COMPRESSOR DESIGN FOR INTENSE ELECTRON RINGS*

J.M. Hauptman, L.J. Laslett, W.W. Chupp, and D. Keefe

Lawrence Berkeley Laboratory
University of California
Berkeley, California

Introduction

An electron ring compressor is being designed for the purpose of obtaining very highly-stripped, high-Z ions. This effort may constitute a necessary step for development of a possible heavy ion accelerator, is expected to lead to useful basic ion-source information, and independently will permit obtaining interesting spectroscopic information concerning such ions. The primary design objective is the achievement of a ring of high electron density that must retain this density and remain stable for a sufficient time to develop the desired highly-stripped ions. A density-time product in the vicinity of 10^{13} electron-sec/cm³ may be required. Vacuum requirements become undesirably stringent if the time interval exceeds 1 sec. Synchrotron radiation over such time periods plays a dominant role in the design, and a proper shaping of the magnetic field is presented which employs synchrotron radiation to enhance ring quality while avoiding both single particle resonances and the onset of negative-mass instability.

The attainment in an electron ring device of high electron density extended over a time of order 1. sec. requires the following two conditions. Firstly, the electron beam from a linac must be formed into a large radius ring in a magnetic field, and pulsed to small major radius in order to obtain a necessary degree of adiabatic damping of the minor ring dimensions. Secondly, the magnetic and electric environment of the beam must be such that the beam amplitudes are stable, and remain small, against both transverse and longitudinal instabilities for several tens of synchrotron radiation time constants. Specifically, the radial and axial betatron tunes must not lie near, nor cross, strong resonant values. The requirement that the beam be stable with respect to longitudinal (negative-mass) instability demands that, at all times, the beam maintain an energy spread in excess of a few percent (FWHM). In the present design, we choose to limit the number of circulating electrons only by the brilliance of the injector, and adjust the energy spread in the beam so that the limit imposed by longitudinal stability is always larger.

The long confinement time makes it impractical to support the magnetic field of this device with air-core coils as previously done in the ERA program at LBL. We envision the use of ferromagnetic pole tips to shape the field, and limit the flux density at 3.0 cm radius on the median plane to 15. Kgauss.

We have investigated the performance of such an electron ring device by dividing the entire cycle into three sequential stages of operation. The first is pulsed field compression with

$$B^m r = \text{constant (Stage 1)}$$

along the compression trajectory. Familiar cases are $m = 0$ (betatron), $m = \frac{1}{n}$ (scaling field $B \sim r^{-n}$ compressor), and $m = 1$ (static magnetic field compressor). The second is synchrotron radiation by the ring in a static magnetic field

$$B(r,z,t) = \text{constant, (Stage 2),}$$

and in the final stage the ring is maintained at a constant major radius

$$r(t) = \text{constant. (Stage 3).}$$

The general differential equations governing r,z motions of an electron in a cylindrical magnetic field are

$$\dot{r} = \frac{-r\left(\dot{B}_z\frac{\partial B_r}{\partial z} - \dot{B}_r\frac{\partial B_z}{\partial z}\right) + \frac{\partial B_r}{\partial z}\left(\dot{A} - \frac{U_\gamma}{\beta}\right)}{D} \qquad (1a)$$

$$\dot{z} = \frac{r\left(\dot{B}_z\frac{\partial B_r}{\partial r} - \dot{B}_r\frac{\partial B_z}{\partial r}\right) - \dot{B}_r B_z - \frac{\partial B_r}{\partial r}\left(\dot{A} - \frac{U_\gamma}{\beta}\right)}{D}, \qquad (1b)$$

where

$$D = \frac{\partial}{\partial z}(B_r B_z) + r\left[\left(\frac{\partial B_z}{\partial z}\right)^2 + \left(\frac{\partial B_z}{\partial r}\right)^2\right] \quad . \qquad (1c)$$

U_γ is the rate of electron energy loss by synchrotron radiation,

$$U_\gamma = \frac{C_\gamma P^4}{r^2}, \qquad (2)$$

with $C\gamma$ an electromagnetic constant

$$C\gamma = \frac{2}{3}\frac{r_o c}{(m_e c^2)^3} \cong .0422365S \text{ (cm}^2\text{/MeV}^3\text{ sec).}$$

It is convenient to define a quantity ξ as the fractional rate of change of momentum,

$$\xi \equiv -\frac{U_\gamma}{P\beta} = \frac{\dot{P}}{P} < 0, \qquad (3)$$

whose negative inverse is an instantaneous decay time constant for the radiation process.

