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AN EXPERIMENTAL DEVICE FOR MEASURING THE MOMENTUM OF DISPERSE GRANULAR MATERIALS*

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ABSTRACT

An experimental device for measuring the time averaged momentum associated with a steady stream of a disperse granular material has been developed. This paper presents the mathematical basis for the device and includes a discussion of using the momentum measurement to compute the local mass or energy fluxes. The analysis considers both nonuniform particle mass and nonuniform velocities for the various constituents of an aggregate material. The results of calibration experiments conducted with a prototype transducer are shown with theoretical predictions of these results.

INTRODUCTION

Consider a quasi-steady, disperse stream of particles intersecting a plane such that a finite amount of mass crosses the plane in some time interval. The particles comprising this mass may be nonuniform in size and in velocity. A significant physical quantity which could be used to characterize such a stream is the time averaged momentum flux. Equivalently, this problem can be posed by considering a collection of particles approaching and colliding with a solid surface, rather than crossing an imagined plane. The corresponding momentum flux is the flux incident of the surface provided that the particles being reflected by the surface do not interact with those of the incoming stream. The local, time averaged flux incident on a surface can, therefore, yield a characterization of the undisturbed stream.

If the mass and velocity distribution of the particles in a stream are known, the momentum flux described above may be used to compute the time-averaged mass flux crossing the plane or incident on the surface. The mass flux, like the momentum flux, is necessarily time averaged. The mass flux of a single translating particle is at best very poorly defined. One particle is either cut by the plane or it is not. As a consequence, the instantaneous mass flux of a number of particles sequentially crossing a plane appears as a series of infinite (or very large) pulses separated by zero values. In many cases, the instantaneous mass flux is not of primary interest. A more useful quantity may be the time averaged flux. This average is obtained by integrating the instantaneous flux over some time interval and dividing the result by the magnitude of the interval.

That the momentum flux incident on a surface is the flux of the undisturbed stream permits characterization of the undisturbed stream by measuring the impaction force of the particles striking the surface. A device based on this principle has been developed and tested. The following sections outline major aspects of the mathematical basis of the device. The results of calibration experiments are presented with the theoretical results.

GOVERNING EQUATIONS

The classic theory of impact is based on the impulse-momentum law for rigid bodies. Governing relations are well known [1]. For linear momentum, in vector notation these are

$$\Delta(m\vec{V}) = m(\vec{V}_1 - \vec{V}_0) = \int_{t_0}^{t_1} \vec{f} \, dt \tag{1}$$

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M is the mass, and V is the linear velocity, $\vec{f}$ is the force acting on the body, and t is time. The subscripts 0 and 1 indicate initial and final conditions. The times t_0 and t_1 indicate the beginning and end of the time interval over which the force is applied. In one dimension, Eq. (1) becomes

$$m(\vec{V}_{1z} - \vec{V}_{0z}) = \int_{t_0}^{t_1} \vec{f}_z \, dt \tag{2}$$

We note that the impulse-momentum relations do not require nor imply conservation of energy. They are valid independent of whether or not the impact is elastic. Furthermore, we note that the equations do not involve material properties as such. Although the contact time,

$$\Delta t_{01} = t_1 - t_0 \tag{3}$$

will depend on material properties, these properties do not enter the equations explicitly. Finally, we note that the times t_0 and t_1 need not be known. Because the external force acting on the particle is clearly zero when the particle is not in contact with the surface, the limits of integration in the impulse-momentum equations may be extended to plus or minus infinity. For example, Eq. 2 could be written

$$m(\vec{V}_{1z} + \vec{V}_{0z}) = \int_{-\infty}^{+\infty} \vec{f}_z \, dt \tag{4}$$

The result in either case is the same.

The simple relations presented above can be extended into a more useful domain by considering a particle impacting on a surface that is part of a classic spring-mass-damper system. A diagram of such a system is shown in Fig. 1. The problem now becomes one comprised of an impulse coupled to a system response or "ringing".

