alpha_2 = 0.4$, we can see an increase in the rate of H_0 for squared returns.

Structural Breaks in μ with α =0.05
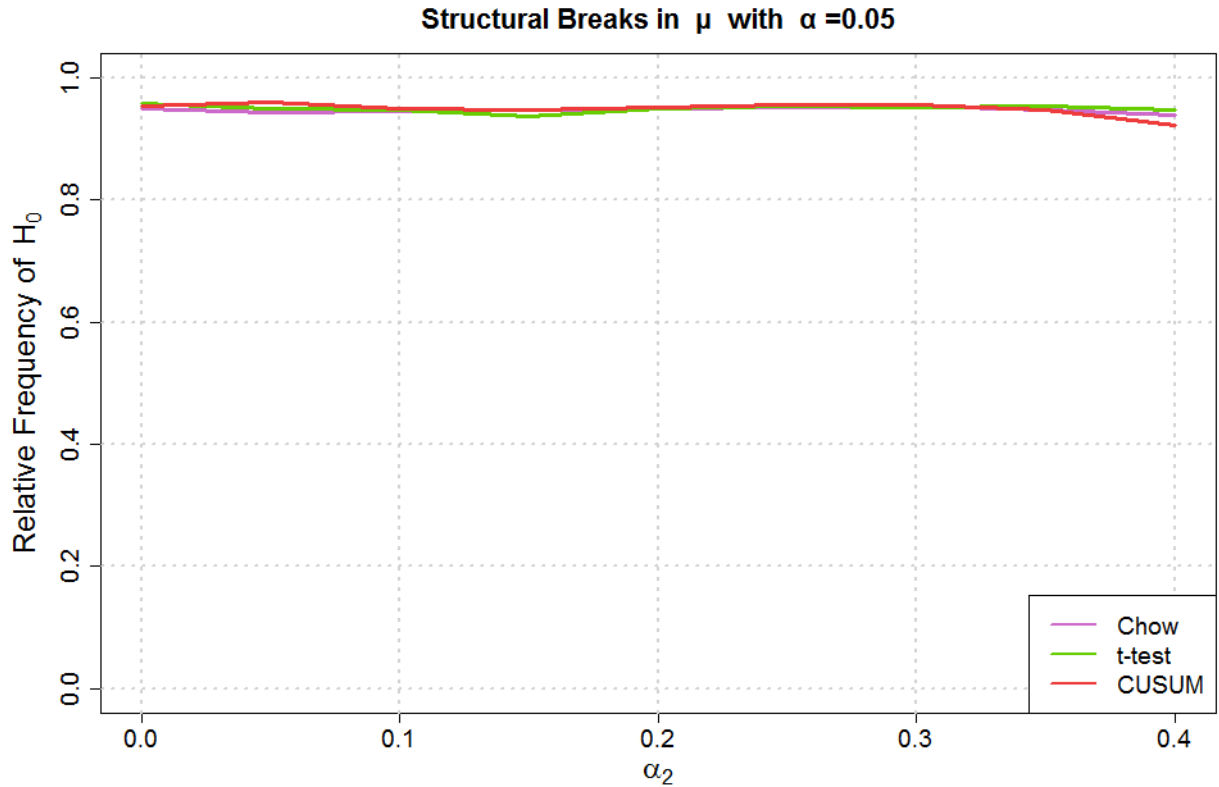
Figure 3.13.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

Structural Breaks in σ with α =0.05

Figure 3.14.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*
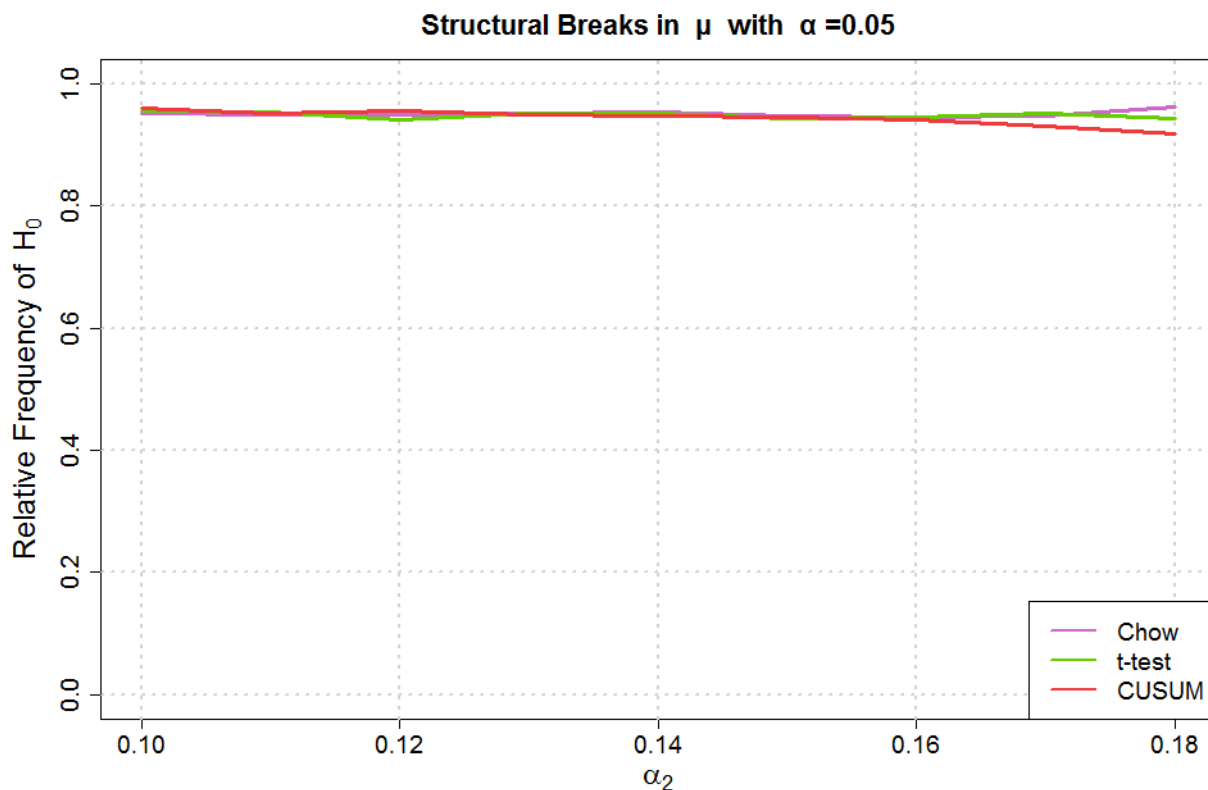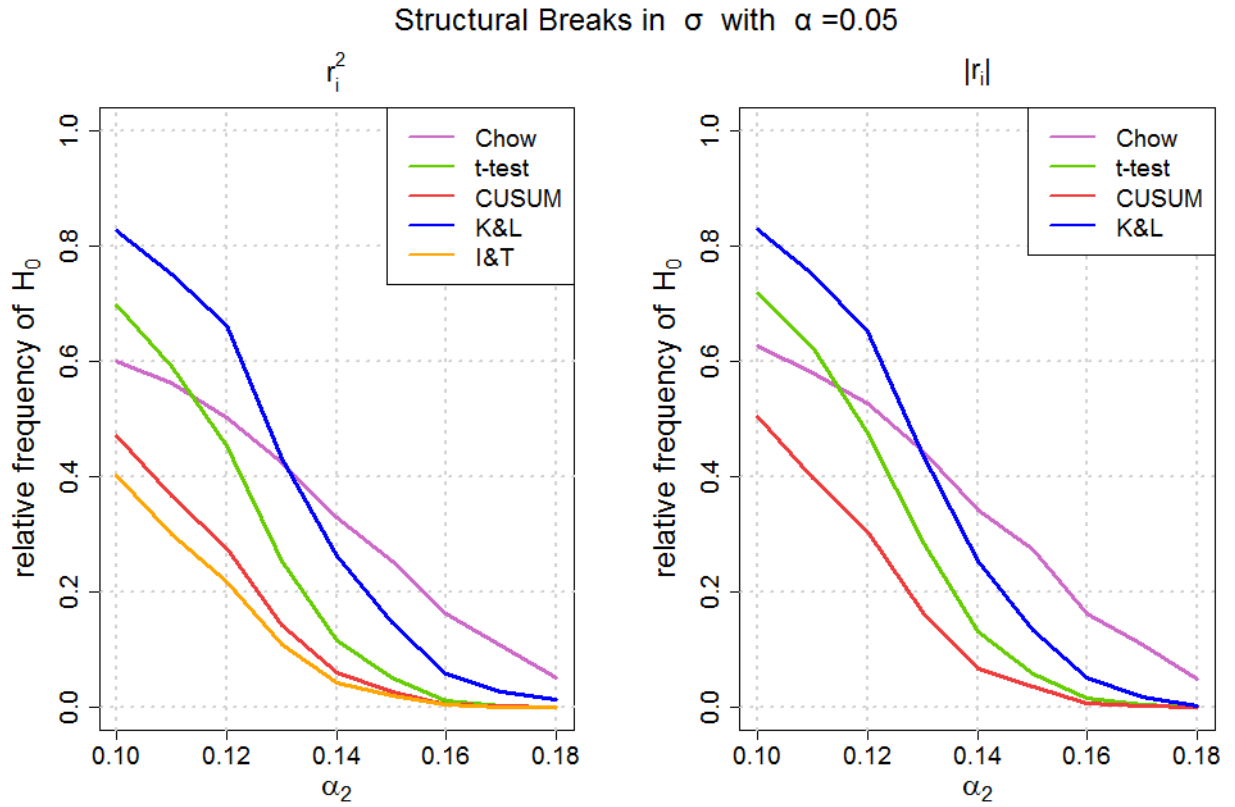
Figure 3.15.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.8$, $\beta_2 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

The second simulation for changing α_2 shall examine whether the results change with a higher autocorrelation effect. To this end, the base level of β_1 will be increased and the simulation will be accomplished with

$$\mu_1 = \mu_2 = 0,$$
$$\omega_1 = \omega_2 = 0.4,$$
$$\alpha_1 = 0.1,$$
$$\beta_1 = \beta_2 = 0.8,$$
$$n = 1000, \ n_1 = 0.5,$$
$$\varepsilon_1 = \varepsilon_2 = 1.$$

For the first moment, pictured in figure 3.15, we have similar results as in the previous simulation. A changing α_2 has no impact on the rejection rate of H_0 and the α level will be kept. Only for $\alpha_2 = 0.18$, some differences between the tests are found. While the t-test keeps the same level, the OLS-CUSUM test falls and the Chow test rises. Next, let us take a look at the second moment in figure 3.16. The only parameter we changed compared to the previous simulation is the base level of $\beta_1 = \beta_2$. This leads to a higher autocorrelation effect. If we compare the two simulations in figure 3.14 and in figure 3.16, many differences can be seen. In the simulation results it can be detected that under H_0,
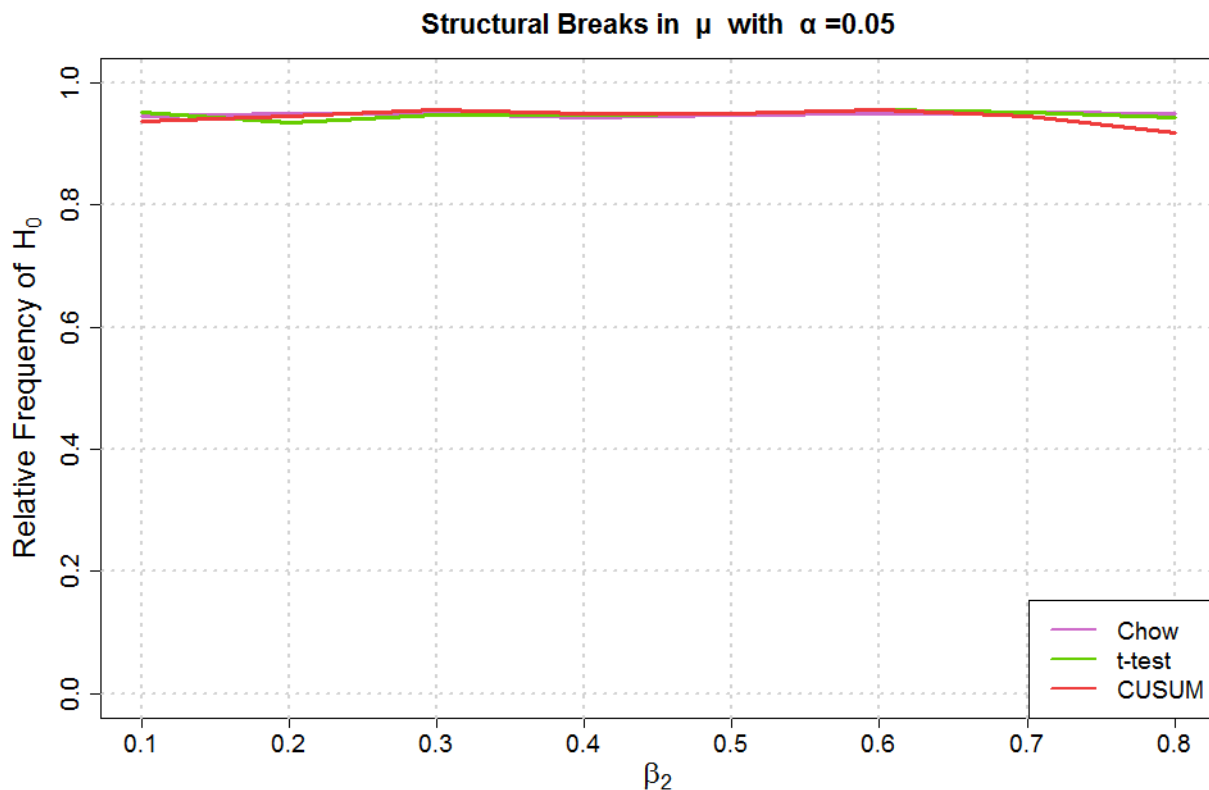
Figure 3.16.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.8$, $\beta_2 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

$\alpha_2 = 0.1$, the specificity gets worse compared to the previous simulation. In contrast to that, the power gets better and with an $\alpha_2 = 0.16$ the t-test, the OLS-CUSUM test and the Chow test reject the null hypothesis by nearly 100 percent. The best result for this simulation delivers the t-test and the K&L test whose specificity is higher and whose power falls steeper as for the other tests.

Another structural break in the dynamics of the volatility can be created through $\beta_1 \neq \beta_2$. Firstly, we want to check the influence of such a break with a low base level for β_1. To this end, we adjust our simulation parameters to

$$\mu_1 = \mu_2 = 0,$$
$$\omega_1 = \omega_2 = 0.4,$$
$$\alpha_1 = \alpha_2 = 0.1,$$
$$\beta_1 = 0.5,$$
$$n = 1000, \; n_1 = 0.5,$$
$$\varepsilon_1 = \varepsilon_2 = 1$$

where β_2 changes. The results are very similar to the previous simulations with a change in α_2. In figure 3.17 the outcome for the first moment is pictured. We can not discover

Figure 3.17.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*
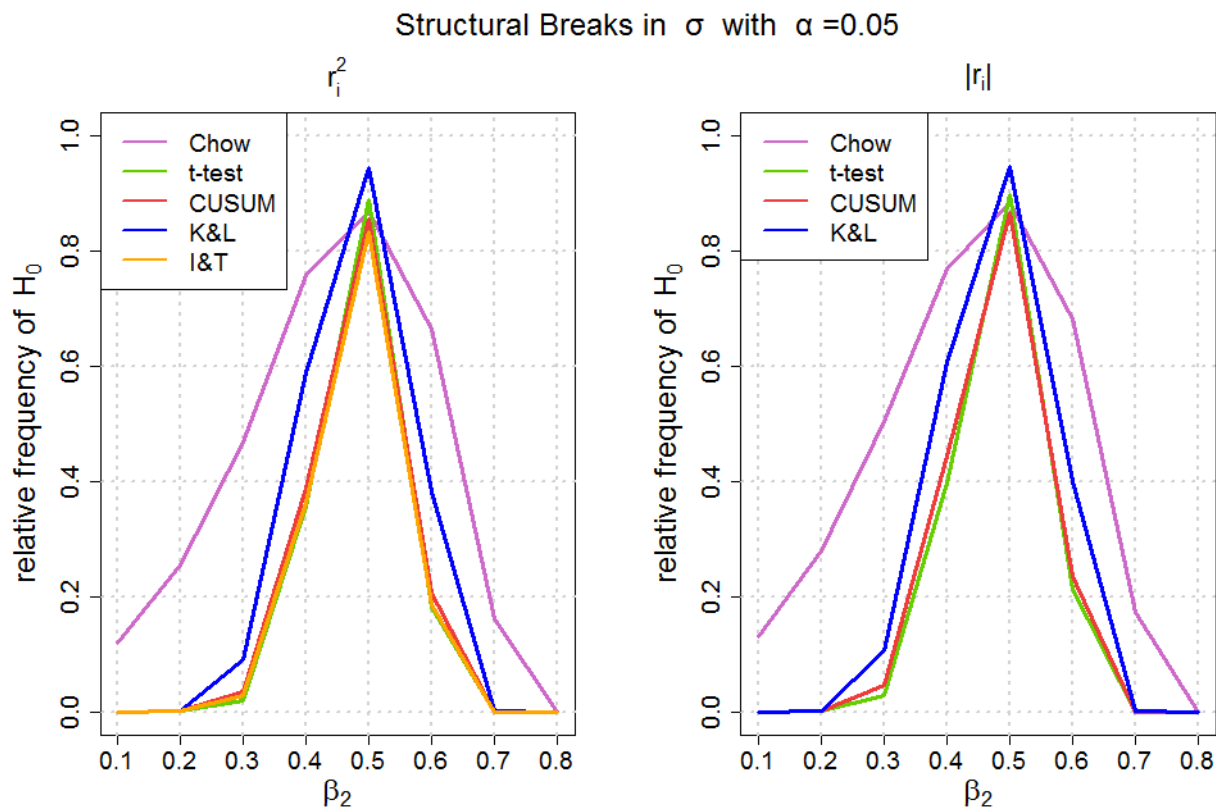
Figure 3.18.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

any effect for the test results with a break in $\beta_1 \neq \beta_2$. On the other hand, an influence on the test decision of the second moment, given in figure 3.18, can be detected. The K&L test has again the best specificity, followed by the t-test. The Chow test has the worst results for the power, as in previous simulations. The t-test is marginally better than the OLS-CUSUM test and the I&T test.

A second simulation with breaks in $\beta_1 \neq \beta_2$ should examine the effect of autocorrelation. To this end, the base level of β_1 will be increased and a new simulation with

$$\mu_1 = \mu_2 = 0,$$
$$\omega_1 = \omega_2 = 0.4,$$
$$\alpha_1 = \alpha_2 = 0.1,$$
$$\beta_1 = 0.8,$$
$$n = 1000, \, n_1 = 0.5,$$
$$\varepsilon_1 = \varepsilon_2 = 1$$

and changing β_2 will be accomplished. In figure 3.19 the results for the first moment are displayed. Here, no impact on the test results and the results resemble the previous simulations can be found. For the second moment, pictured in figure 3.20, it can be seen the same effect as we did before for a break in $\alpha_1 \neq \alpha_2$. With increasing autocorrelation effect, the specificity of all tests becomes worse, while the power gets better. Again, the Chow test has the worst results, whereas the t-test and the K&L test have steeper fall offs and can be classified as better tests.

The last option for structural breaks in the volatility is $\varepsilon_1 \neq \varepsilon_2$. This means a change in the variance of the random error. In a last simulation with structural breaks in the volatility, we use

$$\mu_1 = \mu_2 = 0,$$
$$\omega_1 = \omega_2 = 0.4,$$
$$\alpha_1 = \alpha_2 = 0.1,$$
$$\beta_1 = \beta_2 = 0.5,$$
$$n = 1000, \, n_1 = 0.5,$$
$$\varepsilon_1 = 1$$

where ε_2 changes. The results for the first moment, displayed in figure 3.21, show the same influence on the test decision as in the previous simulation. A break in the dynamics of the volatility has no impact on the test decision for the first moment. In figure 3.22 the outcome of the tests for the second moment is pictured. As we can see, the specificity for

Structural Breaks in μ with α =0.05
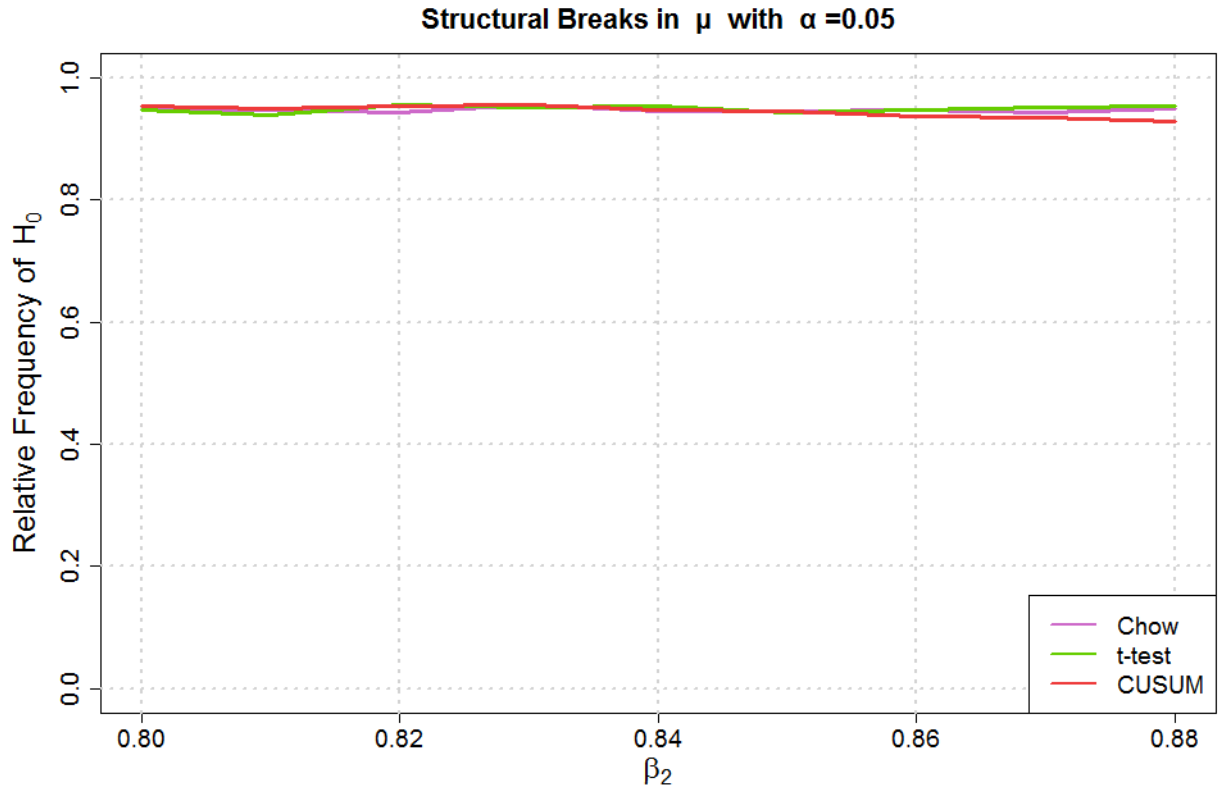
Figure 3.19.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

