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NUCLEAR GAMMA AND BETA DECAY

by

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In this paper some aspects of nuclear beta and gamma decay will be discussed and the places where additional experimental information or where theoretical problems still exist will be stressed. The material will be divided into two parts: first those rules which are geometrical in origin will be discussed and second results arising from isospin will be considered. It will be shown that a measurement of the ${}^{209}_{81}Tl_{128}$ beta decay to the $\frac{1}{2}^+$ state in ${}^{209}_{82}Pb_{127}$ could yield important information about a possible difference in the radii of the neutron and proton single particle potential wells. Further, the problem of the anomalously fast $\Delta T = 0$ E1 transitions in 4n-nuclei will be discussed.

1. Geometrical Rules

It is well known (1) that for a given model space, once the single particle energies and matrix elements of the residual two-body force are known the energy eigenvalues of the multi-nucleon system can be calculated in a purely geometrical manner - that is from a knowledge of Racah coefficients and coefficients of fractional parentage. In this section some gamma and beta decay results that arise from similar purely geometrical considerations will be discussed.

(a) Gamma Decay

The simplest geometrical results occur for M1 decays involving identical nucleons in the configuration j^n. (By identical is meant either all protons or all neutrons.) The matrix element governing this decay is given by

$$ME = \langle (j^n)_{I_f M_f} | (M1)_\lambda | (j^n)_{I_i M_i} \rangle$$

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where the notation $(j^n)_{IM}$ stands for n-nucleons in the single particle orbit j coupling their spins to total angular momentum I and z-component M. The usual form for the λ^{th} spherical component of the MI operator, $(MI)_\lambda$, is [2]

$$(MI)_\lambda = \sqrt{\frac{3}{4\pi}}\left(\frac{e\hbar}{2mc}\right)\sum_i\{[\mu_p\sigma_\lambda(i) + \ell_\lambda(i)][\frac{1-\tau_z(i)}{2}] + \mu_n\sigma_\lambda(i)[\frac{1+\tau_z(i)}{2}]\} \qquad (1)$$

where $\tau_z(i)$ is the z-component of the isospin operator for the i^{th} particle and has the eigenvalue +1 (-1) when operating on a neutron (proton). $\sigma_\lambda(i)$ and $\ell_\lambda(i)$ are the Pauli spin and orbital angular momentum operators for the i^{th} nucleon and μ_p and μ_n are the magnetic moments of the proton and neutron respectively (μ_p = 2.79, μ_n = -1.91 for free nucleons).

In general, the matrix element of <u>any single particle operator</u> $T^1_\lambda(i)$ which is a spherical tensor of rank one, satisfies the Wigner-Eckart theorm

$$<\chi^{j'}_{m'}(i)|T^1_\lambda(i)|\chi^j_m(i)> = (j1m\lambda|j'm')<\chi^{j'}(i)||T^1(i)||\chi^j(i)> \qquad (2)$$

where $(j1m\lambda|j'm')$ is the Clebsch-Gordan coefficient and the double-barred quantity is the reduced matrix element which is independent of m, m' and λ. If j = j' it follows from Eq (2) that

$$<\chi^j_{m'}(i)|T^1_\lambda(i)|\chi^j_m(i)> = \frac{<\chi^j(i)||T^1(i)||\chi^j(i)>}{<\chi^j(i)||\tilde{j}(i)||\chi^j(i)>}<\chi^j_{m'}(i)|\tilde{j}_\lambda(i)|\chi^j_m(i)>$$

$$= \alpha(i)<\chi^j_{m'}(i)|\tilde{j}_\lambda(i)|\chi^j_m(i)> \qquad (3)$$

where $\tilde{j}_\lambda$ is the single particle angular momentum operator and $\alpha(i)$, the ratio of the reduced matrix elements, is independent of the z-components of angular momentum. If matrix elements of $\sum_i T^1_\lambda(i)$ are calculated within the configuration j^n, $\alpha(i) = \alpha$ – that is since all nucleons are in the same single particle orbit $\alpha(i)$ is independent of which nucleon we discuss.