During stages 2 and 3 we consider the possibility of employing a flux bar through the ring such that $\frac{\partial A}{\partial t} \neq 0$ on the equilibrium orbit. If we define f to be the fraction of momentum lost to radiation which is restored by the flux bar (FB) on the equilibrium orbit,

$$f \equiv -\frac{\dot{A}_{FB}}{\xi P}, \qquad (4a)$$

then the flux change at a radius r is

$$\dot{A}_{FB}(r) = -f\xi_o P_o\left(\frac{r_o}{r}\right)^\alpha, \qquad (4b)$$

where the subscript (o) refers to the equilibrium orbit, and $\alpha = 1$ if all flux succeeds in threading the orbit. During stage 3 a magnetic field varying at a rate

$$\frac{\dot{B}}{B} = \lambda\frac{\dot{P}}{P} = \lambda\xi \qquad (5a)$$

will produce a flux change at r given by

* Work supported by the U.S. Atomic Energy Commission.

$$\dot{A}_{FM}(r) = \frac{\partial}{\partial t}\left[\frac{1}{r}\int_{R_L}^{r} RB_o\left(\frac{R}{r_o}\right)^{-n} dR\right]$$

$$= \frac{\lambda\xi_o P_o}{2-n}\left[1 - \left(\frac{R_L}{r}\right)^{2-n}\right] \equiv \xi_o P_o \,\kappa, \quad (5b)$$

where the lower limit R_L reflects the deviation from an r^{-n} field at small r. We estimate that $R_L \cong 1.0$ cm.

A. Pulsed Field Compressor

A compression scheme in which the relationship between ring radius r and axial magnetic field B is specified to be $B^m r$ = constant along the compression trajectory yields the following scaling laws[1] for the beam momentum and synchrotron (a_s) and betatron (a_β, b_β) amplitudes. Let subscript i denote quantities at injection; then for compression from radius r_i to r

$$a_s = a_{s_i}\left(\frac{1-n_i}{1-n}\right)\left(\frac{r}{r_i}\right)^{\frac{1}{m}-1} \quad (6a)$$

$$a_\beta = a_{\beta_i}\left(\frac{1-n_i}{1-n}\right)^{1/4}\left(\frac{r}{r_i}\right)^{\frac{1}{2m}} \quad (6b)$$

$$b_\beta = b_{\beta_i}\left(\frac{n_i}{n}\right)^{1/4}\left(\frac{r}{r_i}\right)^{\frac{1}{2m}} \quad (6c)$$

and

$$P = P_i\left(\frac{r}{r_i}\right)^{1-\frac{1}{m}}. \quad (6d)$$

The injected electron current and the initial betatron amplitudes are taken to be those realized in the present ERA compressor at LBL.

B. Radiation in a Static Magnetic Field

The median plane symmetry of the compressor and the static field maintained in stage 2 requires, respectively, that

$$\frac{\partial B_z}{\partial z} = -\frac{\partial B_r}{\partial r} = 0 \text{ and } \dot{B}_z = \dot{B}_r = 0. \quad (7)$$

Hence from (1b)

$$\dot{z} = 0,$$

and from (1a) we are left with

$$\dot{r} = \frac{\frac{\partial B_r}{\partial z}\left(\dot{A} - \frac{U_\gamma}{\beta}\right)}{B_z\frac{\partial B_z}{\partial r} + r\left(\frac{\partial B_z}{\partial r}\right)^2}$$

$$= \left(\dot{A} - \frac{U_\gamma}{\beta}\right)\frac{-nB_z/r}{B_z(-nB_z/r) + r(-nB_z/r)^2},$$

