REQUEST FOR DECLASSIFICATION AND/OR RELEASE

(Allow 3 – 5 Days For Clearance)

Submit copies 1–7 of form, properly executed to Laboratory Records Department with (a) reproducibles or (b) 3 Xerox copies of document if no reproducible exists.

1. REQUEST: ☒ Classification and Patent Release ☐ Declassification OF THE ATTACHED ☐ Abstract ☒ Paper

2. TITLE: (Show in full unless classified)

"Toroidal Field Ripple Effects in TNS Design"

3. AUTHOR: (List All Authors; if not an ORNL employee, indicate address on a separate line)

N. A. Uckan, J. D. Callen, K. T. Tsang, and J. R. Jim Moore

4. PURPOSE: DO NOT ABBREVIATE Titles of Meetings, Journals, Universities

(a) **Oral Presentation** (Give name, place, and exact dates of meeting)

Seventh Symposium on Engineering Problems of Fusion Researcy by the Nuclear and Plasma Science Society of the IEEE, Knoxville, TN, 10/25-28/77.

To be published in program, booklet, brochure, etc. ☒ Yes ☐ No ☐ Not Known

Copies of this document will be distributed ☐ before, ☒ after, ☐ during the meeting, or ☐ no distribution will be made.

(b) **Publication** (Proceedings of a meeting, give place and dates; Journal, give publication date; Book, give editor and publisher)

To be published in the proceedings of the meeting.

(c) **Thesis** (Give University, place, degree)

none known.

COPYRIGHT DISCLAIMER

To be issued as a report ☒ No ☐ Yes (Number) ________

To be condensed and published in open literature ☒ No ☐ Yes (Where?) ________

5. INVENTIONS: (List Page No. of any item appearing to have significant novelty; indicate if none.)

none known.

6. PREVIOUSLY CLEARED DOCUMENT: (Paper, abstract, progress report, etc., containing similar information; indicate if classified.)

Abstract cleared 8/2/77.

DEADLINE DATE 10/28/77 DIVISION Fusion Energy AUTHOR ________

Associate Laboratory Director Review Required ☐ Yes ☒ No - Determined by

Division Director ________ Project Director ________ Program Director ________

APPROVALS

O B Morgan	10/27/77	R S Baker	10/31/77
Division or Program Director	Date	Classification Officer	Date
		NOV 1 1977	NOV 1 1977
Associate Laboratory Director	Date	Patent Office	Date

Distribution of Form

1. Classification Office File
2. Patent Office File
3. Keypunch
4. Approved, Return to Division Office
5. Laboratory Records Dept. – Reprints
6. Public Information Office
7. ERDA – Technical Information Center
8. Suspense Copy – Division File
9. Suspense Copy – Author's File

Distribution of Documents

1. Classification Office File
2. Patent Office File (*if requested*)
3. ERDA – TIC (*if speech*)

OCT 31 1977

N-87A
9-75)

CONF-771029--64

TOROIDAL FIELD RIPPLE EFFECTS IN TNS DESIGN[*]

N. A. Uckan, J. D. Callen, K. T. Tsang, and J. R. Moore
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37830

Summary

In the design of TNS, the choice of the number of TF coils has been made on the basis of trade-off studies among the plasma physics considerations and engineering design requirements. The theory of the magnetic field ripple effects has been studied to include the effects of noncircular cross sections such as those encountered in high-β equilibria. A computer simulation model (RIPPLE) was developed to examine the field ripple effects on plasma transport and scaling. The magnetic field computer program BOVAL was used to calculate the magnetic field due to current flowing in a noncircular coil of finite rectangular cross section. The number of toroidal field (TF) coils is then determined by calculating the maximum magnetic field ripple that can be tolerated from plasma physics considerations.

Introduction

The discrete nature of toroidal magnetic field coils of a tokamak introduces a small, but finite, amplitude modulation on the toroidal magnetic field. These small modulations, which are called "ripples," destroy the axisymmetry of an ideal tokamak and lead to large particle excursions and hence to particle and energy losses.

In the design of TNS (The Next Step), the choice of the number of toroidal field (TF) coils has been made on the basis of trade-off studies among the plasma physics considerations[1-3] and the access space requirements. Increasing the number of TF coils reduces the field ripple but makes access more difficult. Increasing the number of coils also, of course, involves increased costs. Criteria for the analysis of maximum tolerable ripple from the plasma physics considerations include: (1) plasma energy transport, (2) the loss of fast ions, i.e., injected particles and/or alpha particles, and (3) injection angle in the case of near perpendicular injection.