Thus

$$\sum_i T^1_\lambda(i) = \alpha\sum_i \tilde{j}_\lambda(i) = \alpha J_\lambda \tag{4}$$

where J_λ is the λ^{th} component of the total angular momentum operator. Thus

$$<(j^n)_{I_f M_f}|\sum_i T^1_\lambda(i)|(j^n)_{I_i M_i}> = \alpha<(j^n)_{I_f M_f}|J_\lambda|(j^n)_{I_i M_i}>$$

$$= \alpha(I_i 1 M_i \lambda|I_f M_f)<(j^n)_{I_f}||J||(j^n)_{I_i}>\delta_{I_i I_f} \tag{5}$$

where the $\delta_{I_i I_f}$ arises because the total angular momentum operator cannot change the angular momentum of a nuclear state. Thus we arrive at the selection rule: Within the identical nucleon configuration j^n there can be no M1 transitions.

In Table 1 we give some emperical evidence supporting this selection rule. The Weisskopf estimate for the mean lifetime of the state is based on the relationship (3)

$$\frac{1}{\tau_W(M1)} = 3.2 \times 10^{13}\ E^3 \tag{6}$$

where E is the gamma-ray energy measured in MeV. Normally (4) for nuclei with $A \lesssim 50$

$$\frac{\tau_W(M1)}{\tau_{expt}(M1)} \simeq 0.1\ .$$

Thus as seen from Table 1 these configuration forbidden M1's are inhibited by an extra factor of from 10 to 100.

Table 1

Comparison of lifetimes of forbidden M1 transitions with those computed using the Weisskopf estimate, Eq (5)

Nucleus	Transition	Gamma-Ray Energy in MeV	Mean life in picoseconds	$\frac{\tau_W(M1)}{\tau_{expt}(M1)}$
$^{19}_{8}O_{11}$	$(\nu d^3_{5/2})_{3/2} \rightarrow (\nu d^3_{5/2})_{5/2}$	0.096	1890	1.9×10^{-2}
$^{43}_{20}Ca_{23}$	$(\nu f^3_{7/2})_{5/2} \rightarrow (\nu f^3_{7/2})_{7/2}$	0.373	50	1.2×10^{-2}
	$(\nu f^5_{7/2})_{3/2} \rightarrow (\nu f^3_{7/2})_{5/2}$	0.221	380	6.8×10^{-3}
$^{45}_{20}Ca_{25}$	$(\nu f^5_{7/2})_{5/2} \rightarrow (\nu f^5_{7/2})_{7/2}$	0.1745	577	1.0×10^{-2}
$^{51}_{23}V_{28}$	$(\pi f^3_{7/2})_{5/2} \rightarrow (\pi f^3_{7/2})_{7/2}$	0.320	289	3.4×10^{-3}

A second interesting result for M1's emerges when we consider transitions within the multiplet formed by the configuration $[(\pi j^n)_{I_P} \times (\nu j_1^m)_{I_N}]_I$, where I_P is the angular momentum of the n-protons in the single particle orbit j, I_N is that of the m-neutrons in j_1 and $[\ \times\ ]_I$ stands for vector coupling to resultant spin I. An example of such a multiplet is provided by the states with angular momentum 2^+, 3^+, ..., 7^+ in the nucleus $^{92}_{41}Nb_{51}$ (see fig. 1). These states are thought to arise from the configuration $[\pi g_{9/2} \times \nu d_{5/2}]_I$. The same angular momenta are seen in the nucleus $^{96}_{41}Nb_{55}$ and presumably arise from the configuration $[\pi g_{9/2} \times \nu d^{-1}_{5/2}]_I$. If these states in the two nuclei do indeed come from these configurations their spectra are related by the equation [5]

$$E_I(jj_1^{-1};jj_1^{-1}) = -\sum_K (2K+1)W(jj_1j_1j;IK)E_K(jj_1;jj_1) \qquad (7)$$