Structural Breaks in σ with α =0.05
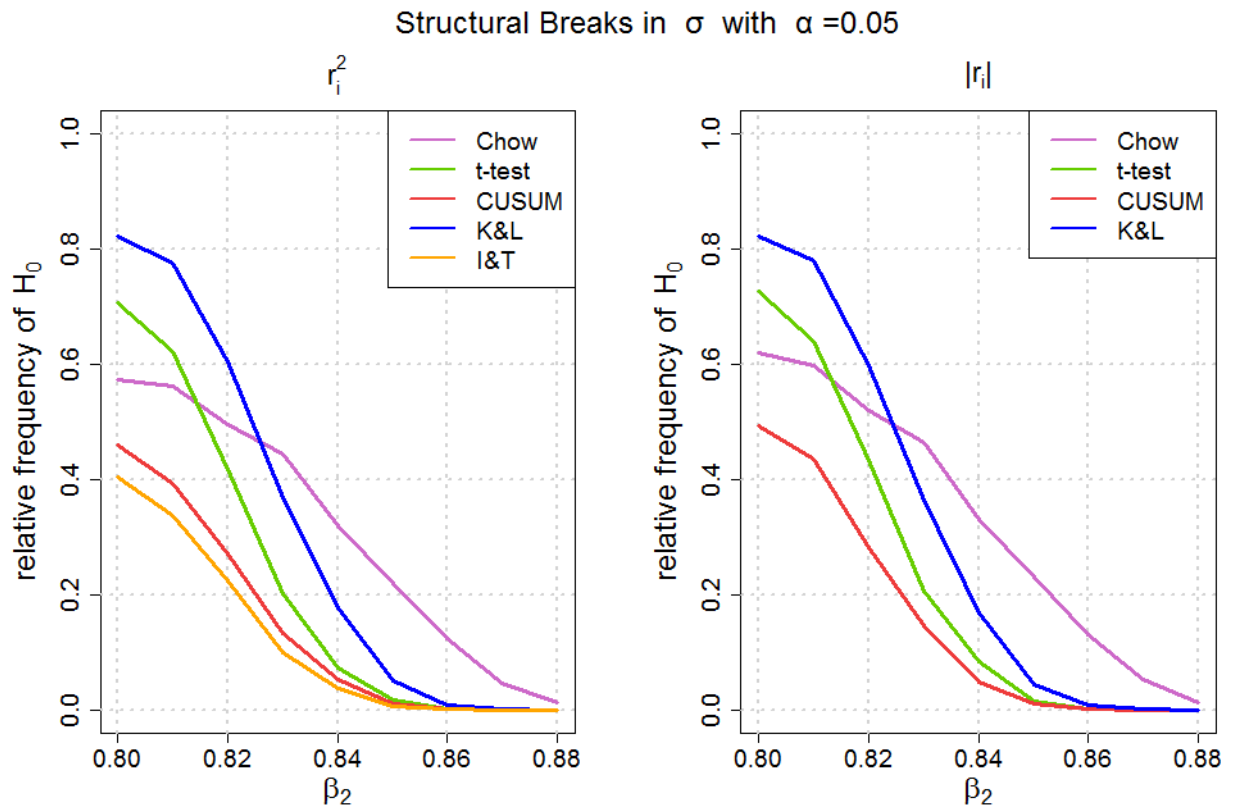
Figure 3.20.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*
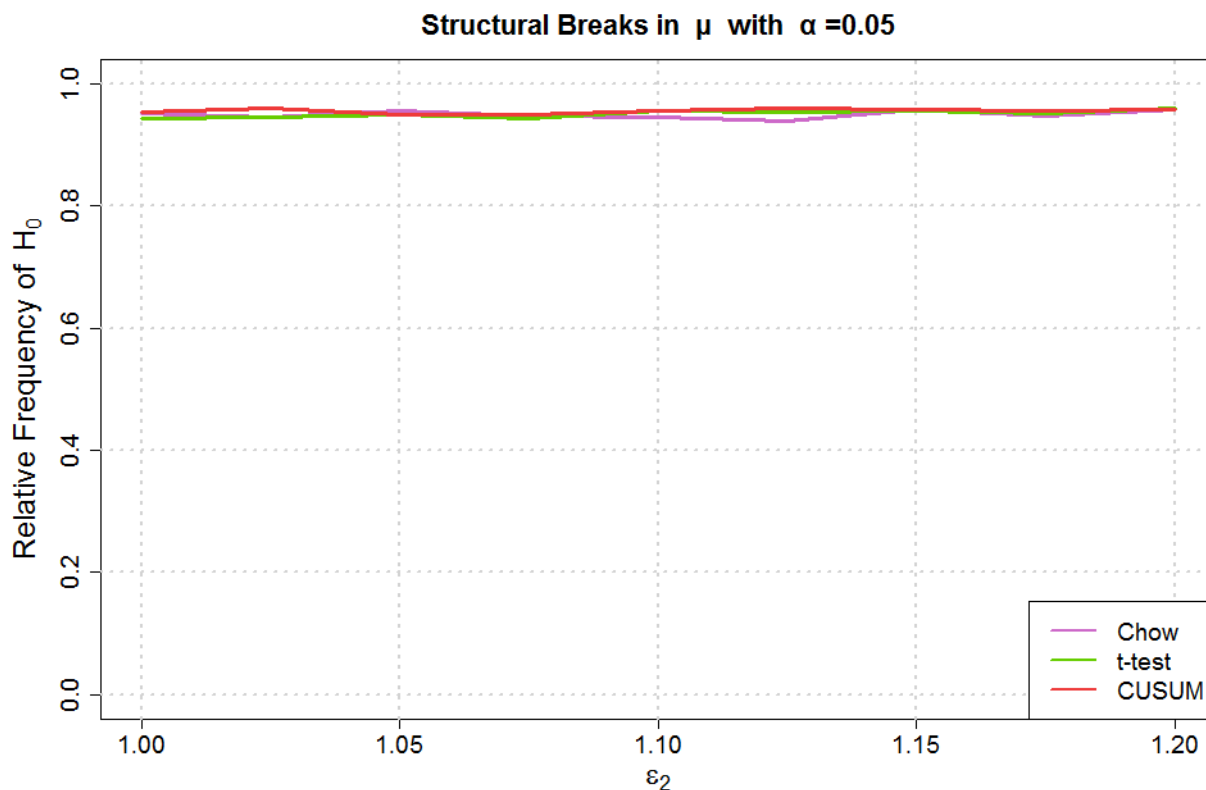
Figure 3.21.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = 1$.*

the K&L test is once again better than for the remaining tests and keeps the α level. The Chow test has the worst result for detecting structural breaks in this simulation. The t-test is slightly better than the OLS-CUSUM test and the I&T test. The results are similar to previous simulations.

To summarize, it can be concluded that the K&L test has the best specificity regarding the identification of structural breaks in the dynamics of the volatility. It is followed by the t-test, which delivers slightly better results than the OLS-CUSUM test and the I&T test. The Chow test scores worst. Furthermore, it does not matter whether squared returns or absolute returns are used. Additionally, a negative impact of autocorrelation effects on the specificity of the tests was detected. For the first moment all tests have similar results and keep the α level.

Figure 3.22.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = 1$.*
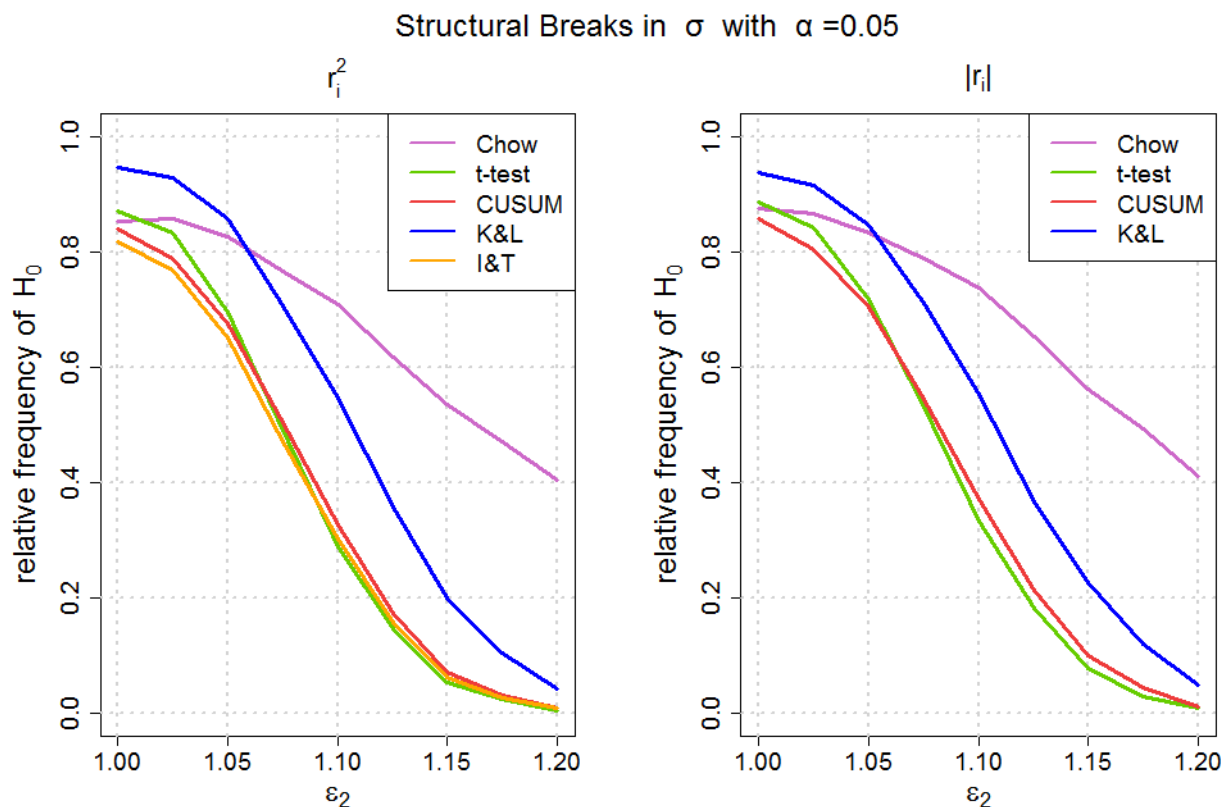
3.4.4. Changing Points of Structural Breaks

A last simulation checks if the breaking point has an influence on the test decision. To this end, we make a new simulation under H_1 with

$$\mu_1 = 0, \ \mu_2 = 0.1,$$
$$\omega_1 = 0.4, \ \omega_2 = 0.5,$$
$$\alpha_1 = 0.1, \ \alpha_2 = 0.2,$$
$$\beta_1 = 0.5, \ \beta_2 = 0.6,$$
$$n = 500,$$
$$\varepsilon_1 = \varepsilon_2 = 1$$

where n_1 changes. Firstly, the first moment in figure 3.23 is considered. As we can see, the power of the t-test is better than the power of the OLS-CUSUM test and the power of the Chow test. Nevertheless, none of the tests can really reliably detect the structural break, although the rejection rate of the null hypothesis gets higher when n_1 converges against 0.5. We can say that the power of all three tests becomes better if the breakpoint is in the middle of the data. In the second moment, displayed in 3.24, especially the K&L

Figure 3.23.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n = 500$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

test has problems if the breakpoint is not in the middle of the data. The t-test has the best results and is able to detect the breaks nearly irrespectively of the position of the breaking point.
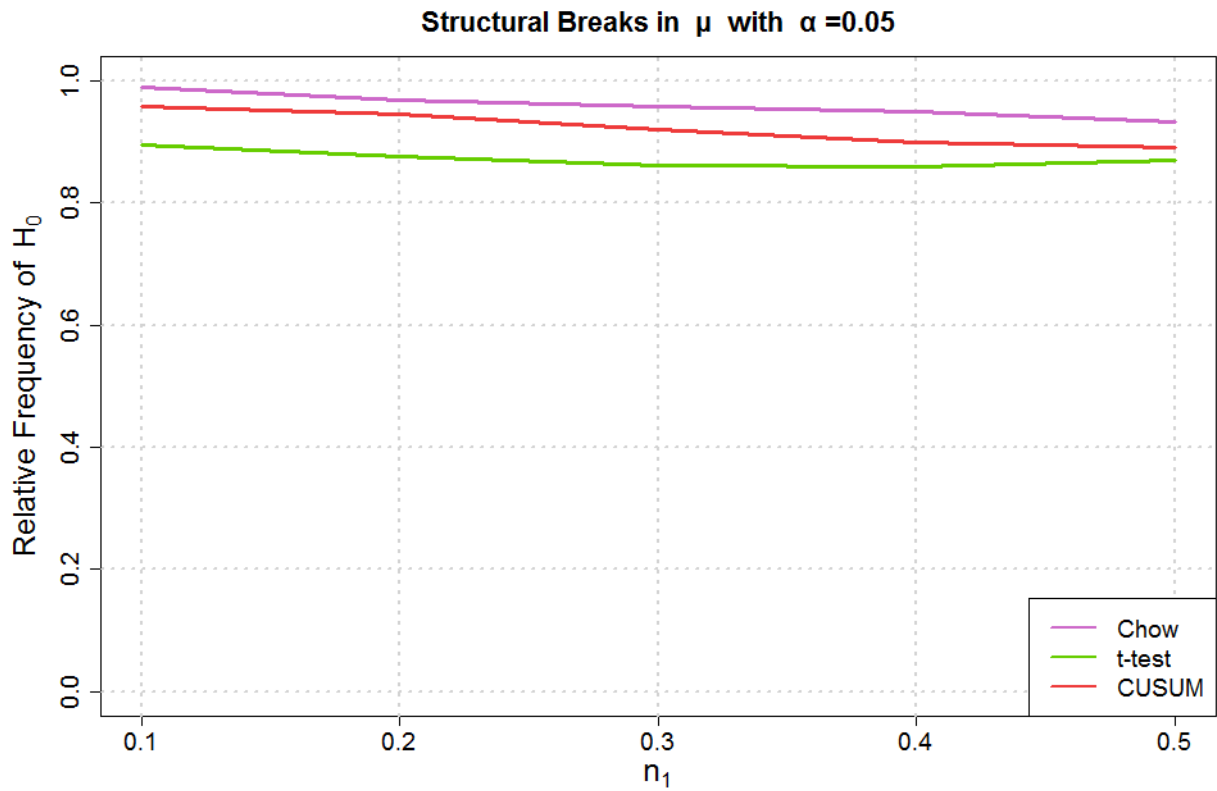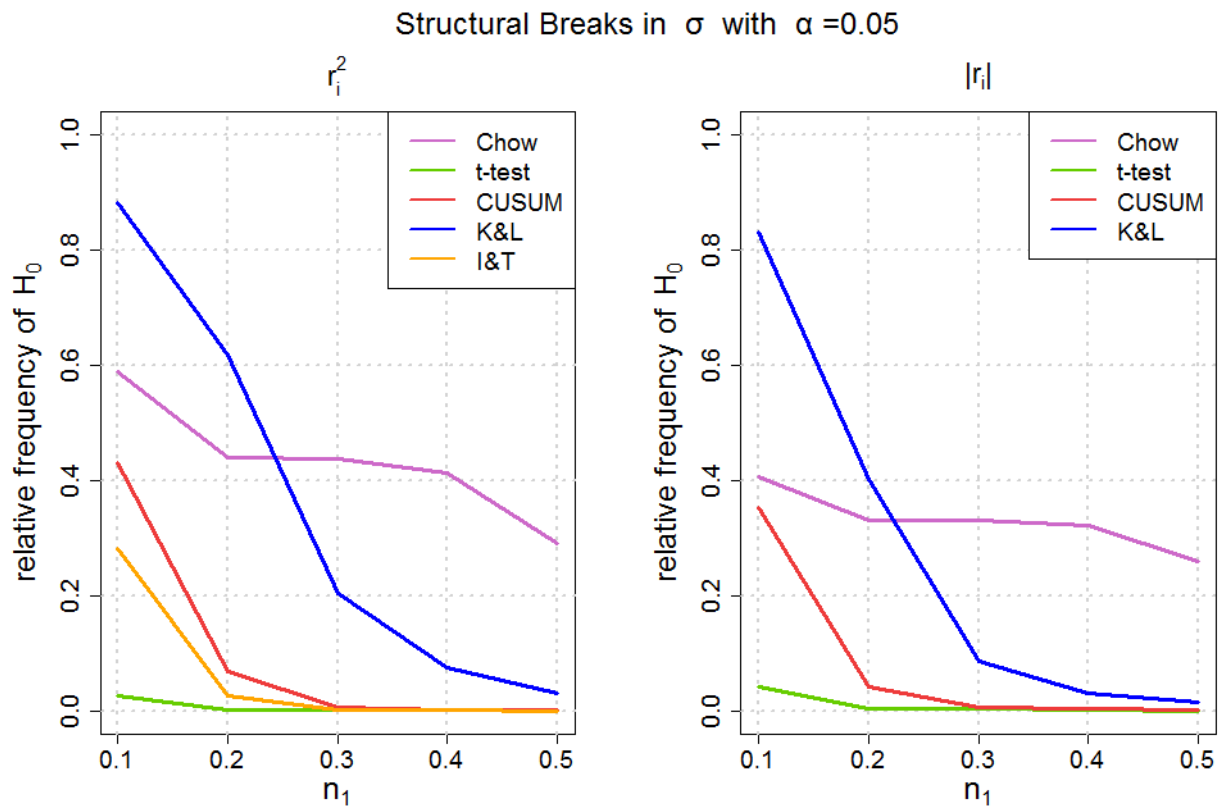
**Figure 3.24.:** *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n = 500$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

4. Data

After having tested the influence of the several tests, those tests are the basis for a trading strategy. To check the reliability of these trading strategies, they shall be applied to some real financial data. Therefore, all stocks of the DAX and the Dow Jones will be analyzed. Figure 4.1 depicts the prices and the percentage log returns of the DAX from 1 January, 2000, until 30 September, 2016. As one can see, there are trends in the prices. From year 2000 until 2003 there was a negative trend, followed by a positive trend until 2008 and again a negative trend until 2009. From this point on, the prices can be categorized in a positive trend with a short crash in year 2012 and 2015. The idea for the followed trading strategies is to detect these trends as early as possible and to gain by buying or selling the stock when the used test detects a breakpoint in the returns. In figure 4.2 the prices and percentage log returns for the Dow Jones are portrayed. By looking at the prices, one can see that the prices and the returns of the Dow Jones are very similar to the prices and the returns of the DAX. It can be recognized that both indices's have volatility clusters. By comparing them to the prices, it can be observed that especially in phases with high volatility there is a negative trend in the prices. This leads to a second approach for creating a trading strategy. It shall be found a test which can detect a breakpoint in the dynamics of the volatility, and if that happens, this leads to a trading signal for buying or selling the underlying stock. Because a small number of shares is too few for checking the utility of a trading strategy, those trading strategies will be applied to all of the actually included stocks of the both indices's. Thereby, the DAX and the Dow Jones have 30 included stocks each. Not every stock has the same timespan as the DAX or the Dow Jones because not every company had already existed on 1 January, 2000.
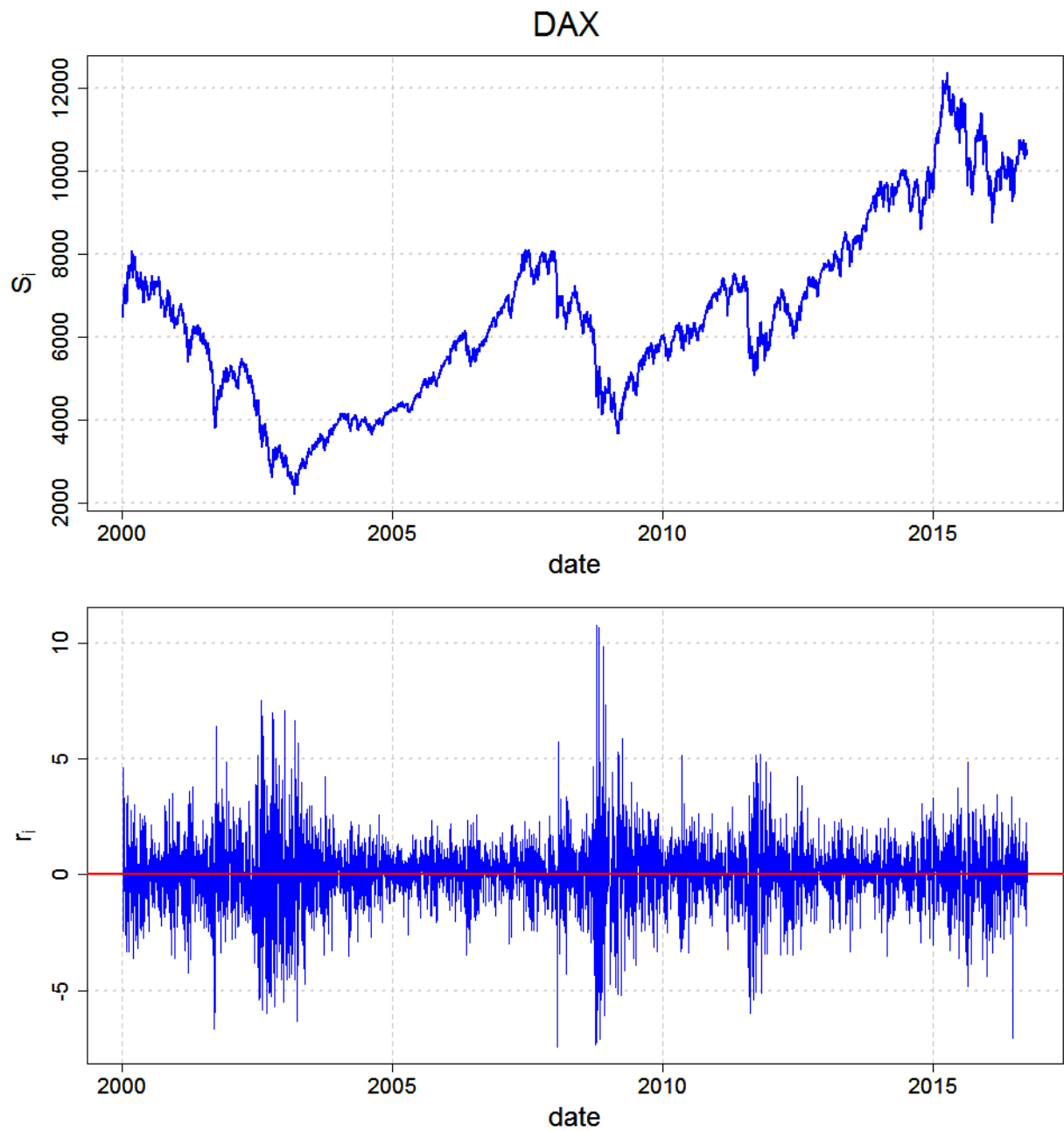
Figure 4.1.: *Prices and log returns in % of the DAX from 1 January, 2000 until 30 September, 2016.*
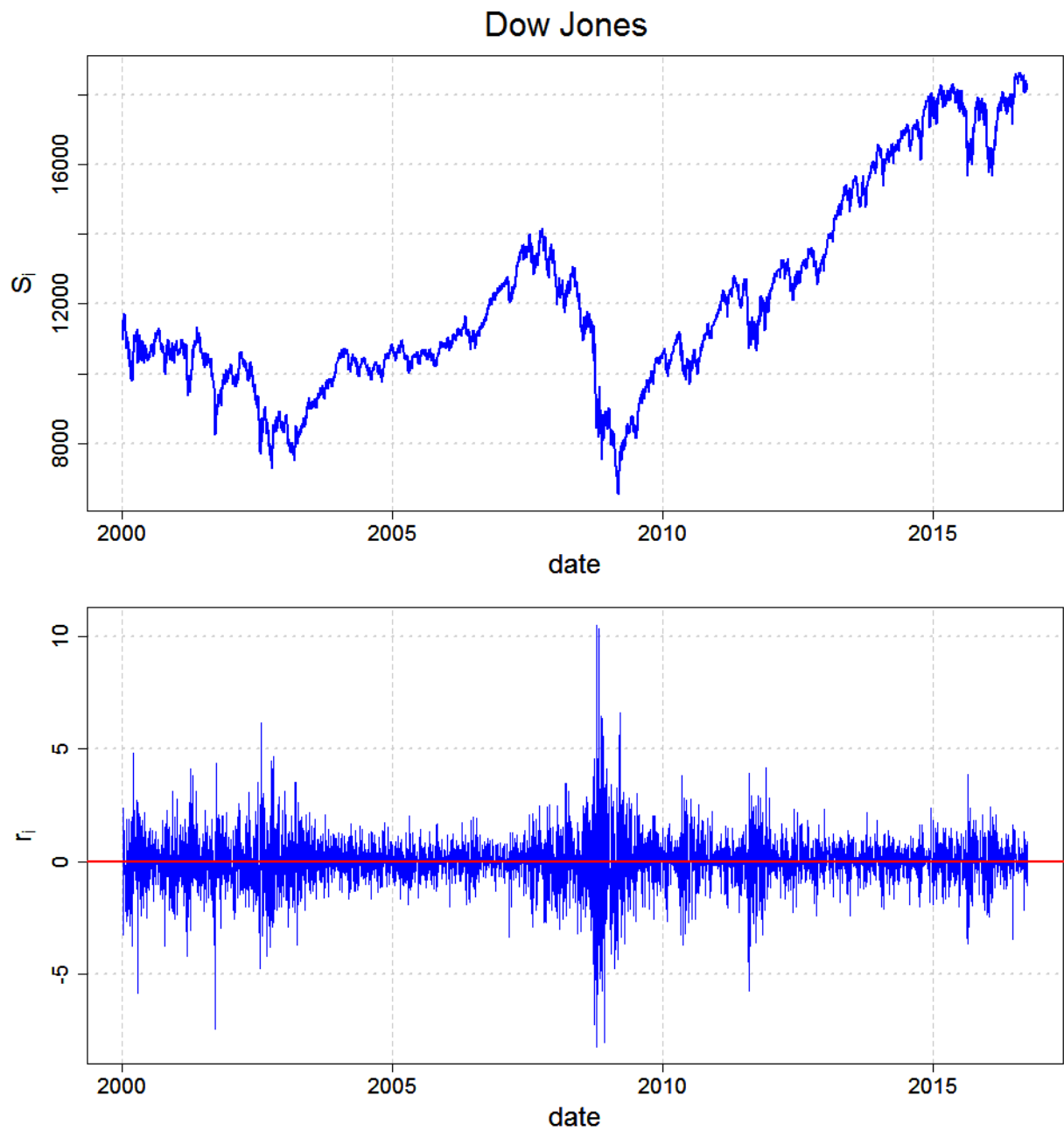
Figure 4.2.: *Prices and log returns in % of the Dow Jones from 1 January, 2000 until 30 September, 2016.*

