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# FUSION REACTIONS IN COLLIDING BEAMS

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N O T I C E

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## Abstract

A system of beams of deuterons and tritons travelling in the same direction and focused by a relativistic electron beam is shown to be capable of yielding kilowatts of fusion power per meter of beam path.

## 1. Energy Released in Colliding Beams

Initially we assume colliding beams, as shown in Fig. 1, of deuterons travelling with velocity $v_1$ and tritons travelling with velocity $-v_2$. Both beams are assumed to have cross-sectional area A, and to include current I (or $6 \times 10^{18}$ I ions per second).

In one meter of the triton beam there are $6 \times 10^{18}\ I/v_2$ ions. These present to the deuteron beam an area $6 \times 10^{18}\ I\sigma/v_2$, where $\sigma$ is the fusion reaction cross section; hence the probability that one deuteron will make a collision in one meter of the triton beam is

$$\frac{6 \times 10^{18}\ I\sigma}{v_2 A} .$$

The rate of arrival of deuterons in the triton reference system is

$$\frac{6 \times 10^{18}\ I(v_1 + v_2)}{v_1} .$$

Hence the number of collisions per second, per meter is

$$\frac{36 \times 10^{36}\ I^2\sigma}{A} \frac{v_1 + v_2}{v_1 v_2} .$$

The fusion energy liberated per second, per meter is

$$\frac{36 \times 10^{36}\ I^2\sigma E}{A} \frac{v_1 + v_2}{v_1 v_2} \text{ joules} , \quad (1)$$

where I is the energy liberated per collision. In this case, and the cases that follow, it will be assumed that the relative velocity of deuterons and tritons is $v_o = 3 \times 10^6$ m/sec corresponding to a relative energy of about

100 keV. At this energy the D-T cross section for fusion $\sigma$ is about $5 \times 10^{-28}$ $m^2$. The energy liberated in the fusion reaction is about 17 MeV or $2.7 \times 10^{-12}$ joules. Hence the fusion power developed per meter is

$$\frac{1.46 \quad 10^5 \; I^2}{v_1 v_2} \quad \text{watts} \quad . \tag{2}$$

If, for example, I = 1000 amperes, A = 100 $cm^2$ or $10^{-2}$ $m^2$ and $v_1 = v_2 = 1.5 \times 10^6$ m/sec, then the power developed per meter is 6.5 watts. This seems a depressingly small amount from beams carrying many megawatts of power. Moreover, several weaknesses exist in the system. The worst lies in the fact that the beams described will blow up in a short distance because of space-charge forces. We will make two changes to solve this problem:

a) We will make the two beams travel in the same direction at much higher energy.

b) The beams will be focused by an electron beam travelling in the opposite direction.

By these means the beam will be concentrated in a very small cross section and the power level will be increased to more useful levels.

## 2. Two Beams in the Same Direction

It will be required that the deuterium and tritium ions differ in velocity by $3 \times 10^6$ m/sec as in the previous section. Also the ions will be required to have the same energy so that both beams can come from the same ion source and so that the unreacted ions in both beams can be retarded by the same field to regain their kinetic energy.

These two requirements set velocities of $1.6 \times 10^7$ and $1.3 \times 10^7$ m/sec for deuterons and tritons respectively; both will have an energy of about 2 MeV.

Using the parameters of the preceding section the power developed per meter is now about $7 \times 10^{-2}$ watts/m.

## 3. Space-Charge Forces

The repulsive radial force in the ion beam will be

$$E_r - vB_\theta = \frac{2Ir}{\epsilon_o v r_o^2} \; (1 - \beta^2) \text{ volts/ m} \quad , \tag{3}$$

where $\varepsilon_o$, the dielectric constant of free space = 1.1 x $10^{-10}$,

v is a mean velocity of about 1.5 x $10^7$ m/sec,

$\beta = v/c = 0.05$,

$r_o$ is the beam radius, (for A = 100 $cm^2$, $r_o$ = 5.6 cm),

whence

$$E_r - vB_\theta \cong 1.2 \times 10^3 \; Ir/r_o^2 \quad \text{volts/m} \quad .$$

The effective potential across the beam will be given by

$$600 \; I(r/r_o)^2 \quad \text{volts} \quad .$$

For a 2000 ampere circular beam the effective potential at the beam axis will be higher by 1.2 MV than the potential at the beam edge.

