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Efficient Design of Two-Dimensional *Recursive Digital Filters*

MASTER

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(Final Report)

A. INTRODUCTION

In recent years, there has been a rapid increase in the number of applications requiring the digital processing of two-dimensional (2-D) signals. Typical areas of application have included; 1) satellite-borne remote photography for the monitoring of environmental effects, earth resources, and urban land use; 2) processing of geological and seismological data in the exploration for oil and natural gas; 3) 3-D imaging of the brain using multiple 2-D projection techniques; and 4) the processing of medical and industrial radiographs.

In most 2-D signal processing applications, the goal is to somehow extract some desired information from a 2-D data array by performing appropriate operations on that data. Perhaps the most common method of extracting the desired information is via 2-D digital filtering. As in 1-D, there are two filtering implementations in the 2-D case: nonrecursive and recursive. Nonrecursive (FIR) 2-D digital filters can be characterized by the difference equation

$$y(m,n) = \sum_{k} \sum_{\ell} h(k,\ell)\, x(m-k,n-\ell) \tag{1}$$

where $\{y(m,n)\}$ is the output array, $\{h(k,\ell)\}$ is the filter array (finite extent), and $\{x(m,n)\}$ is the input array. The more general recursive (IIR) filters are characterized by the equation

$$\sum_{k} \sum_{\ell} b(k,\ell) y(m-k,n-\ell) = \sum_{k} \sum_{\ell} a(k,\ell) x(m-k,n-\ell) \tag{2}$$

where the arrays $\{b(k,\ell)\}$ and $\{a(k,\ell)\}$ describe the recursive filter. Both the nonrecursive and the recursive 2-D filters can alternately be characterized in the frequency domain by the digital transfer function $H(z_1,z_2)$ defined as

$$H(z_1,z_2) = \frac{\mathcal{Z}\{y(m,n)\}}{\mathcal{Z}\{x(m,n)\}} \tag{3}$$

There are basically two steps in the development of a 2-D digital filter: design and implementation. The design step involves the determination of a transfer function $H(z_1,z_2)$ meeting the specified, deterministic space domain or frequency domain requirements within an acceptable error bound. If a FIR filter is desired, $H(z_1,z_2)$ has to be a finite degree polynomial in z_1^{-1} and z_2^{-1}. On the other hand, if an IIR filter is desired, $H(z_1,z_2)$ has to be expressed as a ratio of two polynominals,

$$A(z_1,z_2) = \sum_k \sum_\ell \; a(k,\ell) z_1^{-k} z_2^{-\ell} \tag{4}$$

$$B(z_1,z_2) = \sum_k \sum_\ell \; b(k,\ell) z_1^{-k} z_2^{-\ell} \tag{5}$$

with real coefficients. A problem associated with the design of IIR filters (and not the FIR case) is that of stability. This involves the necessity of ensuring that $H(z_1,z_2)$, when implemented, will lead to a stable filter for which the output sequence will remain bounded for all possible bounded input sequences. After a suitable transfer function has been determined, the corresponding 2-D filter is usually implemented on a general purpose computer. This implementation is based on the recursive difference equation of (2) for an IIR filter or the convolution sum of (1) for a FIR filter.

Our investigations over the grant period have been concerned with the development of efficient techniques for the design and implementation of two-dimensional (2-D) recursive digital filters satisfying prescribed frequency response specifications. The primary emphasis in this work involved the general class of 2-D recursive filters known as half-plane filters as described by Eq. (2).

This report outlines the progress of our activities during the grant period (8/78 - 7/79). This work can be divided into seven basic project areas, each of which is described below. Project 1 deals with a comparative study of 2-D recursive and nonrecursive digital filters. The second project addresses a new design technique for 2-D half-plane recursive filters, and projects 3-5 deal with implementation issues. The sixth project presents our recent study of the applicability of array processors to 2-D digital signal processing. The final project involves our investigation into techniques for incorporating symmetry constraints on 2-D recursive filters in order to yield more efficient implementations.