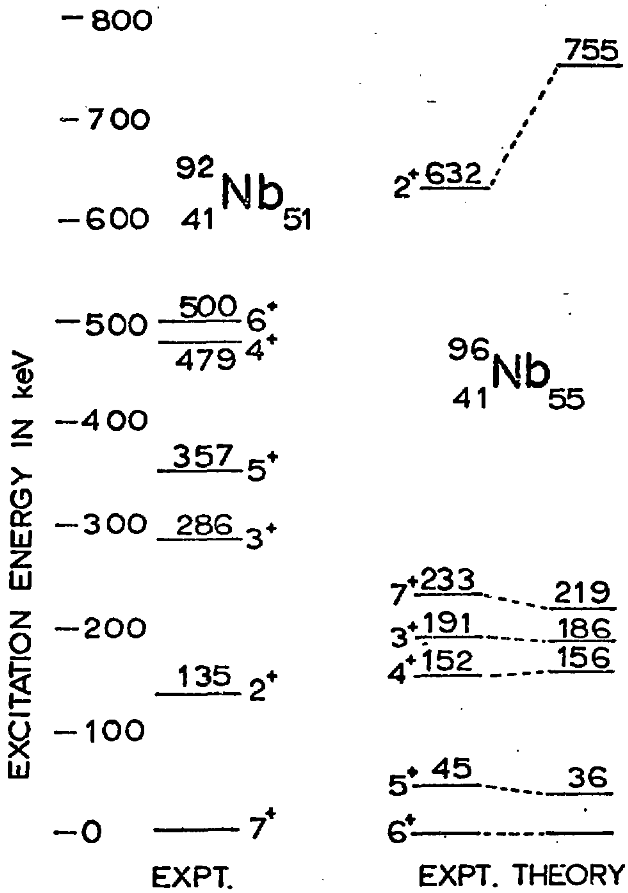

Figure 1. Experimental spectrum of ${}^{92}_{41}\mathrm{Nb}_{51}$ and ${}^{96}_{41}\mathrm{Nb}_{55}$. The theoretical predictions for ${}^{96}_{41}\mathrm{Nb}_{55}$ are computed by use of Eq (7).

where $E_K(jj_1;jj_1)$ are the energies in the particle-particle nucleus (${}^{92}_{41}\mathrm{Nb}_{51}$), $E_I(jj_1^{-1};jj_1^{-1})$ are the energies in the particle-hole nucleus (in this case ${}^{96}_{41}\mathrm{Nb}_{55}$) and W is the Racah coefficient. In fig. 1 the theoretical and experimental spectra of ${}^{96}_{41}\mathrm{Nb}_{55}$ are compared. As is seen, the agreement is remarkable – the rms error in any one of the predicted excitation energies in ${}^{96}_{41}\mathrm{Nb}_{55}$ is only 56-keV. Thus as far as spectra are concerned the 2^+, 3^+,... ......7^+ states in ${}^{92}_{41}\mathrm{Nb}_{51}$ and ${}^{96}_{41}\mathrm{Nb}_{55}$ act as though they came from the configurations $[\pi g_{9/2} \times \nu d_{5/2}]_I$ and $[\pi g_{9/2} \times \nu d^{-1}_{5/2}]_I$ respectively.

A more stringent test of these assignments is provided by a measurement of the M1 gamma decays within either of these multiplets. Since the M1 operator is the sum of a proton part and a neutron part and since the protons (neutrons) are confined to the configuration $(\pi j^n)_{I_p}$ $((\nu j_1^m)_{I_n})$ it follows that

$$(M1)_\lambda = \sum_{\text{protons}} \pi_\lambda^1(i) + \sum_{\text{neutrons}} \upsilon_\lambda^1(i) = \alpha(J_p)_\lambda + \beta(J_n)_\lambda$$

$$= (\frac{\alpha+\beta}{2})(J_p + J_n)_\lambda + (\frac{\alpha-\beta}{2})(J_p - J_n)_\lambda \qquad (8)$$

where $\pi_\lambda^1(i)$ is the M1 operator for the i^{th} proton and $\nu_\lambda^1(i)$ is the M1 operator for the i^{th} neutron. In writing the second line of this equation use has been made of Eq (3), that is

$$\alpha = \frac{\langle \overset{j}{\chi}(i) || \overset{1}{\pi}(i) || \overset{j}{\chi}(i) \rangle}{\langle \overset{j}{\chi}(i) || \tilde{j}(i) || \overset{j}{\chi}(i) \rangle}$$

$$\beta = \frac{\langle \chi^{j_1}(i) || \upsilon^1(i) || \chi^{j_1}(i) \rangle}{\langle \chi^{j_1}(i) || \tilde{j}_1 || \chi^{j_1}(i) \rangle}$$

Finally J_p and J_n are the total angular momentum operators for the protons and neutrons respectively. Since $(J_p + J_n)_\lambda = \tilde{I}_\lambda$, the total angular momentum operator, it follows that the $(\alpha+\beta)/2$ term in Eq (8) does not contribute to gamma decay. Therefore, within a multiplet all M1 decays are proportional to a single number, $(\alpha-\beta)/2$. By straight forward Racah algebra one easily shows that

$$B(M1;I_i \rightarrow I_f) = \left(\frac{2I_f+1}{2I_i+1}\right) \left| \langle [(\pi j^n)_{I_p} \times (\nu j_1^m)_{I_n}]_{I_f} || M1 || [(\pi j^n)_{I_p} \times (\nu j_1^m)_{I_n}]_{I_i} \rangle \right|^2$$

$$= (\alpha-\beta)^2 I_p(I_p+1)(2I_p+1)(2I_f+1)W^2(1I_pI_fI_n;I_pI_i) \quad (9)$$

Consequently the ratios of B(M1)'s within a multiplet depend only on geometrical factors and the precise form of the M1 operator (provided only that it is a sum of single particle operators) is not important.