where we have substituted the n-value, $n = -\frac{r}{B_z}\frac{\partial B_z}{\partial r}$, and employed the curl condition $\nabla \times B = 0$ to obtain $\frac{\partial B_z}{\partial r} = \frac{\partial B_r}{\partial z}$. Then

$$\dot{r} = \left(\dot{A} - \frac{U_\gamma}{\beta}\right)\frac{1}{P}\frac{r}{1-n} \quad (8b)$$

or

$$\frac{\dot{r}}{r} = \frac{1}{1-n}(\xi + \dot{A}/P) = \frac{\xi}{1-n}(1-f).$$

The rate of change of momentum is

$$\frac{\dot{P}}{P} = \xi + \frac{\dot{A}}{P} = \xi(1-f). \quad (9)$$

C. Radiation at Constant Ring Radius

Applying the same symmetry requirement to (1a) as before, we obtain

$$\dot{r} = \frac{-r\dot{B}_z\left(\frac{\partial B_z}{\partial r}\right) + \left(-\frac{\partial B_z}{\partial r}\right)(\dot{A} + \xi P)}{B_z\left(\frac{\partial B_z}{\partial r}\right) + r(-nB_z/r)\left(\frac{\partial B_z}{\partial r}\right)} \quad (10)$$

or

$$\frac{\dot{r}}{r} = \frac{1}{1-n}\left[\xi + \frac{\dot{A}}{P} - \frac{\dot{B}_z}{B_z}\right].$$

The total flux change $\dot{A}$ is due to both the flux bar and the time-varying magnetic field in the ferromagnet,

$$\dot{A} = \dot{A}_{FB} + \dot{A}_{FM} = -f\xi P + \kappa\xi P = \xi P(-f + \kappa). \quad (11)$$

Noting that we have defined λ such that $\frac{\dot{B}_z}{B_z} = \lambda\xi$,

$$\frac{\dot{r}}{r} = \frac{\xi}{1-n}\{1 - f + \kappa - \lambda\}, \quad (12)$$

which is zero as desired if

$$\lambda = \frac{(1-f)(2-n)}{(2-n) - \left[1 - \left(\frac{R_L}{r}\right)^{2-n}\right]}, \quad (13)$$

and where we have employed the definition of κ in (5b). Finally, we have in stage 3 that

$$\frac{\dot{P}}{P} = \xi + \frac{\dot{A}}{P} = \xi\{1 - f + \kappa\} \quad (14)$$

on the equilibrium orbit.

D. Radiation Damping of the Betatron Oscillations

The axial betatron displacement z, according to Bruck[2] (eqn. 23.8), satisfies the damped harmonic oscillator equation

$$\ddot{z} + \left[\frac{U_\gamma}{E} + \frac{\dot{E}}{E}\right]_o \dot{z} + \omega_z^2 z = 0, \quad (15)$$

where E is the total energy, U_γ the rate of energy loss, and the bracket is to be evaluated on the equilibrium orbit. The amplitude of these oscillations damps as

$$b_\beta \propto \exp\left\{-\frac{1}{2}\int\left[\frac{U_\gamma}{E} + \frac{\dot{E}}{E}\right]_o dt\right\}, \quad (16a)$$

so that in the absence of radiation ($U_\gamma = 0$)

$$b_\beta \propto \exp\left\{-\frac{1}{2}\int\frac{\partial}{\partial t}\ln E\, dt\right\} = \frac{1}{\sqrt{E}}. \quad (16b)$$

There is additional damping due to changes in the oscillation frequency $\omega_z^2 = \beta c \nu_z / r$, so that for $\nu_z = \sqrt{n}$ = constant

$$b_\beta \propto \frac{1}{\sqrt{B}}. \quad (16c)$$

Then the axial betatron amplitude has a time-dependence

$$\frac{\dot{b}_\beta}{b_\beta} = -\frac{1}{2}\left[\frac{U_\gamma}{E} + \frac{1}{B}\frac{\partial B}{\partial t}\right]_0 = \frac{1}{2}\left[\xi_0 - \frac{1}{B}\frac{\partial B}{\partial t}\right]. \quad (17)$$

This reduces to $\frac{\dot{b}_\beta}{b_\beta} = \frac{\xi_0}{2}$, as required in a constant field betatron ($\lambda = 0$) with RF cavity flux compensation ($f = 1$, $\alpha = 1$).