5. Trading Strategies

In this chapter, we want to create a trading strategy and test it for all stocks of the DAX and the Dow Jones. After the comparison of the introduced structural break tests, we will apply the t-test and the OLS-CUSUM test for detecting breaks in μ and the K&L test and the t-test for detecting breaks in σ. These tests had the best results in the previous simulations. The way we create a trading strategy will be simplified and is always the same: Firstly, the length of the tested data and the α will be determined. For the first moment and day i the tested data are given by

$$r^{(i)} = r_1^{(i)}, r_2^{(i)}, ..., r_n^{(i)} = r_{i-n+1}, r_{i-n+2}, ..., r_i,$$

and for the second moment and day i by

$$r^{(i)} = r_1^{(i)}, r_2^{(i)}, ..., r_n^{(i)} = r_{i-n+1}^2, r_{i-n+2}^2, ..., r_i^2.$$

Thus, the tests will be accomplished via moving window. If the test detects a structural break in the data, a signal for buying or selling the stock will be created. Whether we buy or sell will be described later. There are two options for our investment status: fully invested or not invested at all. If we are fully invested, we can just sell the stock, and if a signal for buying the stock will be created, nothing will happen. If we are not invested at all, we can just buy the stock, and if a signal for selling the stock will be created, nothing will happen. For better comparison of the various stocks, only the relative returns will be calculated and trading costs are not taken into account. Because the t-test presumes that the point of break is known, we will set $n_1 = 0.5$ and assume a break in $n_1 n$.

Thus, with our formulas from the t-test, 2.2.1 and 2.2.2, we get for day i

$$X_1, ..., X_{n/2} = r_1^{(i)}, ..., r_{n/2}^{(i)},$$
$$Y_1, ..., Y_{n/2} = r_{n/2+1}^{(i)}, ..., r_n^{(i)}.$$
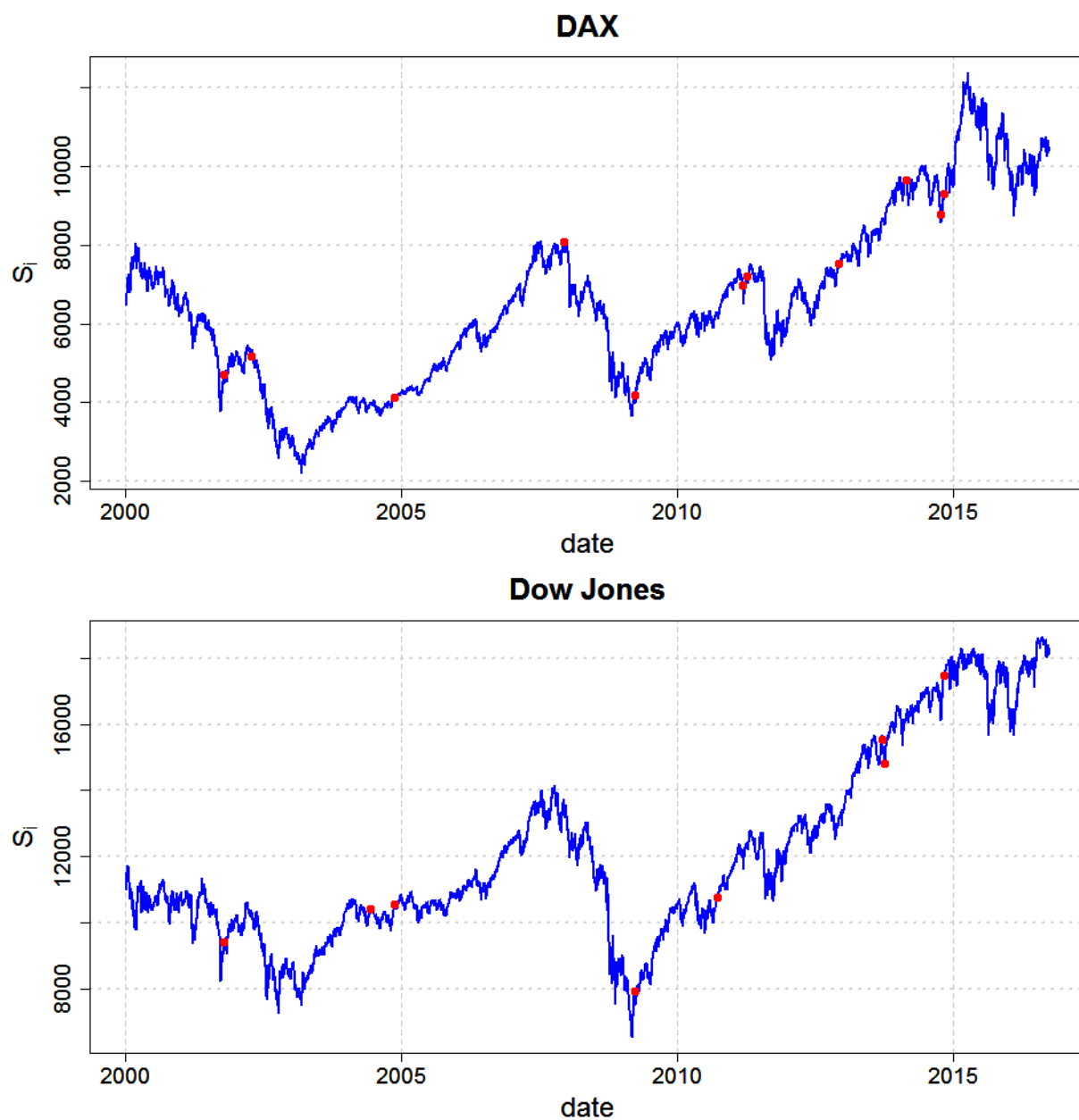
Figure 5.1.: *Prices of the DAX and the Dow Jones from 1 January, 2000 until 30 September, 2016. Additionally, the red points signal where the t-test detects a structural break in r_i with $\alpha = 0.01$ and $n = 30$. After the test has detected a structural break, there cannot be another break for 15 days.*

The OLS-CUSUM test and the K&L test do not need this assumption and it does not matter where the structural break happens. For the OLS-CUSUM test, with formula 2.3.1, and day i, we get

$$X_1, X_2, ..., X_n = 1, 1, ..., 1,$$
$$Y_1, Y_2, ..., Y_n = r^{(i)}.$$

For the K&L test, with formula 2.4.1, and day i, we get

$$X_1, X_2, ..., X_n = r^{(i)}.$$

Due to the fact that the test result of day i depends on the test result of day $(i-1)$, there will be no trading for $0.5n$ days after a trade has been made. In figure 5.1 the structural breaks for the DAX and the Dow Jones are given, detected by the t-test. If the test signals a structural break, we have two different strategies. We call the first strategy momentum strategy and the second one contrarian strategy. Firstly, we calculate

$$\bar{r}_1^{(i)} = \frac{1}{n/2} \sum_{t=1}^{n/2} r_t^{(i)},$$
$$\bar{r}_2^{(i)} = \frac{1}{n/2} \sum_{t=n/2+1}^{n} r_t^{(i)}.$$

By using the momentum strategy, we buy the stock if $\bar{r}_1^{(i)} < \bar{r}_2^{(i)}$ and sell the stock if $\bar{r}_1^{(i)} > \bar{r}_2^{(i)}$. By using the contrarian strategy, we buy the stock if $\bar{r}_1^{(i)} > \bar{r}_2^{(i)}$ and sell the stock if $\bar{r}_1^{(i)} < \bar{r}_2^{(i)}$. The idea of a momentum strategy is to recognize a positive trend as early as possible and to participate in the positive trend by buying the stock as long as possible. The approach of the contrarian strategy is, as the name says, contrary. We assume that the market overreacts and a turnaround in the trend will happen. By testing for breaks in the second moment, we assume an impact of the volatility on the prices. If we use the momentum strategy, we suppose that a higher volatility leads to a positive trend in the prices. If we use the contrarian strategy, we suppose that a higher volatility leads to a negative trend in the prices. As we could see in figure 4.1 and figure 4.2, the contrarian strategy is more realistic for the second moment. It would be also possible to create a trading strategy by trading volatility via options. While all tests are constructed for a two-sided test, both trading strategies adjust the trading signal to an one-sided application. For checking if the trading strategy works, the strategy will be compared to a buy-and-hold strategy. This means that we calculate the returns from day 1 until the last day of the observed data.

r_i	t-test				CUSUM			
	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$
$\alpha = 0.02$	127.43	-123.67	13.67	0.47	151.72	-99.37	2.43	0.43
$\alpha = 0.04$	120.28	-130.81	30.33	0.50	149.54	-101.56	7.03	0.47
$\alpha = 0.06$	64.40	-186.70	45.90	0.30	130.37	-120.73	13.47	0.40
$\alpha = 0.08$	65.40	-185.69	59.13	0.37	117.95	-133.15	19.90	0.47
$\alpha = 0.1$	86.93	-164.16	70.27	0.33	53.75	-197.34	26.77	0.30
$\alpha = 0.12$	77.61	-173.49	81.70	0.30	75.24	-175.85	33.00	0.33
$\alpha = 0.14$	126.44	-124.65	91.60	0.40	100.50	-150.60	39.07	0.33
$\alpha = 0.16$	154.72	-96.38	100.33	0.37	139.04	-112.05	45.83	0.47
$\alpha = 0.18$	171.92	-79.17	108.10	0.33	122.78	-128.31	52.87	0.40
$\alpha = 0.2$	143.12	-107.98	114.67	0.40	156.43	-94.66	59.60	0.40

Table 5.1.: *Momentum Trading Strategy with r_i for the DAX with $n = 30$: average discrete returns of a momentum trading strategy in %, the difference of a momentum trading strategy to a buy-and-hold strategy in % and the average number of trades for a momentum strategy. Additionally, the average number of stocks where the momentum strategy is better than the buy-and-hold strategy is depicted. The applied tests are the t-test and the CUSUM test for the DAX with $n = 30$.*

5.1. Structural Breaks in μ

Our first trading strategies are based on the first moment. Therefore, the t-test and the OLS-CUSUM test will be applied to the data of the DAX and the Dow Jones. In table 5.1 a momentum strategy for the DAX with changing α is tabled. As we can see, for every α the buy-and-hold strategy has higher returns than the momentum strategy. Interesting is the fact that the t-test signals more trades than the OLS-CUSUM test. This is because the power of the t-test is better than the power of the OLS-CUSUM test, as analyzed in figure 3.9. The t-test has the best results for $\alpha = 0.18$, while the OLS-CUSUM test has the best result for $\alpha = 0.2$. To verify these results, the same strategy for the stocks of the Dow Jones should be checked, tabled in 5.2. We can see quickly that the buy-and-hold strategy is better than the momentum strategy for every α and given $n = 30$. Next, we use the approach of a contrarian trading strategy. In table 5.3 and table 5.4 the results for the DAX and the Dow Jones are given with $n = 30$. The results are similar to the momentum trading strategy. For every α our returns are higher by using a buy-and-hold strategy than by using a contrarian strategy. The approach for using $n = 30$ was to detect a break as soon as possible. But as we could see in the simulation studies, breaks in μ will be detected more likely if the number of tested returns becomes higher. To this end, we set $\alpha = 0.1$ and applicate both trading strategies for various n. In table 5.5, 5.6, 5.7 and 5.8 the results are represented. Nevertheless, no trading strategy by using structural break tests for detecting breaks in the first moment can create higher returns than the buy-and-hold strategy.

r_i	t-test				CUSUM			
	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$
$\alpha = 0.02$	180.20	-220.04	12.43	0.30	105.57	-294.67	1.83	0.30
$\alpha = 0.04$	145.67	-254.56	27.97	0.10	166.40	-233.83	5.47	0.23
$\alpha = 0.06$	182.09	-218.15	42.20	0.27	203.86	-196.38	11.67	0.23
$\alpha = 0.08$	101.46	-298.78	57.30	0.17	111.52	-288.71	17.27	0.20
$\alpha = 0.1$	96.59	-303.65	69.07	0.10	111.95	-288.29	23.57	0.07
$\alpha = 0.12$	82.03	-318.21	81.90	0.07	107.75	-292.49	29.60	0.13
$\alpha = 0.14$	86.21	-314.02	91.30	0.10	109.15	-291.09	36.83	0.13
$\alpha = 0.16$	130.49	-269.75	100.17	0.20	101.46	-298.78	44.43	0.20
$\alpha = 0.18$	100.13	-300.11	109.53	0.10	102.51	-297.72	50.80	0.13
$\alpha = 0.2$	101.31	-298.93	116.27	0.17	116.64	-283.59	55.70	0.13

Table 5.2.: *Momentum Trading Strategy with r_i for the Dow Jones with $n = 30$: average discrete returns of a momentum trading strategy in %, the difference of a momentum trading strategy to a buy-and-hold strategy in % and the average number of trades for a momentum strategy. Additionally, the average number of stocks where the momentum strategy is better than the buy-and-hold strategy is depicted. The applied tests are the t-test and the CUSUM test for the Dow Jones with $n = 30$.*

r_i	t-test				CUSUM			
	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$
$\alpha = 0.02$	74.56	-176.53	13.47	0.33	151.26	-99.83	2.47	0.43
$\alpha = 0.04$	107.42	-143.67	30.33	0.40	101.34	-149.76	7.03	0.37
$\alpha = 0.06$	164.15	-86.95	45.97	0.30	100.94	-150.16	13.60	0.37
$\alpha = 0.08$	155.98	-95.12	58.93	0.43	59.13	-191.96	20.10	0.20
$\alpha = 0.1$	119.25	-131.85	70.40	0.37	106.54	-144.55	26.57	0.30
$\alpha = 0.12$	116.10	-134.99	81.70	0.33	117.61	-133.49	33.00	0.27
$\alpha = 0.14$	77.86	-173.23	91.40	0.37	81.26	-169.84	39.20	0.27
$\alpha = 0.16$	82.69	-168.40	100.07	0.23	85.49	-165.61	45.83	0.23
$\alpha = 0.18$	112.88	-138.22	107.77	0.33	119.01	-132.09	52.73	0.27
$\alpha = 0.2$	115.27	-135.83	114.47	0.30	71.72	-179.38	59.27	0.20

Table 5.3.: *Contrarian Trading Strategy with r_i for the DAX with $n = 30$: average discrete returns of a contrarian trading strategy in %, the difference of a contrarian trading strategy to a buy-and-hold strategy in % and the average number of trades for a contrarian strategy. Additionally, the average number of stocks where the contrarian strategy is better than the buy-and-hold strategy is depicted. The applied tests are the t-test and the CUSUM test for the DAX with $n = 30$.*

r_i	t-test				CUSUM			
	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$
$\alpha = 0.02$	111.97	-288.27	12.17	0.17	98.42	-301.82	1.87	0.20
$\alpha = 0.04$	145.46	-254.78	28.03	0.33	220.32	-179.92	5.37	0.20
$\alpha = 0.06$	109.94	-290.30	42.40	0.13	133.54	-266.70	11.73	0.37
$\alpha = 0.08$	155.26	-244.98	57.23	0.17	213.37	-186.86	17.33	0.20
$\alpha = 0.1$	159.32	-240.92	69.13	0.23	148.33	-251.91	23.77	0.17
$\alpha = 0.12$	181.10	-219.14	81.90	0.20	169.68	-230.55	29.93	0.13
$\alpha = 0.14$	166.41	-233.83	91.50	0.20	158.58	-241.65	37.10	0.23
$\alpha = 0.16$	143.95	-256.29	100.30	0.17	167.67	-232.57	44.43	0.20
$\alpha = 0.18$	177.43	-222.81	109.67	0.30	164.18	-236.05	50.87	0.20
$\alpha = 0.2$	178.41	-221.82	116.40	0.20	124.62	-275.62	55.63	0.23

Table 5.4.: *Contrarian Trading Strategy with r_i for the Dow Jones with $n = 30$: average discrete returns of a contrarian trading strategy in %, the difference of a contrarian trading strategy to a buy-and-hold strategy in % and the average number of trades for a contrarian strategy. Additionally, the average number of stocks where the contrarian strategy is better than the buy-and-hold strategy is depicted. The applied tests are the t-test and the CUSUM test for the Dow Jones with $n = 30$.*

r_i	t-test				CUSUM			
	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$
n=20	83.70	-167.40	94.83	0.37	156.55	-94.54	26.57	0.40
n=30	86.93	-164.16	70.27	0.33	53.75	-197.34	26.77	0.30
n=50	123.55	-127.55	48.00	0.37	121.36	-129.73	20.60	0.37
n=100	123.73	-127.36	24.80	0.37	183.65	-67.45	11.60	0.40
n=300	242.16	-8.94	7.83	0.53	126.11	-124.98	4.60	0.37

Table 5.5.: *Momentum Trading Strategy with r_i for the DAX with $\alpha = 0.1$: average discrete returns of a momentum trading strategy in %, the difference of a momentum trading strategy to a buy-and-hold strategy in % and the average number of trades for a momentum strategy. Additionally, the average number of stocks where the momentum strategy is better than the buy-and-hold strategy is depicted. The applied tests are the t-test and the CUSUM test for the DAX with $\alpha = 0.1$.*

r_i	t-test				CUSUM			
	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$
n=20	115.14	-285.09	91.57	0.13	117.09	-283.15	22.17	0.10
n=30	96.59	-303.65	69.07	0.10	111.95	-288.29	23.57	0.07
n=50	158.21	-242.03	46.27	0.20	136.65	-263.59	17.80	0.23
n=100	327.81	-72.42	23.07	0.17	313.97	-86.27	10.67	0.10
n=300	210.26	-189.98	7.33	0.20	183.04	-217.20	3.50	0.17

Table 5.6.: *Momentum Trading Strategy with r_i for the Dow Jones with $\alpha = 0.1$: average discrete returns of a momentum trading strategy in %, the difference of a momentum trading strategy to a buy-and-hold strategy in % and the average number of trades for a momentum strategy. Additionally, the average number of stocks where the momentum strategy is better than the buy-and-hold strategy is depicted. The applied tests are the t-test and the CUSUM test for the Dow Jones with $\alpha = 0.1$.*

r_i	t-test				CUSUM			
	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$
n=20	105.59	-145.51	95.03	0.33	81.33	-169.77	26.63	0.30
n=30	119.25	-131.85	70.40	0.37	106.54	-144.55	26.57	0.30
n=50	111.76	-139.33	47.93	0.27	96.30	-154.80	20.73	0.33
n=100	122.28	-128.82	24.80	0.40	99.70	-151.40	11.80	0.40
n=300	66.44	-184.65	7.87	0.40	157.83	-93.26	4.57	0.50

Table 5.7.: *Contrarian Trading Strategy with r_i for the DAX with $\alpha = 0.1$: average discrete returns of a contrarian trading strategy in %, the difference of a contrarian trading strategy to a buy-and-hold strategy in % and the average number of trades for a contrarian strategy. Additionally, the average number of stocks where the contrarian strategy is better than the buy-and-hold strategy is depicted. The applied tests are the t-test and the CUSUM test for the DAX with $\alpha = 0.1$.*

r_i	t-test				CUSUM			
	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$
n=20	141.41	-258.83	91.70	0.27	119.71	-280.53	22.30	0.13
n=30	159.32	-240.92	69.13	0.23	148.33	-251.91	23.77	0.17
n=50	101.51	-298.72	45.73	0.10	128.76	-271.47	17.73	0.30
n=100	135.29	-264.95	23.07	0.37	168.67	-231.56	10.60	0.43
n=300	108.17	-292.07	7.27	0.30	123.10	-277.13	3.53	0.23

Table 5.8.: *Contrarian Trading Strategy with r_i for the Dow Jones with $\alpha = 0.1$: average discrete returns of a contrarian trading strategy in %, the difference of a contrarian trading strategy to a buy-and-hold strategy in % and the average number of trades for a contrarian strategy. Additionally, the average number of stocks where the contrarian strategy is better than the buy-and-hold strategy is depicted. The applied tests are the t-test and the CUSUM test for the Dow Jones with $\alpha = 0.1$.*

r_i^2	K&L				t-test			
	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$
$\alpha = 0.02$	3.64	-247.45	1.00	0.27	61.93	-189.16	15.27	0.30
$\alpha = 0.04$	8.27	-242.83	1.03	0.27	127.96	-123.14	31.57	0.23
$\alpha = 0.06$	50.73	-200.37	2.43	0.33	128.12	-122.98	48.90	0.40
$\alpha = 0.08$	18.19	-232.90	8.93	0.27	144.88	-106.22	62.80	0.40
$\alpha = 0.1$	56.26	-194.83	18.00	0.30	165.58	-85.51	75.90	0.40
$\alpha = 0.12$	159.43	-91.67	26.77	0.40	180.65	-70.45	87.77	0.40
$\alpha = 0.14$	179.68	-71.42	36.33	0.33	114.61	-136.48	96.90	0.37
$\alpha = 0.16$	146.22	-104.87	44.50	0.40	106.49	-144.60	104.83	0.30
$\alpha = 0.18$	138.77	-112.32	53.33	0.37	106.52	-144.58	111.70	0.37
$\alpha = 0.2$	113.66	-137.43	62.87	0.30	119.25	-131.84	118.30	0.27

Table 5.9.: *Momentum Trading Strategy with r_i^2 for the DAX with $n = 30$: average discrete returns of a momentum trading strategy in %, the difference of a momentum trading strategy to a buy-and-hold strategy in % and the average number of trades for a momentum strategy. Additionally, the average number of stocks where the momentum strategy is better than the buy-and-hold strategy is depicted. The applied tests are the K&L test and the t-test for the DAX with $n = 30$.*