In Table 2 the recent results obtained for $^{92}_{41}Nb_{51}$ by Brenner *et al.*[7] are listed. From the B(M1) ratios alone, it is apparent that a substantial amount of configuration mixing outside the $\pi g_{9/2}$ $\nu d_{5/2}$ model space is needed in the 6^+ and/or 7^+ to explain the transition. However, the other three M1's act as if they could be attributed to transitions between the assumed model space states.

Table 2

*M1 transition rates in $^{92}_{41}Nb_{51}$. In the theoretical calculation of B(M1) g_π and g_ν were taken to be 1.37 and −0.52 respectively. The unit for B(M1) is μ_N^2 where μ_N is the nuclear magne*ton.

Transition	B(M1) Ratios		B(M1) in μ_N^2	
	Experiment	Theory	Experiment	Theory
$3^+ \rightarrow 2^+$	$1.28^{+0.81}_{-0.57}$	0.72	10^{+6}_{-4}	2.79
$4^+ \rightarrow 3^+$	0.85 ± 0.21	0.88	6.6 ± 1	3.41
$4^+ \rightarrow 5^+$	1.00	1.00	7.8 ± 1.5	3.88
$6^+ \rightarrow 7^+$	0.11 ± 0.03	0.41	0.89 ± 0.13	1.58

By looking at the absolute magnetudes of the B(M1)'s one can also check how closely the effective M1 operator in the odd-odd nuclei resembles the magnetic moment operator in the odd-even system. The quantities α and β that appear in Eqs (8) and (9) are related to the nuclear g-factors,

$$\alpha = \sqrt{\frac{3}{4\pi}}\left(\frac{e\hbar}{2mc}\right) g_\pi (j=9/2)$$

$$\beta = \sqrt{\frac{3}{4\pi}}\left(\frac{e\hbar}{2mc}\right) g_\nu (j_1=5/2)$$

The value of g_ν can be obtained directly from the measured magnetic moment[8] of ${}^{91}_{40}Zr_{51}$, $g_\nu(j_1 = 5/2) = -0.52$. There is no data on ${}^{91}_{41}Nb_{50}$. However, the measured moment of the ground state of ${}^{93}_{41}Nb_{52}$ and the moments of the 8^+ states[9] in ${}^{90}_{40}Zr_{50}$ and ${}^{92}_{42}Mo_{50}$ give values of $g_\pi(j = 9/2) = 1.37$, 1.355 and 1.409 respectively. In the theoretical estimates given in the last column of Table 2 we have taken $g_\pi(j = 9/2) = 1.37$. Clearly the configuration mixing effects that lead to an effective operator for the magnetic moments in the odd A nuclei are different from those that are needed to explain the transition rates in the even A-nucleus.

A similar situation exists in ${}^{40}_{19}K_{21}$ – the B(M 1)'s within the $(\pi d^{-1}_{3/2} \times \nu f_{7/2})$ multiplet do not have the values predicted by use of the ${}^{41}_{20}Ca_{21}$ and ${}^{39}_{19}K_{20}$ g-factors. In this case the predicted value for the M1 transition between the two highest spin members of the multiplet $(4^- \rightarrow 5^-)$ is again much larger than observed experimentally[10] ($B(M1)_{expt} = 0.065\ \mu_N^2$) whereas $B(M1)_{theory} = 0.157\ \mu_N^2$). On the other hand, the two measured magnetic moments of states of the multiplet are in excellent agreement with the predictions made by use of the g-factors of the odd A nuclei. For the 4^-, $\mu_{expt} = -1.298\ \mu_N$ whereas $\mu_{theory} = -1.25\ \mu_N$ and a recent experiment[11] on the 3^- level gave $\mu_{expt} = -1.29 \pm 0.09\ \mu_N$ while $\mu_{theory} = -1.368\ \mu_N$. Consequently a measurement of the g-factors for states of the Niobium nuclei would be interesting to see whether the same situation prevails. For a state of angular momentum I the g-factor for either ${}^{92}_{41}Nb_{51}$ or ${}^{96}_{41}Nb_{55}$ is

$$g = \tfrac{1}{2}\left\{(g_\pi + g_\nu) + (g_\pi - g_\nu)\left[\frac{I_p(I_p+1) - I_n(I_n+1)}{I(I+1)}\right]\right\} \tag{10}$$

