LA-5458-MS

INFORMAL REPORT

1/8 1-4-74

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UNITED STATES
ATOMIC ENERGY COMMISSION
CONTRACT W-7405-ENG. 36

MASTER

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Printed in the United States of America. Available from
National Technical Information Service
U. S. Department of Commerce
5285 Port Royal Road
Springfield, Virginia 22151
Price: Printed Copy $4.00; Microfiche $1.45

LA-5458-MS
Informal Report
UC-34
ISSUED: November 1973

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RELATIONSHIP OF SELF-FOCUSING TO SPATIAL INSTABILITY MODES

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A. J. Campillo, S. L. Shapiro, and B. R. Suydam

ABSTRACT

The spatial distribution of focal spots formed when laser beams self-focus in materials is shown to originate from the growth of certain instability modes. These modes are determined by a simple mathematical relationship derived from instability theory, which has been verified experimentally. Because of these instabilities, the threshold power for self-focusing is inversely proportional to the self-focusing length for high-power laser beams.

We have shown recently[1] that high-power laser beams break up into cells whose size is in accord with theory[2,3]. Experimentally this was demonstrated[1] by allowing a diffraction pattern to self-focus, and then letting the beam divide into cells whose size was determined by the fastest growing instability mode. This dominate mode outgrew all the other modes when amplitude and phase fluctuations on the beam were due only to, e.g., refractive index inhomogeneities, dust, or spontaneous noise. However, a less swiftly growing mode can dominate if the initial beam profile has a large amplitude component corresponding to the frequencies of that mode. Linear stability theory predicts that an initial disturbance δE_o, of the field E_o, of transverse wave number vector[4] $k = (k_x, k_y)$, will grow as $\exp(\alpha z)$ as it travels in the z direction, and the growth rate α is given by

$$\alpha = \frac{c}{2\omega n_o} k(2\gamma|E_o|^2 - k^2)^{1/2} ; \quad k^2 \equiv k_x^2 + k_y^2 ,$$

$$\gamma \equiv \frac{3\omega^2}{2c^2} n_o n_2 , \qquad (1)$$

where n_2 is the usual coefficient of the nonlinear part of the refractive index, e.g., $n = n_o + n_2|E|^2$. According to the linear theory all modes with the same value of k^2 grow at the same rate, and those modes for which

$$k^2 = k_m^2 \equiv \gamma|E_o|^2 ; \quad \alpha_m = ck_m^2/(2\omega n_o) \qquad (2)$$

grow the fastest, and will normally outstrip all others. Once the perturbation becomes large, it starts to interact with itself. During this nonlinear phase its growth becomes more rapid than exponential, and the most compact modes, i.e., those for which $k_x = k_y$, grow fastest[2]. This rapid nonlinear growth leads to self-focusing in a finite distance z_f, which our theory gives as[2]

$$z_f = (1/\alpha_m) \log(3/\delta) = \frac{10^{-7} c n_o}{6\pi n_2} \left(\frac{c/\omega}{I}\right) \log(3/\delta), \qquad (3)$$

where I is the intensity of the beam in watts/cm^2 and δ measures the initial perturbation, i.e., $\delta \equiv |\delta E_o|/|E_o|$, or alternatively δ is the phase perturbation in radians. It is the purpose of this letter to show that when a Gaussian beam of many critical powers propagates through a medium, it indeed breaks up into cells of wave number vector k_m, provided there is no gross perturbation on the initial beam. Each cell of the break-up pattern produces a self-focus and from the pattern of self-focal spots one can measure separately k_x and k_y and hence determine k^2.

The mode with maximum growth rate depends on k_x and k_y, and both must be measured to verify Eqs. 1 and 2. One method of verifying these equations is to experimentally select a k_x by amplitude modulation of the beam in the x-direction, allowing k_y to be freely determined by the instabilities and to be measured from the spacing of the spots formed due to self-focusing. This artifice aids in accurately determining k_x and k_y by forcing the beam to break up into cells of rectangular shape. Then the dependence of k^2 on the intensity can be determined experimentally. As the intensity of the beam is increased, the self-focusing length decreases, and many sample cells of different length must therefore be used to determine the relationship between k and intensity. Note that there is an instability mode with a k-vector that grows the fastest whether or not the beam is modulated in the x-direction. Because the mode with $k_x = k_y$ ultimately grows fastest, it is advantageous to choose k_x^2 close to $\gamma A_0^2/2$, bearing in mind that this modulation only aids in determining the cell shape. Furthermore the beam should not be modulated spatially at wavelengths comparable to the beam diameter, nor at such short wavelengths that the growth rate α is negligible or imaginary. Moreover, the beam should not be modulated in both the x- and y-directions because the pattern formed at the end of the cell would result from a competition between the superimposed mode and the mode with maximum growth rate[5] which would make it difficult to interpret and analyze the data.

