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ESTIMATION OF NEUTRAL-BEAM-INDUCED FIELD REVERSAL IN MFTF BY AN APPROXIMATE SCALING LAW

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ABSTRACT

Scaling rules are derived for field-reversed plasmas whose dimensions are common multiples of the ion gyroradius in the vacuum field. These rules are then applied to the tandem MFTF configuration, and it is shown that field reversal appears to be possible for neutral beam currents of the order of 150 amperes, provided that the electron temperature is at least 500 eV.

1. Introduction

This report was originally intended as a rough draft for a more comprehensive document on all conceivable field-reversal experiments in the tandem MFTF. That larger document, however, is not ready. In this report I discuss some scaling concepts that I think are particularly useful for planning field-reversal experiments, and I apply them to neutral beam experiments in the tandem MFTF. As described in the summary, the results are encouraging, even after some discounting of the inherent optimism of the scaling model.

2. Some General Scaling Rules for Field-Reversed Plasmas

The field-reversed mirror plasma is assumed to be stabilized by the effects of the finite gyroradius of the plasma ions. Thus, for scaling purposes, it is useful to consider plasmas whose dimensions are low multiples of an average ion gyroradius a_i:

$$a_i = \frac{1}{B_o}\left(\frac{2\,k\,T_i}{R_i}\right)^{1/2} \tag{1}$$

where B_o is the initial magnetic field, T_i is the ion temperature, and R_i is the classical radius of the hydrogen ion:

$$R_i = e^2/AM_Hc^2 \tag{2}$$

where A is the atomic weight and M_H is the proton mass.

Various analytical models of the distribution of density and field over the field-reversed mirror are possible, such as the "Hills' vortex" formulation.[1] However we wish to establish scaling relations that are more general than a particular model, and this is done by defining a basic unit of length sa_i, where s is the "size factor". We do assume axisymmetry based on the cylindrical coordinates r, Z.

Define the dimensionless scaling coordinates ρ and ζ in terms of the basic length scale sa_i:

$$\rho \equiv r/sa_i \qquad\qquad \zeta \equiv Z/sa_i \tag{3}$$

and then consider the plasma volume V

$$V \equiv \int_0^{R(Z)} \int_{-L_o}^{L_o} 2\pi r\,dr\,dZ \tag{4}$$

where R(Z) is the radius of the separatrix between open and closed field lines, and where L_o is the half-length of the plasma. Assuming symmetry about the midplane, and substituting from equation (1) and (3), one finds:

$$V = S_v\, s^3\, T_i^{\;3/2}/B_o^3 \tag{5}$$

where S_v is the "volume shape factor".

$$S_v = 4\pi \left(\frac{2k}{R_i}\right)^{3/2} \int_0^{\rho(\zeta)} \int_0^{l_o} \rho\,d\rho\,d\zeta \tag{6}$$

where $\rho(\zeta)$ is the separatrix radius in the dimensionless coordinates and l_o is the plasma half-length in those coordinates. Thus, the volume scales with size s, temperature T_i and field B_o, as shown in equation (5), independent of the particular field-reversal model adopted.

Next, consider the maximum plasma density n_o at the null point:

$$n_o = \left(\frac{\beta}{1+T_e/T_i}\right) \left(\frac{1}{8\pi k}\right) \left(\frac{B_o^{\;2}}{T_i}\right) \tag{7}$$

where the plasma β and the electron/ion temperature ratio T_e/T_i are constants of the chosen plasma model. One can immediately see that the maximum density is a function of T_i and B_o, but is independent of s.

Now consider the total number of ions N:

$$N \equiv \int n dV = n_o \int f(r,z)\ dV \qquad (8)$$

where f(r,z) is the density ratio n/n_o throughout the plasma volume V. One then proceeds in just the same manner as for equation (3) through (6), obtaining:

$$N = S_N\ s^3\ T_i^{1/2}/B_o \qquad (9)$$

where S_N is the corresponding shape factor:

$$S_N = \frac{(2k)^{1/2}}{R_i^{3/2}} \left(\frac{\beta}{1 + T_e/T_i}\right) \int_0^{\rho(\zeta)} \int_0^{1_o} f(\rho,\zeta)\ \rho d\rho d\zeta \qquad (10)$$

Next, the total plasma energy W is given by:

$$W = \frac{3}{2}\, k\ T_i(1 + T_e/T_i)\ N = S_w\ s^3\ T_i^{3/2}/B_o \qquad (11)$$

where

$$S_w = \frac{3}{2}\, k\ (1 + T_e/T_i)\ S_N \qquad (12)$$

The field-reversed flux ψ_R is obtained by integration at z=0 from the ring axis out to the magnetic axis radius R_o

$$\psi_R = \int_0^{R_o} (B_z)\, 2\pi r dr \tag{13}$$

Define the model-dependent field ratio $b_z(\rho) \equiv B_z(r,o)/B_o$; and then one finds:

$$\psi_R = S_\psi\, s^2\, T_i/B_o \tag{14}$$

where the corresponding shape factor is:

$$S_\psi = 4\pi\, \frac{k}{R_i} \int_0^{\rho o} b(\rho)\, \rho d\rho \tag{15}$$

The ring current I is another integral:

$$I = \int_0^{n(z)} \int_{-L_o}^{L_o} J_\theta \, drdz \tag{16}$$

where J_θ is the azimuthal current density. This can then also be written in a similar way:

$$I = S_I \; s \; T_i^{1/2} \tag{17}$$

where the shape factor S_I is:

$$S_I = \frac{1}{\pi} \left(\frac{k}{2R_i}\right)^{1/2} \int_0^{\rho(\zeta)} \int_0^{l_o} \left(\frac{\partial b_r}{\partial \zeta} - \frac{\partial b_z}{\partial \zeta}\right) d\rho d\zeta \tag{18}$$

where we have made use of the relation $\underline{\nabla} \times \underline{B} = 4 \pi \underline{J}$. Note that the magnetic field B_o cancels out for this particular parameter.