When the body mass M_b is much greater than the particle mass m, it is reasonable to assume that the force exerted on the body by the particle is that which would be exerted on a rigidly held surface $\vec{f}_z$. Therefore, the equation governing the system response is

$$F - c\dot{z} - M_b \ddot{z} = 0 \tag{5}$$

where a dot denotes differentiation with respect to time and

$$F = -kz \tag{6}$$

subject to some bounded initial conditions at time t_1. For any such initial conditions, the solution of Eq. 5 exhibits the properties [2].

$$z \to 0 \quad \text{as} \quad t \to \infty \tag{7}$$

and

$$\dot{z} \to 0 \quad \text{as} \quad t \to \infty \tag{8}$$

Integrating Eq. 5 once with respect to time, taking the limit as t goes to infinity and making use of Eq. 7 and Eq. 8 gives

$$\lim_{t\to\infty} \int_{t_0}^{t} F \, dt = \lim_{t\to\infty} \int_{t_0}^{t} \vec{f}_z \, dt \tag{9}$$

It is now possible to express the net change in linear momentum of a single particle in terms of the force measured in a spring-mass-damper system.

$$\Delta(m\vec{V}_z) = \int_{t_0}^{t_1} \vec{f}_z \, dt = \int_{t_0}^{t_2} F \, dt \tag{10}$$

provided that t_1 and t_2 are sufficiently large such that z and $\dot{z}$ are small.

Now consider an isolated multi-pulse series resulting from a finite number of particles impacting on a surface. The impact force due to any particle will add linearly with that of any other particles simultaneously in contact with the surface. Therefore, the net change in linear momentum of all i particles of a multiple particle series is simply the integral of the impact force over the total time interval of the series. The resulting impulse-momentum relation for an isolated multiple impact series is

$$\sum_i \Delta(m_i \vec{V}_{zi}) = \int_{t_0}^{t_1} \vec{f}_z \, dt = \int_{t_0}^{t_1} F \, dt \tag{11}$$

Dividing each side of Eq. 11 by the time interval Δt_{01} , the two integral terms are recognized as the time averages of $\vec{f}_z$ and F. Now, let N_j denote the number of particles whose size [3] lies in the range d_{aj} to d_{aj+1} which impact in the time interval Δt_{01}. Assuming that ΔV_{zi} and m_i are smooth, continuous functions of d_{ai}, Eq.11 can be summed over j as

$$\frac{1}{\Delta t_{01}} \sum_j N_j \, m_j \, \Delta\vec{V}_{zj} = \overline{F} \tag{12}$$

where a bar denotes time averaging and where m_j and ΔV_{zj} are evaluated at d_{aj}. The total mass impacting in the time interval Δt_{01} is simply

$$\sum_i m_i \equiv \sum_j N_j \, m_j \tag{13}$$

and the mass fraction of particles in the range d_{aj} to d_{aj+1} which impact in the time interval is

$$\beta_j \equiv N_j \, m_j / \sum_j N_j \, m_j \tag{14}$$

With these two definitions, Eq. 12 can be written in terms of the time averaged mass flow rate $\overline{\dot{M}}$.

$$\overline{\dot{M}} \sum_j \beta_j \, \Delta\vec{V}_{zj} = \overline{F} \tag{15}$$

For a steady, fully-developed disperse stream of particles the time averaged mass fraction of particles in a given size range passing a plane or impacting on a surface is the same as the bulk mass fraction in that range (i.e., as determined by sieve analysis). That is,

$$\beta_j = \eta_j = \phi m \, (d_{aj+1}) - \phi m \, (daj) \tag{16}$$

where η_j is the bulk mass fraction of the particles which lie in the size range d_{aj} to d_{aj}+1 and d_m is the cumulative distribution by mass.

With Eq. 16, the form of Eq. 15 suggests an integral. Taking the limit as $d_{aj+1} - d_{aj}$ approaches zero gives,

$$\overline{\dot{M}} = \overline{F} \int_0^\infty fm \, (d_a) \, \vec{V}_{0z} \, (d_a) \, dd_a \tag{17}$$

The frequency distribution by mass f_m is. by definition,

$$fm \, (d_a) \equiv \lim_{\Delta d_{aj} \to 0} \eta_j / \Delta d_{aj} \tag{18}$$

This function is well-defined for most particle systems and for any usual granular material.