4. Focusing by an Electron Beam

The repulsive forces discussed in the preceding section, tending to blow up the ion beam can be cancelled by the fields of an electron beam coaxial with the ion beam but travelling in the opposite direction.

An electron beam of cross-sectional area A carrying a current $I_e$ of electrons of velocity $\beta_e c$ will exert a radial focusing force on the ion beam of

$$\frac{2I_e r}{\varepsilon_o \beta_e c r_o^2} (1 + \beta\beta_e)$$
$$= \frac{60 \; I_e r}{\beta_e \; r_o^2} (1 + \beta\beta_e) \quad \text{volts/m} \quad . \tag{4}$$

If this is to cancel the outward force given by Eq. (3) we must satisfy

$$(I_e/\beta_e)(1 + \beta\beta_e) = (I/\beta)(1 - \beta^2) \quad \text{amperes} \quad . \tag{5}$$

This equation suggests that, to keep $I_e$ low, $\beta_e$ should be small. This is because, for a given $I_e$ there will be more charge per unit distance in a beam of low $\beta_e$. If, for example, $\beta_e$ is set equal to $\beta$ of 0.05 the electron current required will be equal to I. The electron energy will be only 750 electron volts. However, it seems probable that the intense electron beams required will, of necessity, be formed at relativistic energies. If we assume $\beta_e \cong 1$, Eq. (5) becomes

$$I_e = (I/\beta)(1 - \beta) \quad \text{amperes} \quad . \tag{6}$$

For the parameters quoted above ($I$ = 2000 amperes, $\beta$ = 0.05),

$$I_e = 38{,}000 \quad \text{amperes} \quad .$$

The forces on the electron beam can be evaluated in the same fashion. They are given by

$$F_e = \frac{2I_e r}{\epsilon_o \beta_e c r_o^2}(1 - \beta_e^2) - \frac{2Ir}{\epsilon_o \beta c r_o^2}(1 + \beta\beta_e) \quad \text{volts/m} \; . \tag{7}$$

From (5)

$$F_e = \frac{2rI}{\epsilon_o c r_o^2 \beta}\left\{\frac{(1 - \beta^2)(1 - \beta_e^2)}{1 + \beta\beta_e} - (1 + \beta\beta_e)\right\} \quad \text{volts/m} \; . \tag{8}$$

If $\beta_e \cong 1$

$$F_e \cong - \frac{2rI}{\epsilon_o c r_o^2} \quad \text{volts/m} \quad . \tag{9}$$

The minus sign indicates that the net force on the electron beam is focusing. For a 2000-ampere ion beam and a 38,000-ampere electron beam there will be a focusing force on the electrons corresponding to an effective potential minimum of 60,000 volts on the beam axis.

For beam stability it will probably be desirable that there be roughly equal inward forces on both the ion and the electron beam. This would result if

$$\frac{2Ir}{\epsilon_o \beta c r_o^2}(1 - \beta^2) - \frac{2I_e r}{\epsilon_o \beta_e c r_o^2}(1 + \beta\beta_e)$$

$$= \frac{2I_e r}{\epsilon_o \beta_e c r_o^2}(1 - \beta_e^2) - \frac{2Ir}{\epsilon_o \beta c r_o^2}(1 + \beta\beta_e) \tag{10}$$

whence

$$(I/\beta)(2 + \beta\beta_e - \beta^2) = (I_e/\beta_e)(2 + \beta\beta_e - \beta_e^2) \; . \tag{11}$$

If $\beta_e \cong 1$

$$I_e = (I/\beta)(2 - \beta) \; . \tag{12}$$

For $I = 2000$ amperes, $\beta = 0.05$, $I_e = 78{,}000$ amperes. The net force (inward) on each beam is now

$$-\frac{2Ir(1+\beta)}{\epsilon_o \beta c r_o^2}$$

corresponding to an effective potential minimum on the axis of 1.2 MV.