To gain some insight into the degree of impurity needed to explain the Nb data one can look at the situation in ${}^{38}_{17}Cl_{21}$ (i.e. the $(\pi d_{3/2} \times \nu f_{7/2})$ multiplet). In this case to explain experiment one needed about a 20% admixture from other configurations[12]. Since the pure configuration results are worse in ${}^{92}_{41}Nb_{51}$ than they are in ${}^{38}_{17}Cl_{21}$ one would expect at least this degree of impurity in the Niobium states.

(b) Beta Decay

The theoretical ft value for allowed beta decay, $(I_i T_i) \rightarrow (I_f T_f)$ is given by

$$ft = \frac{6250}{[T_i(T_i+1) - T_{z_i} T_{z_f}]\delta_{T_i T_f}\delta_{I_i I_f} + 1.51\left(\frac{2I_f+1}{2I_i+1}\right)|\langle\Psi^{I_f}||\sum_i \tau_\pm(i)\sigma(i)||\Psi^{I_i}\rangle|^2} \qquad (11)$$

where $\tau_+(i)$ is the operator which changes a proton to a neutron and $\tau_-(i)$ does just the reverse. The selection rules for this decay follow immediately from the form of the operators - that is, $\Delta I = 0, \pm 1$ (no parity change), $\Delta T = 0, \pm 1$; whereas $0 \rightarrow 0$ $\Delta T \neq 0$ transitions are forbidden. In addition there is another selection rule which is usually of only academic interest - namely Δn, the change in the number of radial nodes in the wave function, must be zero. Thus for example, the decay of a $3s_{\frac{1}{2}}$ proton to a $4s_{\frac{1}{2}}$ neutron would vanish because

$$\mathcal{I} = \int R^*_{3s_{\frac{1}{2}}}(r)\, R_{4s_{\frac{1}{2}}}(r)\, r^2 dr \qquad (12)$$

is zero if Coulomb effects are neglected.

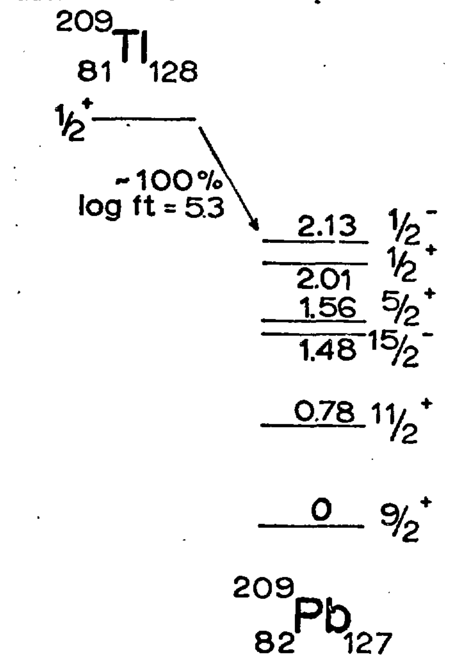

Figure 2. Beta decay of ${}^{209}_{81}Tl_{128}$ and the level sequence of ${}^{209}_{82}Pb_{127}$.

This selection rule seems to be observed in one instance - namely the decay of ${}^{209}_{81}Tl_{128}$ whose ground state is described as a $3s_{\frac{1}{2}}$ proton hole [13]. As shown in Fig. 2 there is no transition observed to the $4s_{\frac{1}{2}}$ neutron state in $^{209}_{82}Pb_{127}$ at 2.032 MeV; instead the decay goes entirely to the $\frac{1}{2}^-$ $p_{\frac{1}{2}}$ hole state at 2.151 MeV. At first glance this seems like rather nice confirmation of the Δn rule. However, further reflection indicates, as we shall now show, that the result is too nice.