If there is a toroidal field B_θ, then the toroidal flux ψ_θ inside the separatrix is defined

$$\psi_\theta = \int_0^{R(z)} \int_{-L_o}^{L_o} B_\theta \, drdz \tag{19}$$

In the same way, one derives the result:

$$\psi_\theta = S_\theta \; s^2 \; T_i / B_o \tag{20}$$

where

$$S_\theta = \frac{2k}{R_i} \int_0^{\rho(\zeta)} \int_0^{l_o} b_\theta \, d\rho d\zeta \tag{21}$$

An important parameter for the calculation of the efficiency of neutral beam absorption is the line density:

$$ndl = n_0 \; sa_i \int_{-\xi}^{+\xi} f(\xi)\, d\xi \tag{22}$$

where $f(\xi)$ is the density ratio (see equation 8), and ξ is the dimensionless scaling coordinate along the path l of the neutral beam. For scaling purposes, using equations (1) and (7), we obtain:

$$\int ndl = S_{nl} \; s \frac{B_o}{T_i^{1/2}} \tag{23}$$

where

$$S_{nl} = \frac{1}{4\pi} \left(\frac{1}{2k \; R_i} \right)^{1/2} \left(\frac{\beta}{1 + T_e/T_i} \right) \int_{-\xi}^{+\xi} f(\xi)\, d\xi \tag{24}$$

Looking ahead to the reactor implications of field-reversed plasmas, one should consider the fusion power rate P_F. For the case of a deuterium-tritium reactor, this rate can be written in the form:

$$P_F = E_F \; \overline{\sigma v} \; (1-f_T) \; (f_T) \; n_o^2 \int f^2 \, dV \tag{25}$$

where E_F is the fusion energy release, f_T is the tritium fraction, and $\overline{\sigma v}$ is the reaction rate, which is a known function of the ion temperature. As for previous parameters, this can be written:

$$P_F = S_F \, s^3 \frac{\overline{\sigma v}}{T_i^{1/2}} B_o \tag{26}$$

where the shape factor S_F is:

$$S_F = \frac{(1-f_T)(f_T \, E_F)}{8\pi} \left(\frac{2}{k}\right)^{1/2} \left(\frac{1}{R_i}\right)^{3/2} \left(\frac{\beta}{1 + T_e/T_i}\right)^2 \int_0^{\rho(\zeta)} \int_0^{1_0} f^2(\rho,\zeta)\rho d\rho d\zeta \tag{27}$$

In the temperature range 25 keV $\leq T_i \leq$ 60 keV, the ratio $\overline{\sigma v}/T_i^{\ 1/2}$ is almost a constant for the DT reaction. Therefore, one concludes that the fusion power scales with field B and size parameter s.

In summary, these equations provide a framework for scaling a given field-reversed plasma model from one experiment to another. The "shape" of the plasma remains a constant when it is measured in units of the ion gyroradius, and then the only variable parameters are the size, the temperature, and the magnetic field. The validity of this scaling rule depends on the assumption that the principal physical properties of the plasma are primarily a function of the ion gyroradius parameter.

3. Choice of Illustrative Model Geometries

In order to apply the general scaling rule to a specific MFTF example, a simplified version of the field-reversed reactor studies plasma model was used.[2] As illustrated in Figure 1, it consists of a hollow cylinder of major radius R, minor radius a, and half-length L, given by the following equations:

$$\begin{aligned} a &\equiv s a_i \\ R &\equiv S_R\ a = S_R\ s a_i \\ L &\equiv S_L\ a = S_L\ s a_i \end{aligned} \tag{28}$$

where the minor radius a is chosen to be the basic unit of length, and where S_R and S_L are the "shape parameters" of this plasma model. The shaded area in Figure 1 contains plasma at a uniform density n_0, completely excluding the magnetic field. There is no toroidal field anywhere. The plasma beta is assumed to be unity and the electron temperature is assumed small ($T_e \ll T_i$) so that it can usually be neglected. The shape factors for this model are displayed in Table 1.

Three different size plasmas were selected for detailed evaluations; their scaling input parameters are shown in Table II. The "SMALL" plasma is based on estimates made for the BETA-II experiment[1] and on the ideal plasma size derived in calculations that took into account electron and Ohkawa currents.[3] The "MEDIUM" plasma is the one that was used in earlier estimates of field reversal in the MFTF plug magnet.[4] The "LARGE" plasma is taken from the field-reversed mirror reactor study.[2]

The temperatures for the "SMALL" and "MEDIUM" models are lower than for ion temperatures in the corresponding open field line ($\beta < 1$) plasmas. This choice conforms with computations done for the MFTF plug magnet.[4] Also, some "typical" numerical values are shown in Table II for four particular cases.

In Figures 2 and 3 there are plots of plasma radius, energy, reverse flux, and line density versus the magnetic field for the three different size plasmas shown in Table II. Labeled points on these graphs correspond to the four "typical" experiments, where "P" means the MFTF plug, and "C" means the MFTF center cell.

For each plasma model, the size, energy, and flux all scale inversely as the field, and the line density increases linearly with the field. These trends favor high magnetic field experiments, but one should reserve judgement until more plasma parameters are evaluated. It i[illegible] evident that there is a large difference in plasma energy between the projected reactor scale experiment and estimates made for smaller scaled plasma sizes. Present field-reversed theta pinch experiments are in the "SMALL" to "MEDIUM" size range, and there are no experiments being done at the "LARGE" size.