5.2. Structural Breaks in σ

The second approach is to detect structural breaks in the volatility. For detecting structural breaks in the volatility, the K&L test and the t-test with squared returns will be used. Therefore, we will test both trading strategies for various α and for the DAX and the Dow Jones. In table 5.9, 5.10, 5.11 and 5.12 the results are given. As we can see, neither the trading strategy with the K&L test nor the trading strategy with the t-test can beat the buy-and-hold strategy. This is true for the momentum trading strategy as well as for the contrarian strategy. Only the contrarian trading strategy for the Dow Jones with the K&L test and with $\alpha = 0.14$ or $\alpha = 0.18$ can beat the buy-and hold-strategy. But this seems more likely to be a coincidence as the same strategy for the DAX has worse results. Next, we want to verify these results by varying n. In table 5.13, 5.14, 5.15 and 5.16 the results for various n and $\alpha = 0.1$ are given. The outcome is the same as in the previous strategies. No strategy can beat the buy-and-hold strategy except for the momentum strategy for the DAX with $n = 20$. Since the same strategy does not work for the Dow Jones, it is obviously a coincidence. Summarized we can say that no accomplished trading strategy for various n or α can beat the buy-and-hold strategy.

r_i^2	K&L				t-test			
	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$
$\alpha = 0.02$	0.00	-400.24	1.00	0.03	90.42	-309.82	15.13	0.10
$\alpha = 0.04$	32.06	-368.18	1.03	0.07	166.64	-233.59	32.37	0.10
$\alpha = 0.06$	214.72	-185.52	2.87	0.13	109.23	-291.00	47.77	0.17
$\alpha = 0.08$	80.55	-319.68	8.37	0.17	119.70	-280.54	62.17	0.27
$\alpha = 0.1$	108.39	-291.85	15.87	0.13	128.22	-272.02	75.30	0.30
$\alpha = 0.12$	100.15	-300.08	25.17	0.17	132.62	-267.62	87.10	0.27
$\alpha = 0.14$	89.52	-310.72	33.87	0.10	150.95	-249.29	96.60	0.33
$\alpha = 0.16$	83.46	-316.77	44.37	0.13	140.19	-260.05	105.33	0.27
$\alpha = 0.18$	89.84	-310.40	52.70	0.13	132.24	-268.00	112.20	0.30
$\alpha = 0.2$	106.12	-294.12	62.10	0.17	124.30	-275.93	119.40	0.33

Table 5.10.: *Momentum Trading Strategy with r_i^2 for the Dow Jones with $n = 30$: average discrete returns of a momentum trading strategy in %, the difference of a momentum trading strategy to a buy-and-hold strategy in % and the average number of trades for a momentum strategy. Additionally, the average number of stocks where the momentum strategy is better than the buy-and-hold strategy is depicted. The applied tests are the K&L test and the t-test for the Dow Jones with $n = 30$.*

r_i^2	K&L				t-test			
	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$
$\alpha = 0.02$	0.00	-251.10	1.00	0.27	111.21	-139.88	14.73	0.43
$\alpha = 0.04$	45.02	-206.08	1.00	0.30	112.49	-138.60	31.63	0.40
$\alpha = 0.06$	162.05	-89.05	2.20	0.43	74.46	-176.63	49.23	0.20
$\alpha = 0.08$	225.82	-25.28	8.67	0.57	98.89	-152.21	63.00	0.27
$\alpha = 0.1$	149.30	-101.80	17.73	0.43	84.30	-166.80	75.97	0.27
$\alpha = 0.12$	126.43	-124.67	26.37	0.37	87.99	-163.10	87.70	0.33
$\alpha = 0.14$	136.24	-114.85	35.87	0.27	138.12	-112.98	96.83	0.27
$\alpha = 0.16$	78.89	-172.21	44.23	0.17	171.21	-79.89	104.77	0.30
$\alpha = 0.18$	63.21	-187.88	53.00	0.17	213.09	-38.01	111.57	0.40
$\alpha = 0.2$	81.58	-169.52	62.67	0.23	187.28	-63.82	118.23	0.40

Table 5.11.: *Contrarian Trading Strategy with r_i^2 for the DAX with $n = 30$: average discrete returns of a contrarian trading strategy in %, the difference of a contrarian trading strategy to a buy-and-hold strategy in % and the average number of trades for a contrarian strategy. Additionally, the average number of stocks where the contrarian strategy is better than the buy-and-hold strategy is depicted. The applied tests are the K&L test and the t-test for the DAX with $n = 30$.*

r_i^2	K&L				t-test			
	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$
$\alpha = 0.02$	0.00	-400.24	1.00	0.03	189.95	-210.29	15.20	0.13
$\alpha = 0.04$	26.58	-373.65	1.10	0.07	156.71	-243.53	32.63	0.17
$\alpha = 0.06$	118.56	-281.67	2.67	0.23	140.74	-259.50	47.97	0.10
$\alpha = 0.08$	136.06	-264.18	8.37	0.17	182.17	-218.06	62.03	0.13
$\alpha = 0.1$	286.56	-113.68	15.73	0.23	204.79	-195.45	75.17	0.13
$\alpha = 0.12$	179.57	-220.67	25.03	0.13	240.87	-159.36	86.97	0.13
$\alpha = 0.14$	445.21	44.97	33.93	0.17	200.42	-199.81	96.53	0.13
$\alpha = 0.16$	223.16	-177.07	44.37	0.20	151.91	-248.32	105.20	0.13
$\alpha = 0.18$	513.56	113.32	52.50	0.23	200.48	-199.76	111.93	0.13
$\alpha = 0.2$	308.64	-91.59	61.97	0.17	204.32	-195.91	119.20	0.13

Table 5.12.: *Contrarian Trading Strategy with r_i^2 for the Dow Jones with $n = 30$: average discrete returns of a contrarian trading strategy in %, the difference of a contrarian trading strategy to a buy-and-hold strategy in % and the average number of trades for a contrarian strategy. Additionally, the average number of stocks where the contrarian strategy is better than the buy-and-hold strategy is depicted. The applied tests are the K&L test and the t-test for the Dow Jones with $n = 30$.*

r_i^2	K&L				t-test			
	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$
n=20	347.21	96.12	5.40	0.43	99.38	-151.72	86.60	0.37
n=30	56.26	-194.83	18.00	0.30	165.58	-85.51	75.90	0.40
n=50	54.69	-196.40	21.27	0.37	81.33	-169.77	53.47	0.30
n=100	55.02	-196.08	17.97	0.20	55.24	-195.86	30.70	0.20
n=300	25.41	-225.69	8.47	0.27	21.31	-229.79	10.70	0.20

Table 5.13.: *Momentum Trading Strategy with r_i^2 for the DAX with $\alpha = 0.1$: average discrete returns of a momentum trading strategy in %, the difference of a momentum trading strategy to a buy-and-hold strategy in % and the average number of trades for a momentum strategy. Additionally, the average number of stocks where the momentum strategy is better than the buy-and-hold strategy is depicted. The applied tests are the K&L test and the t-test for the DAX with $\alpha = 0.1$.*

r_i^2	K&L				t-test			
	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$	r_{mom}	$r_{mom} - r_{b\&h}$	trades	$r_{mom} > r_{b\&h}$
n=20	98.03	-302.20	5.50	0.07	130.02	-270.22	82.83	0.23
n=30	108.39	-291.85	15.87	0.13	128.22	-272.02	75.30	0.30
n=50	84.56	-315.68	22.13	0.23	162.11	-238.13	55.63	0.33
n=100	58.46	-341.78	16.20	0.10	66.64	-333.60	29.43	0.17
n=300	37.80	-362.44	9.13	0.10	89.69	-310.54	10.80	0.10

Table 5.14.: *Momentum Trading Strategy with r_i^2 for the Dow Jones with $\alpha = 0.1$: average discrete returns of a momentum trading strategy in %, the difference of a momentum trading strategy to a buy-and-hold strategy in % and the average number of trades for a momentum strategy. Additionally, the average number of stocks where the momentum strategy is better than the buy-and-hold strategy is depicted. The applied tests are the K&L test and the t-test for the Dow Jones with $\alpha = 0.1$.*

r_i^2	K&L				t-test			
	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$
n=20	99.78	-151.31	5.13	0.47	134.87	-116.23	86.53	0.33
n=30	149.30	-101.80	17.73	0.43	84.30	-166.80	75.97	0.27
n=50	163.33	-87.77	21.27	0.33	140.16	-110.93	53.47	0.33
n=100	169.11	-81.98	18.43	0.47	152.11	-98.98	31.30	0.47
n=300	203.33	-47.77	8.67	0.60	167.86	-83.24	10.97	0.57

Table 5.15.: *Contrarian Trading Strategy with r_i^2 for the DAX with $\alpha = 0.1$: average discrete returns of a contrarian trading strategy in %, the difference of a contrarian trading strategy to a buy-and-hold strategy in % and the average number of trades for a contrarian strategy. Additionally, the average number of stocks where the contrarian strategy is better than the buy-and-hold strategy is depicted. The applied tests are the K&L test and the t-test for the DAX with $\alpha = 0.1$.*

r_i^2	K&L				t-test			
	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$	r_{con}	$r_{con} - r_{b\&h}$	trades	$r_{con} > r_{b\&h}$
n=20	169.14	-231.10	5.60	0.23	158.23	-242.00	82.57	0.20
n=30	286.56	-113.68	15.73	0.23	204.79	-195.45	75.17	0.13
n=50	246.54	-153.69	22.27	0.20	119.16	-281.07	55.37	0.10
n=100	315.26	-84.98	17.00	0.27	323.66	-76.58	29.97	0.30
n=300	326.31	-73.93	9.73	0.33	212.49	-187.75	11.33	0.30

Table 5.16.: *Contrarian Trading Strategy with r_i^2 for the Dow Jones with $\alpha = 0.1$: average discrete returns of a contrarian trading strategy in %, the difference of a contrarian trading strategy to a buy-and-hold strategy in % and the average number of trades for a contrarian strategy. Additionally, the average number of stocks where the contrarian strategy is better than the buy-and-hold strategy is depicted. The applied tests are the K&L test and the t-test for the Dow Jones with $\alpha = 0.1$.*

6. Conclusion and Outlook

We discussed various tests for detecting structural breaks in μ or in σ. The t-test had the best result for the first moment, while the K&L test and the t-test had the best results for the second moment. It was shown in the analysis that various parameters of the GARCH model have an influence on the test results. Additionally, the autocorrelation effect in the data worsens the results of the test decisions. Since stocks normally have a slow decay of autocorrelation, this could be the reason why the various trading strategies do not work. Moreover, by using daily data, it may takes too long for detecting a structural break. Further studies can try to use intraday returns for detecting a structural break earlier. This could lead to a better trading strategy especially with a momentum signal. Thus, a positive trend of a stock maybe can be detected earlier and a longer participation in the positive trend is possible. Furthermore, if a structural break occurs and a positive trend changes into a negative trend, a test which uses intraday data could detect this faster and can possibly avoid shrinking returns. Another possible improvement for the outcome of the trading strategies would be a sequential trading. As example, the α level of the test will be adjusted every day. Moreover, an adjustment of n is imaginable. Since the positive trend of the prices exists longer than a negative trend but the negative trend is steeper, compare figure 4.1 and figure 4.2, a sequential trading with changing n could improve the results. In phases with a negative trend, we could use a larger n to ensure that the negative trend is over, and in phases with a positive trend, we could use a smaller n to detect the crash faster. A simultaneously use of tests for detecting breaks in the first moment and in the second moment is an option too. Due to the fact that volatility is higher in phases with a crash, we could try to detect crashes by testing for breaks in the second moment. Then we buy if a test for a break in the first moment signals a break with following positive trend. Additionally, a change of the signals is possible. In the applied strategies it does not matter if the break changes from positive to negative or the other way. We could change the signal in a way that a trade will only be made if the trend changes the sign. For the trading strategies based on the second moment trading volatility instead of the stock would also be possible. This could happen for instance by using a straddle strategy.

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8. List of Figures

9. List of Tables

A. Appendix

A.1. Tables and Figures
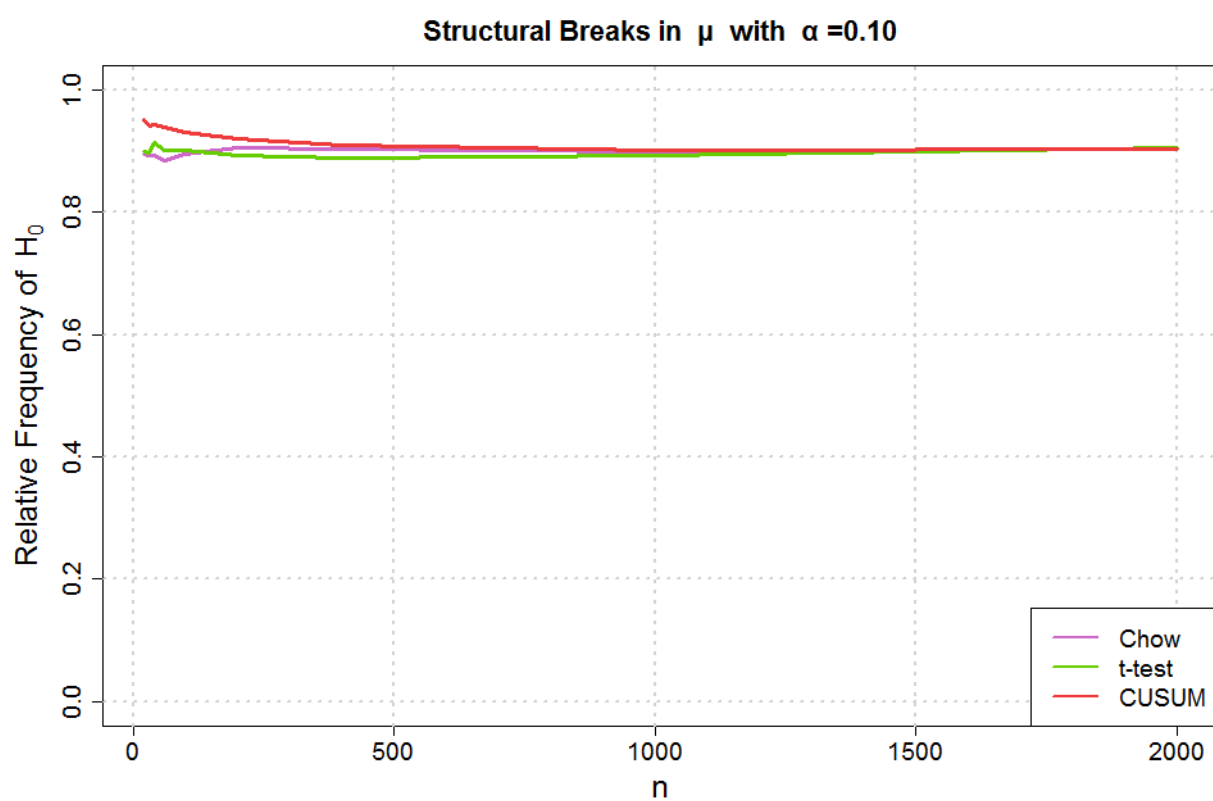
Figure A.1.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.5$, and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i	$\alpha = 0.05$			$\alpha = 0.10$		
	Chow test	t-test	CUSUM	Chow test	t-test	CUSUM
n=20	0.95	0.94	0.99	0.90	0.90	0.95
n=30	0.94	0.95	0.98	0.89	0.90	0.94
n=40	0.95	0.95	0.98	0.89	0.91	0.94
n=60	0.94	0.95	0.97	0.88	0.90	0.94
n=100	0.95	0.95	0.96	0.90	0.90	0.93
n=200	0.95	0.95	0.96	0.91	0.89	0.92
n=400	0.95	0.95	0.95	0.90	0.89	0.91
n=1000	0.94	0.95	0.95	0.90	0.89	0.90
n=2000	0.96	0.95	0.96	0.90	0.91	0.90

Table A.1.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.5$, and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i^2	$\alpha = 0.05$					$\alpha = 0.10$				
	Chow test	t-test	CUSUM	K&L	I&T	Chow test	t-test	CUSUM	K&L	I&T
n=20	0.92	0.95	0.98	1.00	0.97	0.86	0.89	0.95	0.99	0.95
n=30	0.91	0.94	0.96	1.00	0.95	0.84	0.86	0.91	0.98	0.91
n=40	0.91	0.92	0.95	0.99	0.94	0.84	0.85	0.90	0.96	0.89
n=60	0.90	0.91	0.93	0.99	0.92	0.81	0.84	0.87	0.95	0.86
n=100	0.88	0.90	0.90	0.98	0.89	0.80	0.83	0.83	0.93	0.82
n=200	0.87	0.89	0.88	0.96	0.86	0.81	0.81	0.80	0.91	0.78
n=400	0.87	0.89	0.88	0.96	0.87	0.80	0.83	0.79	0.91	0.78
n=1000	0.86	0.89	0.85	0.95	0.84	0.78	0.82	0.76	0.90	0.74
n=2000	0.87	0.87	0.83	0.94	0.80	0.78	0.80	0.74	0.88	0.72

Table A.2.: *Relative frequencies of H_0 by using the introduced tests with r_i^2: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.5$, and $\varepsilon_1 = \varepsilon_2 = 1$.*

Figure A.2.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.5$, and $\varepsilon_1 = \varepsilon_2 = 1$.*

$|r_i|$	$\alpha = 0.05$				$\alpha = 0.10$			
	Chow test	t-test	CUSUM	K&L	Chow test	t-test	CUSUM	K&L
n=20	0.92	0.93	0.97	1.00	0.85	0.87	0.94	0.99
n=30	0.91	0.92	0.95	1.00	0.84	0.86	0.90	0.97
n=40	0.91	0.92	0.94	0.99	0.84	0.85	0.89	0.95
n=60	0.90	0.91	0.93	0.99	0.83	0.84	0.87	0.94
n=100	0.88	0.90	0.90	0.98	0.81	0.84	0.83	0.93
n=200	0.88	0.89	0.88	0.96	0.81	0.82	0.79	0.90
n=400	0.88	0.90	0.89	0.96	0.81	0.84	0.81	0.91
n=1000	0.87	0.90	0.87	0.95	0.79	0.84	0.77	0.90
n=2000	0.88	0.88	0.84	0.94	0.79	0.81	0.75	0.88

Table A.3.: *Relative frequencies of H_0 by using the introduced tests with $|r_i|$: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.5$, and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i	$\alpha = 0.05$			$\alpha = 0.10$		
	Chow test	t-test	CUSUM	Chow test	t-test	CUSUM
n=20	0.94	0.95	0.99	0.90	0.90	0.96
n=30	0.95	0.94	0.97	0.89	0.90	0.94
n=40	0.95	0.96	0.98	0.90	0.90	0.94
n=60	0.95	0.95	0.97	0.89	0.89	0.93
n=100	0.95	0.95	0.95	0.90	0.90	0.92
n=200	0.95	0.94	0.95	0.91	0.89	0.90
n=400	0.95	0.95	0.96	0.90	0.91	0.92
n=1000	0.95	0.95	0.95	0.90	0.89	0.89
n=2000	0.95	0.96	0.95	0.90	0.90	0.90

Table A.4.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.85$, and $\varepsilon_1 = \varepsilon_2 = 1$.*

Figure A.3.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.85$, and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i^2	$\alpha = 0.05$					$\alpha = 0.10$				
	Chow test	t-test	CUSUM	K&L	I&T	Chow test	t-test	CUSUM	K&L	I&T
n=20	0.94	0.94	0.98	1.00	0.98	0.89	0.87	0.94	0.99	0.95
n=30	0.92	0.89	0.95	1.00	0.94	0.87	0.81	0.89	0.96	0.90
n=40	0.90	0.84	0.90	0.98	0.89	0.82	0.75	0.82	0.91	0.84
n=60	0.85	0.77	0.82	0.94	0.81	0.76	0.68	0.73	0.84	0.73
n=100	0.75	0.69	0.66	0.86	0.66	0.64	0.60	0.55	0.73	0.56
n=200	0.57	0.62	0.50	0.76	0.46	0.48	0.54	0.39	0.63	0.37
n=400	0.49	0.58	0.36	0.66	0.31	0.40	0.50	0.27	0.55	0.23
n=1000	0.39	0.54	0.20	0.62	0.13	0.31	0.46	0.13	0.49	0.09
n=2000	0.36	0.55	0.15	0.66	0.09	0.30	0.47	0.08	0.54	0.04

Table A.5.: *Relative frequencies of H_0 by using the introduced tests with r_i^2: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.85$, and $\varepsilon_1 = \varepsilon_2 = 1$.*

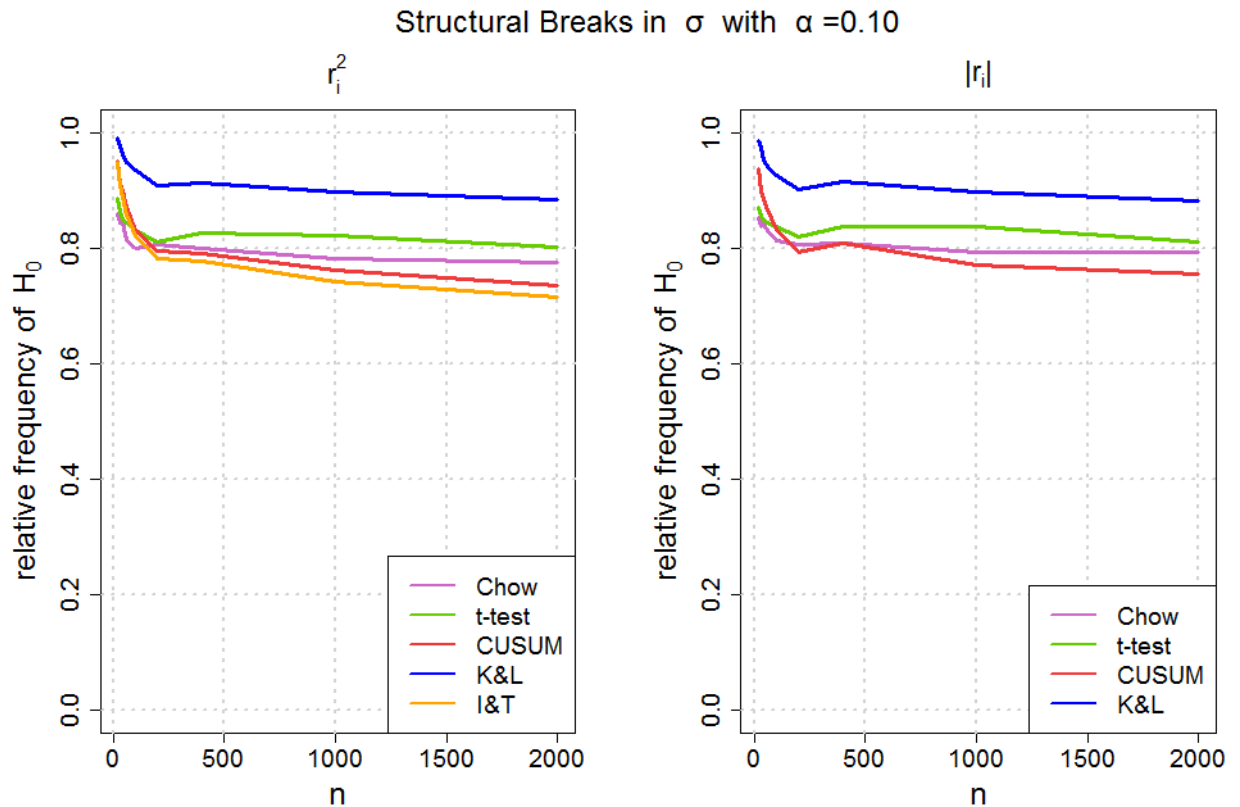| $|r_i|$	$\alpha = 0.05$				$\alpha = 0.10$			
	Chow test	t-test	CUSUM	K&L	Chow test	t-test	CUSUM	K&L
n=20	0.94	0.92	0.97	1.00	0.88	0.85	0.94	0.99
n=30	0.93	0.88	0.94	1.00	0.87	0.80	0.87	0.96
n=40	0.89	0.84	0.88	0.97	0.82	0.75	0.81	0.90
n=60	0.85	0.77	0.80	0.93	0.77	0.69	0.72	0.82
n=100	0.74	0.70	0.67	0.85	0.65	0.62	0.57	0.72
n=200	0.57	0.64	0.49	0.73	0.49	0.56	0.40	0.61
n=400	0.49	0.59	0.38	0.66	0.41	0.52	0.27	0.54
n=1000	0.40	0.55	0.22	0.60	0.33	0.47	0.14	0.48
n=2000	0.39	0.58	0.17	0.67	0.32	0.51	0.10	0.54

