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MOMENTUM FLUX HYDRAULIC TERMS IN DECOMPRESSION CODES

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ABSTRACT

A one-dimensional transient flow equation was developed that includes an approximation to the momentum flux portion of the fundamental fluid momentum equation that describes the effects of fluid compressibility and flow area changes. Calculations from a version of the RELAP3 computer code modified to include the momentum flux terms were obtained and compared with experimental data. These comparisons showed that the accuracy of the calculations in regions of one-dimensional flow where large density gradients may exist can be increased by including the momentum flux terms in computer codes designed for the hydraulic analysis of postulated loss-of-coolant accidents in light-water nuclear reactors.

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Analysis of transient fluid processes, such as decompression of a hydraulic loop, usually involves the use of system describing computer codes such as RELAP3[1]. The fundamental equations describing such processes are numerically approximated to obtain a solution. In part, the accuracy of the current computer codes depends upon these assumptions and approximations.

Most computer codes used for transient hydraulic analysis neglect the momentum flux term which appears as part of the fundamental fluid flow equation. The usual assumption is that the major effect of this term can be represented by an equivalent friction term. Computer codes resulting from this simplification of the mathematics are easier and more economical to develop and use. A computer code that includes an explicit form of the momentum flux term, although more complex, does eliminate the need for some of the empirical coefficients that are used in the simpler mathematical approximations. The impetus for a closer examination of the assumption of neglecting the momentum flux terms was provided by a calculational anomaly.

The published version of the RELAP3 computer code used Moody's steady-state choked flow model[2] to describe two-phase fluid flow at a break between the high pressure fluid system and the low pressure sink. An empirical coefficient is used as a multiplier to make Moody's model applicable to actual experimental systems. As shown, this empiricism can be eliminated in some cases by the inclusion of the momentum flux term in the fluid flow equation.

The published version of the RELAP3 computer code occasionally displays a calculational difficulty referred to as "mass depletion". This condition arises when all the mass in one of the several control volumes is exhausted. The occurrence of this condition is obvious to the code user because the calculation is terminated when non-physical conditions such as negative mass occurs.

The exact cause of the mass depletion effect is not obvious. Several interrelated items are known to be effective in removing this calculational difficulty. For example, in certain cases, mass depletion occurs when the time-step size exceeds the numerical stability limit. Sometimes a change in the nodalization (control volume size) by the code user eliminates mass depletion because the numerical stability is related to nodalization.

In this paper, the particular case of the mass depletion effect is used as an example to illustrate the effect of correcting the flow equation by inclusion of the momentum flux terms. The following text presents a development of the flow equation that includes both compressible flow and area change effects applicable to one-dimensional homogeneous flow. Comparisons of experimental data with calculations from the published RELAP3 code and the version of RELAP3 modified with this new flow equation are presented. The momentum flux approach presented in this paper is being included as one of several modifications to the new RELAP4 code, which is to be released in the near future.

The following mathematical development for the momentum flux term is restricted to one-dimensional, homogenous flow. For simplicity, one-dimensional stream mixing effects such as occur in jet-pumps are neglected in this development.

MOMENTUM FLOW EQUATION

A generally accepted form of the fluid momentum conservation equation is[3]

$$\frac{\partial}{\partial t}(\rho \vec{v}) + \nabla \cdot (\rho \vec{v}\vec{v}) = -\nabla P - \nabla \cdot \vec{\tau} + \rho \vec{g} \quad (1)$$

Rate of increase of momentum per unit volume	Rate of momentum gain by convection per unit volume	Pressure force on element per unit volume	Rate of momentum gain by viscous transfer per unit volume	Gravitational force on element per unit volume
Accumulation term	Momentum flux term	Pressure term	Stress term	Gravitational term

where

t = time

ρ = fluid density

∇ = vector differential operator

v = fluid velocity

P = fluid pressure

$\vec{\tau}$ = fluid shear stress

$\vec{g}$ = gravitational acceleration.

For the purposes of this development, a simplified form of Equation (1) restricted to one-dimensional, stream-tube flow[4]

$$\frac{1}{A}\frac{\partial W}{\partial t} = -\frac{1}{A}\frac{\partial}{\partial x}(vW) - \frac{\partial P}{\partial x} - \frac{1}{A}\frac{\partial F_K}{\partial x} + \frac{\partial(\rho g z)}{\partial x} \quad (2)$$

where

W = mass flow ($\rho v A$)

A = flow area

F_K = frictional force term

x = distance along the fluid stream path

z = vertical distance.

The particular stream tube geometry assumed for this development is shown in Figure 1. The flow path consists of two constant area sections of lengths $l_1/2$ and $l_2/2$ that are joined together at the station labeled j_n. The inlet flow plane, Station 1, has a boundary pressure, p_1, and an inlet flow, $\rho_1 v_1 A_1$; likewise, the outlet flow plane, Station 2, has a boundary pressure, P_2, and an outlet flow, $\rho_2 v_2 A_2$.