The radial displacement is similarly governed by the equation (Bruck 23.12)

$$\ddot{x} + \left[\frac{U_\gamma}{E} + \frac{\dot{E}}{E}\right]_0 \dot{x} + \omega_x^2 x = -\ddot{x}_R , \quad (18)$$

where $\ddot{x}_R$ expresses the shrinkage of the orbit due to radiation and also possibly flux change acceleration. We calculate the radial acceleration in the following manner. From Bruck eqns. 23.16 and 23.17, we have

$$\dot{x}_R = -\frac{1}{1-n}\frac{r_0}{P_0}[\dot{P}(1+\frac{x}{r_0}) - \dot{P}_0] , \quad (19)$$

where $\dot{P}$ and $\dot{P}_0$ are the rates of momentum loss on, respectively, a betatron excursion of amount x and the equilibrium orbit. The above expression gives the radial velocity due to the differential rates of azimuthal momentum loss. Since $\dot{P}_0 = P_0\xi_0$, $\dot{P}$ and $\dot{P}_0$ are for small excursions from the equilibrium orbit

$$\dot{P} = \xi_0 P_0(1-2n\frac{x}{r_0}) - f\xi_0 P_0(1-\alpha\frac{x}{r_0}) + \varkappa\xi_0 P_0\left[1+(1-n)\frac{x}{r_0}\right]$$

$$= \xi_0 P_0\left[(1-f+\varkappa) + (-2n+f\alpha+\lambda(1-n))\frac{x}{r_0}\right] \quad (20a)$$

and

$$\dot{P}_0 = \xi_0 P_0[1-f+\varkappa]. \quad (20b)$$

Substitution into (19) yields

$$\dot{x}_R = \xi_0\left[\frac{(1-f+\varkappa) - 2n + f\alpha + \lambda(1-n)}{1-n}\right]x, \quad (21)$$

where the entire bracket is a constant so that $\ddot{x}_R \propto \dot{x}$. Finally the radial equation becomes

$$\ddot{x} + \left[\frac{U_\gamma}{E} - \frac{U_\gamma}{E}\left\{\frac{(1-f+\varkappa) - 2n + f\alpha + \lambda(1-n)}{1-n}\right\} + \frac{\dot{E}}{E}\right]\dot{x} + \omega_x^2 x = 0. \quad (22)$$

Hence the damping rate for the radial betatron amplitude is

$$\frac{\dot{a}_\beta}{a_\beta} = \frac{1}{2}\left[\xi_0\left\{\frac{f(1-\alpha) - 2\varkappa + (1+\varkappa)n}{1-n}\right\} - \frac{1}{B}\frac{\partial B}{\partial t}\right], \quad (23)$$

which reduces to

$$\frac{\dot{a}_\beta}{a_\beta} = \frac{\xi_0}{2}\frac{n}{1-n}$$

as required for the usual betatron with RF cavity ($f = 1$, $\alpha = 1$) in a static magnetic field ($\lambda = 0$).

E. Radiation Damping of the Momentum Spread in the Beam

Consider an electron on the equilibrium orbit r_0 with momentum P_0. An achromatic electron of momentum P *has an equilibrium orbit at*

$$r = r_0\left[1 + \frac{1}{1-n}\frac{\Delta P}{P}\right] = r_0\left[1 + \frac{1}{1-n}\left(\frac{P-P_0}{P}\right)\right]. (24)$$

The rate of change of momentum spread is just

$$\frac{\partial}{\partial t}\left(\frac{\Delta P}{P}\right) = \frac{P}{P_0}\left[\frac{\dot{P}}{P} - \frac{\dot{P}_0}{P_0}\right] ,$$

where

$$\frac{\dot{P}}{P} = -\frac{C_\gamma}{\beta}\frac{P^3}{r^2} + \frac{\dot{A}(r)}{P}$$

$$= \xi_0\left[(1-f+\varkappa) + \left(\frac{1-3n}{1-n} - \frac{1}{\gamma^2} + \frac{f\alpha}{1-n} + f\right)\frac{\Delta P}{P}\right]$$

and

$$\frac{\dot{P}_0}{P_0} = -\frac{C_\gamma}{\beta_0}\frac{P_0^3}{r_0^2} + \frac{\dot{A}(r_0)}{P_0} = \xi_0[1-f+\varkappa] .$$