Table A.6.: *Relative frequencies of H_0 by using the introduced tests with $|r_i|$: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.85$, and $\varepsilon_1 = \varepsilon_2 = 1$.*

Figure A.4.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = \mu_2 = 0$, $\omega_1 = \omega_2 = 0.4$, $\alpha_1 = \alpha_2 = 0.1$, $\beta_1 = \beta_2 = 0.85$, and $\varepsilon_1 = \varepsilon_2 = 1$.*
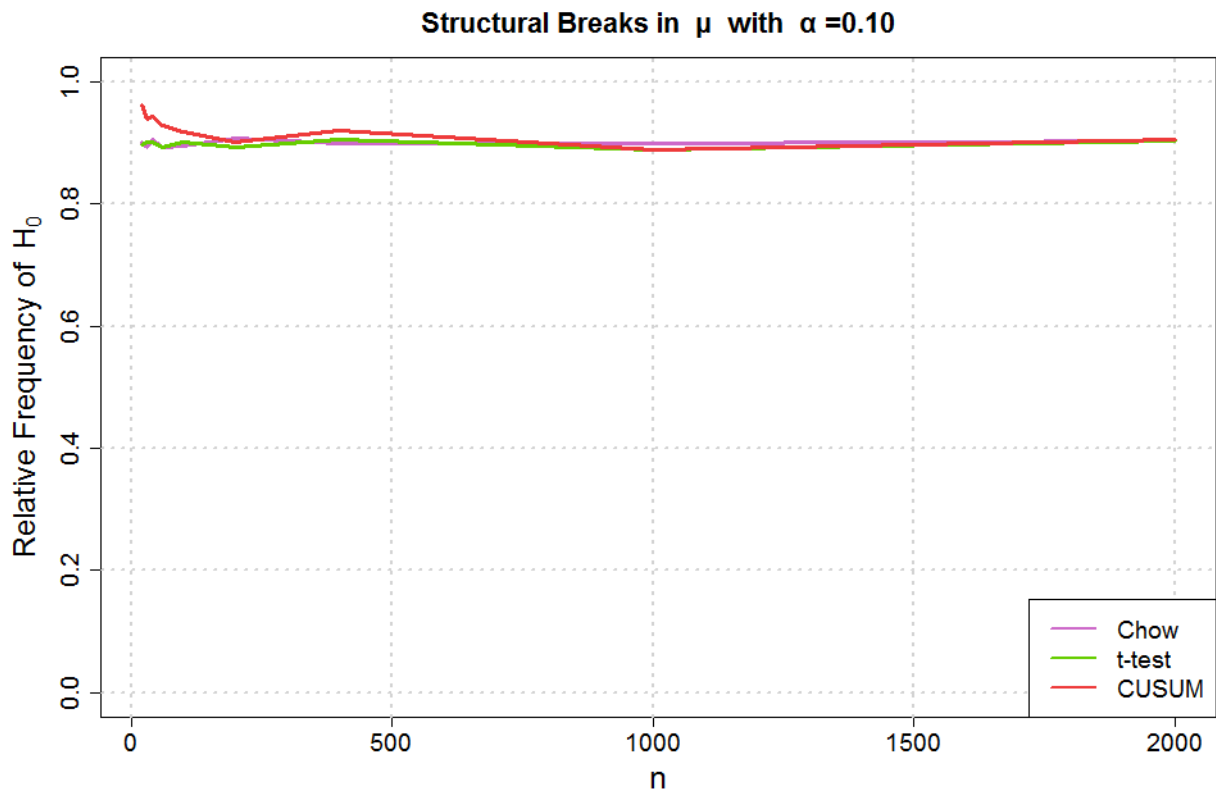
r_i	$\alpha = 0.05$			$\alpha = 0.10$		
	Chow test	t-test	CUSUM	Chow test	t-test	CUSUM
n=20	0.95	0.95	0.98	0.91	0.90	0.95
n=30	0.95	0.94	0.96	0.91	0.88	0.92
n=40	0.95	0.94	0.96	0.91	0.89	0.91
n=60	0.95	0.94	0.96	0.90	0.89	0.92
n=100	0.94	0.92	0.93	0.90	0.87	0.88
n=200	0.95	0.93	0.94	0.90	0.87	0.89
n=400	0.94	0.88	0.90	0.89	0.81	0.84
n=1000	0.91	0.78	0.82	0.86	0.68	0.72
n=2000	0.91	0.62	0.69	0.83	0.50	0.58

Table A.7.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i^2	$\alpha = 0.05$					$\alpha = 0.10$				
	Chow test	t-test	CUSUM	K&L	I&T	Chow test	t-test	CUSUM	K&L	I&T
n=20	0.93	0.88	0.93	1.00	0.90	0.86	0.76	0.85	1.00	0.83
n=30	0.90	0.78	0.86	1.00	0.79	0.83	0.66	0.75	0.94	0.70
n=40	0.88	0.70	0.76	0.97	0.68	0.80	0.55	0.64	0.86	0.59
n=60	0.82	0.52	0.59	0.90	0.53	0.72	0.40	0.47	0.75	0.44
n=100	0.74	0.30	0.37	0.74	0.31	0.64	0.22	0.27	0.56	0.24
n=200	0.60	0.07	0.10	0.39	0.08	0.49	0.04	0.06	0.24	0.05
n=400	0.37	0.00	0.01	0.06	0.01	0.26	0.00	0.00	0.03	0.00
n=1000	0.06	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00
n=2000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table A.8.: *Relative frequencies of H_0 by using the introduced tests with r_i^2: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*
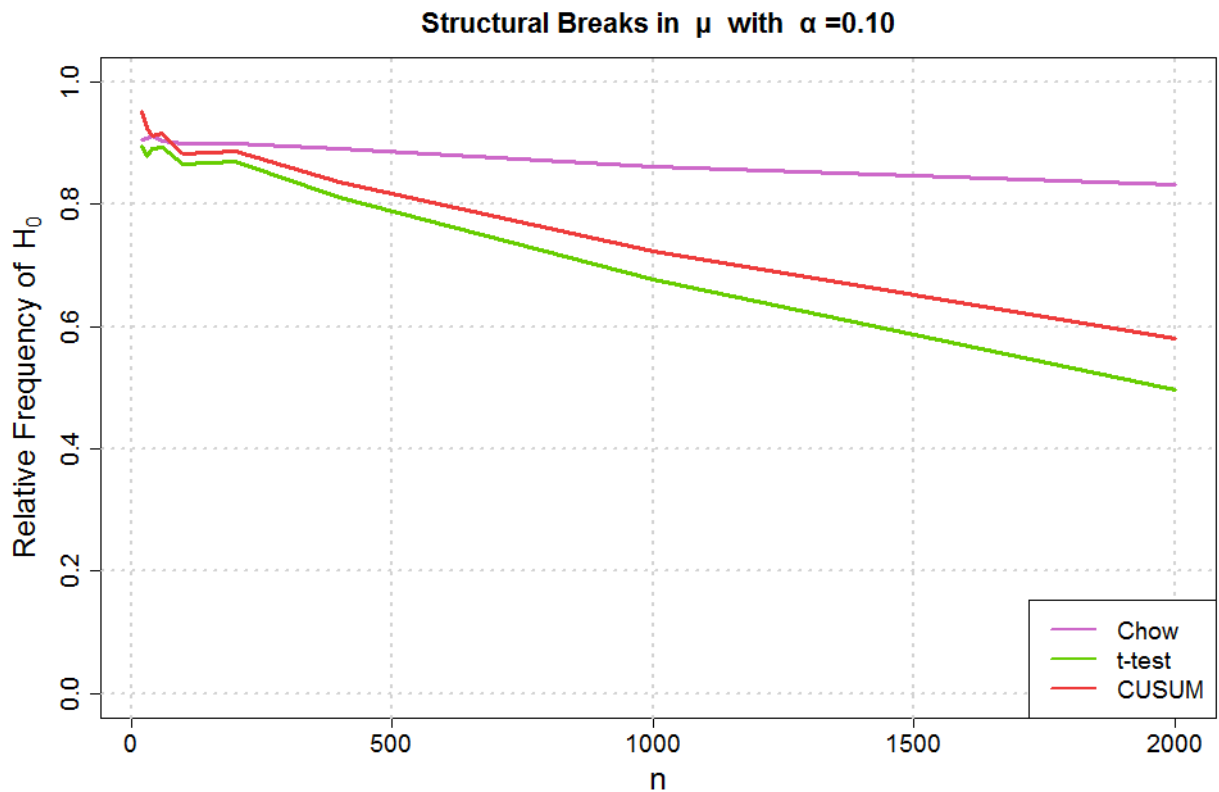
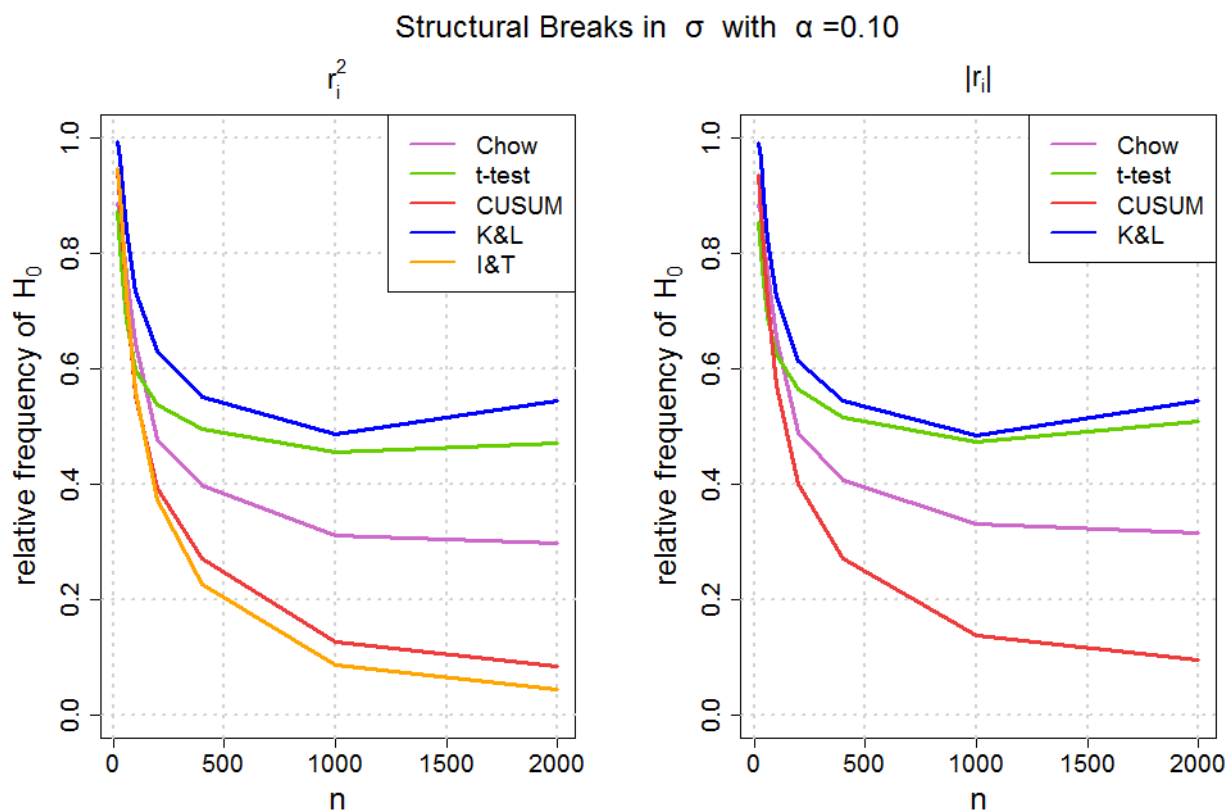**Figure A.5.:** *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

$\lvert r_i \rvert$	$\alpha = 0.05$				$\alpha = 0.10$			
	Chow test	t-test	CUSUM	K&L	Chow test	t-test	CUSUM	K&L
n=20	0.90	0.83	0.91	1.00	0.83	0.74	0.83	0.99
n=30	0.89	0.74	0.83	1.00	0.82	0.65	0.72	0.93
n=40	0.86	0.67	0.72	0.95	0.77	0.56	0.62	0.82
n=60	0.80	0.52	0.57	0.87	0.71	0.42	0.47	0.70
n=100	0.73	0.32	0.37	0.68	0.62	0.24	0.28	0.51
n=200	0.57	0.09	0.12	0.33	0.47	0.05	0.08	0.20
n=400	0.34	0.01	0.01	0.04	0.26	0.00	0.01	0.02
n=1000	0.05	0.00	0.00	0.00	0.03	0.00	0.00	0.00
n=2000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table A.9.: *Relative frequencies of H_0 by using the introduced tests with $\lvert r_i \rvert$: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

Figure A.6.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $\lvert r_i \rvert$: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i	$\alpha = 0.05$			$\alpha = 0.10$		
	Chow test	t-test	CUSUM	Chow test	t-test	CUSUM
$\mu_2 = -0.4$	0.19	0.00	0.00	0.12	0.00	0.00
$\mu_2 = -0.3$	0.44	0.00	0.01	0.33	0.00	0.00
$\mu_2 = -0.2$	0.73	0.12	0.19	0.61	0.07	0.12
$\mu_2 = -0.1$	0.89	0.65	0.73	0.83	0.52	0.61
$\mu_2 = 0$	0.95	0.95	0.95	0.90	0.91	0.91
$\mu_2 = 0.1$	0.90	0.63	0.72	0.82	0.51	0.61
$\mu_2 = 0.2$	0.73	0.12	0.20	0.61	0.07	0.12
$\mu_2 = 0.3$	0.46	0.00	0.01	0.32	0.00	0.00
$\mu_2 = 0.4$	0.18	0.00	0.00	0.11	0.00	0.00

Table A.10.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

| r_i^2	$\alpha = 0.05$					$\alpha = 0.10$				
	Chow test	t-test	CUSUM	K&L	I&T	Chow test	t-test	CUSUM	K&L	I&T
$\mu_2 = -0.4$	0.79	0.48	0.51	0.71	0.49	0.68	0.38	0.40	0.58	0.38
$\mu_2 = -0.3$	0.83	0.72	0.72	0.87	0.70	0.75	0.63	0.61	0.78	0.60
$\mu_2 = -0.2$	0.85	0.84	0.82	0.94	0.80	0.77	0.75	0.72	0.87	0.69
$\mu_2 = -0.1$	0.87	0.89	0.84	0.95	0.83	0.79	0.81	0.76	0.90	0.73
$\mu_2 = 0$	0.86	0.89	0.86	0.95	0.84	0.78	0.82	0.77	0.91	0.75
$\mu_2 = 0.1$	0.87	0.88	0.86	0.95	0.84	0.80	0.82	0.77	0.90	0.75
$\mu_2 = 0.2$	0.88	0.85	0.82	0.94	0.79	0.80	0.76	0.72	0.88	0.70
$\mu_2 = 0.3$	0.82	0.72	0.71	0.86	0.69	0.74	0.62	0.61	0.78	0.59
$\mu_2 = 0.4$	0.77	0.49	0.52	0.70	0.49	0.68	0.38	0.39	0.58	0.38

Table A.11.: *Relative frequencies of H_0 by using the introduced tests with r_i^2: $\mu_1 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

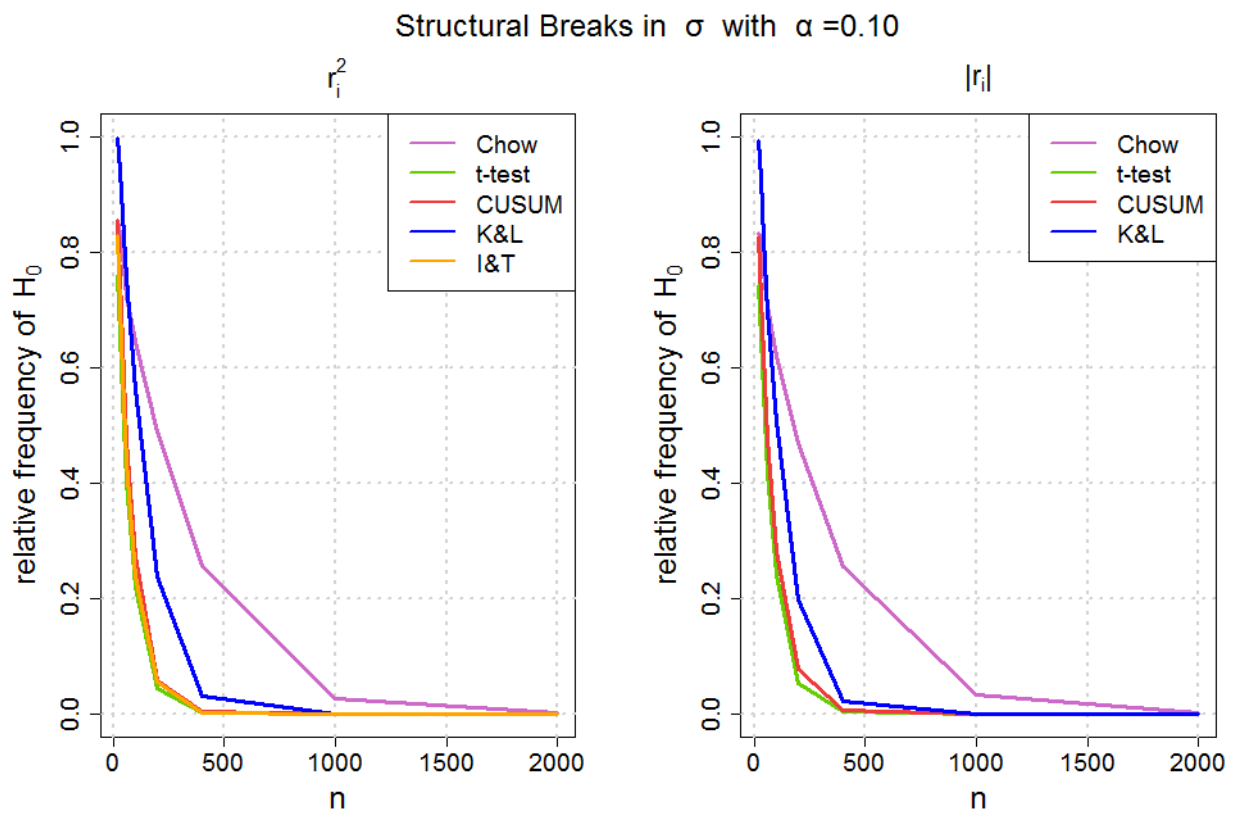| $|r_i|$	$\alpha = 0.05$				$\alpha = 0.10$			
	Chow test	t-test	CUSUM	K&L	Chow test	t-test	CUSUM	K&L
$\mu_2 = -0.4$	0.80	0.51	0.54	0.70	0.70	0.39	0.42	0.59
$\mu_2 = -0.3$	0.85	0.75	0.74	0.86	0.76	0.66	0.64	0.79
$\mu_2 = -0.2$	0.88	0.85	0.84	0.93	0.79	0.77	0.74	0.87
$\mu_2 = -0.1$	0.88	0.90	0.86	0.95	0.80	0.83	0.78	0.90
$\mu_2 = 0$	0.88	0.90	0.86	0.95	0.80	0.83	0.78	0.90
$\mu_2 = 0.1$	0.88	0.89	0.87	0.95	0.81	0.83	0.78	0.89
$\mu_2 = 0.2$	0.88	0.87	0.83	0.93	0.80	0.79	0.74	0.87
$\mu_2 = 0.3$	0.84	0.75	0.74	0.87	0.76	0.65	0.63	0.78
$\mu_2 = 0.4$	0.79	0.51	0.53	0.69	0.69	0.40	0.42	0.58

Table A.12.: *Relative frequencies of H_0 by using the introduced tests with $|r_i|$: $\mu_1 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*
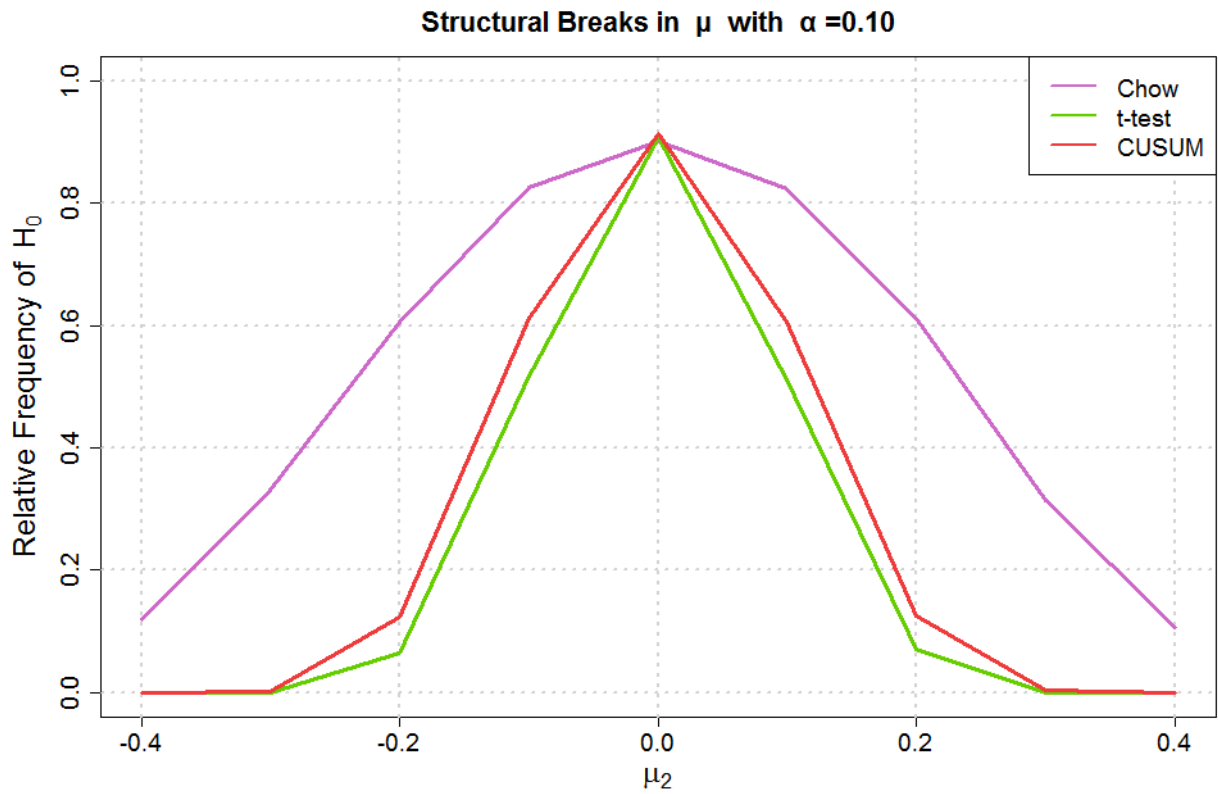
Figure A.7.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i	$\alpha = 0.05$			$\alpha = 0.10$		
	Chow test	t-test	CUSUM	Chow test	t-test	CUSUM
$\omega_2 = 0.1$	0.94	0.96	0.94	0.90	0.91	0.88
$\omega_2 = 0.2$	0.94	0.96	0.95	0.90	0.92	0.91
$\omega_2 = 0.3$	0.95	0.96	0.96	0.90	0.91	0.91
$\omega_2 = 0.4$	0.95	0.95	0.95	0.89	0.90	0.91
$\omega_2 = 0.5$	0.95	0.95	0.96	0.90	0.90	0.90
$\omega_2 = 0.6$	0.95	0.96	0.96	0.90	0.91	0.91
$\omega_2 = 0.7$	0.95	0.95	0.95	0.90	0.90	0.89
$\omega_2 = 0.8$	0.95	0.95	0.95	0.90	0.90	0.90