We note the fact that this is not a real potential minimum since it includes a pseudo-potential derived by integration of magnetic forces. To make clear the composition of the actual forces on the two beams we include the following table for an ion beam of 1000 amperes of deuterons and 1000 amperes of tritons and a 78,000-ampere electron beam:

Forces on the ion beam: (all in volts/meter)
Self electrostatic force: $2.4 \times 10^6\ r/r_o^2$
Self magnetic force: $-0.006 \times 10^6\ r/r_o^2$
Electrostatic force due to electron beam: $-4.7 \times 10^6\ r/r_o^2$
Magnetic force due to electron beam: $-0.2 \times r/r_o^2$
for a net force of $-2.5 \times 10^6\ r/r_o^2$ volts/m . (13)

Forces on the electron beam:
Self electrostatic force: $4.7 \times 10^6\ r/r_o^2$
Self magnetic force: $-4.7 \times 10^6\ r/r_o^2$
Electrostatic force due to ion beam: $-2.4 \times 10^6\ r/r_o^2$
Magnetic force due to ion beam: $-0.1 \times 10^6\ r/r_o^2$
for a net force of $-2.5 \times 10^6\ r/r_o^2$ volts/m . (14)

## 5. Collapse of the Composite Beam

Under the forces discussed in the preceding section the combined ion and electron beams will collapse to a small diameter.

How small will be the final beam radius is difficult to estimate precisely. The first step is evaluation of the interaction between emittance and restoring forces.

To obtain order of magnitude estimates we suppose that the emittance of both ion and electron beams is 10 $\pi$ cm·mrad or $10^{-4}$ $\pi$ m·rad.

If the restoring force is $-k^2 r/r_o^2$ the particle motion is governed by

$$m\ddot{r} + ek^2 r/r_o^2 = 0 \tag{15}$$

whence

$$r = r_1 \sin\left(\sqrt{\frac{e}{m}}\ \frac{k}{r_o}\ t\right) = r_1 \sin\left(\sqrt{\frac{e}{m}}\ \frac{k}{r_o}\ \frac{z}{\beta c}\right) \tag{16}$$

$$r' = \sqrt{\frac{e}{m}}\ \frac{kr_1}{r_o \beta c}\ \cos\left(\sqrt{\frac{e}{m}}\ \frac{k}{r_o}\ \frac{z}{\beta c}\right) . \tag{17}$$

The beam emittance will be given by

$$\text{emittance} = 10^{-4}\ \pi \sqrt{\frac{e}{m}}\ \frac{kr_1^2 \pi}{r_o \beta c} . \tag{18}$$

We choose

$$\sqrt{\frac{e}{m}} = \sqrt{0.48 \times 10^8} = 0.7 \times 10^4 \quad \text{(for deuterons)}$$

$$\beta = 0.05$$

$$k = 1.6 \times 10^3 \quad \text{[from (13) or (14)]}$$

and obtain

$$\frac{r_1^2}{r_o} = 1.4 \times 10^{-4}\ \text{m} . \tag{19}$$

For the electron beam assuming the same emittance and using

$$\sqrt{\frac{e}{m}} = \sqrt{1.76 \times 10^{11}} = 4 \times 10^5$$

$$\beta_e = 1$$

$$k = 1.6 \times 10^3$$

we obtain

$$\frac{r_1^2}{r_o} = 5 \times 10^{-5}\ \text{m} . \tag{20}$$

From (19) or (20) it seems reasonable to assume that $r_1 \cong r_o \cong 10^{-4}$ m.

The area of the beam has collapsed from an area of 100 $cm^2$ to an area of $3 \times 10^{-4}$ $cm^2$; the area has decreased by a factor of $3 \times 10^5$.

## 6. Performance of the Collapsed Beam

First we note that the fusion power output, given by (2), has increased to 20 kW/m.

The fusion reaction is removing about $10^{16}$ ions per meter per second from the beam, or about one ion in $10^6$ is lost per meter.

The fields in the collapsed beam are very high. They result in betatron wavelengths of about $8 \times 10^{-4}$ m for the ions and about $3 \times 10^{-4}$ m for the electrons. The maximum value of $r'$ in the betatron oscillation is about 0.7.