Hence (defining $\varepsilon \equiv \Delta P/P$) the damping rate for momentum spread is

$$\frac{\dot{\varepsilon}}{\varepsilon} = \xi_0\left[\frac{(1+f(\alpha+1)) - (3+f)n}{1-n} - \frac{1}{\gamma^2}\right]$$

Quantum fluctuations are negligible in our domain of interest. The time-dependence of the synchrotron amplitude in a constant n field is

$$\frac{\dot{a}_s}{a_s} = \frac{\dot{r}}{r} + \frac{\dot{\varepsilon}}{\varepsilon} .$$

For the usual betatron case ($f = 1$, $\alpha = 1$, $\lambda = 0$), we obtain

$$\frac{\dot{a}_s}{a_s} = \xi_0\frac{3-4n}{1-n} ,$$

which is twice the rate in a betatron. Our case, however, differs from a betatron in that the electron trajectory is the envelope, rather than an oscillatory trajectory within the envelope. The usual sum rule for radiation damping rates does not maintain in the present case.

F. Longitudinal (Negative-mass) Instability

The number of circulating electrons allowed for longitudinal stability has been calculated by L.J. Laslett to be

$$N_e < 1.57 \times 10^{12}\,\gamma(1-n)\frac{a_s^2}{\Delta R_n} , \quad (28)$$

where $\gamma = E/m_ec^2$ for the electrons and ΔR_M is the axial displacement of electric side plates about the beam. We have chosen $\Delta R_M = 2b_\beta$ at injection, and taper the side-plates according to

$$\Delta R_M \propto \frac{1}{\sqrt{B}} \propto r^{n/2}. \quad (29)$$

As previously stated, we have chosen the energy spread of the injected beam so that the quantity of eqn. (28) is just larger than the brilliance limit of the injector. A smaller value of (28) at any time will limit the number of circulating electrons whereas a larger value of (28) implies an unnecessarily large value of a_s, and a consequent reduction in electron density. The most favorable circumstance is where the limit (28) always remains slightly larger than the number injected. The conditions under which the limit (28) is constant are straightforward to obtain in a constant n field. In stage 2, $N_e \propto \gamma a_s^2/\Delta R_M \propto p^{3/2}r^{3/2}\varepsilon^2$, so that

$$\frac{\dot{N}_e}{N_e} = \frac{3}{2}\left[\frac{\dot{P}}{P} + \frac{\dot{r}}{r}\right] + 2\frac{\dot{\varepsilon}}{\varepsilon}$$

$$= \frac{\xi}{2(1-n)}\begin{bmatrix}(10+f(4\alpha-2) - 4/\gamma^2) \\ -(15+f-4/\gamma^2)n\end{bmatrix}, \quad (30)$$

using (8), (9), and (26).

Then the number of circulating electrons allowed for longitudinal stability is constant if n is a "critical value",

$$n_c = \frac{10 + f(4\alpha-2) - 4/\gamma^2}{15 + f - 4/\gamma^2} \quad \text{(Stage 2)}, \quad (31)$$

which reduces to $n_c = 2/3$ in the absence of a flux bar and for γ large. Similarly for stage 3, $N_e \propto \gamma a_s^2 \propto P\varepsilon^2$ and

$$\frac{\dot{N}_0}{N_e} = \frac{\dot{P}}{P} + 2\frac{\dot{\varepsilon}}{\varepsilon}$$

$$= (1 - f + \kappa) + 2\left[\frac{1 + f(\alpha+1)-(3+f)n}{1-n} - \frac{1}{\gamma^2}\right], \quad (32)$$

The stage 3 condition that r = constant requires that we take

$$\kappa = \frac{(1-f)\left[1 - \left(\frac{R_L}{r}\right)^{2-n}\right]}{(2-n) - \left[1 - \left(\frac{R_L}{r}\right)^{2-n}\right]} \quad . \quad (33)$$

The resulting equation for n_c is complicated if $R_L \neq 0$ and $f \neq 1$. The critical n-value in stage 3 ranges from $n_c = 3/4$ ($f=1$) down to $n_c \cong 0.48$ ($f = 0$, $R_L = 1.5$ cm). Figure 1 displays the dependence of n_c on the strength of the flux bar, f, during stages 2 and 3.