Experimentally, a beam from a single-mode ruby laser[1] is allowed to traverse cells of carbon disulfide (CS_2). Before entering the CS_2 cells, the beam is amplitude-modulated in one direction by means of a glass wedge of small angle. A glass wedge produces an intensity modulation of 16% of the form sin ($k_x x$), and by using several wedges any spatial perturbation k_x can be applied to the beam. The modulated intensity profile is allowed to self-focus in five CS_2 cells which range in length from 50 to 150 cm. For each CS_2 cell the laser intensity is adjusted by means of different attenuating cells of copper sulfate until self-focusing is produced at the exit face of each CS_2 cell. These self-focusing threshold intensities are measured by means of a photodiode connected to an oscilloscope, and are determined by the onset of stimulated Raman scattering at the end of each cell which is detected by a second photodiode. Beam energies are measured with a calorimeter. These self-focusing threshold intensities, along with photographs of the focal spots at the end of each CS_2 cell from which k_x and k_y are measured, yield the relationship between the intensity and k.

A photograph of a self-focusing pattern that developed at the exit face of a CS_2 cell is shown in Fig. 1. The k-vector of the instability mode is determined by dividing 2π by the spacing between the focal spots in the x- and y- directions yielding

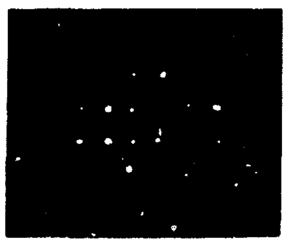

Fig. 1. A self-focusing pattern photographed at the exit face of a CS_2 cell. This pattern originates because of the rapid growth of a spatial perturbation whose k-vector components, k_x and k_y, can be determined by measuring the spacings between the focal spots.

k_x and k_y, respectively. For the case shown in Fig. 1, k_x and k_y are nearly equal and each k-vector cell contains one focal spot. At higher powers than threshold, the cell can divide further and more focal spots are formed.

The absolute magnitude of the k vector which develops at the end of each CS_2 cell is plotted as a function of threshold intensity in Fig. 2. Error bars are mainly because the intensity near the peak of the beam is not uniform, but Gaussian, leading to a k vector dependence with position, and because of a nonlinear lens effect due to the Gaussian profile. These effects are easily observed experimentally as the focal spots are compressed closer near the peak of the beam, and, rather than coinciding exactly with the peaks of the modulation which originally followed straight lines, the focal spots lie on lines that bend toward the center of the beam. The Gaussian intensity profile causes a 2 to 10% correction in the k-vector (depending on components of the k-vector), while nonlinear lens effects are estimated to cause a variation of 3%. In some cases a reading error in the position of the focal spots can lead to a 5% error. Figure 2 shows that the intensity fits a k^2 dependence as shown by the solid line. The absolute coefficient between I and k^2 as determined experimentally differs by a factor of 1.6 from the theoretical coefficient calculated by using the values of $\lambda = 6943$ Å, $n_o = 1.62$, and $n_2 = 1.8 \times 10^{-11}$ esu. The agreement with the absolute coefficient is satisfactory as the theory is only meant to apply to beams of uniform intensity.

Finally the threshold power for self-focusing is plotted as a function of the cell length in Fig. 3. The threshold power for self-focusing is inversely proportional to the cell length, a result predicted by Eq. 3 for high-power laser beams. This is a different result than that given by the theory of Kelley[6], which predicts that the self-focusing length is inversely dependent on the square root of the power minus one critical power--a result which applies to low-power laser beams. Note that Chilingarian[7] demonstrated that the self-focusing

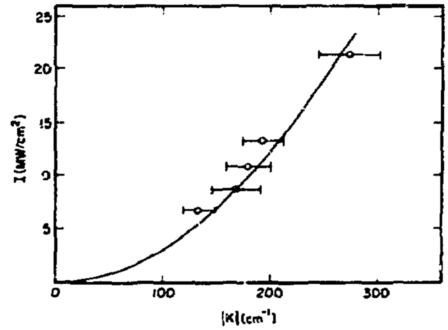

Fig. 2. The threshold intensity for self-focusing is plotted as a function of |k|. The solid line shows that the intensity is dependent upon k^2 in agreement with instability theory.

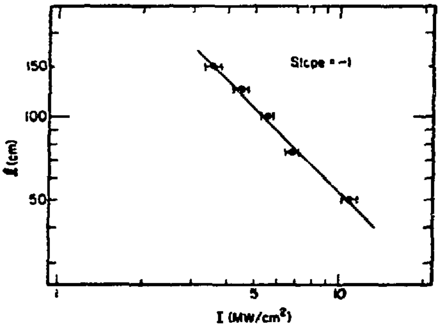

Fig. 3. The self-focusing length is plotted as a function of an average intensity. A slope of minus one indicates that the self-focusing length is inversely dependent on the intensity. The peak intensity is twice the average intensity plotted.

length for inhomogeneous laser beams goes inversely with the power. In his case inhomogeneities were already present in the beam at the outset, and those with certain favorable characteristic frequencies self-focused easily, while in our case spatial inhomogeneities grow from the instabilities. Experimental data reported in this letter substantially verifies the theoretical model of Suydam.[2]

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4. Note that k is the wave number vector of the perturbation, which we presume to have been Fourier analysed in space; it is not ω/c.
5. When the beam was modulated experimentally in both directions, there was a strong tendency for the mode with the highest growth rate to dominate over any such superimposed mode.
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