The "SMALL" size plasma is of particular interest because it approximates the ideal size for the initial production of field reversal by neutral beam injection.[3] Thus, in Figures 4 and 5, this model has been used to plot the same four parameters (radius, energy, flux, and line density) versus temperature at four different magnetic field values. As one should expect, all of these parameters (except the line density) are increasing functions of the temperature. This plasma model will be used f[illegible] further discussions of buildup and diffusion in the next section.

4. Neutral Beam Buildup Requirements

For buildup purposes, the trapped ion current I_T must exceed the steady state ion rate loss rate from the plasma:

$$I_T = q \frac{N}{\tau} = q \frac{n^2 V}{\overline{n\tau}} \qquad (29)$$

where q is the ion charge and τ is the ion loss time. For each loss process, an estimate of $\overline{n\tau}$ leads to a corresponding minimum requirement for the trapped ion current I_T.

Before reaching the field-reversed condition, one must satisfy open field line requirements, as described by the 2XIIB experimental group.[5]

$$n\tau \approx 1.39 \times 10^{12} \; T_e^{3/2} \qquad (30)$$

Then, equations (29) and (30) can be rewritten in terms of the parameters of the model as follows:

$$I_{TM} = S_{TM} \; s^3 \; B_o/T_i^{1/2} \qquad (31)$$

where I_{TM} stands for the mirror-trapped plasma current, and where the shape factor S_{TM} is given by:

$$S_{TM} = \frac{q}{4\pi}\left(\frac{2}{k}\right)^{1/2}\left(\frac{1}{R_i}\right)^{3/2}\left(\frac{\beta}{1+T_e/T_i}\right)^2 \frac{10^{-12}}{1.39\ T_e^{3/2}}\ S_R\ S_L \qquad (32)$$

Strictly speaking, the values of β and the plasma shape may change during field line closure, but these differences are neglected here.

Baldwin and Fowler assessed the neutral beam field requirements, and found that adding an impurity to the plasma can prevent current cancellation by electrons near the field null.[3] This effect then should permit the plasma to become field-reversed near $B = 0$ ($\beta = 1$), provided the trapped current exceeds a minimum value I_{T0}, given by:

$$I_{T0} = q\ N\ a_i\ (\ R\ \tau_{sk}\,\alpha) \qquad (33)$$

where equation (33) is an approximation to Table I of the above report[3], where the impurity fraction α is defined by:

$$\alpha \equiv (n_Z/n)\ Z\ (Z-1) \qquad (34)$$

where the "skin-time" τ_{sk} is given by:

$$\tau_{sk} = 4\pi\ R^2/\eta \qquad (35)$$

where η is the classical collisional plasma resistivity:

$$\eta = \gamma_{ei}/r_o\ n_e \approx 5 \times 10^3/T_e^{\ 3/2} \tag{36}$$

where γ_{ei} is the electron collision frequency, and r_o is the classical electron radius. The numerical approximation assumes $\ln \Lambda \approx 15$.

Next, we solve equations (33-34) for I_{TO} and write the result in the form of our scaling notation:

$$I_{TO} = S_{TO}\ B_o/T_i^{\ 1/2} \tag{37}$$

where the "shape factor" S_{TO} also involves the impurity fraction α and the electron temperature T_e:

$$S_{TO} = 5 \times 10^3 \frac{q}{4\pi} \left(\frac{2}{k\ R_i}\right)^{1/2} \left(\frac{S_L}{S_R^{\ 2}}\right) \left(\frac{1}{\alpha T_e^{\ 3/2}}\right) \tag{38}$$

The factor s has canceled out; the current minimum I_{TO} is independent of this scaled size parameter.

Now compare the open field line trapped current I_{TM} with the impurity-trapping condition I_{TO}. Both of these currents scale as $(B_o/T_i^{\ 1/2}\ T_e^{\ 3/2})$ and thus their ratio is:

$$\frac{I_{TM}}{I_{TO}} = s^3 \frac{S_{TM}}{S_{TO}} = 1.87\ (s\ S_R)^3 \alpha \tag{38}$$

For this model, the two currents are equal for rather small plasmas. For example, at $\alpha = 0.1$, we find s $S_R = 1.75$, corresponding to s = 0.70 and a plasma radius $R_p = R + a \approx 2.5\ a_i$. This is somewhat smaller than the previous estimate[3] of $R_p \approx 3.5\ a_i$, but the agreement is probably close enough considering the roughness of both models.

Figure 6 is a plot of these two currents; it shows that I_{TM} is the governing minimum current requirement for most plasmas, such as the SMALL model of Table I, where s ≈ 1.2. Figure 7 displays this trapped current requirement versus field B_o and ion temperature T_i for the BETA-II environment (T_e = 0.1 keV) and the MFTF environment (T_e = 0.5 keV). The lower values for MFTF are a vivid illustration of the importance of the electron temperature. Also note that higher magnetic field experiments require higher currents, even though the total particle number N is lower.

Another important consideration is the time scale to approach the field-reversed state[3], which is of the order of the skin time (equa. 35). In terms of the previous scaling parameters, this is:

$$\tau_{SK} \approx S_{SK}\ s^2\ T_i / B_o^{\ 2} \tag{39}$$

where the shape factor S_{SK} is:

$$S_{SK} = \frac{4\pi}{5 \times 10^3} \left(\frac{2k}{R_i}\right) S_R^{\ 2}\ T_e^{\ 3/2} \tag{40}$$

Note the strong dependence of this time scale on the size s and the field B_0. Figure 8 is a plot of the skin time for the same range of conditions as the trapped current results of Figure 7. Comparison of the results shows that as one tries to find plasma conditions permitting a lower trapped current, one increases the time scale. In fact, the total required charge Q scales as:

$$Q \equiv I_{TM}\, \tau_{SK} \sim s^5\, T_i^{1/2}/B_0 \tag{41}$$

Note that Q scales as the fifth power of the plasma size.

