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Informal Report

INCOHERENT TUNE SHIFTS FOR THE ISA

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N O T I C E

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ABSTRACT

The vertical tune shifts due to space charge and due to images are computed for two possible sets of ISA magnet parameters. A circular geometry corresponds to a design suitable for a synchrotron with very high field magnets, ~ 40 kG. This is referred to as the cold case. A second possibility is a larger radius synchrotron with low field magnets, ~ 13 kG. The main feature in this case, referred to as the warm case, is a very small gap height in the magnets. Parallel plate geometry is assumed. The warm case, as presented here, is clearly unacceptable, leading to an incoherent ν-shift of almost 3 full units. Given the following three assumptions: 1. that the injection energy is 30 GeV so that the AGS can be used as an injector to the ISA; 2. that the tune be about 20 in order that the transition energy be sufficiently below the injection energy; and 3. that the circulating current not be less than 15 A, so that sufficient luminosity be attainable when the beams of the intersecting rings collide; then, with these three assumptions, a simple relation connecting the tune shifts for the cold and warm cases is obtained. It is concluded that, in order to bring the image tune shift for the warm case to an acceptable value, the radius must be decreased and the magnet gap height must be increased. In fact, the required changes tend to lead the design strongly in the direction of the original cold case.

1. Introduction

We compute the incoherent vertical tune shifts for two possible sets of ISA[1] design parameters. We consider first a circular geometry which corresponds to a design suitable for a synchrotron with very high field magnets, ~ 40 kG. This is referred to as the cold case. A second possibility is a larger radius synchrotron with low field magnets,[2] ~ 13 kG. The main feature in this case, referred to as the warm case, is a very small gap height in the magnets. Parallel plate geometry is assumed. There are essentially two contributions to the incoherent vertical tune shift. First, there is the space-charge tune shift, which is dependent on the beam dimensions. If one includes the effects of metallic boundaries, i.e., the vacuum chamber wall, and ferromagnetic boundaries, i.e. the magnetic pole surfaces, then one obtains the image contributions to the tune shift. These depend on the chamber and magnet geometry and only inconsequentially on the beam dimensions. As we will see, it is the image contributions which produce the design limitations.

2. Vertical Incoherent Tune Shift

The vertical incoherent tune shifts, $\Delta\nu_{sc}$ due to space charge, and $\Delta\nu_{im}$ due to images, can be written[3]

$$\Delta\nu_{sc} = - \frac{Nr_p R}{\pi\beta^2\gamma^3\nu b(a + b)} \quad , \tag{1}$$

and

$$\Delta\nu_{im} = - \frac{Nr_p R}{\pi\beta^2\gamma\nu} \left[\frac{\epsilon_1}{h^2} + \beta^2 \frac{\epsilon_2}{g^2} \right] \quad , \tag{2}$$

where
- N = total number of particles in the ring,
- R = average synchrotron radius,
- β = particle velocity in units of c,
- γ = energy in units of proton mass,
- ν = vertical tune,
- b = vertical half-size of beam,
- a = horizontal half-size of beam,
- r_p = classical proton radius, = 1.54×10^{-18} m,
- h = chamber half-height, and
- g = magnetic gap half-height.

The quantities ϵ_1 and ϵ_2 are geometry dependent terms and relate to the presence of chamber and magnet pole boundaries, respectively. They are easily evaluated[3] for circular and parallel plate geometry. For the circular chamber, $\epsilon_1 = 0$, while for the narrow height chamber, $\epsilon_1 = \pi^2/48$. The ϵ_2 term is not sensitive to the geometry[3] and we assume $\epsilon_2 = \pi^2/24$.

We can thus compute the tune shifts for various designs.