G. Compressor Design

The above solutions have been employed to calculate the performance expected of an electron ring device with the intent of maximizing the density-time product

$$\int_0^{1.} \frac{N_e}{\text{Volume}}\, dt = \text{figure-of-merit (FM)},$$

where the volume is calculated from the ring parameters as

$$\text{Volume} = 2\pi^2\, r b_\beta \sqrt{a_s^2 + a_\beta^2} \quad .$$

It will be convenient, and probably necessary in an operational device, to maintain a constant n-value at the orbit radius everywhere during stages 2 and 3. The ring will then never cross a betatron resonance line (where the growth rates are much larger than the typical rates of change ~ξ in this device), and the rates of change (30) or (32) of the longitudinal stability limit can be kept very near zero for the duration of the stage by an appropriate choice of f.

Two cases have been considered: case A without use of a flux bar, and case B with a flux bar. A case without a flux bar is attractive experimentally due to its simplicity. In regard to case A, it is evident from Figure 1 that if $f = 0$ at all times, one cannot have the rate (30) zero in stage 2 and the rate (32) zero in stage 3 without the ring crossing a single particle resonance at $n = 0.64$. Hence we choose a case A with only stages 1 and 3, and an n-value everywhere constant and equal to $n_c = 0.526$ (for the choice $R_L = 0.75$cm). Results of this calculation are tabulated in Table I and displayed in Figure 2. The figure-of-merit for this case is 2.59×10^{12} e^-sec/cm^3.

A case employing a flux bar during stage 3 only, if it is to maintain the rate (30) zero during stage 2, must have $n = 2/3$ by (31) By inspection of Figure 1, this demands that $f \cong 0.625$ during stage 3 so that the rate (32) is zero. This is case B, from which relevant quantities are listed in Table II and displayed in Figure 3. The figure-of-merit for case B is 3.97×10^{12} e^-sec/cm^3

Finally, there are many possible cases of interest in which a flux bar is employed during both stages 2 and 3. The operating n-value would then be chosen in the range $2/3 < n < 3/4$ for this device.

The successive ionization of Argon, for example, in a ring described by case B of this paper is displayed in Figure 4. The Argon gas was injected into the compressor at the beginning of stage 2. The calculation of the ionization progression is given by A. Salop.[3]

References

1. Electron Ring Accelerator Proceedings, L.J. Laslett, p. 270 (1968).

2. Introduction to the Theory of Circular Particle Accelerators, H. Bruck (1966).

3. Multi-ionization of Neon, Argon, and Xenon and their Ions by High Energy Electron Impact, A. Salop, LBL-2440 (unpublished), December 1973. (Submitted to Phys. Rev. A).

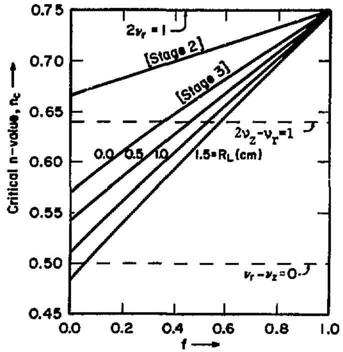

Fig. 1 - Critical n-values as function of flux bar strength, f, for stages 2 and 3. In stage 3, four choices of R_L are shown. The horizontal dotted lines are locations of prominent single particle betatron resonances.