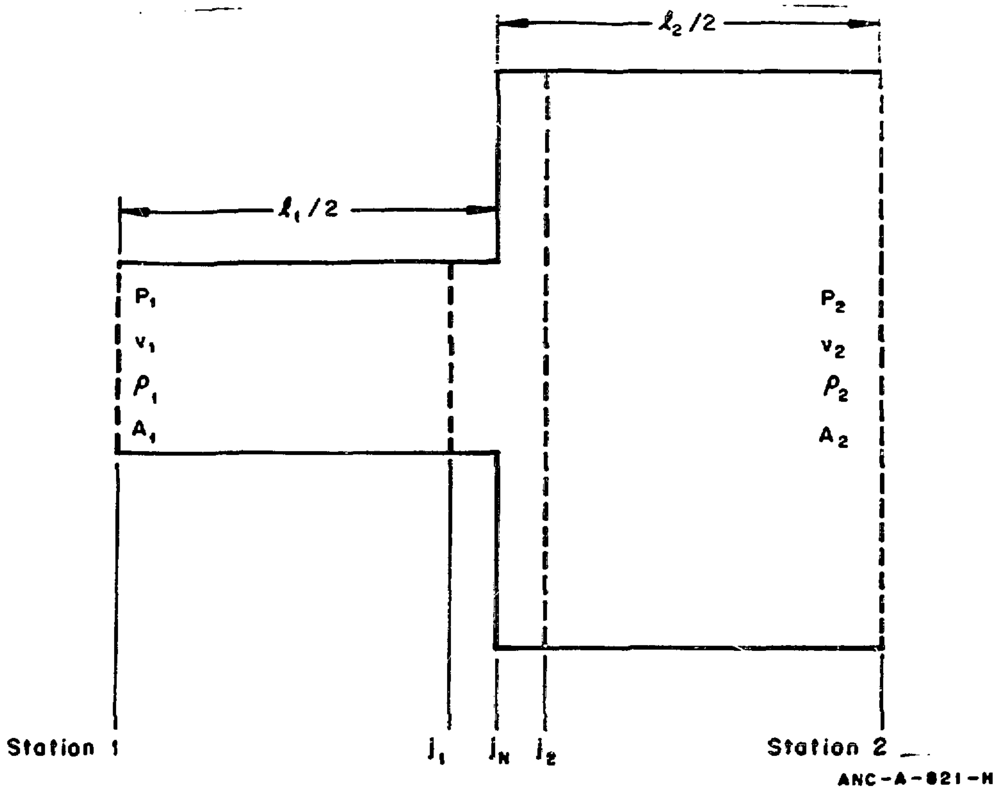

FIG. 1 CONTROL VOLUME FOR INTEGRATION OF FLOW EQUATIONS.

The term on the left side of Equation (1) is integrated with respect to x from Station 1 to Station 2 by first interchanging the order of differentiation and integration in the following manner:

$$\int_1^2 \frac{1}{A} \left(\frac{\partial W}{\partial t} \right) dx = \frac{\partial}{\partial t} \int_1^2 \frac{W}{A} \, dx \, . \tag{3}$$

By assuming an average value of W, the integral reduces to

$$\frac{\partial \overline{W}}{\partial t} \int_1^2 \frac{dx}{A}$$

where the bar denotes an average value of W and its location in space has not been specified.

The integration between Stations 1 and 2 can be split into two parts:

$$\frac{\partial \overline{W}}{\partial t} \int_1^2 \frac{dx}{A} = \frac{\partial \overline{W}}{\partial t} \left\{ \frac{1}{A_1} \int_1^{j_n} dx + \frac{1}{A_2} \int_{j_n}^2 dx \right\} \tag{4a}$$

$$= \frac{\partial \overline{W}}{\partial t} \left\{ \frac{\ell_1}{2A_1} + \frac{\ell_2}{2A_2} \right\} . \tag{4b}$$

By defining

$$I = \frac{\ell_1}{2A_1} + \frac{\ell_2}{2A_2}$$

then

$$\int_1^2 \frac{1}{A} \frac{\partial W}{\partial t} dx = I \dot{\overline{W}}.$$

where $\frac{\partial W}{\partial t}$ is defined as $\dot{\overline{W}}$.

In order to integrate the momentum flux term, the region of integration is split into three parts as

$$-\int_1^2 \frac{1}{A} \frac{d(vW)}{dx} dx = -\int_1^2 \frac{1}{A} d(vW) \tag{5a}$$

$$= -\left\{ \int_1^{j_1} \frac{1}{A} d(vW) + \int_{j_1}^{j_2} \frac{1}{A} d(vW) + \int_{j_2}^2 \frac{1}{A} d(vW) \right\}. \tag{5b}$$

Each term is now integrated by parts and the resulting terms are

$$-\int_1^2 \frac{1}{A} \frac{d(vW)}{dx} dx = -\frac{v_2 W_2}{A_2} + \frac{v_1 W_1}{A_1} + \int_{j_1}^{j_2} vW\, d(\frac{1}{A}). \tag{6}$$

The rightmost term may be further simplified by assuming that the distance between j_1 and j_2 is infinitesimal and thus W is constant between j_1 and j_2, such that

$$\int_{j_1}^{j_2} vWd(\frac{1}{A}) = \int_{j_1}^{j_2} vWd(\frac{v\rho}{W}) = \int_{j_1}^{j_2} vd(\rho v) \tag{7a}$$

$$= \int_{j_1}^{j_2} \rho v dv + \int_{j_1}^{j_2} v^2 \, d\rho \tag{7b}$$

$$= \int_{j_1}^{j_2} \frac{\rho}{2} dv^2 + \int_{j_1}^{j_2} v^2 \, d\rho. \tag{7c}$$

Additionally, integration of the rightmost term by parts yields

$$\int_{j_1}^{j_2} vWd(\frac{1}{A}) = \int_{j_1}^{j_2} \frac{\rho}{2} dv^2 + (v^2\rho)_{j_2} - (v^2\rho)_{j_1} - \int_{j_1}^{j_2} \rho dv^2 \tag{8a}$$

$$= (v^2\rho)_{j_2} - (v^2\rho)_{j_1} - \int_{j_1}^{j_2} \frac{\rho}{2} dv^2 . \tag{8b}$$

At this point, two key assumptions are necessary to complete the integration. The first assumption is that the properties immediately downstream from the area change can be approximated by the equations describing an isentropic steady state expansion where the friction neglected in the isentropic expansion is accounted for separately as an additional friction term. Secondly, the sonic velocity is assumed constant throughout the area change.