3. Field-Reversal Possibilities for the Proposed MFTF-B Configuration

It has recently been proposed[6,7] to expand the MFTF project from a single mirror experiment to a tandem mirror configuration. Assuming that this proposal is accepted, one must re-examine the revised configuration for field-reversal possibilities.

Each mirror plug of the tandem configuration would have the same magnetic field configuration as the early design, and almost as many neutral beams will be available (20 startup and 23 sustaining beams). Consequently, the previously-described MFTF field-reversal experiment[4] could still be done. For the plug, we take B_0 = 20 kg, average neutral beam energy E_B = 60 kV, ion temperature T_i = 40 keV and electron temperature T_e = 0.5 keV.[4,8] Then the beam absorption efficiency η_p = 0.7, as given by equation (A12) of the Appendix, where $\sigma_i = \sigma_i' = 2.39 \times 10^{-16}$ and $\sigma_x = \sigma_x' = 4.0 \times 10^{-16}$. For the SMALL plasma model (at s=1.2), the open field line trapped current (equa. 32 or Fig. 6) then must exceed its minimum value of 120 amp, and the corresponding beam current should be $120/.7 \approx 190$ amperes.

This beam current estimate is low by comparison with the 2XIIB experiment.[3] However, in that case, one had $B_0 \approx 7$ kg, $T_i \approx 10$ keV, and $T_e \simeq .1$ keV, which corresponds to a trapped current [see equa. (31) - (32)] which is more than 8 times larger, or ~980 amperes. From equation (A12) of the Appendix, one finds $\eta_p = .82$, so the minimum beam requirement is estimated to be ≃ 1200 amperes. The actual experiments were in the range 400–500 amperes of beam current, and did not achieve field reversal.

The major reason for the lowered current requirement in the MFTF experiment is the higher electron temperature. It should also be emphasized that these MFTF plug plasma parameters pose a severe requirement for the neutral beam focusing. The outer plasma radius R_p for the SMALL plasma is:

$$R_p = R+a = s(S_R+1)\ a_i = 4.2\ a_i \tag{42}$$

which is about 8.5 cm for $B = 20$ kg, $T_i = 40$ keV. However, the minimum width of the beam, as given in the original MFTF proposal[9], is 15 cm. Thus, only for head-on aiming would the beam even approach the maximum line density which was used in our beam efficiency estimates. Tangential aiming would be even less efficient.

Next, consider the time scale for field-reversal buildup, as given by equations (34) – (40), using this same SMALL plasma model. Using the MFTF plug parameters, one finds a time constant $\tau_{SK} \approx 33$ millseconds, which is much less than the half-second time duration of the sustaining beam pulse.

For the *2XIIB parameters, one obtains* $\tau_{SK} \approx 6$ milliseconds, which is of the same order of magnitude as the duration of that experiment. Thus, both the beam current and time constant comparisons favor the MFTF experiment.

A previous trapped current estimate for the plug[4] was in the range of 600 amperes for a 10 kG experiment, but this estimate was based on the MEDIUM size plasma model, for which the plasma volume is about 5.5 times larger than the SMALL model, for the same field and temperature. Using this volume factor, and correcting for the lower field, one obtains 280 amperes for the trapped current. The remaining differences with the previous estimate are a more optimistic procedure for estimating $n\tau$, and a lower adopted value for the plasma beta. *In* addition to the higher current requirement, the MEDIUM size plasma at the lower field would have a much longer time scale for field reversal. Again using equations (39) - (40), one estimates $\tau_{SK} \approx 500$ milliseconds for this case, which is equal to the sustaining beam pulse duration.

Previous current requirement estimates for field reversal in the TMX central cell[10] were in the range of 50-200 amperes Using the SMALL plasma model with $B_o = 2$ kG, $T_i = 10$ keV, and $T_e = 0.2$ keV, this scaling theory gives a trapped current estimate of 290 amperes. The plasma radius at this low field, however, is 42 cm, and the beam absorption efficiency ($\eta_p \approx 0.3$) corresponds to a beam current requirement of 960 amperes. These results are higher than the earlier estimates[10] because they were made with a higher magnetic field, and a larger plasma radius.

A more attractive place for a field reversal experiment than any of the above locations is the center cell of the proposed MFTF tandem.[6,7] It has a high magnetic field (10 kg), large radius, and high energy ion beams. It offers the possibility of better control of the boundary conditions at the separatrix of the field-reversed region -- both by better plasma control and by better magnetic field control. The plasma on the open field lines is confined (at least partially) by the potential barriers at the plugs. Even if this confinement is not as ideal as in a conventional tandem mirror experiment, any increase in plasma density or pressure at the separatrix should lead to less drastic gradients there. This effect is sometimes described as immersing the field-reversed ring in a "bath" or "soup" of additional plasma. The magnetic field of the center cell is more controllable than in the plugs because the individual coil currents can be separately adjusted, and there is room inside them to insert auxiliary apparatus such as Ioffe coils, stellerator-like windings, or conducting walls.

The neutral beam current requirements for the central cell resemble the plug case; the principal difference being the lower magnetic field of 10 kg. Using the same beam energy and temperatures, the SMALL plasma model estimate is then 60 amperes for the trapped current. However, the beam absorption efficiency is lower, about $\eta \approx 0.45$, so the corresponding beam current should be 133 amperes, which is not very different from the plug current estimate. As emphasized above, this low estimate for the required trapped current is based on the assumption that the electron temperature is high (and the $n\tau$ trapping time is long) for the open field lines. In addition, one must remember that the neutral beams will focus into a fairly small volume; from equation (41) one finds that the outer radius $R_p \approx 17$ cm. Thus, the 15 cm wide beam has more aiming flexibility than it did for the plug plasma (at 20 kG). The field reversal time scale is also greater than for the plug; $\tau_{SK} \approx 130$ milliseconds, which is less than the 500 millisecond neutral beam pulse time.