A computer model (RIPPLE) for ripple analysis has been developed to study the magnetic field ripple effects on plasma transport and scaling. This model includes comparison with various energy transport scaling laws such as ion neoclassical transport, trapped particle instabilities, and empirical scaling. Calculations include both radial and poloidal variations in the magnitude of the field ripple as well as the effects of high-β operation. The magnetic field code BOVAL[4] was used to calculate magnetic field due to current flowing in a coil of rectangular cross section. The coil lies in a plane and may take any shape except one where a multivalued function is necessary to describe the path of the current.

Physics Considerations

The nonaxisymmetric toroidal field ripple caused by the discrete coils can form an additional trap for particles, loss regions, and plasma transport. There are three types of particles in such a magnetic field geometry: those trapped in the ripples, which are called "ripple-trapped" particles; those trapped in the toroidal field modulation, which are called "bananas" because of the shape of their orbits; and those untrapped in both magnetic wells. The particle and energy losses by ripple-trapped particles have been studied widely.[1,2] The particle and energy losses through the effect of ripples on the banana particles are considered in Refs. 3 and 5. The early results from Ref. 3 indicated that ripple-induced banana drift losses are larger than the losses due to ripple-trapped particles. Recent calculations by Tsang,[5] which are contrary to Ref. 3, show that the predictions of Ref. 3 correspond to the limit where the collisionality of the plasma is large. In a reactor regime, this loss mechanism is shown to be unimportant.

Considering a simple model of tokamak confinement geometry, the resulting magnetic field can be written as

$$\underline{B} = \frac{B_o}{h}\left[\frac{B_r}{B_\phi},\ \Theta,\ 1 - \delta(r,\theta)\cos N\phi\right], \tag{1}$$

where r and θ are polar coordinates in the minor cross section of the torus and ϕ is the angular coordinate along the magnetic axis; $h = 1 + (r/R_o)\cos\theta = 1 + \epsilon\cos\theta$; $\delta(r,\theta) = (B_{max} - B_{min})/(B_{max} - B_{min}) << r/R_o$ is the depth of the ripples and can approximately be modeled to be[2] $\delta \sim \delta(r)\ g(\theta) \sim (r/a)^m \exp(-\beta\theta^2)$; N is the number of TF coils; and $\Theta = B_\theta/B_\phi$ is the ratio of poloidal to toroidal field. Ignoring the terms of order δ^2, the field strength is approximately

$$B \simeq B_c\left[1 - \epsilon\cos\theta + \tfrac{1}{2}\left(\frac{\epsilon}{q}\right)^2 + \frac{\epsilon A}{2}\cos^2\theta - \delta(r,\theta)\cos N\phi\right], \tag{2}$$

where

$$A = \frac{\epsilon}{q^2}(\sigma^2 - 1), \tag{3}$$

with σ being elongation of the plasma cross section in the vertical direction and q being the safety factor. Following Ref. 2, we find the effective ripple well depth along the field line to be

$$\begin{aligned}\Delta(r,\theta;\sigma) &= \frac{B_{max} - B_{min}}{B_o}\\ &= 2\delta(r,\theta)\Big(\sqrt{1 - \alpha^2(r,\theta)\sin^2\theta\,(1 - A\cos\theta)^2}\\ &\quad - \alpha(r,\theta)|\sin\theta(1 - A\cos\theta)|\\ &\quad \times \left\{\frac{\pi}{2} - \sin^{-1}[\alpha(r,\theta)|\sin\theta(1 - A\cos\theta)|]\right\}\Big),\end{aligned} \tag{4}$$

where $\alpha(r,\theta) = \epsilon/Nq\delta(r,\theta) = \epsilon/Nq\delta(r)\ g(\theta) = \alpha(r)/g(\theta)$.

The expression for the ion heat conductivity coefficient for ripple trapping is[2]

$$\chi_i^\delta \simeq 46.5\ \frac{\delta(r)^{3/2}\ G(\alpha,\beta;\ \sigma)}{\nu_{ei}}\left(\frac{m_i}{m_e}\right)^{1/2} V_D^2 \tag{5}$$

and for banana drift is[2,3,5]

$$\chi_i^{BD} \simeq 9.23\ \epsilon^{3/2}\ \delta^2(r)\ \frac{N^2}{(m + 2)^2}\ \frac{q^2Q^2}{\nu_{ei}}\left(\frac{m_i}{m_e}\right)^{1/2} V_D^2 \tag{6}$$

---

[*] Research sponsored by the Department of Energy under contract with Union Carbide Corporation.