The integral in Eq. 17 is, for steady-state conditions, a constant. Thus, the time averaged mass flow rate has been related to the time averaged impact force on the particles or the force as measured in a spring-mass-damper system. If the out-going velocity of each particle is nearly zero (either slightly positive or negative), then the relation depends only on their incoming velocities. Based on the force measured in a spring-mass-damper system, we then have

$$\bar{\dot{M}} = \bar{F} \int_0^\infty fm\ (d_a)\ \vec{V}_{0z}\ (d_a)\ dd_a \tag{19}$$

For particles in free-fall, the vertical velocity, as a function of size, can readily be computed. The right side of Eq. 19, except $\bar{F}$, can therefore be known in terms of properties of the particle system or granular material only. We note that the integral term in Eq. 19 is the equivalent momentum velocity [4] defined as

$$\bar{V}_p \equiv \int_0^\infty fm\ (da)\ V\ (d_a)\ dd_a \tag{20}$$

Thus, the time averaged momentum transport rate is

$$\bar{\dot{P}} = \bar{\dot{M}}\ \bar{V}_p = \bar{F} \tag{21}$$

Similarly, the equivalent energy velocity and corresponding energy transport rate can be defined.

$$\bar{V}_e \equiv \left[\int_0^\infty fm\ (d_a)\ V^2\ (da)\ dd_a\right]^{1/2} \tag{22}$$

$$\bar{\dot{E}} = 1/2\ \bar{\dot{M}}\ \bar{V}_e^{\,2} \tag{23}$$

THE DEVICE

A drawing of the instrument is shown in Fig. 2. It consists of two cylinders, one inside the other. The inner cylinder is supported on a hydraulic piston/cylinder attached to the outer cylinder. A pressure transducer is located at the bottom of the fluid reservoir. By measuring the fluid pressure, the vertical force applied to the inner cylinder can be determined.

$$\overline{F} = P_f AP \tag{24}$$

P_f is the fluid pressure and A_p is the area of the piston. For the prototype, $Ap = .11\ in^2$ [.71 cm^2]. The electric output of the transducer is related to the pressure by its sensitivity.

$$\varepsilon = S\ P_f \tag{25}$$

where S = 2.0 mV/PSI [.29 mV/KPA] for the prototype.

The inner cylinder contains two baffles; a conic section along the center line of the cylinder and a beveled ring along its inner perimeter. These are arranged so that any particle entering the top of the cylinder will strike one or both of the baffles and then exit through the bottom with a negligible velocity. Thus, the change in velocity of any particle, as it passes through the device, is equal to its initial incoming velocity. All of its momentum is transferred to the inner cylinder.

With these definitions and previous results, the theoretical electric output can be computed from equations 21, 24, and 25.

$$\varepsilon = S\ \bar{\dot{M}}\ \bar{V}p/Ap \tag{26}$$

or

$$\bar{\dot{M}} = (\varepsilon Ap)/(S\bar{V}p) \tag{27}$$

Equation 27 describes the total time averages mass flow through the device and not the mass flux. The flux is obtained by dividng each side by the cross-section area of the inner cylinder A_c. A_c = 25 in^2 [160 cm^2] for the prototype. When the mass flow rate or flux are known, the corresponding momentum or energy can be computed from equations 21 or 23.

CALIBRATION AND TESTS

The prototype device was tested using LLNL coarse fill (pea gravel) [3,5]. The material was dispensed from a container, through an orifice, 23 ft. [7.0 M] above the device. For free-fall from this elevation, the equivalent momentum velocity for LLNL coarse fill is [4] V_p = 34 ft/s [10m/s]

The time-averaged mass flow rate was determined from the total time of a test, the volume of material used and its bulk density. Using various size orifices, mass flow rates from about .25 to 3.0 lbm/s [.11 to 1.4 kg/s] were achieved.