Table I. Case A (no flux bar)

	Stage 1		Stage 2	Stage 3
	Injection	End of Compression		
t(sec)	0.	.001		1.0
r(cm)	40.	3.0		3.0
B(Kg)	.0842	15.0	(none)	3.396
KE(MeV)	.621	12.96		2.536
n-value	5/9	.525		.526
a_s(cm)	1.350	.095		.200
a_β(cm)	1.495	.110		.074
b_β(cm)	1.115	.085		.029
$\frac{\Delta P}{P}$	.015	.015		.0316
$\rho(e^-/cm^3)$	6.09×10^8	1.42×10^{12}		2.80×10^{12}

Table II. Case B(with flux bar)

	Stage 1		Stage 2	Stage 3
	Injection	End of Compression		
t(sec)	0.	.001	.0202	1.00
r(cm)	40.	4.999	3.000	3.00
B(kg)	.167	10.678	15.009	4.978
KE(MeV)	1.553	15.50	12.997	3.995
n-value	2/3	2/3	2/3	2/3
a_s(cm)	2.100	.262	.262	.457
a_β(cm)	1.607	.201	.162	.090
b_β(cm)	1.066	.133	.103	.029
$\frac{\Delta P}{P}$	.0175	.0175	.029	.051
$\rho(e^-/cm^3)$	1.75×10^9	8.98×10^{11}	2.07×10^{12}	4.84×10^{12}

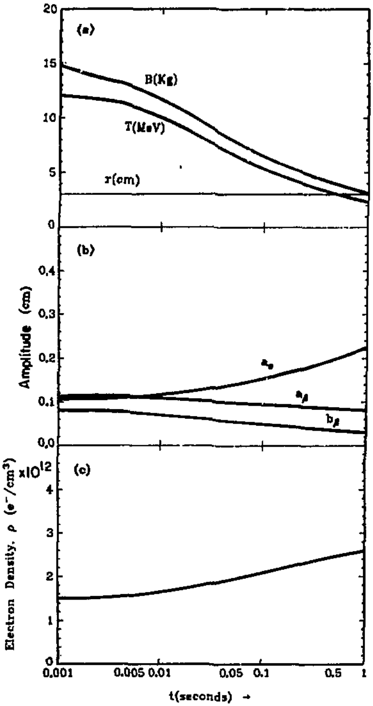

Fig. 2 - Ring parameters for Case A. (a) equilibrium orbit radius, r(cm), magnetic field at r, B(kg), and kinetic energy of electrons at r, T(MeV); (b) synchrotron, a_s, radial betatron, a_β, and axial betatron, b_β, amplitudes; and (c) electron density as function of time. The figure-of-merit for this case is ≅ 2.59 × 10^{12} e^-sec/cm^3.

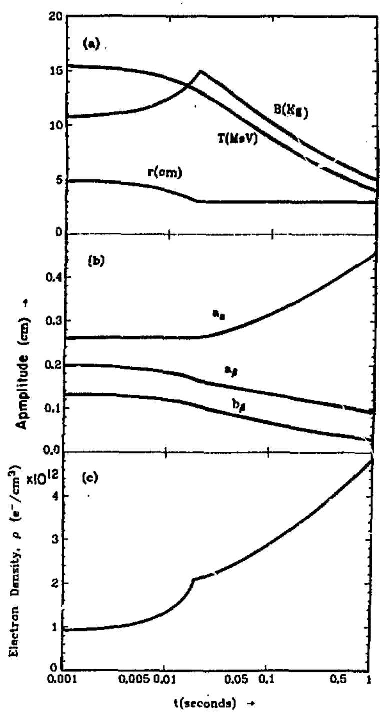

Fig. 3 - Ring parameters for Case b; the figure-of-merit is ≅ 3.97 × 10^{12} e^-sec/cm^3.

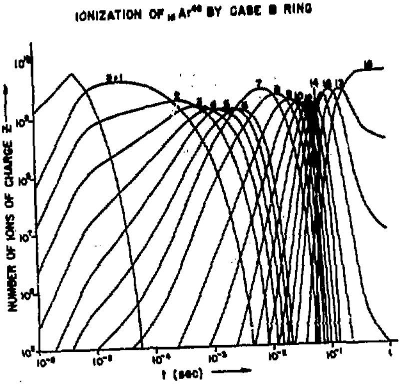

Fig. 4 - Ionization of $_{18}Ar^{40}$ by the ring of case B.