28

$$\times \left[\left(\frac{0.4\,\nu_{ei}R_o}{Np V_D(Q - \frac{1}{2})}\right)^2 \left(\lambda_3 - \frac{\lambda_2^2}{\lambda_1}\right)\right], \tag{6}$$

where $Q = d\ell nq/d\ell nr$, $V_D = cT_i/eB_oR_o\sigma$, and

$$G(\alpha,\beta;\,\sigma) = \frac{1}{\pi}\int \left[\frac{\Delta(r,\theta;\,\sigma)}{2\delta(r)}\right]^{3/2} \sin^2\theta(1 - A\cos\theta)^2 d\theta$$

$$= \frac{2}{\pi}\int_c d\theta\,\sin^2\theta(1 - A\cos\theta)^2\,[g(\theta)]$$

$$\times \Bigg(1 - \alpha^2(r)\sin^2\theta(1 - A\cos\theta)^2/g^2(\theta)$$

$$- \frac{\alpha(r)}{g(\theta)}\,|\sin\theta(1 - A\cos\theta)|\Bigg\{\frac{\pi}{2} -$$

$$\sin^{-1}\left[\frac{\alpha(r)}{g(\theta)}\,|\sin\theta(1 - A\cos\theta)|\right]\Bigg\}\Bigg)^{3/2}, \tag{7}$$

with $\int_c^\theta = \int_0^{\theta_1} + \int_{\theta_2}^{\pi}$, where θ and θ_2 are the roots of

$$1 - \alpha^2(r)\sin^2\theta(1 - A\cos\theta)^2/g^2(\theta) = 0 \tag{8}$$

The term in square bracket in Eq. (6) is the modification term to Ref. 3, where[5]

$$\lambda_n = \sum_{j=1}^{\infty} \int_0^{\infty} \frac{dx\; x^4 \xi(x) e^{-x^2} x^{2n}}{x^4 + \nu_{ii}^2\alpha_j^2[\xi(x)]^2} \tag{9}$$

with

$$\nu_{ii} = \frac{3}{4}\sqrt{2\pi}\,\nu_{ei}\,,$$

$$\xi(x) = \frac{1}{x^3}\left[\frac{e^{-x^2}}{\sqrt{\pi}x} + \left(1 - \frac{1}{2x^2}\right)\mathrm{Erf}(x)\right] \tag{10}$$

and α_j is the jth zero of Bessel function, $J_0(x)$.

The actual relationship of δ to r and θ depends substantially on the coil design and on the size and shape of the gap between the coils. In a typical case, to find the function $\delta(r,\theta)$, one must carry out computer calculations for the particular magnet design. However, a great number of calculations have shown that it is possible to write an approximate expression for the ripple $\delta(r,\theta)$, which is suitable for the purpose of these calculations, as

$$\delta(r,\theta) = \delta(r)\,g(\theta) \tag{11}$$

$$= [\delta_o + (\delta_a - \delta_o)(r/a)^m]\,\exp(-\beta\theta^2)\,,$$

where a is the plasma radius; δ_o and δ_a are the values of δ at r = 0 and r = a, respectively, with $\delta_o/\delta_a \sim 10^{-3} - 10^{-4}$; and β is a parameter which can be determined from the calculations and closely depends on the shape of the coils and plasma locations -- usually, $0.5 \leq \beta \leq 1$. The poloidal angle variation in the field ripple strength reduces the quantity $G(\alpha,\beta;\,\sigma)$ (depending on TF coil and on plasma shape as well as on the plasma location in the coil) by a significant amount in comparison with the results of $\beta = 0$ cases.

It is well known that deviations from axisymmetry in toroidal devices lead to the formation of loss regions that are related to the fact that particles trapped in the ripple mirrors tend to be localized and, therefore, are subject to an uncompensated ∇B drift motion. For the background plasma, collisions are usually strong enough to scatter the particles out of the ripple-trapping region in velocity space before they drift out of the plasma. At the higher ion energies typical of neutral beam injection into large devices (≥100 keV) or alpha particles produced from D-T fusion (3.5 MeV), collisional effects are much weaker and the fast ions trapped in the ripples can drift out of the plasma before collisional detrapping occurs. The loss for particles diffusing into the ripple loss region has been estimated analytically.[6] The calculations show that most of the alpha particles can be lost during the slowing down process; however, loss occurs primarily in energies on the order of or below 200-300 keV, so it appears that they deposit their energy before they escape from the system.