In Table III most of the results of this section are displayed together. The ion temperatures are hotter than the corresponding plasmas of Table II because they were intended to be more representative of the conditions at the beginning of field reversal, closer to the open field line values. Note that all of these estimates are based only on the SMALL plasma model; no attempt was made to estimate the anticipated increase in plasma size after the initiation of field reversal.

6. Summary

By using some simplified approximate scaling rules, it has been shown that neutral-beam-induced field-reversal experiments are more promising in the MFTF tandem central cell than in the plugs (Yin-Yang coils) or in the TMX or BETA-II experiments. The required neutral beam buildup current is the lowest, the plasma radius is not too small for the width of the beam, and the time scale to achieve field reversal (the "skin time") is shorter than the beam time. In 2XIIB, the current requirement exceeds the installed capability of the experiment. In TMX, the time scale exceeds the neutral beam pulse length. In the MFTF plug, either the plasma is awkwardly small, or else the beam current and pulse length requirements are too close to the design maximum of the installation. Furthermore, the MFTF tandem central cell is large enough to permit the installation of shaped conducting walls (for boundary conditions), and of additional field-shaping coils.

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TABLE I

Shape Factors for Illustrative Model

(See Fig. 1 and Equa. 28 of text)

Volume	$S_V = \left[8\,\pi\,(2k/R_i)^{3/2}\, S_R\, S_L \right]$
Total Ions[a]	$S_N = \left[(k)^{1/2}\,(2/R_i)^{3/2}\, S_R\, S_L \right]$
Plasma Energy[a]	$S_W = \left[3/2\,(2k/R_i)^{3/2}\, S_R\, S_L \right]$
Reverse Flux	$S = \left[2\,\pi\,(k/R_i)\,(S_R - 1)^2 \right]$
Current	$S_I = \left[\frac{1}{4\pi}\,(2k/R_i)^{1/2}\,(\sqrt{S_L{}^2 + (S_R-1)^2} + \sqrt{S_L{}^2 + (S_R+1)^2} \right]$
Toroidal Flux	$S_\theta = 0$
Line Density[a][b]	$S_{nl} = \left[(1/\pi)\,(2/kR_i)^{1/2} \right]$
Fusion Power[a]	$S_F = \left[(1/4\pi)\,(2/k)^{1/2}\,(1/R_i)^{3/2}\,(E_f)\,(f_T)\,(1-f_T)\,S_R\,S_L \right]$

(a) $\beta = 1.0$ and $T_e \ll T_i$

(b) Maximum line density normal to the axis at the midplane.

TABLE II

Field-Reversed Plasma Model Parameters

Parameter	Symbol	Plasma Model			
		SMALL	MEDIUM		LARGE
Scaled Size	$s = a/a_i$	1.2	3		5
Major Radius Ratio	$S_R = R/a$	2.5	2		2
Half-length Ratio	$S_L = L/a$	6.25	3		6
Ion Temperature (keV)	T_i	7.5	20		64
Plasma Beta	β	1.0	1.0		1.0
Atomic Number	A	2.	2.		2.
"Typical" Experiment		βII	MFTF		Reactor
			Plug (P)	CTR (C)	
Maximum Field (kG)	B	10	20	10	40
Minimum Radius (cm)	R+a	7.5	12.5	25.	19.
Minimum Volume (liters)	V	3.75	12.	98.	81
Minimum Energy (kjoules)	W	2.24	29.	58.	775.
Ring Current (megamp)	I	0.23	0.51	0.51	2.6

TABLE III

Estimates of Field Reversal Neutral Beam Buildup Parameters

Using the SMALL Plasma Model

	2XIIB and BETA-II	TMX (CTR)	MFTF (PLUG)	MFTF (CTR)
B_o (kG)	7.	2.	20.	10.
T_i (keV)	10.	10.	40.	40.
T_e (keV)	0.1	0.2	0.5	0.5
$\bar{E}_B$ (keV)	15.	15.	60.	60.
R_p (cm)	12.	42.	8.5	17.
τ_{SK} (msec)	6.	210.	33.	130.
τ_{BEAM} (msec)	5.	20.	500.	500.
I_{TRAP} (amp)	980.	290.	120.	60.
I_{BEAM} (amp)	1200.	960.	190.	133.

GEOMETRY OF ILLUSTRATIVE MODEL

Similar to Ref. 2, but without end cap corrections and with uniform density inside shaded volume.