The steady state isentropic flow equation is

$$dP + \rho v dv = 0 = c^2 d\rho + \rho v dv = c^2 d\rho + \frac{\rho}{2} dv^2 \qquad (9a)$$

where c is the sonic velocity

or

$$\frac{\rho}{2} dv^2 = - c^2 d\rho . \qquad (9b)$$

Utilizing this relationship and assuming a constant sonic velocity allows the following integration:

$$-\int_{j_1}^{j_2} \frac{\rho}{2} dv^2 = c^2 \int_{j_1}^{j_2} d\rho = c^2 (\rho_{j_2} - \rho_{j_1}) . \qquad (10)$$

Therefore, Equation (7a) reduces to

$$\int_{j_1}^{j_2} v\, d\,(\rho v) = (v^2\rho)_{j_2} - (v^2\ )_{j_1} + c^2 (\rho_{j_2} - \rho_{j_1}) \qquad (11a)$$

$$= \rho_{j_2} (v^2_{j_2} + c^2) - \rho_{j_1} (v^2_{j_1} + c^2). \qquad (11b)$$

By designating the inlet (j_1) with the subscript o and dropping the subscript on the outlet (j_2) and allowing the velocity to go either direction, the preceding equation becomes

$$\int_{j_1}^{j_2} vd\ (\rho v) = s\ [\rho (v^2 + c^2) - \rho_0\ (v_0^2 + c^2)] \qquad (12)$$

where s = ± 1 depending on the direction of the velocity.

The previous equation is now rearranged to give

$$\int_{j_1}^{j_2} vWd\left(\frac{1}{A}\right) = \frac{\frac{sW^2}{2\rho_0} \left\{ \frac{1}{A^2} - \frac{1}{A_0^2} \right\} \left\{ \frac{2\rho}{\rho_0} \left(\frac{M^2}{M_0^2} + \frac{1}{M_0^2}\right) - \left(1 + \frac{1}{M_0^2}\right) \right\}}{A_0^2 \left[\frac{1}{A^2} \quad \frac{1}{A_0^2} \right]} \qquad (13)$$

where M is the Mach number.

If the flow were incompressible, the value of the integral would simply be

$$\int_{j_1}^{j_2} vWd\ \left(\frac{1}{A}\right) = \frac{W^2}{2\rho_0} \left(\frac{1}{A^2} - \frac{1}{A_0^2}\right) . \qquad (14)$$

As can be seen by examination of Equations (13) and (14), the compressible form is the product of the incompressible form and another term; that is,

$$\int_{j_1}^{j_2} vWd\left(\frac{1}{A}\right) = \frac{W^2}{2\rho_0}\left(\frac{1}{A_2^{\,2}} - \frac{1}{A_1^{\,2}}\right) F_j \tag{15}$$

where

$$F_j = \frac{2\left[\frac{\rho}{\rho_0}\left(\frac{M^2}{M_0^{\,2}} + \frac{1}{M_0^{\,2}}\right) - \left(1 + \frac{1}{M_0^{\,2}}\right)\right]}{A_0^{\,2}\left[\frac{1}{A^2} - \frac{1}{A_0^{\,2}}\right]} \quad . \tag{16}$$

The relationship between the inlet and outlet properties can be determined from the classic solutions to the isentropic flow equation:

$$\frac{\rho}{\rho_0} = e^{-\left(\frac{M^2 - M_0^{\,2}}{2}\right)} \tag{17}$$

and

$$\left(\frac{A_0}{A}\right)^2 = \frac{M^2}{M_0^{\,2}} \quad e^{-(M^2 - M_0^{\,2})} \quad . \tag{18}$$

The remainder of the terms in the flow equation were integrated directly in the following manner:

$$-\int_1^2 dP = P_1 - P_2 \quad , \tag{19}$$

$$-\frac{1}{A}\int_1^2 dF = -\frac{1}{A}\int_1^{j_1} dF - \frac{1}{A}\int_{j_1}^{j_2} dF - \frac{1}{A}\int_{j_2}^{2} dF \tag{20a}$$

$$= -\frac{s_1}{\pi r_1^{\,2}}\left[2\pi r_1 \left(\frac{\ell_1}{2}\right) f \left(\frac{\rho v_1^{\,2}}{2}\right)\right] - \frac{s_j}{A}\left[kA\frac{\rho v^2}{2}\right] - \frac{s_2}{\pi r_2^{\,2}}\left[2\pi r_2 \left(\frac{\ell_2}{2}\right) f \left(\frac{\rho v_2^{\,2}}{2}\right)\right] \tag{20b}$$

$$= -s_1 \left(\frac{f_1 \ell_1}{D_1}\right) \rho v_1^{\,2} - s_j k \frac{\rho v_j^{\,2}}{2} - s_2 \left(\frac{f_2 \ell_2}{D_2}\right) \rho v_2^{\,2} \tag{20c}$$

where

s_i = ± 1 depending on the direction of flow

f = friction factor

k = loss coefficient

r = hydraulic radius

D = hydraulic diameter

and

$$- \int_1^2 g \rho dz = P_{g_1} - P_{g_2} \qquad (21)$$

where P_g is the gravity head between the center of mass and the junction.

