UCRL -74974
PREPRINT
Conf- 730906 - - 5

LAWRENCE LIVERMORE LABORATORY
University of California/Livermore, California

WAVE CONVERSION AND RESONANCES IN LASER LIGHT SCATTERING

A. L. Peratt

August 16, 1973

This paper was prepared for
XIth International Conference on Phenomena in Ionized Gases
Prague, Czechoslovakia
September 10-14, 1973

WAVE CONVERSION AND RESONANCES IN LASER LIGHT SCATTERING*

A. L. Peratt
Lawrence Livermore Laboratory, University of California
Livermore, California, USA 94550

INTRODUCTION The present work is concerned with the calculation of the differential cross section for the scattering of an electromagnetic wave in an underdense plasma. The method used is based on the derivation of a formal relation between the autocorrelations of current density and dielectric tensor. The formulation leads to expressions that are particularly simple when the plasma is in thermal equilibrium.

An enhancement of the scattering spectra because of the coupling of transverse waves and excitation of longitudinal slow wave resonances is found to result when the Doppler shift exceeds collisional frequencies when the differential scattering vector is aligned primarily perpendicular to the magnetic field.

FORMULATION Our treatment is based upon a first-order solution of the linearized Boltzmann equation. When the system is homogeneous, the spectral distribution of the electron density fluctuations, denoted by $S(\underline{k},\omega)$, is given by the simple relation [1] $S^{+}(\underline{k},\omega) = 2\mathrm{Re}\, S^{+}(\underline{k}, -i\omega)$, where, in the first Born approximation [2],

$$S^{+}(\underline{k}, -i\omega) = \lim_{s\to -i\omega}\left[\frac{(2\pi)^3\delta(\underline{k})n_e}{s} + \int d\underline{v}\; F_e(\underline{k},\underline{v},s)\right], \tag{1}$$

where n_e is the particle density and $F(\underline{k},\underline{v},s)$ is the usual Fourier-Laplace (space-time) transform of the first-order distribution function,

$$F_\alpha(\underline{k},\underline{v},s) = -\int_{\pm\infty}^{\phi} d\phi'\,\frac{G_\alpha}{\omega_{c\alpha}}\left[-\frac{q_\alpha}{m_\alpha}\,\underline{E}(\underline{k},s)\cdot\nabla_v f_0(v) + f_1(\underline{k},\underline{v},t=0)\right]. \tag{2}$$

Here, G_α is the well-known Bernstein magnetic field factor [3] for particle species $\alpha = e, I$ (electrons, ions) and where $\omega_{c\alpha} = g_\alpha B_0/m_\alpha$ will denote the cyclotron frequency. The wave vectors and frequencies of the incident and scattered waves, $(\underline{k}_0, \omega_0)$ and $(\underline{k}_1, \omega_1)$, respectively, are connected by $\underline{k} = \underline{k}_0 - \underline{k}_1$ and $\omega = \omega_0 - \omega_1$.

The Fourier-Laplace transform of the set of Maxwell's equations yields $\underline{E}(\underline{k},\omega) = \underline{\underline{P}}^{-1}(\underline{k},\omega)\cdot\underline{Q}(\underline{k},\omega)$, where

$$\underline{\underline{P}} = \left[-k^2\,\underline{\underline{I}} + \underline{k}\,\underline{k} + \frac{\omega^2}{c^2}\,\underline{\underline{K}}(\underline{k},\omega)\right]. \tag{3}$$

$$\underline{Q} = -\omega\mu_0 e\int d\underline{v}\,\frac{\underline{v}\,f_1(\underline{k},\underline{v},t=0)}{\omega - \underline{k}\cdot\underline{v}} + i\omega\mu_0\,\underline{E}(\underline{k},t=0) - i\mu_0\,\underline{k}\times H(\underline{k},t=0). \tag{4}$$

In the above, $c = (\mu_0 \epsilon_0)^{-1/2}$ is the velocity of light in a vacuum, $\underline{\underline{I}}$ represents the unit dyadic while $\underline{\underline{K}}$ is the equivalent dielectric tensor [4]. We shall assume sufficiently well-behaved initial conditions such that $\underline{Q}$ is analytic. This behavior leads to resonance peaks in the spectral density associated with the oscillatory modes of Eq. (3).

SCATTERING IN A CONSTANT MAGNETIC FIELD We choose B_0 to be aligned in the z-direction while the wave vector $\underline{k}$ lies in the xz plane, making an angle θ with respect to the x-axis. We shall consider the case of a transverse electromagnetic wave whose electric field is polarized transverse to the constant magnetic field. It is convenient to express $\underline{E}(\underline{k}, \omega)$ in terms of fields whose phase velocities are fast and slow such that $\underline{E} = \underline{E}^E + \underline{E}^L$. In this way coupling between the fast incident wave and the fast electromagnetic wave and the subsequent excitation of slow waves may be accounted for. In the long wavelength limit the first term of $\underline{E}$ corresponds to the cold plasma extraordinary mode, whereas the second term corresponds to a field that is essentially longitudinal in the hot plasma model. Since we are concerned with the effect of thermal broadening, we may, to an approximation, retain only short wavelength terms so that $\underline{E} = c^2 \underline{k}\,\underline{k} \cdot \underline{Q}/k^2 \omega^2 K_{xx}$. The above equations yield (upon use of integrals like Eq. (8) to handle initial conditions [1]),

$$S(\underline{k}, \omega) = \frac{2\pi n_e}{k} \left(\left| 1 + \frac{1 - K^e_{xx}}{K_{xx}} \right|^2 F_e + \left| \frac{1 - K^e_{xx}}{K_{xx}} \right|^2 F_I \right), \tag{5}$$