The $\frac{1}{2}^+$ state in $^{209}_{82}Pb_{127}$ has the structure

$$\Psi^{\frac{1}{2}^+}(^{209}_{82}Pb_{127}) = (\nu 4s_{\frac{1}{2}})\Phi_c \tag{13a}$$

where Φ_c is the $^{208}_{82}Pb_{126}$ core. On the other hand, the structure of the $\frac{1}{2}^+$ state in $^{209}_{82}Tl_{128}$ is dominated by

$$\Psi^{\frac{1}{2}^+}(^{209}_{81}Tl_{128}) = \sum_j \alpha_j (\nu j^2)_o \Phi_c(\pi 3s_{\frac{1}{2}}^{-1}) \tag{13b}$$

where $\Phi_c(\pi 3s_{\frac{1}{2}}^{-1})$ is the $^{208}_{82}Pb_{126}$ core with a $3s_{\frac{1}{2}}$ proton hole in it and $\sum_j \alpha_j (\nu j^2)_o$ is the wave function for the two neutrons outside $^{208}_{82}Pb_{126}$ – that is the $^{210}_{82}Pb_{128}$ neutron eigenfunction. With the wave functions of Eq (13) it is straight forward to show that

$$\langle \Psi^{\frac{1}{2}^+}(^{209}_{82}Pb_{127}) || \tau_- \sigma || \Psi^{\frac{1}{2}^+}(^{209}_{81}Tl_{128}) \rangle = \sqrt{3}\alpha_{\frac{1}{2}} \mathcal{L} \tag{13}$$

where $\mathcal{L}$ is given by Eq (12). The coefficient $\alpha_{\frac{1}{2}}^2$ is the probability that the two extra core neutrons are in the $4s_{\frac{1}{2}}$ orbit and according to Herling and Kuo [14] $\alpha_{\frac{1}{2}}$ has a value of about 0.05.

In addition to this contribution to the beta decay matrix element there should also be one that arises from weak configuration mixing effects in $^{209}_{82}Pb_{127}$. The important mixings are those in which a $3s_{\frac{1}{2}}$ proton is excited to the Z = 82-126 shell and at the same time a neutron is excited from one of the N = 82-126 core orbits to say the $4s_{\frac{1}{2}}$ level. This type of admixture

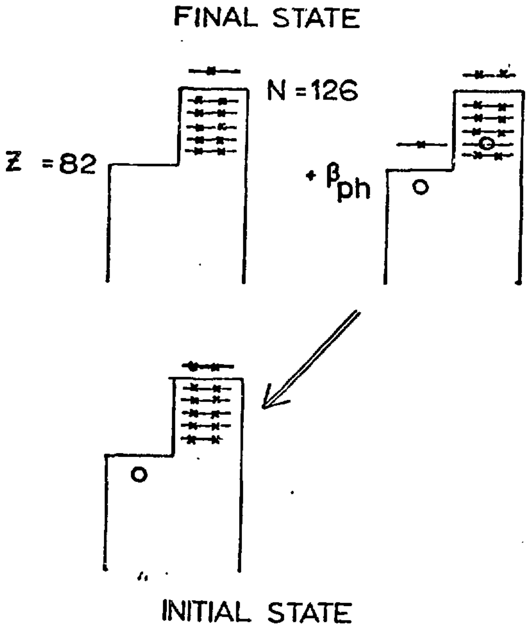

Figure 3. Schematic illustration of the wave functions involved in the beta decay of $^{209}_{81}Tl_{128}$. The coefficient β_{ph}^2 gives the probability of admixture of the particle hole wave functions.

together with the unperturbed initial and final state wave functions of Eq (11) is illustrated schematically in Fig. 3. Clearly if the particle-hole pair in the 82-126 shell couple to spin one a contribution to the beta decay which is linear in the admixture coefficient β_{ph} can arise. Such an admixture is known to give a substantial destructive interference [15] to allowed beta decay when $\Delta n = 0$. Consequently if $\mathcal{L}$, Eq (12) is indeed zero there is nothing with which this collective effect can destructively interfere and hence one would expect a substantial probability for beta decay to the $\frac{1}{2}^+$ state in $^{209}_{82}Pb_{127}$. Clearly the absence of this decay is a problem and it is important that a reliable limit for this branch in the decay of $^{209}_{81}Tl_{128}$ be established.