Collection of all the terms results in the final integrated form of the momentum equation:

$$\underbrace{\frac{1}{2}\left(\frac{\ell_1}{A_1} + \frac{\ell_2}{A_2}\right)\frac{\partial \overline{W}}{\partial t}}_{\text{Accumulation of momentum}} = \underbrace{\left(\frac{v_1 W_1}{A_1}\right)}_{\text{Momentum entering control volume by convection}} + \underbrace{P_1}_{\text{Static pressure at entrance of control volume}} + \underbrace{P_{g_1}}_{\text{Pressure due to gravity at entrance}}$$

$$- \underbrace{\left(\frac{v_2 W_2}{A_2}\right)}_{\text{Momentum leaving control volume by convection}} + \underbrace{P_2}_{\text{Static pressure at exit of control volume}} + \underbrace{P_{g_2}}_{\text{Pressure due to gravity at exit}})$$

$$+ \underbrace{\frac{W_1^2}{2\rho_{j0}}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) \frac{2\left[\frac{\rho}{\rho_0}\left(\frac{M^2}{M_0^2} + \frac{1}{M_0^2}\right) - \left(1 + \frac{1}{M_0^2}\right)\right]}{A_o^2\left[\frac{1}{A^2} - \frac{1}{A_0^2}\right]}}_{\text{Momentum change due to area change}}$$

$$- \underbrace{S_1 f_1 \left(\frac{\ell_1}{D_1}\right)\frac{W_1^2}{\rho_1 A_1^2}}_{\text{Frictional pressure drop due to wall friction in section 1}} - \underbrace{S_2 f_2 \left(\frac{\ell_2}{D_2}\right)\frac{W_2^2}{\rho_2 A_2^2}}_{\text{Frictional pressure drop due to wall friction in section 2}} - \underbrace{S_j \frac{K_j W_j^2}{2\rho_j A_j^2}}_{\text{Frictional pressure drop due to area change}} \qquad (22)$$

The published version of RELAP3 includes the gravity and pressure terms identified in Equation (22). Furthermore, the three friction terms in Equation (22) are combined as an equivalent friction term proportional to the square of junction flow, $s_j W_j^2$. For incompressible flow, the terms describing momentum transfer into and out of the control volume along with the momentum area change term reduce to the classic Bernoulli form, $\frac{W_1^2}{2\rho_j}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right)$. This term, because of the flow squared dependence, can be grouped with the friction term if the flow does not change direction. Thus in RELAP3, the simplified friction term includes several different frictional effects and an approximate Bernoulli effect for undirectional flow.

To test the effects of Equation (22) in comparison to that used in the published RELAP3 code, a special modified version of RELAP3 was produced which includes explicitly all terms appearing in Equation (22). Since both the kinetic energy and momentum flux terms are physically related; for consistency, the energy flow equation was also changed to include kinetic as well as thermodynamic energy. Calculated results from these two version of RELAP3, the published version and the version modified for the momentum flux terms, were compared with results from several simple experiments.

COMPARISONS WITH EXPERIMENTAL DATA

Two different sets of experimental data obtained from simple systems are presented for comparisons in the following text. These systems were chosen for discussion because both emphasize the differences between the calculations from the two RELAP3 versions.

One set, the data of Edwards[5], obtained from a simple pipe blowdown experiment is used to illustrate the mass depletion effect. The second set of data obtained from the semiscale experiment operated by Aerojet Nuclear Company[6] is used to illustrate the differences in the break flow models used in the published and modified RELAP3 codes[a]. Basically, the semiscale system was a horizontal blowdown pipe connected to a vertical vessel.

Edwards' Data

Edwards' blowdown system consisted basically of a straight pipe, 13.44 feet long with an internal diameter of 2.88 inches. Seven pressure measurements were taken along the length of the pipe at stations designated: GS1, GS2, . . .GS7. Station GS1 is closest to the break and Station GS7 is closest to the closed end. The initial conditions for the experimental test were reported as 1000 psig and 467°F. However, experimental saturation pressures indicated that the temperature was considerably lower and varied along the length of the pipe, thus the initial temperatures used for these analytical studies were determined by using the experimental saturation pressures. The resulting temperature profile used for the RELAP3 calculations is shown in Figure 2.

Computer calculations for this system were made using both the published and modified versions of RELAP3. These versions were dimensioned to accommodate a maximum of 20 control volumes. Edwards' system was modeled with 19 control volumes to ensure an adequate description of the density gradients expected during the transient. This nodalization was sufficiently detailed to describe each pipe station of the experiment.

Comparison of pressure histories at selected pipe stations are presented in Figure 3. The calculations from the published version of RELAP3 were prematurely terminated about one-third of the way through the transient due to one of the RELAP3 volumes becoming depleted of mass as mentioned previously. Up to the time of the termination,

[a] Other analytical studies involving RELAP3 analysis of semiscale results are presented in Reference 7.