Three methods of reducing the rough data to obtain a single time-averaged value of the electric output have been used. These were digital time averaging, high frequency analog filtering and simple curve fit through the strip chart record. All three gave nearly identical results in several test cases. The third method was adopted for general use.

The results of calibration tests are shown in Fig. 3. The theoretical response is also shown. The agreement seems very good over the range of flow rates considered. At a mass flow rate of about 4.0 lbm/sec [1.8 kg/s] the instrument begins to accumulate material which eventually fills the inner cylinder. During this transition, the device is weighing the accumulated gravel and measurement of the impact force is no longer accurate. The approximate rate at which this occurs is noted on the plot as choking.

CONCLUSIONS

An experimental device for measuring the momentum associated with a steady stream of a disperse granular material has been developed and tested. The measurement made with this device can be used to compute the mass, momentum or energy flux of such a stream. Tests have shown that measurements made when the mass flow rate is known are in good agreement with theoretical predictions of the response.

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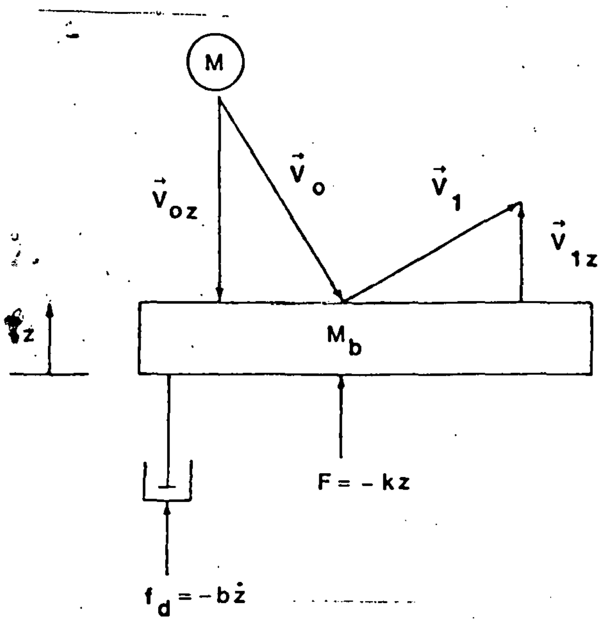