If this branch is indeed small, as it seems to be, the value of $\mathcal{L}$ must be sufficiently large to cancel out the collective contribution. In Table 3 we give values [16] of $\mathcal{L}$, Eq (12), as a function of the radius of the neutron single particle well. The integral is rather insensitive to the diffuseness parameter, a, and consequently values are only given for a = 0.65 fm. Clearly $\mathcal{L}$ is extremely sensitive to the difference in the neutron and proton well radii and essentially vanishes when they are equal. Thus a measurement of this beta decay branching ratio is likely to provide a stringent condition on the allowable difference of these radii in heavy nuclei and it is well known that Coulomb energy differences are very sensitive to this quantity [17].

Table 3

Values of the overlap integral $\mathcal{I}$, Eq (12). The proton well radius was held fixed at $1.2 \times A^{1/3}$ fm and the strength of the Woods-Saxon potential, $V(r) = -V_o/(1 + \exp(r-R_p)/a)$ was adjusted so that the $3s_{\frac{1}{2}}$ proton was bound, in the presence of the Coulomb field of a uniform charge distribution, by 7.367 MeV. For each neutron radius the depth of the neutron well was chosen so that the $4s_{\frac{1}{2}}$ neutron was bound by 1.928 MeV. In all cases the diffuseness parameter, a, was chosen to be 0.65 fm.

Neutron Well radius in fm	$\mathcal{I} = \int_0^\infty R_{3s}(r)R_{4s}(r)r^2dr$
$1.0 \times A^{1/3}$	-0.439
$1.1 \times A^{1/3}$	-0.228
$1.2 \times A^{1/3}$	-0.013
$1.3 \times A^{1/3}$	0.243

2. Isospin Rules

We now turn to selection rules or inhibitions brought about by isospin considerations.

(a) Gamma Decay

There is a stringent isospin selection rule which occurs for E1 transitions - that is T=0 → T=0 transitions are strictly forbidden if isospin is a good quantum number. This rule is easily deduced from the properties of the E1 operator, which has the form

$$(E1)_\lambda = \sqrt{\frac{3}{4\pi}}\, e \sum_i r_\lambda(i)\left[\frac{1-\tau_z(i)}{2}\right]$$

where $\underline{r}(i)$ is the position vector of the i^{th} nucleon and the factor $[1-\tau_z(i)]/2$

insures that only protons contribute to the transition probability. Since

$$\sum_i r_\lambda(i) = AR_\lambda$$

where A is the number of nucleons in the nucleus and $\underline{R}$ is the position vector of the center of mass, it follows that only the $\tau_z(i)r_\lambda(i)$ part of the E1 operator contributes to transition rates. This is true because the center of mass of the nucleus must always be in its ground state and hence $\langle\Psi_{M_f}^{I_f}|R_\lambda|\Psi_{M_i}^{I_i}\rangle = 0$.

Thus

$$(E1)_\lambda = \frac{1}{2}\sqrt{\frac{3}{4\pi}}\, e \sum_i r_\lambda(i)\tau_z(i) \tag{14}$$

Since $\tau_z(i)$ is a tensor operator of rank one it follows that $T=0 \rightarrow T=0$ transitions are forbidden.

An examination of the experimental data [3,18] shows that all E1's between low-lying states are severely inhibited and on the average the isospin allowed transitions have

$$\frac{B(E1)_{\mathrm{expt}}}{B(E1)_{\mathrm{Weisskopf}}} \approx 5 \times 10^{-4}$$

whereas the $T=0 \rightarrow T=0$ transitions show an additional inhibition of a factor of 10-50.

However, there are some isospin forbidden transitions which are anomalously fast and consequently pose an interesting theoretical problem. For example, the 6.95 MeV (1^-, T=0) decay to the 0^+ T=0 ground state in ${}^{40}_{20}Ca_{20}$ has

$$\frac{B(E1)_{\mathrm{expt}}}{B(E1)_{\mathrm{Weisskopf}}} \approx 2 \times 10^{-3}$$

In other words, this transition is faster than usual isospin allowed decays.

In Table 3 we have collected all the 1^- T=0 to 0^+ T=0 transitions which have the above ratio greater than 10^{-4}. From a knowledge of the position[19] of the closest 1^- T=1 state that can mix with the 1^- T=0 level we can estimate the size of <H>, the isospin non-conserving matrix element, which is needed to explain the transition rate. In making this estimate we have always assumed that B(E1) for the admixed state is one Weisskopf unit. Thus once ΔE, the energy difference between the 1^- T=0 and T=1 states is known <H> can be computed from the expression

$$\sqrt{\frac{B(E1)_{expt}}{B(E1)_{Weisskopf}}} = \frac{\langle H \rangle}{\Delta E} \tag{15}$$

The results that emerge from this calculation are listed in the last column of Table 4. In all cases <H> is much larger than would be computed from the Coulomb interaction and in fact is much larger than the values deduced from the beta-gamma-circular-polarization experiments. These latter experiments[20] require <H> between 1 and 40 keV.