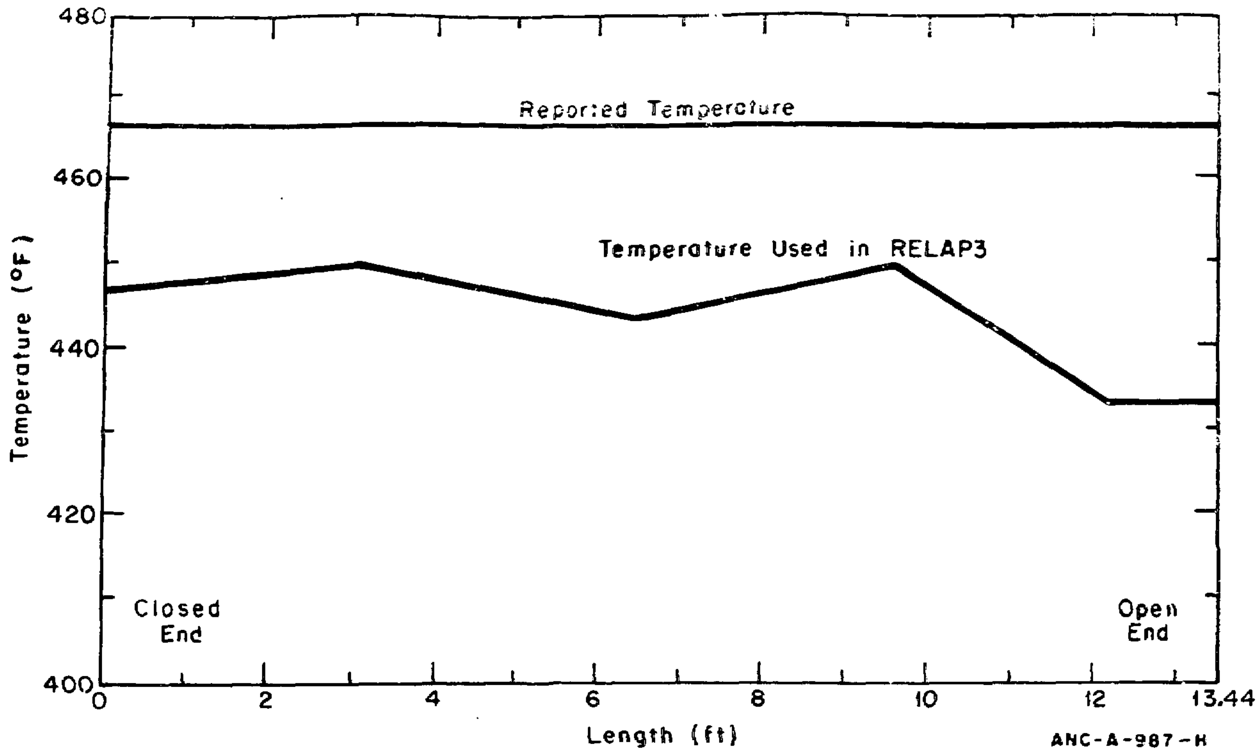

FIG. 2 TEMPERATURE PROFILES IN EDWARDS' EXPERIMENTAL SYSTEM.

agreement between experimental and published RELAP3 calculations was adequate. Break flow in the published version of RELAP3 is calculated from Moody's model with an empirical correction multiplier[2].

Figure 3 also shows the calculations from the version of RELAP3 modified by the addition of the momentum flux terms. As can be seen, the agreement between these calculations and experimental pressures is quite good throughout the entire transient. Since the flow area in the Edwards' system was constant, these results indicate that large density gradients existed during the transient.

Semiscale Test 710 Data

The basic semiscale system is shown in Figure 4. This system was a simple vertical vessel with an attached horizontal pipe. Test 710 used the top horizontal pipe for depressurization with a fully open break area of 0.09 ft^2 that was the same as the pipe flow area. The vessel and the pipe were initially filled with subcooled liquid water at 2350 psia and 540°F. Both one-volume and two-volume models were used to examine the effects of area changes on each version of RELAP3.

In the two-volume RELAP3 model used to describe the system, one volume represented the vessel and the second volume described the pipe. Two calculations were made using the published versions of RELAP3 with Moody-flow multipliers of 0.6 and 1.0. These multipliers are constant factors used to modify the flow predicted by Moody's model. Figure 5 presents the comparisons between the experimental pressure data and the RELAP3 pressure calculations.

As can be seen in Figure 5, the agreement between published versions of RELAP3 and the experimental data is quite good for a multiplier of 0.6, but very poor for a multiplier of 1.0. The calculated results from the modified version of RELAP3 with the momentum flux terms, but without the empirical multiplier, also agree quite well with the experimental pressures.