This means that the ions which undergo Coulomb scattering, the great majority of which are scattered through small angles, will be restored to the beam with negligible increase in beam size. (We note that the effects of Coulomb scattering need to be calculated in detail before the effects of Coulomb scattering can be given quantitative statement.)

Furthermore, because both ion beams are travelling forward with a velocity of about five times their relative velocity, the Coulomb scattering angles are reduced by a factor of five.

## 7. Practical Implementation

Production of thousand-ampere beams of hydrogen ions is within the capability of the present art. Acceleration to 3 MeV and deceleration of unreacted ions should require reasonable quantities of power.

Production of thousands of amperes of electrons presents no real problems, at least in pulsed regimes.

The attainment of megawatt levels of fusion power depends only on further increasing the currents in the ion and electron beams.

A practical configuration for the composite beams, their generators and their terminations is shown in Fig. 2. It is assumed that both the ion beam and the electron beam have energies of 2 MeV. The ion beam is first accelerated to an energy of 4 MeV; its configuration is maintained by a Pierce electrode system (similar in principle to that used in the 750-MeV preinjector accelerating column). It is then decelerated to 2 MeV. The decelerating field serves also to stop the electron beam as indicated in the figure. A similar arrangement at the other end of the beam serves to produce a 2-MeV electron beam while terminating the composite ion beam.

## 8. Conclusion

The purpose of this report has been the preliminary exploration of possible variants of colliding beams for generation of fusion power.

The concepts developed here evidently need much refinement; possible beam instabilities should be investigated and the mechanism of beam collapse requires further study. Nonlinear forces will probably develop in the composite beam; their effects will require further analysis.

## 9. Acknowledgements

The writer would like particularly to express his indebtedness to Bogdan Maglich, the inventor of the migmatron in which colliding beams of hydrogen ions produce fusion reactions. An agreeable period of joint analysis of the migmatron helped to inspire the present study.

It is a pleasure also to acknowledge several helpful discussions with Rena Chasman and Fred Mills.

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DEUTERON BEAM

Velocity $v_1$

Area A

Current I

TRITON BEAM

Velocity $-v_2$

Area A

Current I

Figure 1

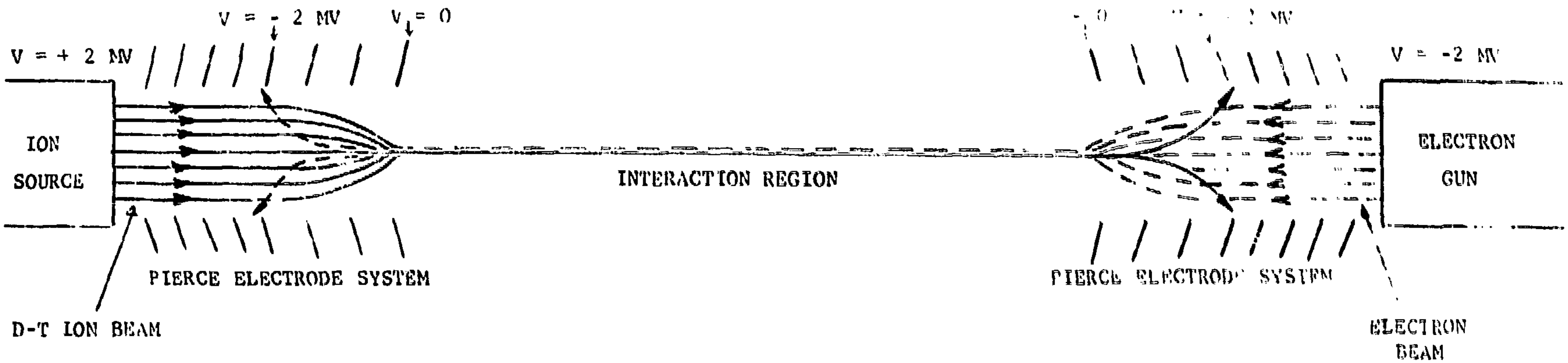
V = - 2 MV
V = 0
V = + 2 MV
ION
SOURCE
INTERACTION REGION
PIERCE ELECTRODE SYSTEM
D-T ION BEAM
V = -2 MV
ELECTRON
GUN
ELECTRON
BEAM

Figure 2