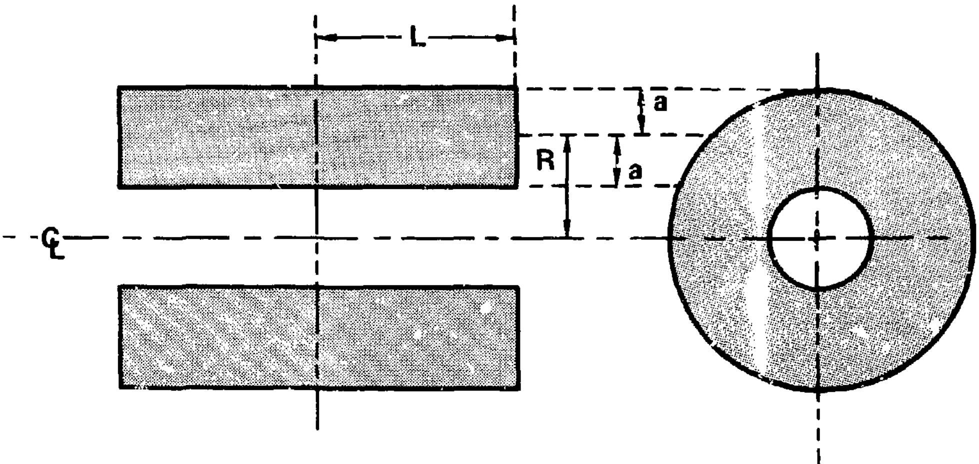


Fig. 1

RADIUS & ENERGY VS MAGNETIC FIELD FOR THREE SCALED PLASMAS

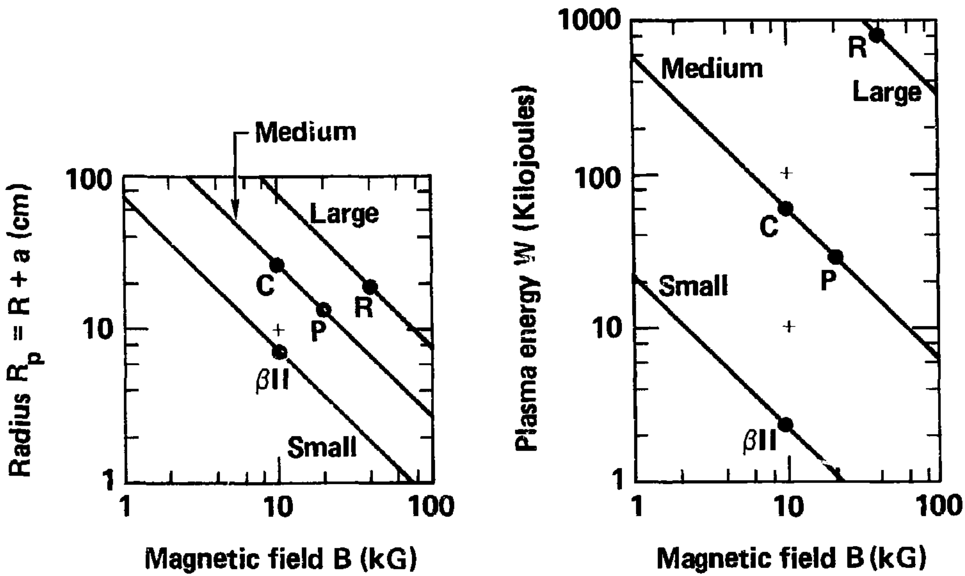


Fig. 2

LINE DENSITY & REVERSED FLUX VS MAGNETIC FIELD FOR THREE SCALED PLASMAS

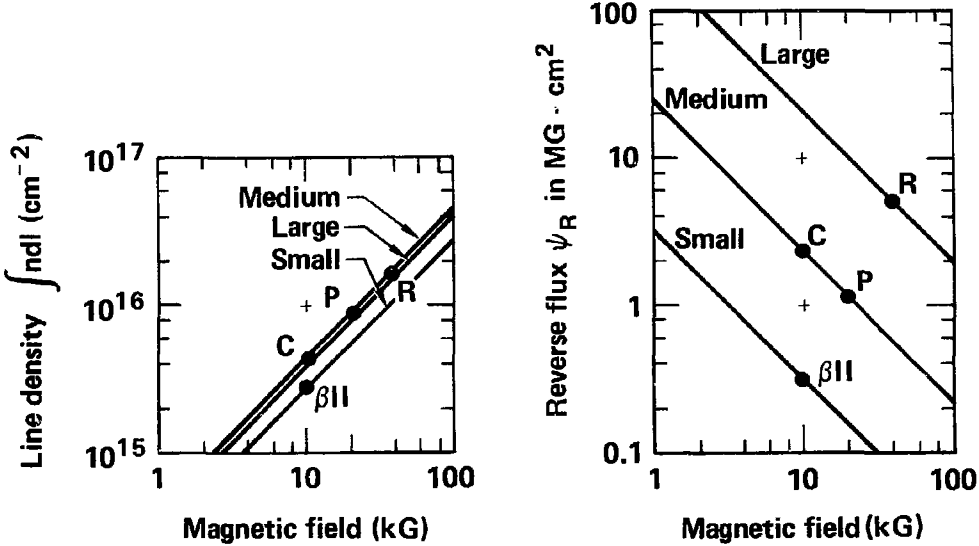


Fig. 3

RADIUS & ENERGY VS TEMPERATURE FOR SMALL PLASMA MODEL