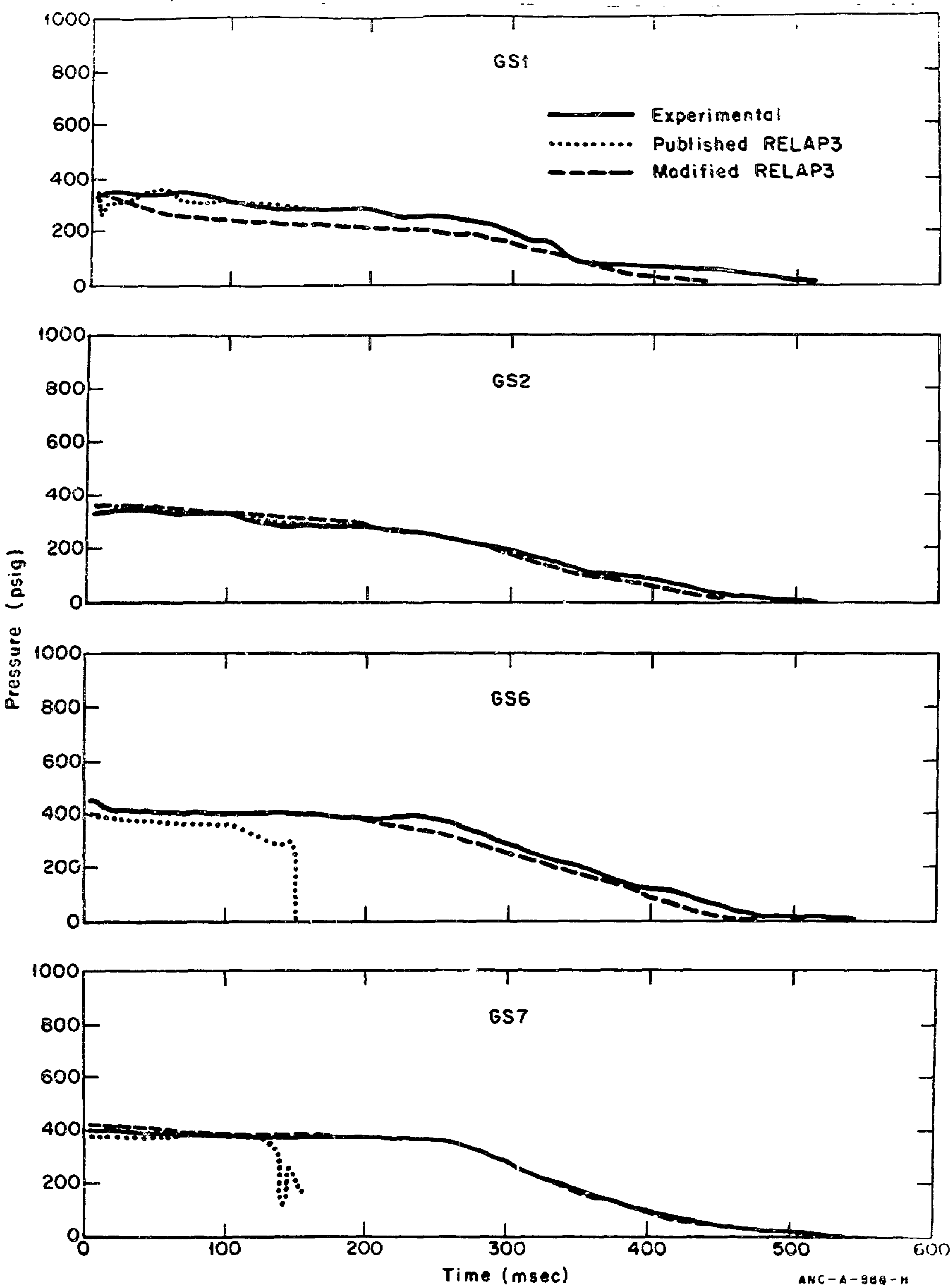

FIG. 3 COMPARISON OF PRESSURE CALCULATIONS FROM PUBLISHED AND MODIFIED VERSIONS OF RELAP3 WITH DATA FROM EDWARDS' EXPERIMENTAL SYSTEM.