$$K_{xx} = 1 - \frac{\omega_p^2 e^{-\lambda}}{\omega \lambda} \sum_{n=-\infty}^{\infty} n^2 I_n(\lambda)/(\omega + i\nu + n\omega_c), \tag{6}$$

and $\lambda = k^2 V_{th}^2/2\omega_c^2$, $V_{th}^2 = 2\kappa T/m$, $\omega_p^2 = n_e q^2/\epsilon_0 m$, κ is the Boltzmann constant and ν is the collision frequency. The summation over α of K_{xx} is implied and we denote only the electron portion as K^e_{xx}. $F_{e,I} = k/\pi \, i n_e I_m$ [Eq. (8)]. It should be noted that since $\underline{k} \perp \underline{B}_0$, cyclotron and Landau damping do not exist. Equation (5) may be generalized to investigate the effect of noncollisional damping by considering values of θ other than $\pi/2$. To do this we replace $k^2 K_{xx}$ in the above with $\underline{k} \cdot \underline{\underline{P}} \cdot \underline{k}/k^2 = k_x^2 K_{xx} + 2k_x k_z k_{xz} + k_z^2 k_{zz}$, where $\underline{k} = \hat{x} k_x + \hat{z} k_z$. This expression is frequently rewritten in the form

$$\underline{k} \cdot \underline{\underline{P}} \cdot \underline{k}/k^4 = 1 + \frac{\omega_p^2}{\omega_c^2} \frac{e^{-\lambda}}{\lambda} \sum_{n=-\infty}^{\infty} I_n(\lambda)[1 + \zeta_0 Z(\zeta_n)], \tag{7}$$

where $\lambda = k_x^2 V_{th}^2/2\omega_c^2$, $\zeta_n = (\omega + i\nu + n\omega_c)/k_z V_{th}$. In deriving Eq. (5) we have followed the details presented in Appendix II of Ref. [3] while making the replacement $\omega \to \omega + i\nu$ to replace the integrals found in Eq. (3) with expressions like

$$\int d\underline{v} \int_{-\infty}^{\phi} G_e f_0(\underline{v}) d\phi' = \frac{i n_e}{k_z V_{th}} e^{-\lambda} \sum_{n=-\infty}^{\infty} I_n(\lambda) Z(\zeta_n). \tag{8}$$

CONCLUSION Figures 1 and 2 illustrate Eq. (5) for the scattering of 6943 Å light in a deuterium plasma in thermal equilibrium. The constant magnetic field is 70 kG. We take $n_e = 10^{16}$ cm^{-3}, $\kappa T_e = \kappa T_i = 5$ eV ($\alpha = 0.47$, $\lambda_D = 0.165$ μm) and $n_e = 10^{17}$ cm^{-3}, $\kappa T = 10$ eV ($\alpha = 1$, $\lambda_D = 0.074$ μm), respectively, where $\alpha \sim \lambda/\lambda_D$ is the Salpeter parameter [5] and λ_D is the Debye length. The advantage obtained in using Eq. (6) is especially important near the Bernstein resonances even though $\nu/\omega << 1$ because of the terms $(\omega^2 - n^2\omega_c^2)^{-1}$ which appear in $S(\underline{k}, \omega)$. Enhanced scattering at the electron cyclotron harmonics is more pronounced when $\lambda < \lambda_D$, a consequence of $\alpha \sim n^{1/2} T^{-1/2}$ while $\nu = \nu_{ee} \sim nT^{-3/2}$ [6]. When $\underline{k}$ is more than a few degrees away from being perpendicular to B_0, noncollisional damping predominates.

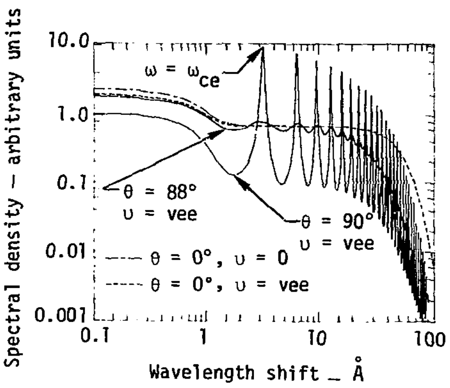

Fig. 1. $S(\underline{k}, \omega)$ versus λ in Angstroms ($\alpha = 0.47$).

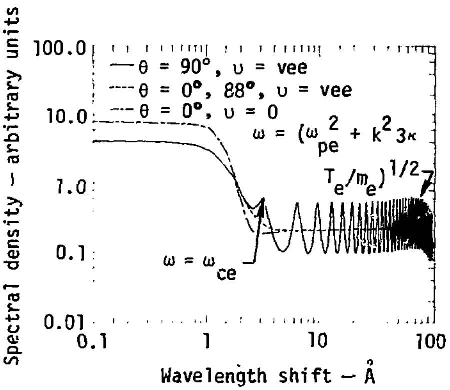

Fig. 2. $S(k, \omega)$ versus λ in Angstroms ($\alpha = 1$).

REFERENCES

1. M. N. Rosenbluth and N. Rostoker, Phys. Fluids 5 776 (1962).
2. J. Weinstock, Phys. Rev. 139 A388 (1965).
3. I. Bernstein, Phys. Rev. 109 10 (1958).
4. B. Fried and S. Conte, The Plasma Dispersion Function (Academic Press, New York, 1961).
5. E. E. Salpeter, Phys. Rev. 120 1528 (1960).
6. L. Spitzer, Physics of Fully Ionized Gases (London, Interscience, 1962).