Numerical Evaluation

The ripple-induced ion heat conduction coefficients (Eqs. 5 and 6) were calculated for the typical ORNL/W TNS parameters given in Table I and compared with the total heat conduction loss due to the neoclassical (NC), trapped particle modes, and pseudoclassical (PS) coefficients as well as empirical (EMP) Alcator scaling (see Ref. 2 for complete list and expressions).

ORNL/DWG/FED-77812

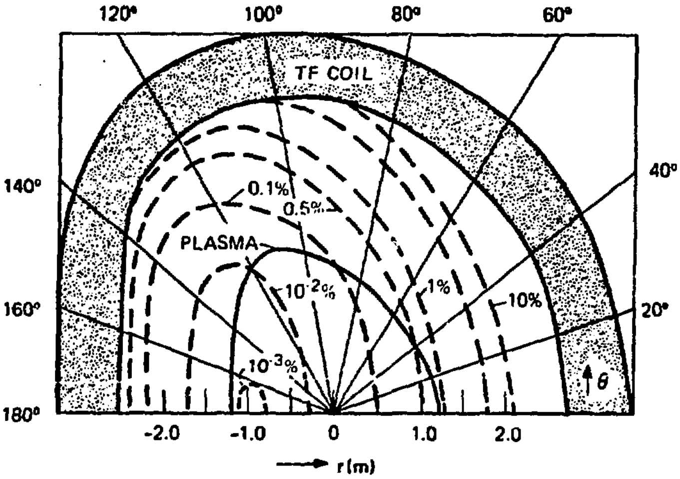

Fig. 1. Heat conduction coefficients vs plasma radius for TNs design.

Figure 1 represents the constant tension D-shaped TF coil configuration with D-shaped plasma cross section. Constant ripple contours are also shown in Fig. 1 for 20 TF coils. Equation 11 satisfactorily approximates the field in the cross section of plasma; thus for N = 20,

$$\delta(r,\theta) = [2.2 \times 10^{-4} + 8.3 \times 10^{-3}(r/a)^3]\,\exp(-0.85\theta^2), \tag{12}$$

and for N = 16

$$\delta(r,\theta) = [1.3 \times 10^{-3} + 2.3 \times 10^{-2}(r/a)^3]\,\exp(-0.7\theta^2). \tag{13}$$

In the calculation, a parabolic temperature profile, $T = T_o(1 - r^2/a^2)^s$, has been assumed with $n = n_o(1 - r^2/a^2)^p$; the safety factor q is calculated assuming a current density profile that varies as $T^{3/2}$ and given by

$$q(r) = q(a)\,\frac{(r/a)^2}{\left[1 - \left(1 - \frac{r}{a}^{2}\right)^{3/2s + 1}\right]} . \qquad (14)$$

From the MHD stability requirement the safety factor q at the plasma boundary must satisfy

$$q(a) > \frac{3}{2}s + 1 . \qquad (15)$$

Modifications to different plasma parameters and scaling relationships in noncircular cross-sections appropriate for large fusion-grade systems are not known; nonetheless estimates were made within the uncertainties posed in Ref. 2. Assuming the forms in Ref. 2, the results are shown in Fig. 2 for 16 and 20 TF coils. In both cases banana drift losses are much smaller than the losses due to ripple trapping. For 20 coils, the ion heat conduction coefficient associated with ripple trapping (RT) is comparable to those calculated from the ion neoclassical and electron pseudoclassical coefficients. The trapped ion and electron mode coefficients which are not shown in the figure are about two orders of magnitude larger than the ripple-induced heat conduction coefficients. Therefore, the overall enhanced heat loss due to ripples for 20 TF coils has a negligible effect on plasma confinement.

ORNL/DWG/FED-77813

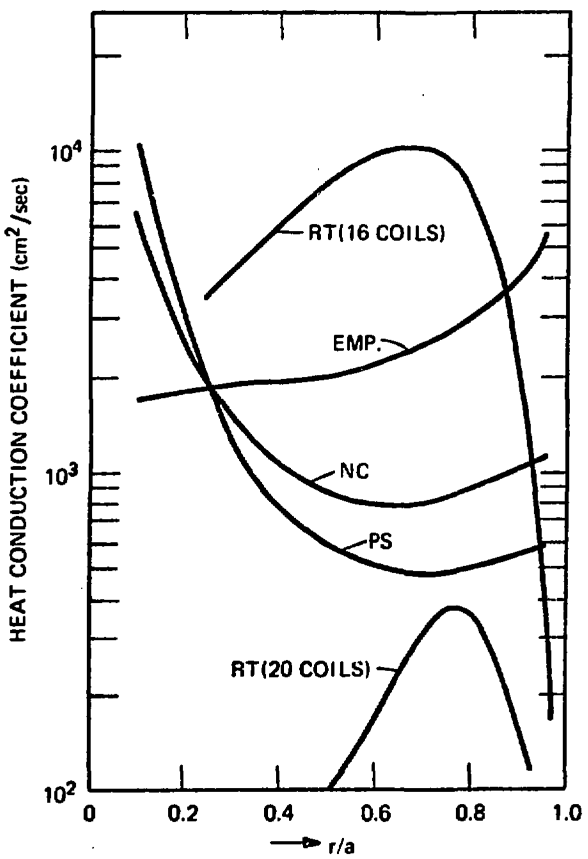

Fig. 2. Constant ripple contours for TNS (20 TF coils) design.