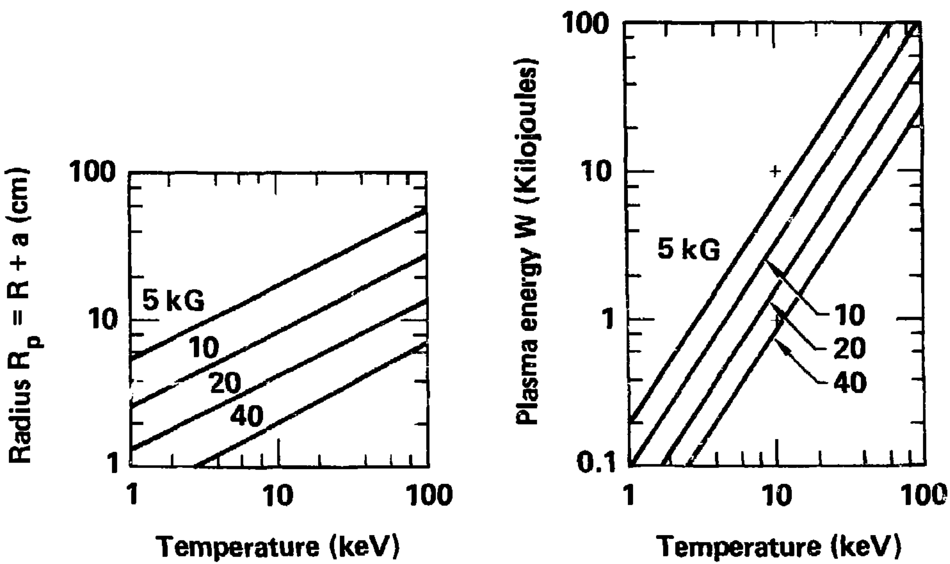


Fig. 4

LINE DENSITY & REVERSED FLUX VS TEMPERATURE FOR SMALL PLASMA MODEL

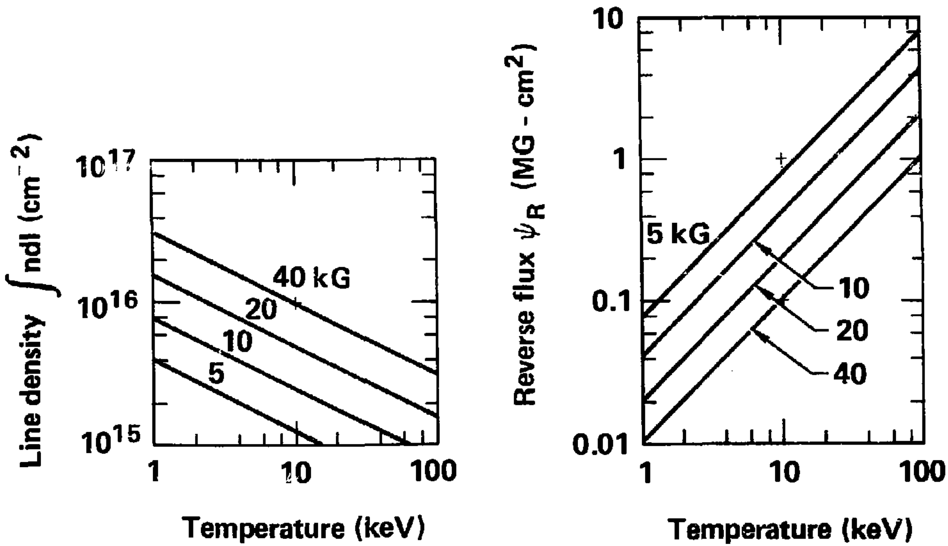


Fig. 5

TOTAL TRAPPED CURRENT VS SIZE PARAMETERS s FOR SMALL PLASMA MODEL

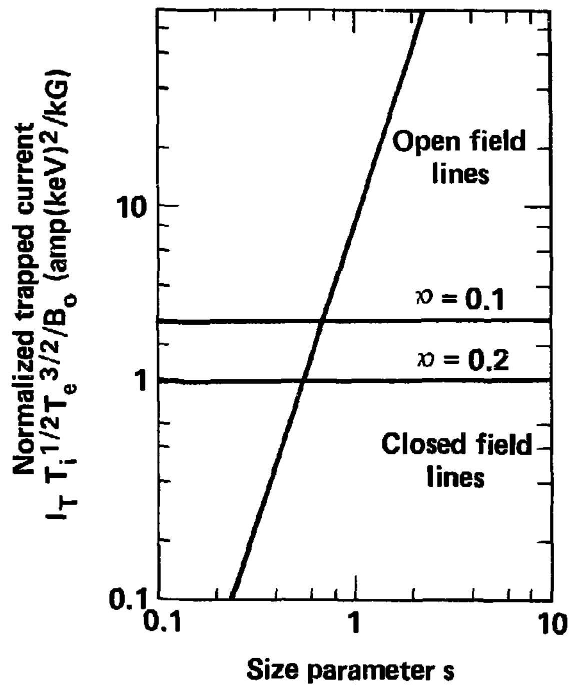
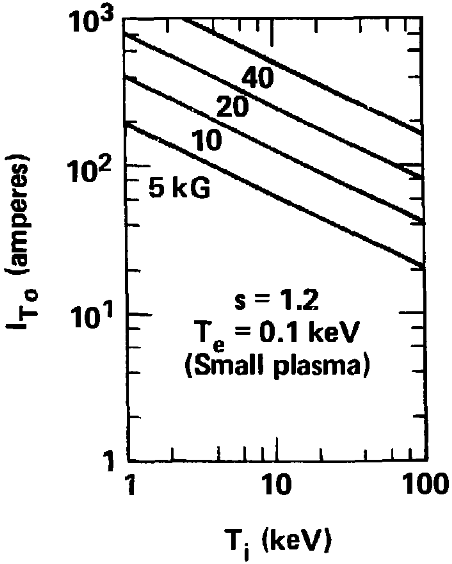