B. SUMMARY OF RESEARCH

Project 1: A Comparative Study of Nonrecursive and Recursive 2-D Digital Filters

Even though the main aim of our investigations has been the design and implementation of 2-D recursive filters, it was felt necessary to undertake a project comparing the recursive approach to the nonrecursive approach of processing 2-D data. A complete study of the comparative capabilities of the numerous 2-D nonrecursive and recursive filter design procedures is a formidable task and beyond the scope of this presentation. Instead, we restricted our attention to a comparison of the half-plane recursive filter design results of [3] and the nonrecursive filter design results, via the generalized McClellan transformation.* A specific practical design example was utilized for comparison purposes and it showed very clearly the ability of 2-D recursive filters to yield excellent responses with fewer coefficients than nonrecursive filters require. This difficulty is seen to be partially overcome by using nonrecursive filters with very efficient implementations. Although the example discussed contains just one nonrecursive and one recursive design procedure, our experience with other examples and other procedures indicates that it is fairly representative of the relative capabilities of recursive and nonrecursive filters in two dimensions. Results of this investigation were presented at an international conference [1].

*R. M. Mersereau, W. F. G. Mecklenbräuker and T. F. Quatieri, Jr., "McClellan Transformations for Two-Dimensional Digital Filtering: I-Design," IEEE Trans. Circuits and Systems, Vol. CAS-23, pp. 405-414, 1976.

Project 2: Design of 2-D Recursive Filters Via a Spectral Factorization Approach

A key project under this grant is to develop a general technique for designing stable, 2-D recursive filters. Our approach involves using a spectral factorization in the following manner. A frequency domain description of the filter is first obtained from its specification (in spatial or frequency domain). This filter function is then decomposed into components which are recursively stable. This decomposition is the so-called spectral factorization step and leads directly to the recursive filter form.

In contrast to the 1-D case, the 2-D spectral factorization has many canonical forms each characterizable by analyticity properties of the factors. One form, when applied to recursive filtering, results in the four factor quarter plane decomposition. Another canonical form results in a new class of half-plane recursive filters. These filters have the property that any 2-D magnitude specification can be approximated arbitrarily well by members of this class. Such a result is not available using a causal quarter-plane factor, as demonstrated by the theory. Unfortunately, 2-D polynomials do not factor into finite order factors. However, spectral factorization in two-dimensions does retain the analyticity properties which result in recursive filter stability. Thus as in 1-D, by incorporating this factorization into a design method which provides the constraint of finite, prescribed filter order, one could have stability ensured.

The main thrust of our design work has thus been to incorporate 2-D spectral factorization into practical design methods for half-plane recursive filters. This is motivated primarily by the fact that the guaranteed analyticity of the factors negates the need for cumbersome and time consuming stability tests at each stage in the design procedure. A problem with this approach is the need to enforce a finite order constraint on the spectral factors.

Our design algorithm is based on a nonlinear optimization procedure with a joint or dual constraint. One constraint in the spatial domain leads to finite order factorizations, the other in the frequency domain leads to satisfying the prescribed filter specification. Our initial development involved designing half-plane filters to a prescribed magnitude function. This avoids problems in defining the complex cepstrum in the spectral factorization. To our knowledge, this is the only general design algorithm yet developed for the design of 2-D recursive digital filters which are guaranteed to be stable. In addition, we have completed the following refinements to the design procedure; 1) inclusion of a numerator polynomial in the technique, and 2) studying alternative optimization methods for the solution of the dual constraint problem. A paper documenting these results has appeared in a journal [3].