Table A.13.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

Figure A.8.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

| r_i^2	$\alpha = 0.05$					$\alpha = 0.10$				
	Chow test	t-test	CUSUM	K&L	I&T	Chow test	t-test	CUSUM	K&L	I&T
$\omega_2 = 0.1$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\omega_2 = 0.2$	0.13	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00
$\omega_2 = 0.3$	0.68	0.17	0.20	0.36	0.18	0.56	0.11	0.14	0.25	0.12
$\omega_2 = 0.4$	0.88	0.89	0.84	0.95	0.83	0.79	0.81	0.76	0.90	0.73
$\omega_2 = 0.5$	0.76	0.37	0.40	0.59	0.37	0.66	0.28	0.30	0.48	0.28
$\omega_2 = 0.6$	0.50	0.03	0.04	0.11	0.04	0.40	0.01	0.02	0.05	0.02
$\omega_2 = 0.7$	0.27	0.00	0.00	0.01	0.00	0.19	0.00	0.00	0.00	0.00
$\omega_2 = 0.8$	0.13	0.00	0.00	0.00	0.00	0.08	0.00	0.00	0.00	0.00

Table A.14.: *Relative frequencies of H_0 by using the introduced tests with r_i^2: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

$|r_i|$	$\alpha = 0.05$				$\alpha = 0.10$			
	Chow test	t-test	CUSUM	K&L	Chow test	t-test	CUSUM	K&L
$\omega_2 = 0.1$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\omega_2 = 0.2$	0.14	0.00	0.00	0.00	0.09	0.00	0.00	0.00
$\omega_2 = 0.3$	0.70	0.19	0.22	0.38	0.59	0.13	0.15	0.26
$\omega_2 = 0.4$	0.88	0.90	0.86	0.95	0.81	0.82	0.77	0.90
$\omega_2 = 0.5$	0.78	0.40	0.43	0.61	0.69	0.32	0.34	0.49
$\omega_2 = 0.6$	0.53	0.04	0.05	0.11	0.41	0.02	0.03	0.06
$\omega_2 = 0.7$	0.28	0.00	0.00	0.01	0.20	0.00	0.00	0.00
$\omega_2 = 0.8$	0.13	0.00	0.00	0.00	0.08	0.00	0.00	0.00

Table A.15.: *Relative frequencies of H_0 by using the introduced tests with $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

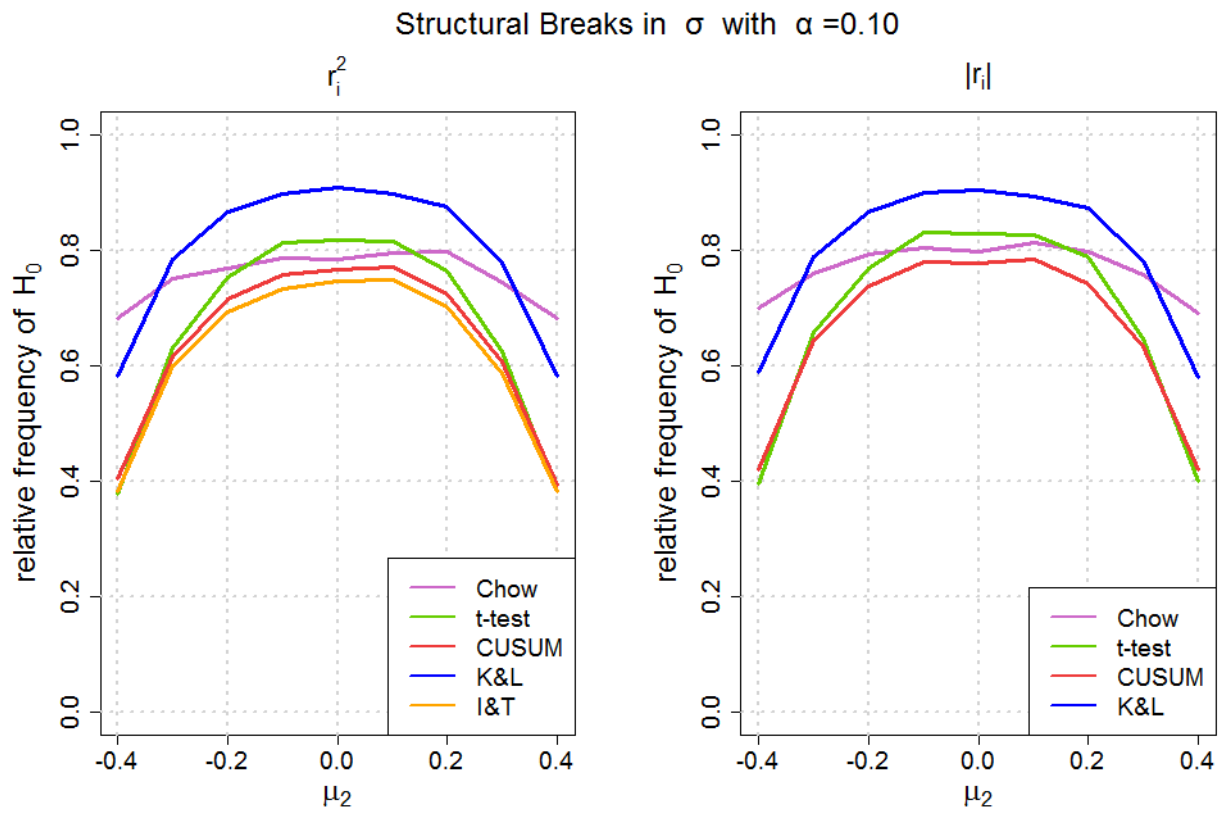**Figure A.9.:** *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i	$\alpha = 0.05$			$\alpha = 0.10$		
	Chow test	t-test	CUSUM	Chow test	t-test	CUSUM
$\alpha_2 = 0$	0.95	0.96	0.95	0.91	0.91	0.91
$\alpha_2 = 0.05$	0.94	0.95	0.96	0.89	0.90	0.91
$\alpha_2 = 0.1$	0.95	0.95	0.95	0.90	0.89	0.90
$\alpha_2 = 0.15$	0.95	0.94	0.95	0.90	0.89	0.90
$\alpha_2 = 0.2$	0.95	0.95	0.95	0.90	0.90	0.90
$\alpha_2 = 0.25$	0.95	0.95	0.96	0.91	0.91	0.91
$\alpha_2 = 0.3$	0.95	0.95	0.96	0.91	0.90	0.90
$\alpha_2 = 0.35$	0.95	0.95	0.95	0.90	0.90	0.90
$\alpha_2 = 0.4$	0.94	0.95	0.92	0.90	0.89	0.87

Table A.16.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

Figure A.10.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

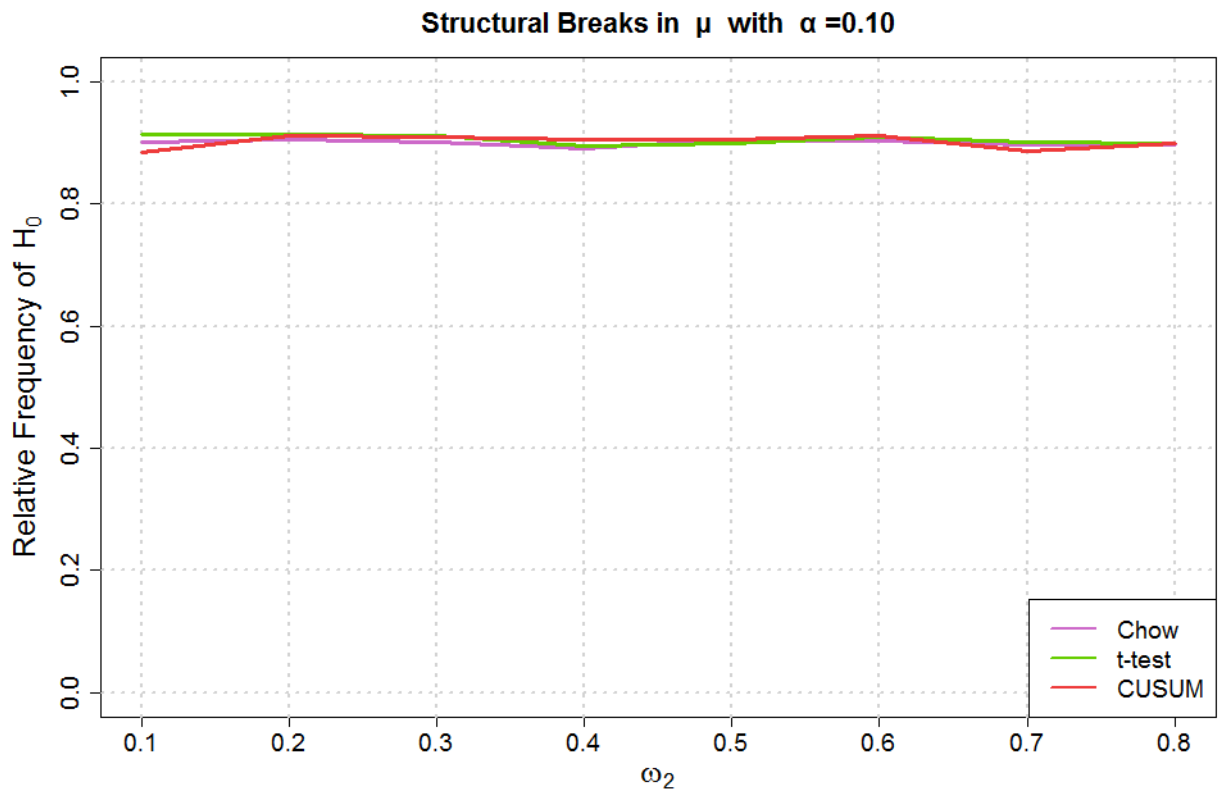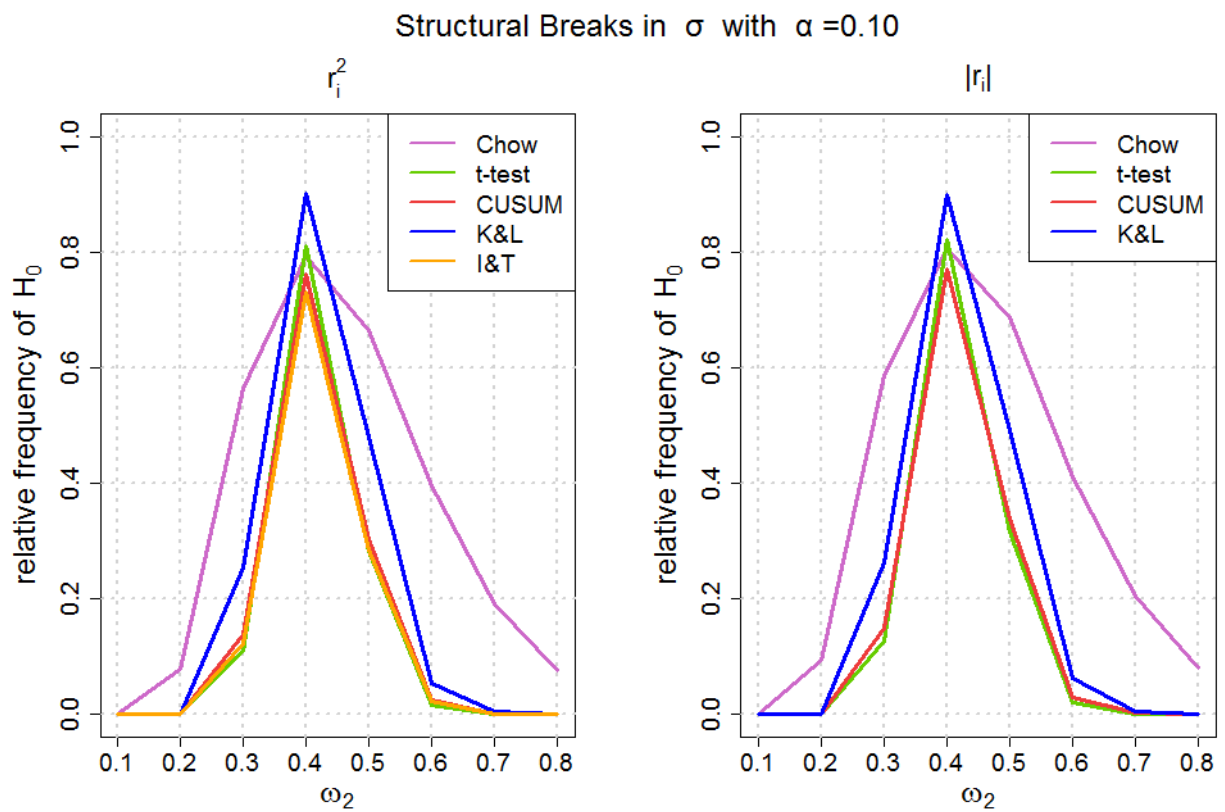| r_i^2	$\alpha = 0.05$					$\alpha = 0.10$				
	Chow test	t-test	CUSUM	K&L	I&T	Chow test	t-test	CUSUM	K&L	I&T
$\alpha_2 = 0$	0.79	0.34	0.41	0.56	0.40	0.69	0.25	0.31	0.42	0.30
$\alpha_2 = 0.05$	0.86	0.72	0.72	0.86	0.71	0.78	0.63	0.63	0.75	0.60
$\alpha_2 = 0.1$	0.86	0.89	0.86	0.96	0.83	0.79	0.82	0.76	0.91	0.73
$\alpha_2 = 0.15$	0.78	0.66	0.63	0.84	0.58	0.70	0.56	0.52	0.75	0.48
$\alpha_2 = 0.2$	0.64	0.26	0.25	0.56	0.21	0.53	0.19	0.18	0.42	0.15
$\alpha_2 = 0.25$	0.47	0.04	0.05	0.25	0.04	0.36	0.03	0.03	0.14	0.03
$\alpha_2 = 0.3$	0.27	0.00	0.00	0.09	0.00	0.19	0.00	0.00	0.04	0.00
$\alpha_2 = 0.35$	0.18	0.00	0.00	0.08	0.00	0.10	0.00	0.00	0.04	0.00
$\alpha_2 = 0.4$	0.12	0.00	0.00	0.10	0.00	0.07	0.00	0.00	0.06	0.00

Table A.17.: *Relative frequencies of H_0 by using the introduced tests with r_i^2: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

$|r_i|$	$\alpha = 0.05$				$\alpha = 0.10$			
	Chow test	t-test	CUSUM	K&L	Chow test	t-test	CUSUM	K&L
$\alpha_2 = 0$	0.81	0.41	0.47	0.60	0.73	0.30	0.36	0.46
$\alpha_2 = 0.05$	0.89	0.76	0.76	0.86	0.80	0.66	0.66	0.78
$\alpha_2 = 0.1$	0.88	0.90	0.87	0.96	0.80	0.84	0.79	0.91
$\alpha_2 = 0.15$	0.80	0.70	0.68	0.85	0.71	0.61	0.56	0.76
$\alpha_2 = 0.2$	0.66	0.32	0.31	0.57	0.56	0.25	0.23	0.44
$\alpha_2 = 0.25$	0.49	0.06	0.07	0.24	0.39	0.04	0.04	0.15
$\alpha_2 = 0.3$	0.28	0.00	0.01	0.06	0.19	0.00	0.00	0.03
$\alpha_2 = 0.35$	0.14	0.00	0.00	0.01	0.09	0.00	0.00	0.00
$\alpha_2 = 0.4$	0.05	0.00	0.00	0.00	0.03	0.00	0.00	0.00

Table A.18.: *Relative frequencies of H_0 by using the introduced tests with $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

Figure A.11.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i	$\alpha = 0.05$			$\alpha = 0.10$		
	Chow test	t-test	CUSUM	Chow test	t-test	CUSUM
$\alpha_2 = 0.1$	0.95	0.96	0.96	0.90	0.90	0.91
$\alpha_2 = 0.11$	0.95	0.95	0.95	0.89	0.90	0.91
$\alpha_2 = 0.12$	0.95	0.94	0.96	0.91	0.90	0.91
$\alpha_2 = 0.13$	0.95	0.95	0.95	0.91	0.91	0.91
$\alpha_2 = 0.14$	0.95	0.95	0.95	0.91	0.90	0.90
$\alpha_2 = 0.15$	0.95	0.94	0.95	0.90	0.90	0.90
$\alpha_2 = 0.16$	0.95	0.95	0.94	0.90	0.89	0.89
$\alpha_2 = 0.17$	0.95	0.95	0.93	0.90	0.90	0.88
$\alpha_2 = 0.18$	0.96	0.94	0.92	0.92	0.89	0.85

Table A.19.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.8$, $\beta_2 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*
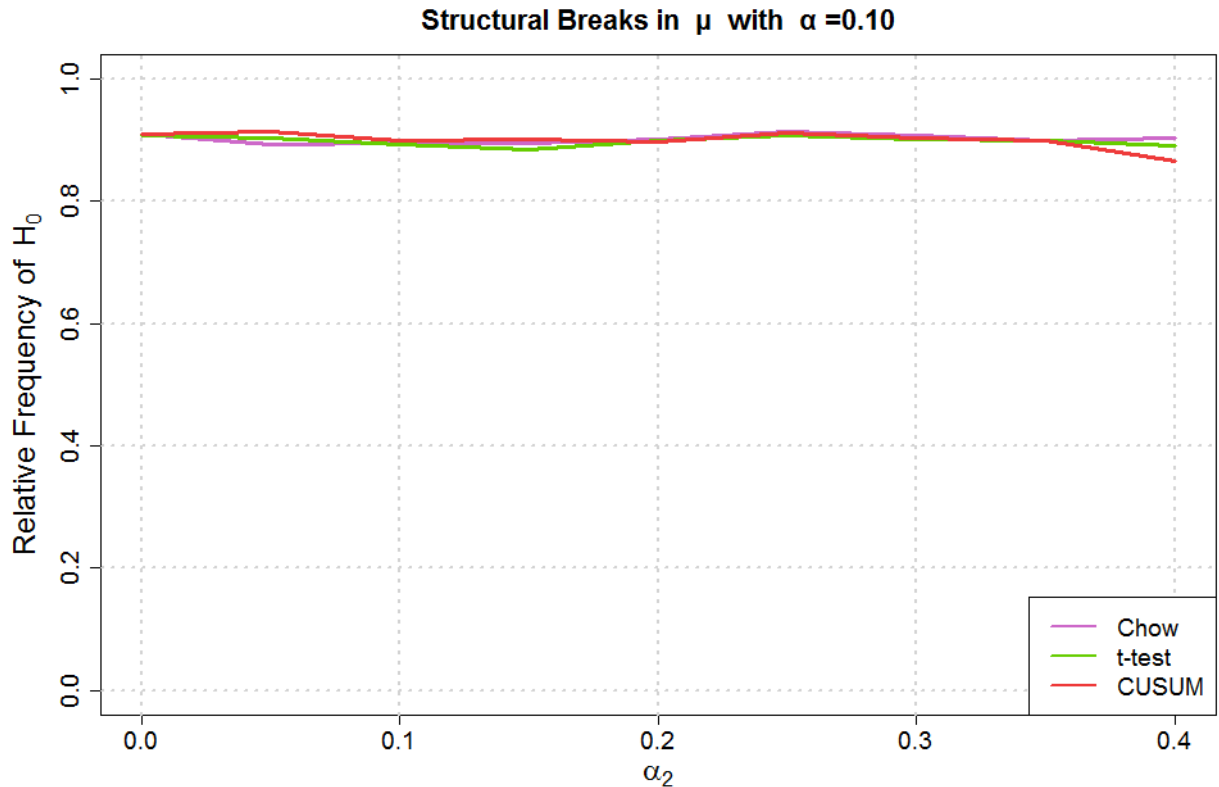

Figure A.12.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

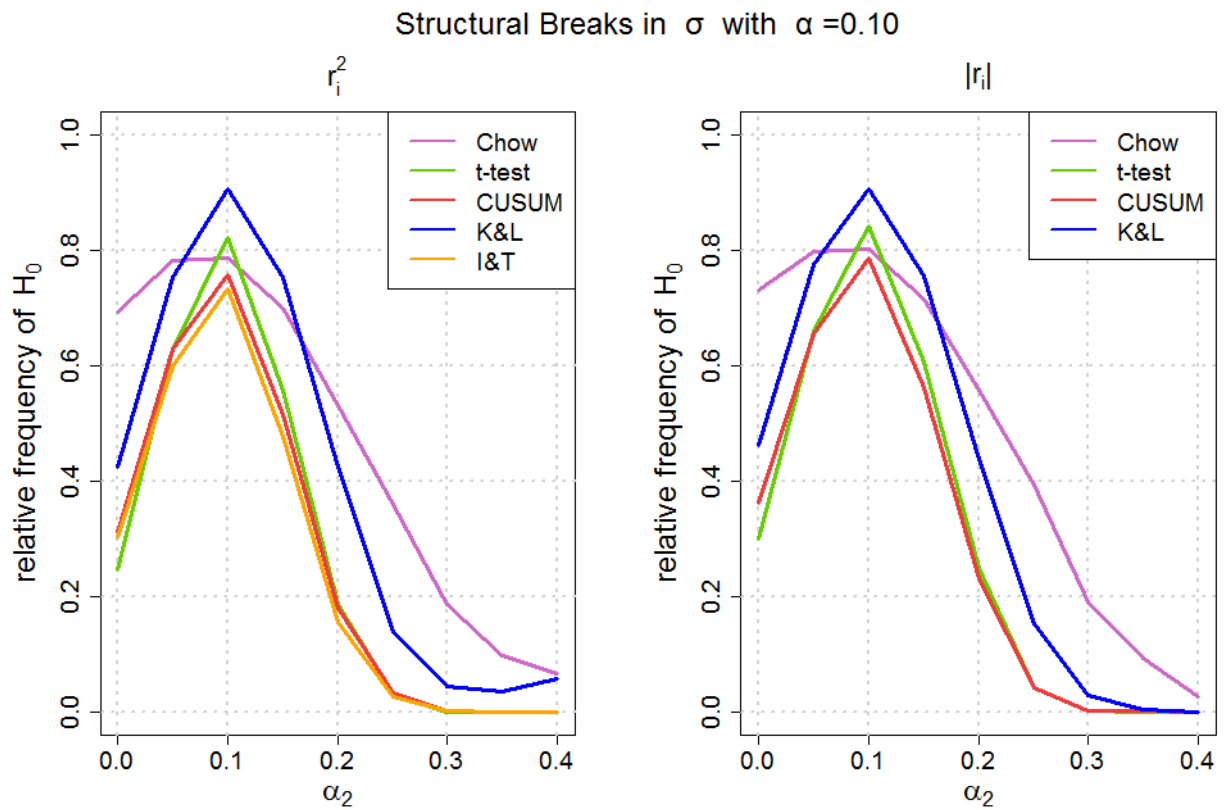| r_i^2	$\alpha = 0.05$					$\alpha = 0.10$				
	Chow test	t-test	CUSUM	K&L	I&T	Chow test	t-test	CUSUM	K&L	I&T
$\alpha_2 = 0.1$	0.60	0.70	0.47	0.83	0.40	0.50	0.61	0.34	0.74	0.29
$\alpha_2 = 0.11$	0.56	0.59	0.37	0.75	0.30	0.48	0.52	0.26	0.64	0.21
$\alpha_2 = 0.12$	0.50	0.45	0.28	0.66	0.22	0.41	0.38	0.19	0.55	0.16
$\alpha_2 = 0.13$	0.42	0.25	0.14	0.43	0.11	0.33	0.21	0.10	0.34	0.07
$\alpha_2 = 0.14$	0.33	0.12	0.06	0.26	0.04	0.26	0.09	0.04	0.18	0.03
$\alpha_2 = 0.15$	0.25	0.05	0.03	0.15	0.02	0.20	0.04	0.02	0.09	0.01
$\alpha_2 = 0.16$	0.16	0.01	0.00	0.06	0.00	0.11	0.01	0.00	0.03	0.00
$\alpha_2 = 0.17$	0.11	0.00	0.00	0.03	0.00	0.07	0.00	0.00	0.01	0.00
$\alpha_2 = 0.18$	0.05	0.00	0.00	0.01	0.00	0.03	0.00	0.00	0.01	0.00