Figure 1

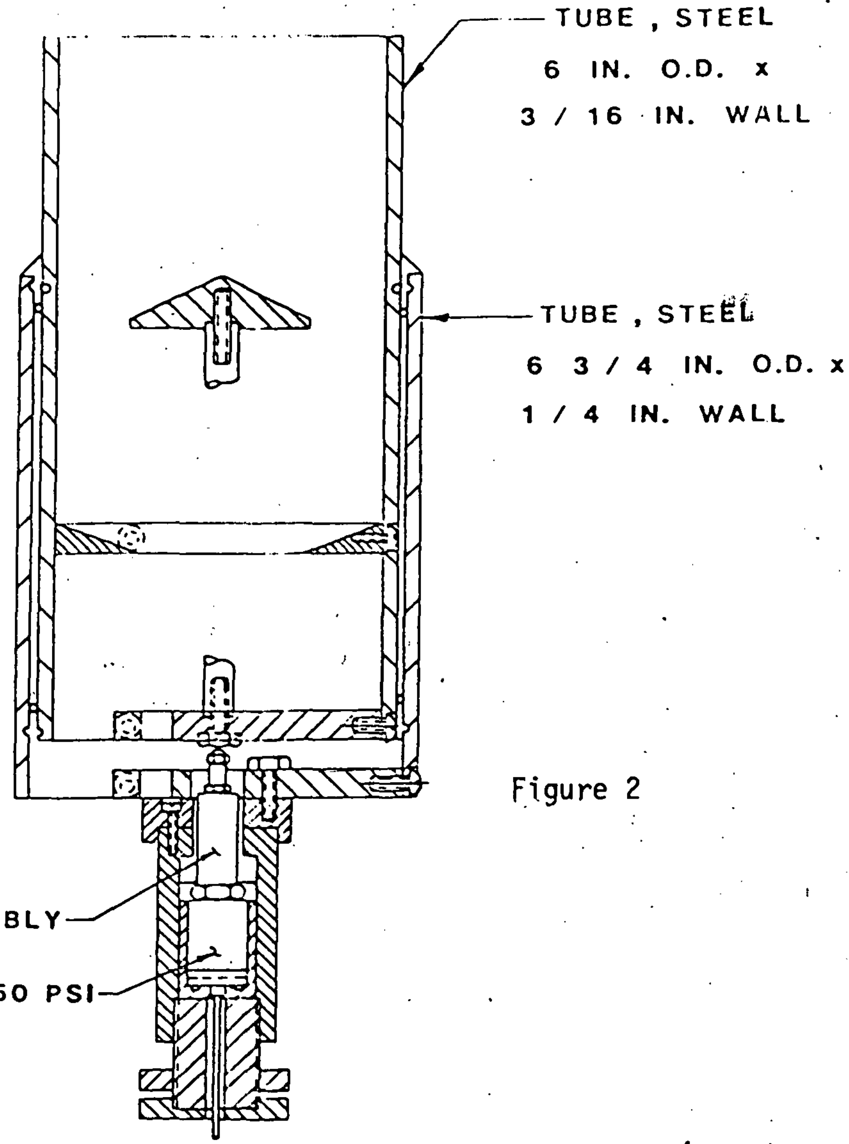

Figure 2

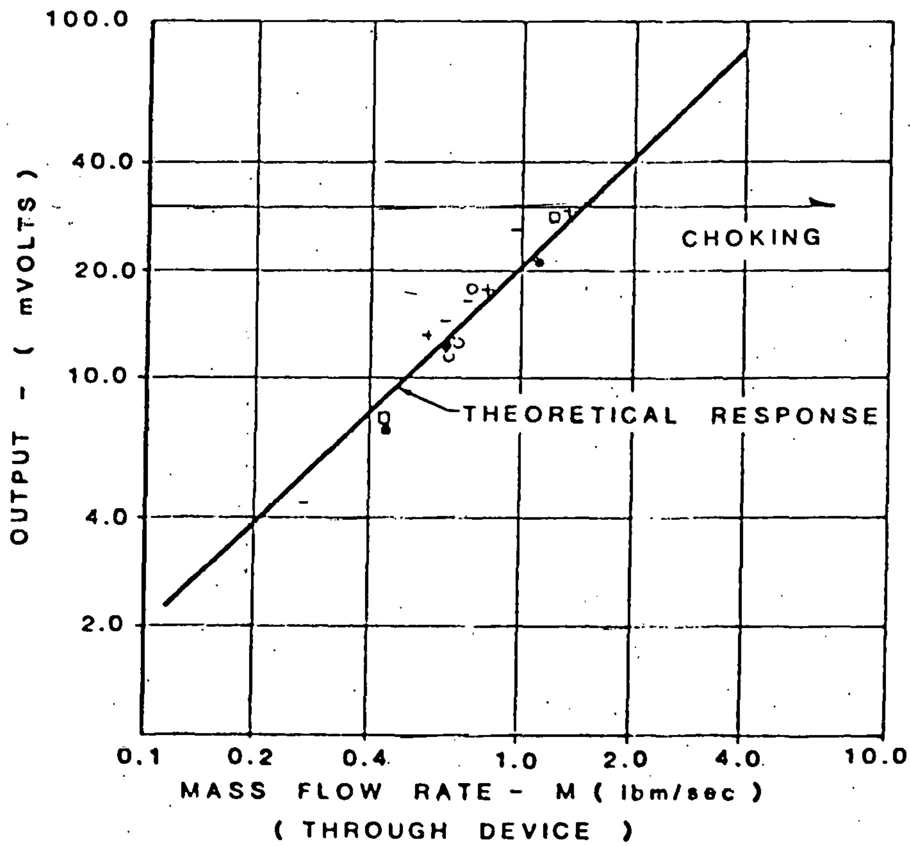

Figure 3

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