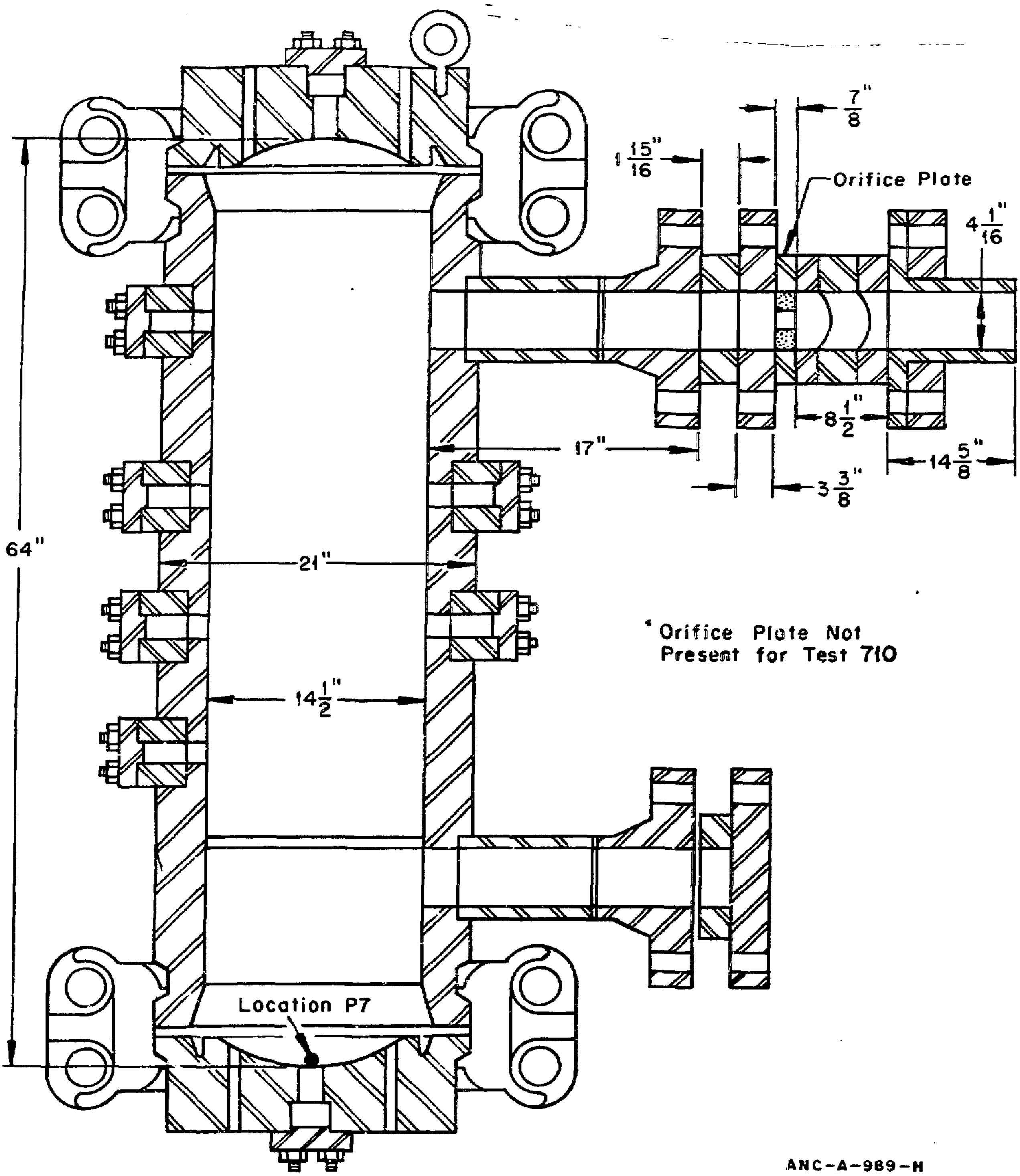

FIG. 4 SEMISCALE VESSEL FOR TEST 710.

The calculations for the one-volume model are presented in Figure 6. The single control volume combined both the pipe and vessel and, therefore, all phenomena occurring at the junction between the vessel and the pipe were neglected.

The agreement between the pressure calculations from the published version of RELAP3 using a multiplier of 0.6 and the experimental data is not as good as that achieved between the two-volume calculations and the experimental data. The pressure calculation from the modified RELAP3 code, also shown in Figure 6, is very different from the experimental pressure. Since the one-volume model omits the junction area description, momentum effects occurring at the junction are also neglected and thus the poor agreement results.

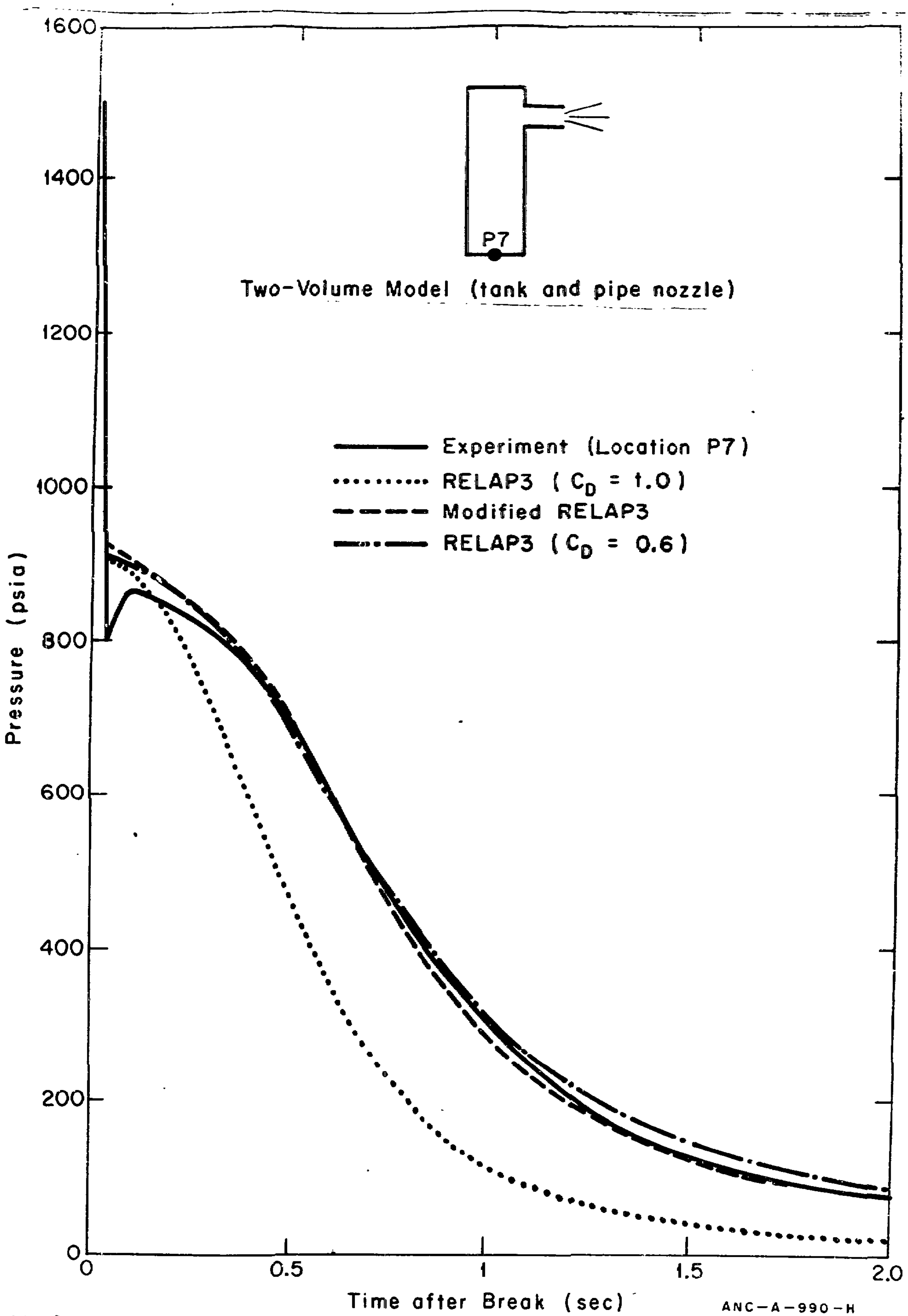

FIG. 5 SEMISCALE TEST 710, VESSEL PRESSURE AS FUNCTION OF BLOWDOWN TIME -- TWO-VOLUME MODEL.