Table A.20.: *Relative frequencies of H_0 by using the introduced tests with r_i^2: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.8$, $\beta_2 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

$|r_i|$	$\alpha = 0.05$				$\alpha = 0.10$			
	Chow test	t-test	CUSUM	K&L	Chow test	t-test	CUSUM	K&L
$\alpha_2 = 0.1$	0.63	0.72	0.50	0.83	0.53	0.64	0.38	0.73
$\alpha_2 = 0.11$	0.58	0.62	0.40	0.75	0.49	0.55	0.29	0.63
$\alpha_2 = 0.12$	0.53	0.48	0.30	0.65	0.44	0.40	0.21	0.55
$\alpha_2 = 0.13$	0.44	0.29	0.16	0.44	0.36	0.24	0.12	0.33
$\alpha_2 = 0.14$	0.34	0.13	0.07	0.25	0.28	0.10	0.04	0.18
$\alpha_2 = 0.15$	0.27	0.06	0.04	0.13	0.21	0.04	0.02	0.09
$\alpha_2 = 0.16$	0.16	0.02	0.01	0.05	0.12	0.01	0.00	0.03
$\alpha_2 = 0.17$	0.11	0.00	0.00	0.02	0.07	0.00	0.00	0.01
$\alpha_2 = 0.18$	0.05	0.00	0.00	0.00	0.03	0.00	0.00	0.00

Table A.21.: *Relative frequencies of H_0 by using the introduced tests with $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.8$, $\beta_2 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*
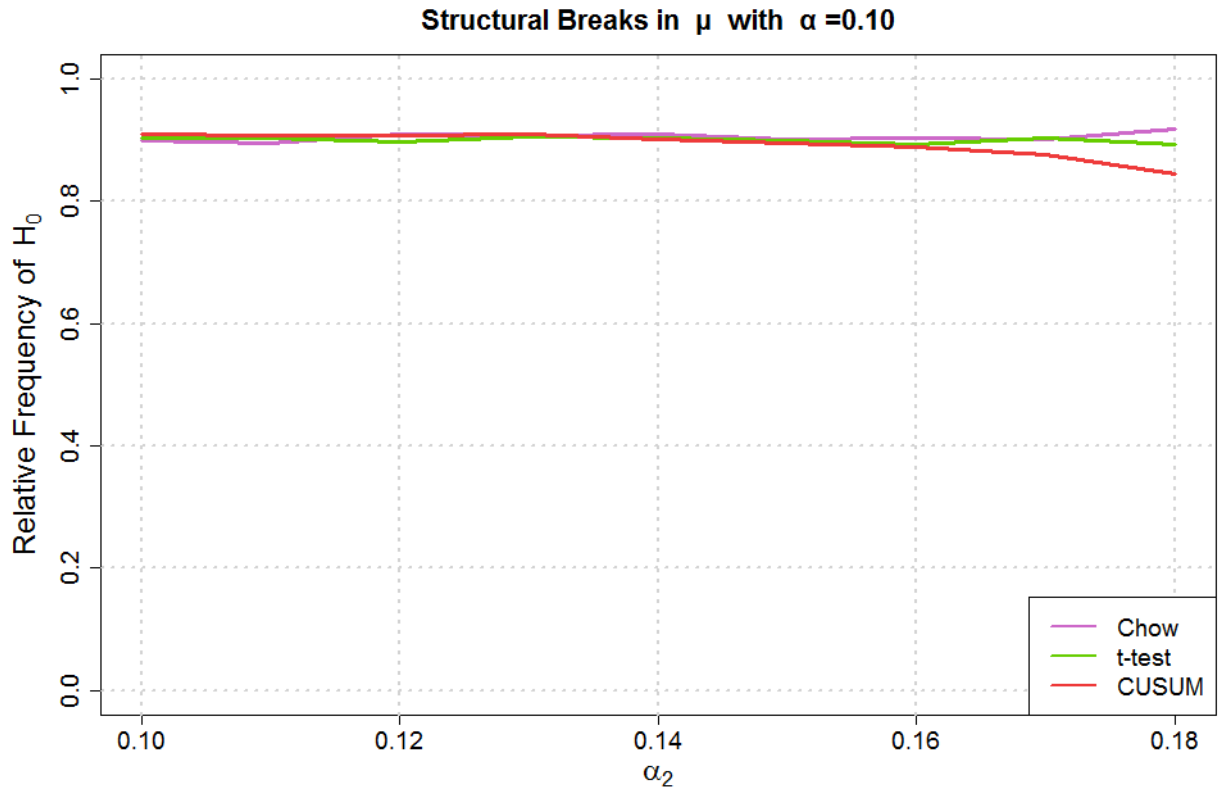
Figure A.13.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.8$, $\beta_2 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i	$\alpha = 0.05$			$\alpha = 0.10$		
	Chow test	t-test	CUSUM	Chow test	t-test	CUSUM
$\beta_2 = 0.1$	0.95	0.95	0.94	0.90	0.90	0.89
$\beta_2 = 0.2$	0.95	0.94	0.95	0.89	0.89	0.89
$\beta_2 = 0.3$	0.95	0.95	0.96	0.90	0.90	0.91
$\beta_2 = 0.4$	0.94	0.95	0.95	0.89	0.90	0.90
$\beta_2 = 0.5$	0.95	0.95	0.95	0.90	0.89	0.91
$\beta_2 = 0.6$	0.95	0.96	0.96	0.90	0.91	0.91
$\beta_2 = 0.7$	0.95	0.95	0.95	0.90	0.90	0.90
$\beta_2 = 0.8$	0.95	0.94	0.92	0.90	0.90	0.85

Table A.22.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

Figure A.14.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\beta_1 = 0.8$, $\beta_2 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

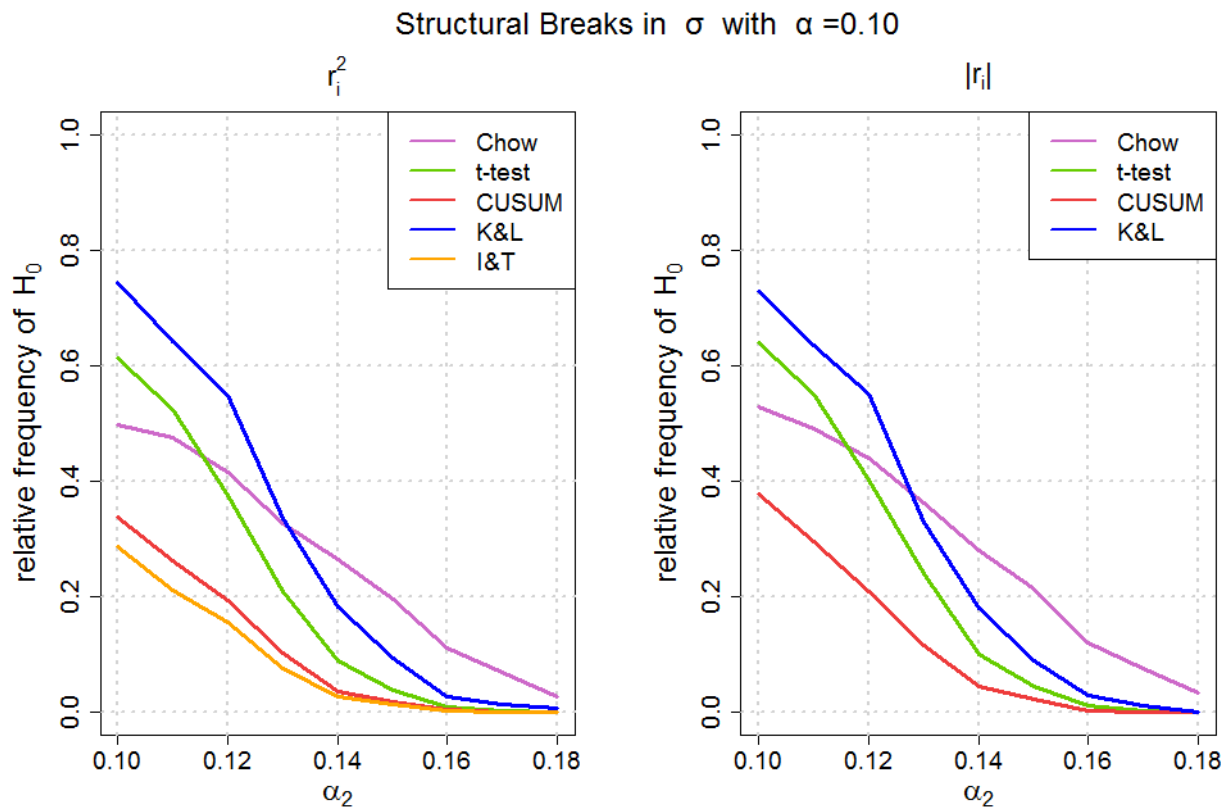| r_i^2	$\alpha = 0.05$					$\alpha = 0.10$				
	Chow test	t-test	CUSUM	K&L	I&T	Chow test	t-test	CUSUM	K&L	I&T
$\beta_2 = 0.1$	0.12	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00	0.00
$\beta_2 = 0.2$	0.26	0.00	0.00	0.00	0.00	0.17	0.00	0.00	0.00	0.00
$\beta_2 = 0.3$	0.47	0.02	0.04	0.09	0.03	0.36	0.01	0.02	0.05	0.02
$\beta_2 = 0.4$	0.76	0.36	0.39	0.59	0.36	0.65	0.27	0.30	0.46	0.28
$\beta_2 = 0.5$	0.87	0.89	0.85	0.94	0.83	0.79	0.82	0.77	0.90	0.75
$\beta_2 = 0.6$	0.66	0.18	0.21	0.38	0.19	0.56	0.12	0.14	0.28	0.13
$\beta_2 = 0.7$	0.16	0.00	0.00	0.00	0.00	0.10	0.00	0.00	0.00	0.00
$\beta_2 = 0.8$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table A.23.: *Relative frequencies of H_0 by using the introduced tests with r_i^2: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

$|r_i|$	$\alpha = 0.05$				$\alpha = 0.10$			
	Chow test	t-test	CUSUM	K&L	Chow test	t-test	CUSUM	K&L
$\beta_2 = 0.1$	0.13	0.00	0.00	0.00	0.08	0.00	0.00	0.00
$\beta_2 = 0.2$	0.28	0.00	0.00	0.00	0.19	0.00	0.00	0.00
$\beta_2 = 0.3$	0.50	0.03	0.05	0.11	0.39	0.02	0.03	0.06
$\beta_2 = 0.4$	0.77	0.40	0.44	0.61	0.68	0.30	0.34	0.49
$\beta_2 = 0.5$	0.88	0.90	0.87	0.95	0.81	0.84	0.79	0.89
$\beta_2 = 0.6$	0.68	0.21	0.24	0.40	0.57	0.14	0.17	0.28
$\beta_2 = 0.7$	0.17	0.00	0.00	0.00	0.12	0.00	0.00	0.00
$\beta_2 = 0.8$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table A.24.: *Relative frequencies of H_0 by using the introduced tests with $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*
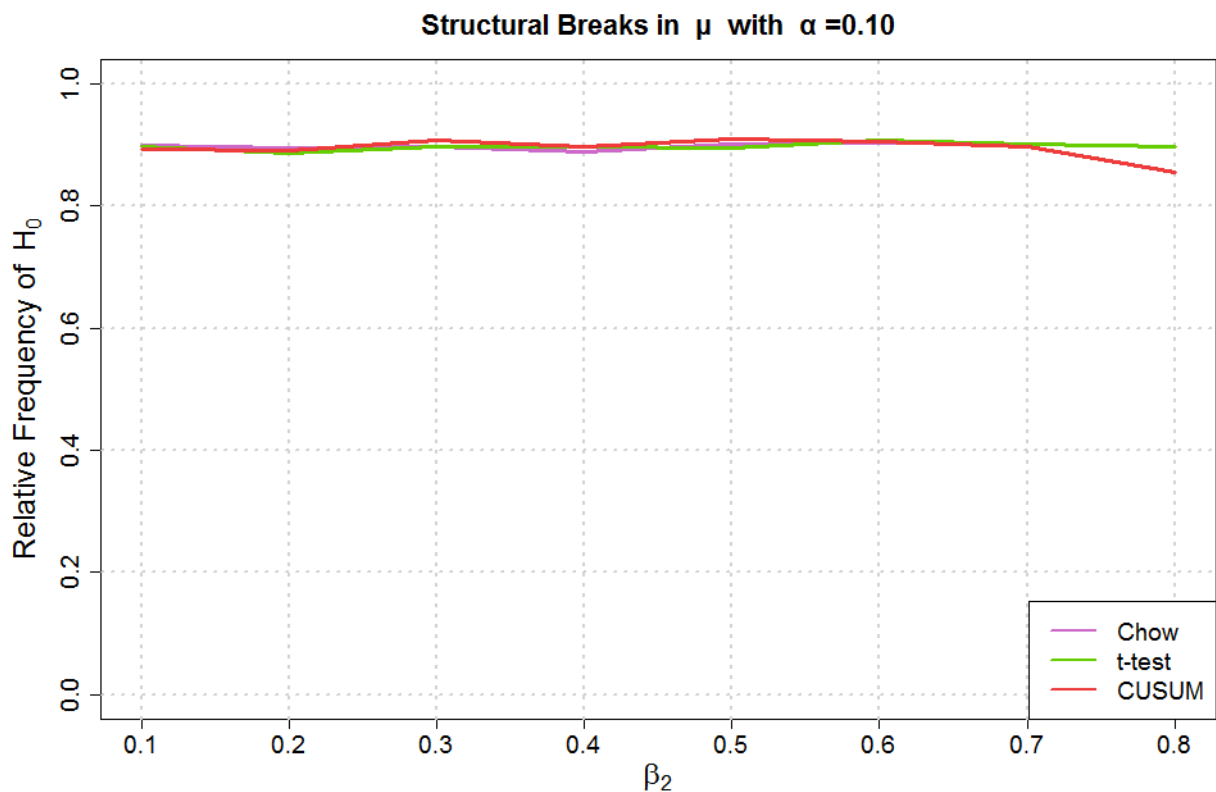

Figure A.15.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

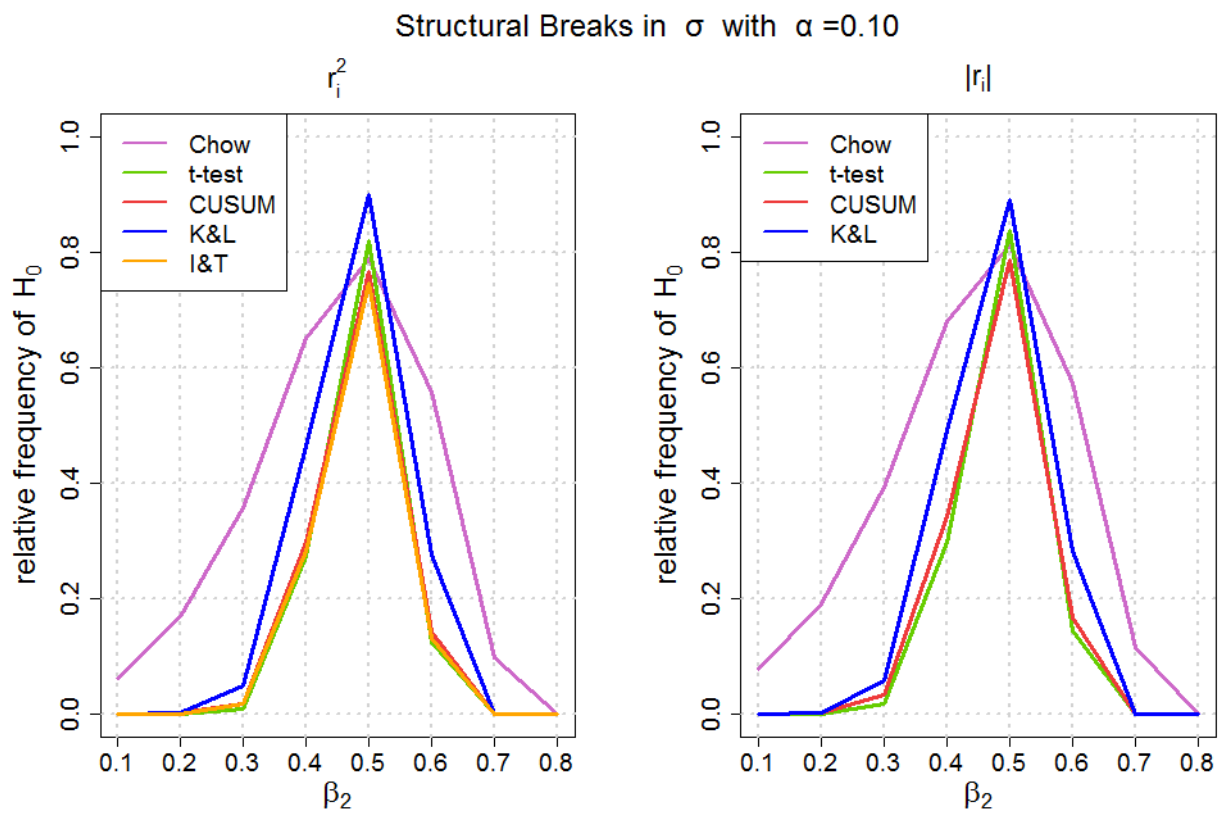| r_i	$\alpha = 0.05$			$\alpha = 0.10$		
	Chow test	t-test	CUSUM	Chow test	t-test	CUSUM
$\beta_2 = 0.8$	0.95	0.95	0.95	0.90	0.89	0.91
$\beta_2 = 0.81$	0.95	0.94	0.95	0.90	0.89	0.91
$\beta_2 = 0.82$	0.94	0.96	0.95	0.90	0.91	0.91
$\beta_2 = 0.83$	0.96	0.95	0.96	0.91	0.90	0.91
$\beta_2 = 0.84$	0.94	0.95	0.95	0.90	0.90	0.90
$\beta_2 = 0.85$	0.95	0.94	0.95	0.90	0.90	0.89
$\beta_2 = 0.86$	0.95	0.95	0.94	0.89	0.91	0.89
$\beta_2 = 0.87$	0.94	0.95	0.93	0.90	0.90	0.88
$\beta_2 = 0.88$	0.95	0.95	0.93	0.92	0.90	0.87

Table A.25.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*
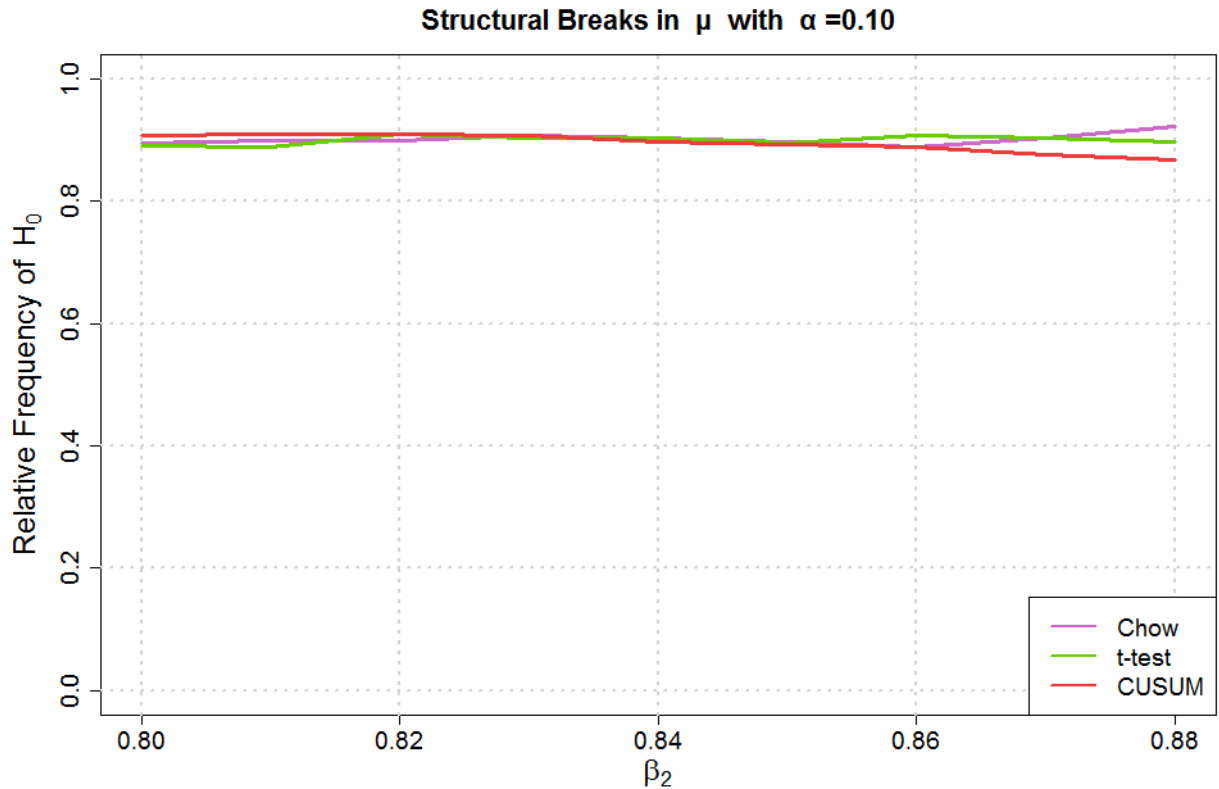

Figure A.16.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i^2	$\alpha = 0.05$					$\alpha = 0.10$				
	Chow test	t-test	CUSUM	K&L	I&T	Chow test	t-test	CUSUM	K&L	I&T
$\beta_2 = 0.8$	0.57	0.71	0.46	0.82	0.40	0.48	0.62	0.35	0.72	0.29
$\beta_2 = 0.81$	0.56	0.62	0.39	0.78	0.34	0.46	0.53	0.29	0.67	0.24
$\beta_2 = 0.82$	0.50	0.42	0.27	0.60	0.22	0.42	0.35	0.20	0.49	0.16
$\beta_2 = 0.83$	0.44	0.20	0.13	0.37	0.10	0.36	0.15	0.09	0.28	0.06
$\beta_2 = 0.84$	0.32	0.07	0.05	0.18	0.04	0.25	0.05	0.03	0.12	0.02
$\beta_2 = 0.85$	0.22	0.02	0.01	0.05	0.01	0.16	0.01	0.01	0.03	0.00
$\beta_2 = 0.86$	0.12	0.00	0.00	0.01	0.00	0.08	0.00	0.00	0.01	0.00
$\beta_2 = 0.87$	0.05	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00
$\beta_2 = 0.88$	0.01	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00

Table A.26.: *Relative frequencies of H_0 by using the introduced tests with r_i^2: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

$|r_i|$	$\alpha = 0.05$				$\alpha = 0.10$			
	Chow test	t-test	CUSUM	K&L	Chow test	t-test	CUSUM	K&L
$\beta_2 = 0.8$	0.62	0.73	0.49	0.82	0.51	0.65	0.37	0.72
$\beta_2 = 0.81$	0.60	0.64	0.43	0.78	0.50	0.56	0.32	0.66
$\beta_2 = 0.82$	0.52	0.43	0.28	0.60	0.43	0.36	0.21	0.48
$\beta_2 = 0.83$	0.46	0.21	0.15	0.36	0.38	0.16	0.10	0.26
$\beta_2 = 0.84$	0.33	0.08	0.05	0.17	0.26	0.06	0.03	0.12
$\beta_2 = 0.85$	0.23	0.01	0.01	0.04	0.17	0.01	0.01	0.03
$\beta_2 = 0.86$	0.13	0.00	0.00	0.01	0.10	0.00	0.00	0.00
$\beta_2 = 0.87$	0.05	0.00	0.00	0.00	0.04	0.00	0.00	0.00
$\beta_2 = 0.88$	0.01	0.00	0.00	0.00	0.01	0.00	0.00	0.00