For 16 coils, the enhanced heat losses due to ripples become larger than those calculated from the empirical scaling but still smaller than that due to trapped particle instabilities. If one assumes Alcator-like empirical scaling for ignition, ripple losses become intolerable for this 16 TF coil case. On the other hand, if full trapped particle losses are assumed, with the profile effects taken into account, ripple losses may be tolerated. However, this problem should be studied in more detail with 1-D transport simulation models in order to see the profile effects (in a self-consistent manner) on ripple-induced losses before making a final statement.

Discussions

A major question that arises in the design of tokamak devices is the choice of the number of TF coils; this number is determined by calculating the maximum toroidal field ripple that can be tolerated from plasma physics considerations without seriously restricting the design of the mechanical support systems and by the necessity for access to the neutral beam injection. From the plasma physics point of view:
(1) The overall enhanced heat loss due to ripples should not seriously affect the plasma confinement.
(2) Fast ion losses due to ripple trapping should be kept at a minimum in order to have an insignificant effect on power balance.
(3) Due to penetration problems in high densities, perpendicular injection becomes necessary and the ripple value at the plasma edge should be kept small. The injection angle is then $\theta > \tan^{-1}\sqrt{2\delta}$.

It should also be noted that enhanced heat loss due to ripples near the plasma edge might help to reduce charge-exchange losses because there is a drop in ion temperature in that region where the neutral population is high. Also note that near the plasma center higher magnetic field ripple might be somewhat desirable for producing a viscous drag on toroidal rotations.[2]

In order to optimize the design with tolerable field ripple, acceptable access space, and reasonable cost, it is possible to change: (1) the number of TF coils, (2) the coil bore dimensions, (3) the shape of the TF coils, or (4) the plasma location in the TF coils. Trade-offs among these alternatives should be studied to provide the information necessary for optimization.

Table 1. Typical TNS Parameters

Plasma minor radius, a(m)	1.2
Major radius, R(m)	5.7
Elongation, σ(-)	1.6
Safety factor, q(a)	4.0
Particle density on axis, n(cm^{-3})	2.2×10^{14}
Ion temperature on axis, T_0(keV)	26
Toroidal field coil major radius, (m)	5.85
Coil inner bore, (m)	5.2 × 7.5
Coil outer bore, (m)	6.7 × 9.0
Number of TF coils, N(-)	16 or 20
Magnetic field on axis, B_T(T)	5.3
Plasma current, I(MA)	4.0

References

1. T. E. Stringer, Nucl. Fusion 12, 689 (1972); J. W. Connor and R. J. Hastie, Nucl. Fusion 13, 221 (1973).
2. N. A. Uckan et al., *Effects of the Poloidal Variation of the Magnetic Field Ripple on Enhanced Heat Transport in Tokamaks*, ORNL/TM-5438, Oak Ridge National Laboratory (June 1976); *Calculation of Magnetic Field Ripple Effects in Circular and Noncircular Tokamaks*, ORNL/TM-5603, Oak Ridge National Laboratory (September 1976); *Proc. of the Sixth Symp. on Eng. Problems of Fusion Research* 1105 (1975).
3. J. N. Davidson, Nucl. Fusion 16, 731 (1977).
4. J. C. Whitson et al., *Magnetic Field of a Flat Coil of Rectangular Cross Section in Arbitrarily Shaped Border*, UCCND-CSD-INF-56, Union Carbide Corporation (January 1976).
5. K. T. Tsang, *Banana Drift Diffusion in a Tokamak Magnetic Field with Ripples*, ORNL/TM-5630, Oak Ridge National Laboratory (November 1976); Nucl. Fusion, 17, 557 (1977).
6. N. A. Uckan, in *Plasma Engineering in a Deuterium-Tritium Fueled Tokamak*, Section 5, eds. D. G. McAlees and F. B. Marcus, ORNL/TM-5573, Oak Ridge National Laboratory (October 1976).