Project 3: A General Realization Scheme for Half-Plane 2-D Digital Transfer Functions

Obtaining the hardware realizations of 2-D digital filters is one of the major goals of 2-D filter researchers, because otherwise, the entire theory becomes an academic exercise. However, because of the inherent impossibility of factorizing arbitrary 2-D polynomials, all realization techniques for the general 2-D digital transfer functions presented so far deal with the realization of such functions as a whole. In this project we have completed the development of a realization scheme which makes use of basic building blocks (such as delay elements, advance elements, multipliers, and adders) and takes into account the restriction of a fixed direction of recursion.

A 2-D half-plane digital filter configuration can be conveniently represented by block diagrams, the basic elements of which are: constant multipliers, multi-input adders, delay elements z_1^{-1} and z_2^{-1}, and advance elements z_1 and z_2. Using these elements, a circuit-theoretic realization of a rational 2-D digital transfer function (with unit numerator) has been developed under this project. The final structure is a single input, single output configuration with no delay-free loops which is admissible in the sense that for a fixed direction of recursion all the internal node variable values in the configuration are uniquely determined by previously computed node signal values.

preliminary results of this project were presented at an international conference [2] and a complete paper is under preparation for submission to a journal [6].

The realization method developed hinges upon a novel extension of the realization method for 2-D quarter-plane recursive digital filters developed under the support of the previous grant (NSF Grant ENG 75-15027) and published in a journal.*

Project 4: Block Implementation of 2-D Recursive Digital Filters

In the conventional implementation of recursive digital filters, the input sequence is processed one sample at a time to generate one sample of output sequence. An alternate implementation approach considered under this project is the processing of a block of input samples at one time to generate a block of output samples. Several new block implementation schemes have been developed and properties of these schemes have been fully investigated. The complete results of this project have been compiled in a paper and submitted to a journal for possible publication [7].

Project 5: Hardware Implementation of 2-D Digital Filters

This project was concerned with the hardware implementation of 2-D digital filters. To this end, two basically different approaches were developed under the support of NSG Grant 75-15027. In one of the approaches, the implementation of a recursive 2-D digital filter using Read-Only-Memory (ROM) was investigated.

In 1974, Peled and Liu** forwarded a new approach to the implementation of 1-D digital filters. This implementation is based on repeated table look-ups

*S. K. Mitra and S. Chakrabarti, "A General Method for Realizing 2-D Digital Transfer Function," IEEE Trans. on Acoustics, Speech, and Signal Processing, Vol. ASSP-27, pp. 544-550, December 1979.

**A. Peled and B. Liu, "A New Hardware Realization of Digital Filters," IEEE Trans. on Acoustics, Speech and Signal Processing, Vol. ASSP-22, pp. 456-462, December 1974.

and additions as the basic operations. This approach is different from other approaches in the sense that it does not employ any multiplier in the implementation. Burks-Goldstine-von Neumann's repeated addition algorithm is employed for multiplication (with data represented by 2's complement code) in which all combinations of partial products are stored in the ROM beforehand. By appropriate addressing of the ROM, a partial sum is selected, added to the partial sum residing in another register, and stored with proper shift. This process is continued as many times as the number of bits used to represent the signal. The main advantage of this method is that it offers significant reduction in cost and power consumption for the same speed of operation as that of existing realizations. Also, the method allows us to reach speeds of operation which cannot be achieved by other existing realization methods.

In this project, we implemented in hardware form a recursive digital filter structure in a manner similar to that described by Peled and Liu, and have verified the feasibility of the ROM-based implementation approach described earlier.*

Project 6: Array Processor Implementation of 2-D Digital Filters

The introduction of the array processor (AP) provides the potential of very efficient, cost-effective implementations of digital signal processing procedures such as the 2-D filtering described by Eqs. (1)-(2). An array processor is a high-speed arithmetic processor which, when connected with a host computer (typically a minicomputer), provides the floating-point computational power of a mega-dollar mainframe computer at a small fraction (a few percent) of the cost. This is achieved via the use of parallel, pipelined floating point units and very fast registers, data memory, and program memory.