1. Cold ISA: $a = 0.76$ cm, $b = 0.76$ cm, $\gamma = 30$, $\nu = 20.25$, $R = 385.5$ m, $N = 7.5 \times 10^{14}$ ($I = 15$ A), $h = 3.5$ cm, $g = 7.5$ cm, $\epsilon_1 = 0$, $\epsilon_2 = \pi^2/24$.

$$\Delta\nu_{sc} = - 2.2 \times 10^{-3}$$

$$\Delta\nu_{im} = - 0.017 \quad .$$

2. Warm ISA: $a = 2.27$ cm, $b = 0.42$ cm, $\gamma = 30$, $\nu = 20.25$, $R = 899.5$ m, $N = 1.75 \times 10^{15}$ ($I = 15$ A), $h = 1.35$ cm, $g = 2.0$ cm, $\epsilon_1 = \pi^2/48$, $\epsilon_2 = \pi^2/24$.

$$\Delta\nu_{sc} = - 1.25 \times 10^{-2}$$

$$\Delta\nu_{im} = - 2.73 \quad .$$

(Note the unacceptable ν-shift due to images.)

3. ISR (Only image contributions): $\gamma = 30$, $\nu = 8.75$, $R = 150$ m, $N = 2.94 \times 10^{14}$ ($I = 15$ A), $h = 2.5$ cm, $g = 3.5$ cm (est.), $\epsilon_1 = \pi^2/48$, $\epsilon_2 = \pi^2/24$.

$$\Delta\nu_{im} = - 0.05 \quad .$$

4. NAL (Only image contributions): $\gamma = 11.66$, $\nu = 20.25$, $R = 1000$ m, $N = 5 \times 10^{13}$ ($I = 0.38$ A), $h = 2.4$ cm (1.8 cm), $g = 2.5$ cm (1.9 cm), $\epsilon_1 = \pi^2/48$, $\epsilon_2 = \pi^2/24$.

$$\Delta\nu_{im} = - 0.10 \; (- 0.19) \quad .$$

(Note the small current here. This is acceptable for an accelerator where particles will ultimately bombard a fixed target, but not acceptable for storage ring design.)

Let us consider the relationship of cold to warm ISA a little more generally. The connection between the image contributions to the tune shifts for the two cases can be made very simple if we assume the following three general requirements for the design. We assume first that the injection energy is 30 GeV. This is clearly related to the natural choice of the AGS as an injector to the ISA. Note, however, that higher energy injection will only decrease the ν-shift due to images linearly. A second assumption will be that the tune should be about 20. This follows from the desire for high ν but transition energy sufficiently below 30 GeV. This means that ν must be sufficiently below 30. And thirdly, we assume that the circulating current should be 15 A; this is to ensure sufficient luminosity when the beams of the two intersecting rings collide. Then, with these assumptions, and using Eq. (2), we have

$$\frac{\Delta\nu_w}{\Delta\nu_c} = \frac{R_w^2}{R_c^2}\,\frac{g_c^2}{h_w^2} \tag{3}$$

where the subscripts w and c refer to the warm and cold cases, respectively; and we have made the simplifying and reasonable assumption that

$$g_w^2 = 2h_w^2 \quad . \tag{4}$$

From Eq. (3), we can see immediately where the large image - tune-shift for the warm case comes from. Using, $R_w = 899.5$ m, $R_c = 385.5$ m, $g_c = 7.5$ cm, $h_w = 1.35$ cm, we have

$$\left(\frac{R_w}{R_c}\right)^2 = 5.4 \;, \; \left(\frac{g_c}{h_w}\right)^2 = 30.9 \quad .$$

This leads to a total value for the warm case of 168 times the cold case.

3. Conclusions

We have computed the vertical tune shifts due to space charge and image terms for two possible ISA designs. The image contributions provide the significant limitation. The warm case as presented here is clearly unacceptable, leading to an incoherent ν-shift of almost 3 full units.* Given our assumptions concerning injection energy ($\gamma_{INJ} = 30$), tune ($\nu \sim 20$, in order that the transition energy be sufficiently below the injection energy), and luminosity ($I \geq 15$ A), we conclude that there are

*This conclusion is derived from the assumption that the beam dimensions play no essential role in determining the image contributions to the tune shift. In fact, this is not true when the width of the beam is larger than the chamber height. It can be shown in the warm case considered here that this effect reduces the tune shift by at least a factor of four. Thus, the conclusions presented in this paper must be somewhat tempered with this fact.

two ways to bring the image-tune-shift for the warm case to an acceptable value. We must decrease the radius and increase the magnet gap. In fact, it seems clear that the required changes will lead us inexorably in the direction of the original cold design.

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