Table A.27.: *Relative frequencies of H_0 by using the introduced tests with $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

Figure A.17.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

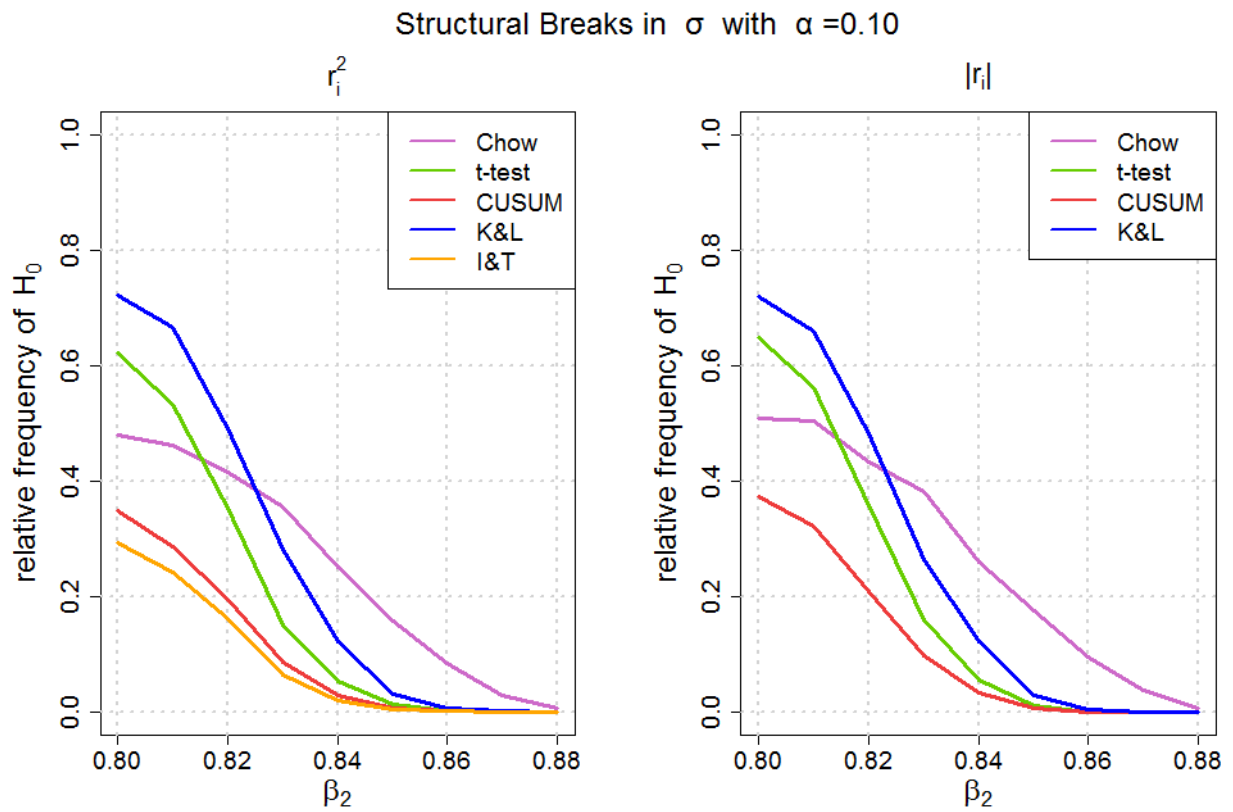| r_i	$\alpha = 0.05$			$\alpha = 0.10$		
	Chow test	t-test	CUSUM	Chow test	t-test	CUSUM
$\varepsilon_2 = 1$	0.95	0.94	0.95	0.90	0.90	0.91
$\varepsilon_2 = 1.025$	0.94	0.95	0.96	0.89	0.90	0.91
$\varepsilon_2 = 1.05$	0.96	0.95	0.95	0.91	0.90	0.91
$\varepsilon_2 = 1.075$	0.95	0.94	0.95	0.90	0.90	0.90
$\varepsilon_2 = 1.1$	0.95	0.96	0.96	0.89	0.90	0.91
$\varepsilon_2 = 1.125$	0.94	0.95	0.96	0.89	0.90	0.91
$\varepsilon_2 = 1.15$	0.96	0.96	0.96	0.91	0.90	0.90
$\varepsilon_2 = 1.175$	0.95	0.95	0.96	0.90	0.89	0.91
$\varepsilon_2 = 1.2$	0.96	0.96	0.96	0.91	0.91	0.91

Table A.28.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = 1$.*
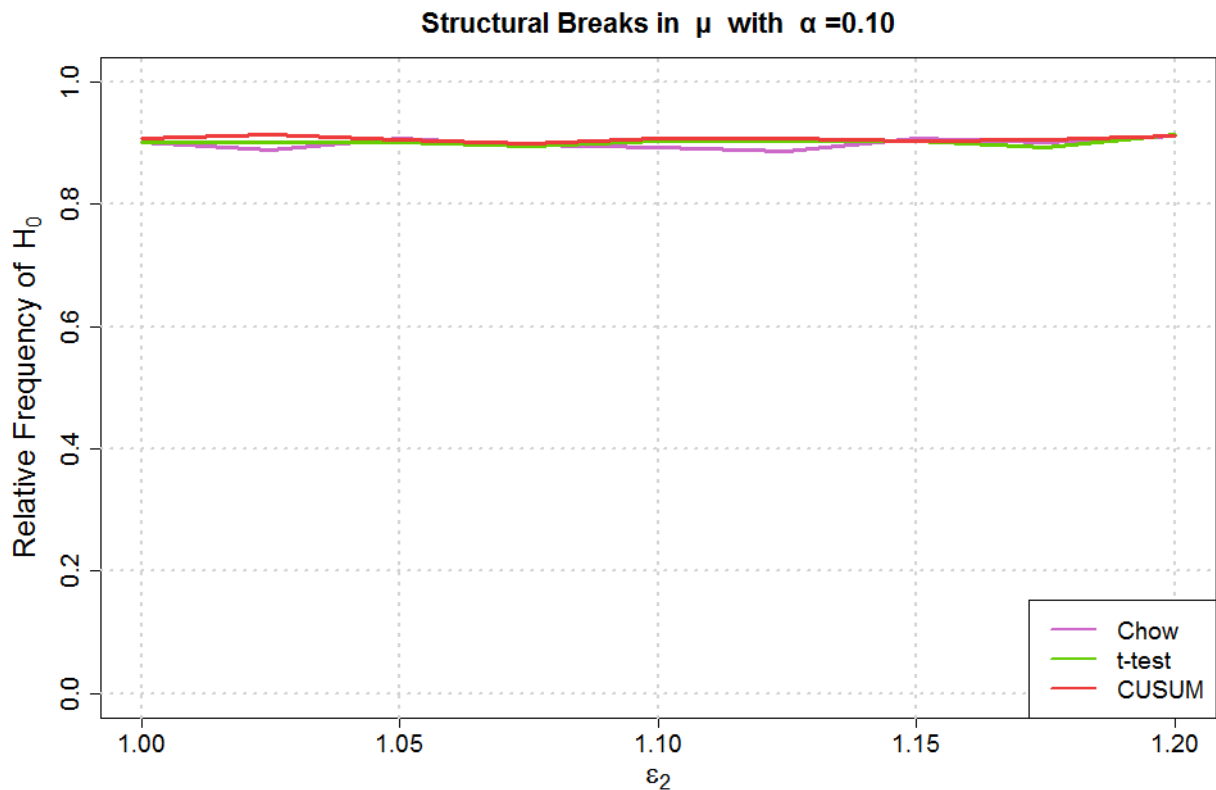

Figure A.18.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.8$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

r_i^2	$\alpha = 0.05$					$\alpha = 0.10$				
	Chow test	t-test	CUSUM	K&L	I&T	Chow test	t-test	CUSUM	K&L	I&T
$\varepsilon_2 = 1$	0.85	0.87	0.84	0.95	0.82	0.78	0.80	0.75	0.89	0.72
$\varepsilon_2 = 1.025$	0.86	0.83	0.79	0.93	0.77	0.78	0.75	0.70	0.85	0.68
$\varepsilon_2 = 1.05$	0.83	0.69	0.68	0.86	0.65	0.73	0.59	0.58	0.76	0.55
$\varepsilon_2 = 1.075$	0.77	0.49	0.50	0.71	0.47	0.67	0.39	0.40	0.58	0.37
$\varepsilon_2 = 1.1$	0.71	0.29	0.33	0.55	0.30	0.61	0.21	0.25	0.42	0.23
$\varepsilon_2 = 1.125$	0.62	0.15	0.17	0.36	0.16	0.52	0.10	0.12	0.24	0.11
$\varepsilon_2 = 1.15$	0.54	0.05	0.07	0.20	0.06	0.43	0.04	0.05	0.12	0.04
$\varepsilon_2 = 1.175$	0.47	0.02	0.03	0.10	0.03	0.36	0.01	0.02	0.06	0.01
$\varepsilon_2 = 1.2$	0.40	0.00	0.01	0.04	0.01	0.29	0.00	0.00	0.02	0.00

Table A.29.: *Relative frequencies of H_0 by using the introduced tests with r_i^2: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = 1$.*

$|r_i|$	$\alpha = 0.05$				$\alpha = 0.10$			
	Chow test	t-test	CUSUM	K&L	Chow test	t-test	CUSUM	K&L
$\varepsilon_2 = 1$	0.88	0.89	0.86	0.94	0.80	0.82	0.77	0.89
$\varepsilon_2 = 1.025$	0.87	0.84	0.80	0.91	0.80	0.76	0.72	0.85
$\varepsilon_2 = 1.05$	0.83	0.72	0.70	0.85	0.74	0.62	0.59	0.76
$\varepsilon_2 = 1.075$	0.79	0.53	0.54	0.71	0.70	0.44	0.43	0.60
$\varepsilon_2 = 1.1$	0.74	0.33	0.37	0.55	0.64	0.24	0.28	0.43
$\varepsilon_2 = 1.125$	0.65	0.18	0.21	0.37	0.55	0.12	0.15	0.27
$\varepsilon_2 = 1.15$	0.56	0.08	0.10	0.23	0.46	0.04	0.06	0.14
$\varepsilon_2 = 1.175$	0.49	0.03	0.04	0.12	0.38	0.01	0.02	0.07
$\varepsilon_2 = 1.2$	0.41	0.01	0.01	0.05	0.31	0.00	0.01	0.02

Table A.30.: *Relative frequencies of H_0 by using the introduced tests with $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = 1$.*
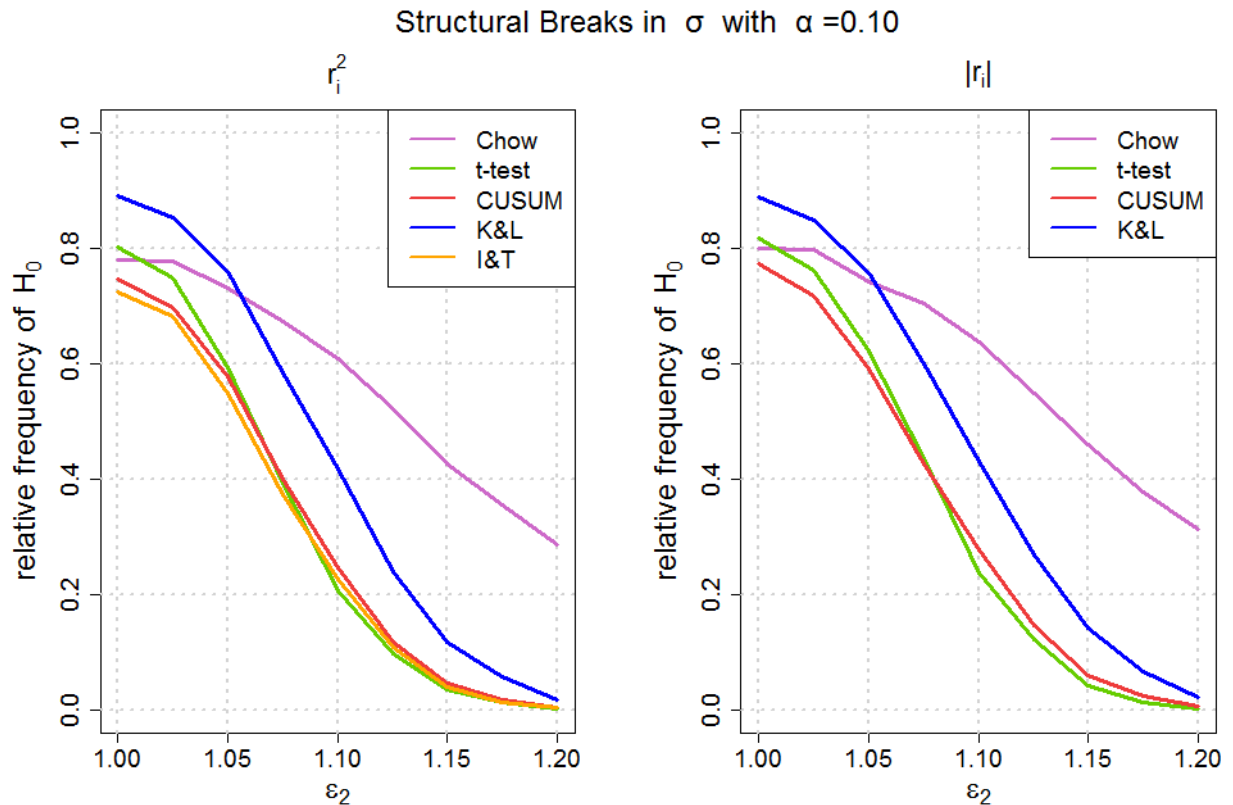

Figure A.19.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = 1$.*

r_i	$\alpha = 0.05$			$\alpha = 0.10$		
	Chow test	t-test	CUSUM	Chow test	t-test	CUSUM
$n_1 = 0.1$	0.99	0.90	0.96	0.97	0.83	0.91
$n_1 = 0.2$	0.97	0.88	0.94	0.94	0.81	0.90
$n_1 = 0.3$	0.96	0.86	0.92	0.92	0.78	0.86
$n_1 = 0.4$	0.95	0.86	0.90	0.90	0.78	0.83
$n_1 = 0.5$	0.93	0.87	0.89	0.88	0.79	0.82

Table A.31.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n = 500$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

| r_i^2	$\alpha = 0.05$					$\alpha = 0.10$				
	Chow test	t-test	CUSUM	K&L	I&T	Chow test	t-test	CUSUM	K&L	I&T
$n_1 = 0.1$	0.59	0.03	0.43	0.88	0.28	0.46	0.02	0.29	0.78	0.18
$n_1 = 0.2$	0.44	0.00	0.07	0.62	0.03	0.35	0.00	0.02	0.41	0.01
$n_1 = 0.3$	0.44	0.00	0.01	0.20	0.00	0.36	0.00	0.00	0.09	0.00
$n_1 = 0.4$	0.41	0.00	0.00	0.07	0.00	0.31	0.00	0.00	0.03	0.00
$n_1 = 0.5$	0.29	0.00	0.00	0.03	0.00	0.18	0.00	0.00	0.01	0.00

Table A.32.: *Relative frequencies of H_0 by using the introduced tests with r_i^2: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n = 500$ and $\varepsilon_1 = \varepsilon_2 = 1$.*
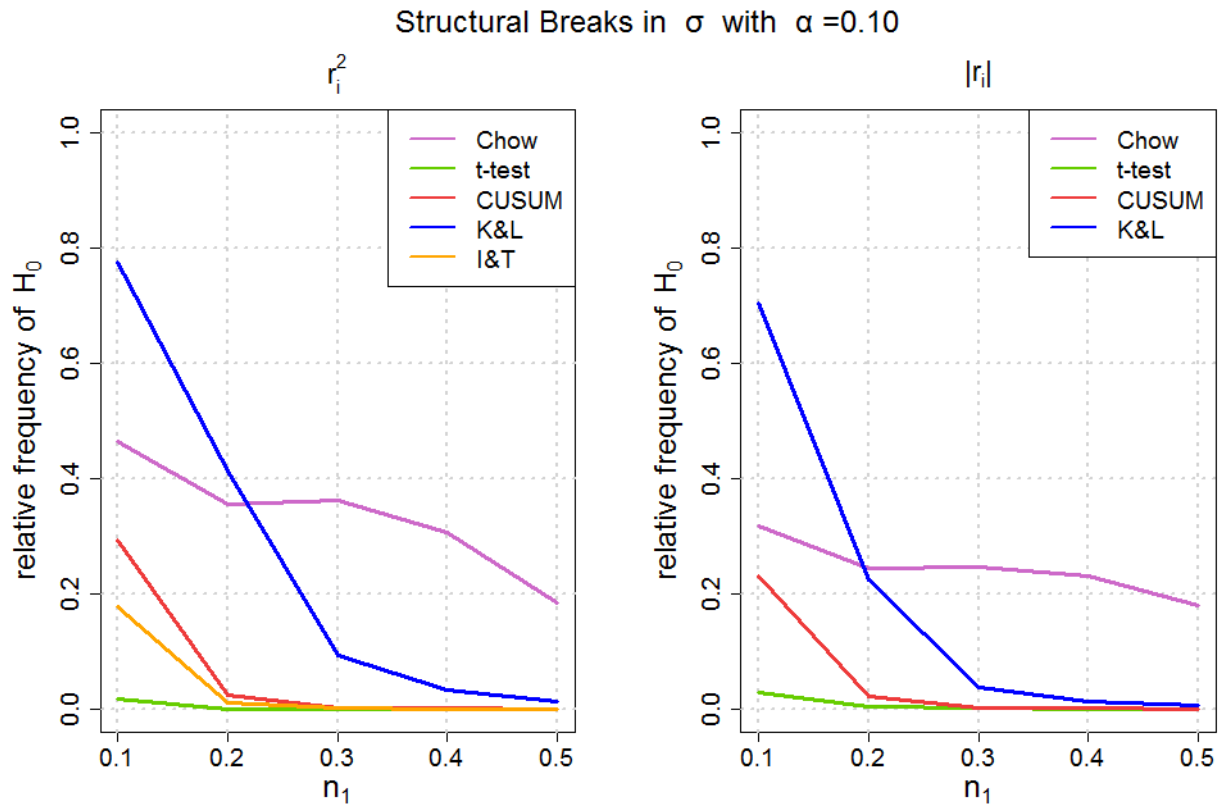

Figure A.20.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0$, $\omega_1 = 0.4$, $\omega_2 = 0.4$, $\alpha_1 = 0.1$, $\alpha_2 = 0.1$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, $n = 1000$, $n_1 = 0.5$ and $\varepsilon_1 = 1$.*

$\lvert r_i \rvert$	$\alpha = 0.05$				$\alpha = 0.10$			
	Chow test	t-test	CUSUM	K&L	Chow test	t-test	CUSUM	K&L
$n_1 = 0.1$	0.41	0.04	0.35	0.83	0.32	0.03	0.23	0.70
$n_1 = 0.2$	0.33	0.00	0.04	0.40	0.24	0.00	0.02	0.22
$n_1 = 0.3$	0.33	0.00	0.01	0.09	0.25	0.00	0.00	0.04
$n_1 = 0.4$	0.32	0.00	0.00	0.03	0.23	0.00	0.00	0.01
$n_1 = 0.5$	0.26	0.00	0.00	0.02	0.18	0.00	0.00	0.01

Table A.33.: *Relative frequencies of H_0 by using the introduced tests with $\lvert r_i \rvert$: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n = 500$ and $\varepsilon_1 = \varepsilon_2 = 1$.*
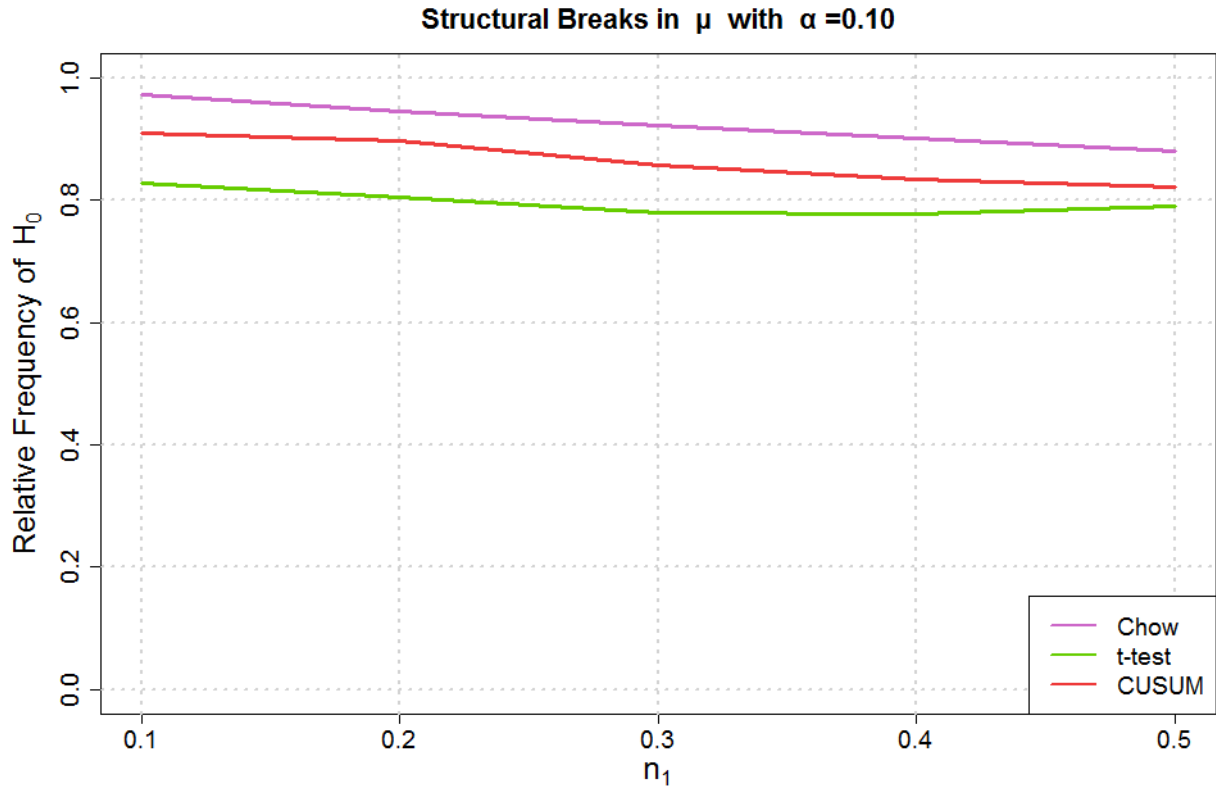
Figure A.21.: *Relative frequencies of H_0 by using the introduced tests with r_i: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n = 500$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

Figure A.22.: *Relative frequencies of H_0 by using the introduced tests with r_i^2 and $|r_i|$: $\mu_1 = 0$, $\mu_2 = 0.1$, $\omega_1 = 0.4$, $\omega_2 = 0.5$, $\alpha_1 = 0.1$, $\alpha_2 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.6$, $n = 500$ and $\varepsilon_1 = \varepsilon_2 = 1$.*

A.2. Contents of the CD

- The folder 'Grafiken' contains all figures of the master thesis.

- The file 'StructuralBreaks_R' contains the R code for all simulations.

- The folder 'Simulationen' contains all simulation results.

- The file 'StructuralBreaks' contains this thesis as pdf file.

A.3. Eidesstattliche Versicherung

Erklärung

Hiermit versichere ich, dass ich meine Masterarbeit selbständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

München, den 07.12.2016: ....

(Unterschrift)