Fig. 6

NEUTRAL BEAM OPEN FIELD LINE CURRENT VS ION TEMPERATURE FOR VARIOUS FIELDS AND ELECTRON TEMPERATURES



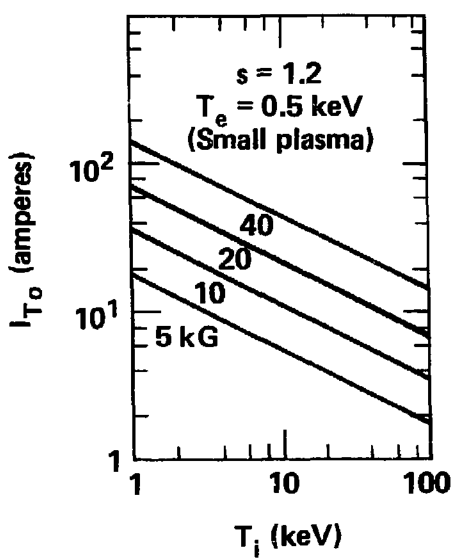


Fig. 7

TIME SCALE ("SKIN TIME") VS T_I AT TWO T_E FOR SMALL PLASMA

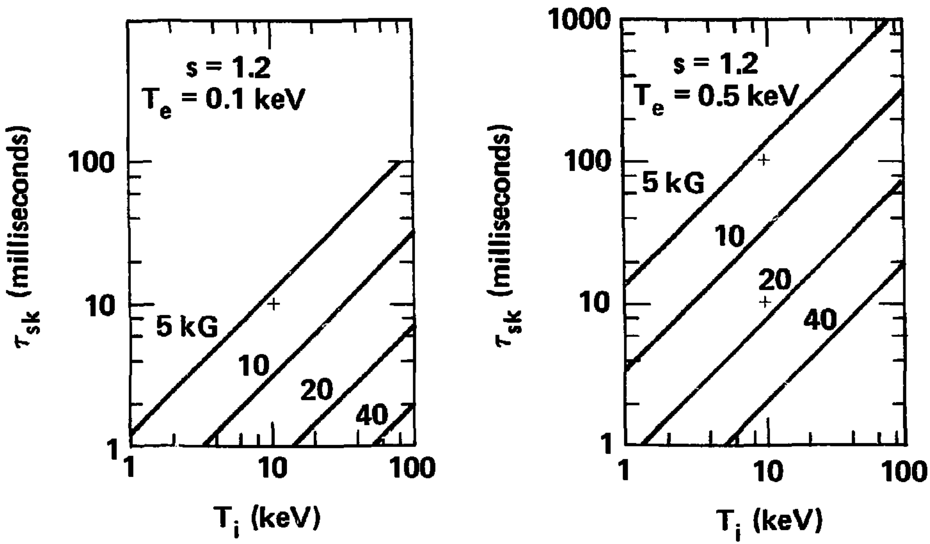


Fig. 8

APPENDIX

An Improved Approximation for Beam-Trapping Efficiency

In earlier work[4], an approximate model for beam trapping efficiency was used which assumed that the incident ion energy of the neutral beam was equal to the average plasma ion energy. This model has been improved by dropping that assumption; it is necessary to do this because these two energies can be far apart, especially in the case of the plasma-gun-produced target.[1] The improved approximation is rederived here.

Consider a neutral beam incident on a plasma target, and define the absorption integral M:

$$M \equiv (\sigma_i + \sigma_x) \int_0^R n\,dl \qquad \text{(A1)}$$

where n is the plasma density, l is the beam path, R is the plasma radius, σ_i is the total ionization cross-section, and σ_x is the charge exchange cross-section. For the usual velocity ordering case:

$$v_e \gg v_B \gg v_i \qquad \text{(A2)}$$

one obtains σ_i and σ_x from the standard tables:

$$\sigma_i = \sigma_{ii}(E_B) + \frac{ve}{vB}\,\sigma_{ie}\,(T_e) \qquad \text{(A3)}$$

$$\sigma_x = \sigma_{xi} \; (E_B) \tag{A4}$$

For a normally incident neutral beam, the total interaction fraction F_M is then given by:

$$F_{2M} = 1 - e^{-2M} \tag{A5}$$

where the factor 2 is needed because the beam goes in and out of the plasma. Let f_1 be the fraction that is ionized:

$$f_i \equiv \frac{\sigma_i}{\sigma_i + \sigma_x} F_{2M} \tag{A6}$$

and let f_x be the fraction that is charge exchanged:

$$f_x \equiv \frac{\sigma_x}{\sigma_i + \sigma_x} F_{2M} \tag{A7}$$

Now consider the "second generation" of neutral particles formed by this charge exchange process. These particles carry the average plasma energy, rather than the beam energy, and they are born inside the plasma. We approximate their interaction fraction $F_M^{'}$ by:

$$F_M^{'} \equiv 1 - e^{-M'} \tag{A8}$$

where no factor of 2 is needed because these particles are already inside the plasma and merely have to get out. Also, all of these secondary particle quantities are primed to distinguish them from the primary beam. Thus, one can write down their ionization fraction.

$$f_i' = \frac{\sigma_i'}{\sigma_i' + \sigma_x'} F_M' \tag{A9}$$

and their charge exchange fraction,

$$f_x' = \frac{\sigma_x'}{\sigma_i' + \sigma_x'} F_M' \tag{A10}$$

Now we are ready to formulate the particle trapping efficiency η_p for the infinite number of generations of neutral particles·

$$\eta_p = f_i + f_x \left\{ f_i' + f_x' \left[f_i' + f_x' (f_i' + \ldots .) \right] \right\}$$

$$= f_i + f_x f_i' \left(\frac{1}{1 - f_x'} \right) \tag{A11}$$

Expressing this equation in terms of the cross-sections, one gets:

$$\eta_p = \frac{F_{2M}}{\sigma_i \times \sigma_x} \left\{ \sigma_i + \sigma_x \left(\frac{\sigma_i' F_M'}{\sigma_i' + \sigma_x' (1-F_M')} \right) \right\} \tag{A12}$$

This is the desired result for the particle trapping efficiency η_p. In the special case where the beam energy equals the plasma ion energy, it yields the earlier approximation.[4]

A separate expression is required for the energy trapping efficiency η_E, when the beam energy differs from the plasma energy. A simple way to derive it is to evaluate the fraction of energy which is lost from the plasma. From the initial beam this loss fraction f_L is:

$$f_L = e^{-2M} = 1 - F_{2M} \tag{A13}$$

In addition, there are secondary neutrals lost, each of which carries away, on the average, the energy ratio E'/E_B where E' is the average plasma energy and E_B is the beam energy. This secondary loss fraction is:

$$f_L' = f_x \frac{E'}{E_B} \left(1 - F_M'\right) \left\{ 1 + f_x' \left(1 + f_x' (1 + \ldots.)\right) \right\} \tag{A14}$$

Then the energy trapping efficiency η_E is:

$$\eta_E = 1 - f_L - f_L'$$

$$= F_{2M} \left[1 - \frac{\sigma_x}{\sigma_i + \sigma_x} \frac{E'}{E_B} \frac{\sigma_i' + \sigma_x'}{\sigma_i' e^{M'} + \sigma_x'} \right] \tag{A15}$$

In the special case where $E' = E_B$, this expression also yields the earlier approximation.[4]