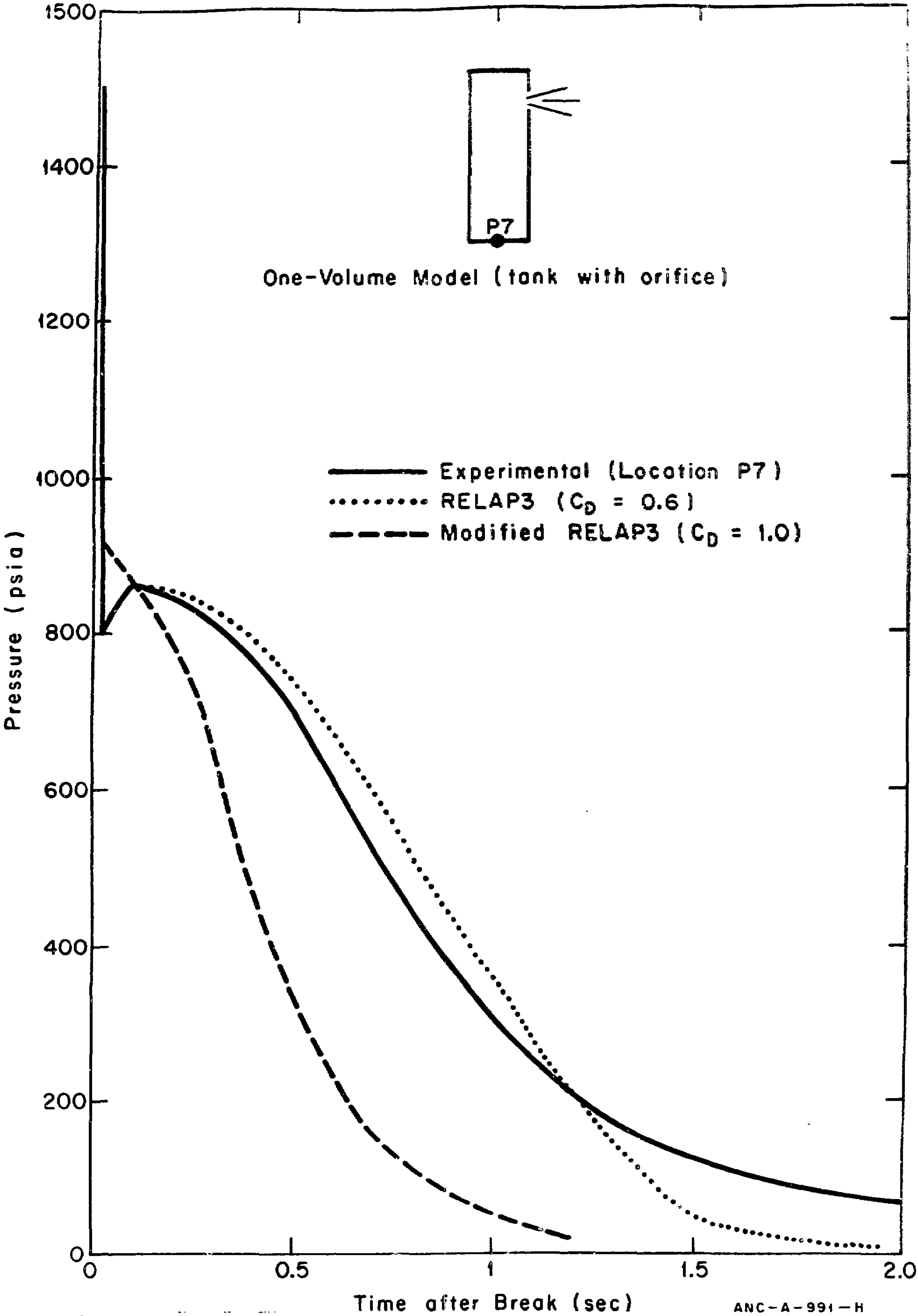

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FIG. 6 SEMISCALE TEST 710, VESSEL PRESSURE AS FUNCTION OF BLOWDOWN TIME -- ONE-VOLUME MODEL.

CONCLUSIONS

The major conclusion based on the results of this study was that the momentum flux terms can improve the accuracy of calculated decompression transients. The addition of the momentum flux terms to RELAP3 produced calculations that compared quite well to the two sets of experimental data presented. The results from the published version of RELAP3 also compared favorably with the experimental data, but with two qualifications: (1) the published version of RELAP3 occasionally exhibits a calculational difficulty in which a volume is depleted of mass, which causes the calculation to be prematurely terminated, and (2) the published version of RELAP3 used a break flow model with an empirical coefficient that was found unnecessary in the momentum flux modified versions. These results also indicate the feasibility of calculating critical flow as an inherent part of the momentum equation, rather than using a separate critical flow model.

That momentum flux terms are applicable to systems with no area changes was shown by the comparison of calculations from the modified RELAP3 code to Edwards' data. The effect of the momentum flux terms in this constant area case was attributed to large density gradients within the experiment during the transient.

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