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*R. Gnanasekaran, K. Mondal and S. K. Mitra, "Hardware Implementations of Two-Dimensional Digital Filters Using ROMs," Proc. 1977 IEEE International Conference on Acoustics, Speech and Signal Processing, Hartford, CT, pp. 519-522, May 1977.

The applicability of array processor technology to 2-D digital signal processing has been investigated in this project. Several key 2-D digital signal processing algorithms were implemented and compared with other conventional processors, indicating that array processors are extremely cost-effective for image processing applications. For efficient implementation using the array processor, both CPU and I/O effects were considered, and architecture considerations such as the amount of memory available in the AP were analyzed with respect to their impact on the 2-D signal processing algorithms.

In the first phase of this project, the implementation of 2-D finite impulse response (FIR) digital filters via the 2-D FFT algorithm, was investigated. Results of this investigation were presented at an international conference [4]. In the next phase, other 2-D image processing algorithms were implemented using the array processing minicomputer facility. These later results were presented at another international conference [5].

Project 7: Symmetry Conditions for 2-D Half-Plant Recursive Filters

The problem of designing a two-dimensional digital filter to approximate a specified frequency response is often simplified by making use of symmetries which appear in the magnitude response. This is because frequency domain symmetries result in symmetries in the space domain as well, thereby substantially reducing the number of filter coefficients which must be determined. Another benefit of the resulting space domain symmetries is that filter realization may be simplified.

Our investigation into the symmetry properties of 2-D half-plane recursive filters, which augments the results of Projects 1 and 2, were summarized in the report [8]. A similar analysis was performed for 2-D nonrecursive filter symmetry conditions, with the results appearing in [9].

In addition, we developed a natural extension of the 2-D spectral factorization results (which are a crucial component of the general design technique developed for project 2) to the M-dimensional case. These results have been published in a journal [10].

C. PUBLICATIONS

[1] M. P. Ekstrom and R. E. Twogood, "Why Filter Recursively in Two Dimensions?" Proc. 1979 IEEE International Conference on Acoustics, Speech, and Signal Processing, Washington, DC, April 1979, pp. 20-23.

[2] S. K. Mitra, K. Mondal and R. E. Twogood, "Design and Implementation of Half-Plane Two-Dimensional Digital Filters," Pro. 1979 International Colloquium on Circuits and Systems, Taipei, Taiwan, July 1979.

[3] M. P. Ekstrom, R. E. Twogood and J. W. Woods, "Two-Dimensional Recursive Filter Design - A Spectral Factorization Approach," IEEE Trans. on Acoustics, Speech, and Signal Processing, Vol. ASSP-28, No. 1, February 1980.

[4] R. E. Twogood, "Array Processor Implementation of 2-D Digital Filters," Proc. 1979 IEEE International Symposium on Circuits and Systems, Tokyo, Japan, July 1979, pp. 531-534.

[5] R. E. Twogood, "2-D Digital Signal Processing with an Array Processor," Proc. 1980 IEEE International Conference on Acoustics, Speech and Signal Processing, Denver, Colorado, April 1980.

[6] K. Mondal, S. K. Mitra, and S. Chakrabarti, "A General Method for the Realization of Half-Plane 2-D Digital Filters," in preparation.

[7] S. K. Mitra and R. Gnanasekaran, "Block Implementation of Two-Dimensional Recursive Digital Filters," IEEE Trans. on Circuits and Systems, submitted.

[8] D. M. Goodman, "Quadrantal Symmetry Conditions for Nonsymmetric Half-Plane Filters," UCRL-82443, March 7, 1979.

[9] D. M. Goodman, "Symmetry Conditions for Two-Dimensional FIR Filters," UCID-18311, September 14, 1979.

[10] D. M. Goodman and M. P. Ekstrom, "Multidimensional Spectral Factorization and Unilateral Autoregressive Models," IEEE Trans. Automatic Control, Vol. AC-25, April 1980.

D. PERSONNEL

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