Table 4

1^- T=0 to 0^+ T=0 transitions with anomalously large B(E1) values

(a) Estimated from the excitation energy (2.10 MeV) of the lowest 1^- state in $^{40}_{19}K_{21}$

(b) The first candidate for 1^- in $^{36}_{17}Cl_{19}$ is at 2.52 MeV

(c) The lowest known 1^- state in $^{32}_{15}P_{17}$ is at 4.04 MeV.

Nucleus	Transition $E_{initial} \rightarrow E_{final}$ (MeV)	$\frac{B(E1)_{expt}}{B(E1)_{Weisskopf}}$	Energy of nearest 1^- T=1 state in MeV	Required value of <H> the isospin mixing matrix element in keV
$^{40}_{20}Ca_{20}$	6.95 → 0	2×10^{-3}	9.76 (a)	126
$^{36}_{18}Ar_{18}$	5.84 → 0	6×10^{-4}	9.13 (b)	80
$^{32}_{16}S_{16}$	5.80 → 0	5×10^{-4}	11.04 (c)	117
$^{16}_{8}O_{8}$	7.12 → 0	3.9×10^{-4}	13.09	118

Since these isospin mixing matrix elements are so large, one must look for other ways to explain these anomalously fast isospin forbidden E1's. Several possibilities present themselves:

(i) Mixing with the E1 Giant Resonance State

To estimate <H> required with this type of mixing we assume the giant resonance is concentrated at 20 MeV excitation energy and has a strength of ten Weisskopf units. In the most favorable case, ${}^{16}_{8}O_{8}$, we need a value of

$$\langle H \rangle = 80 \text{ keV}$$

to explain experiment. Again this matrix element is much too large.

(ii) Departure from the long wave length limit

Since these gamma transitions are of high energy one should check whether the neglected $(kr)^3$ term in the transition matrix element is important. Since

$$j_1(kr) = \frac{kr}{3} - \frac{(kr)^3}{30}$$

one would expect the neglected term to give rise to a matrix element approximately $\frac{(kr)^2}{10}$ times the usual Weisskopf estimate. Thus B'(E1) due to this added term is

$$B'(E1) \simeq \frac{(kR)^4}{100} B(E1)_{Weisskopf}$$

where R is the nuclear radius. For the 6.95 MeV transition in ${}^{40}_{20}Ca_{20}$ this leads to

$$B'(E1) \simeq 4.4 \times 10^{-6} \; B(E1)_{Weisskopf}$$

when R is taken to be $1.2 \times 40^{1/3}$ fm. Thus this gives too small a contribution and is ruled out as a possible explanation.

(iii) Magnetic contributions to the E1 operator

The E1 operator also contains a contribution which is proportional to the proton magnetic moment [2], that is

$$(E1)_\lambda = e\, r\, Y^1_\lambda(\theta,\phi) - i\, \mu_p \frac{e\hbar}{4mc^2} \frac{E}{\hbar} (\sigma \times Y^1(\theta,\phi))_\lambda$$

where E is the energy of the emitted gamma-ray. The ratio of the contributions of these two terms should be approximately

$$\frac{\mu_p E}{4mc^2} = 5.2 \times 10^{-3}$$

Thus in this case B'(E1) would be

$$B'(E1) \simeq 2.6 \times 10^{-5}\ B(E1)_{\text{Weisskopf}}$$

Again this result is too small.

(iv) Possible Spin orbit effect

Since the single particle shell model Hamiltonian

$$H = \frac{p^2}{2m} + V(r) + f(r)\underline{\sigma}\cdot(\underline{r} \times \underline{p})$$

has a strong one-body spin orbit force one should logically take this into account when constructing the electromagnetic operator. Thus for the electric multipole operator, instead of considering $-\frac{e}{mc}\underline{p}\cdot\underline{A}$, one should use

$$-\frac{e}{mc}\underline{p}\cdot\underline{A} - \frac{e}{c} f(r)\sigma\cdot(\underline{r} \times \underline{A})$$

In the dipole limit the vector potential, $\underline{A} = \varepsilon e^{i\underline{k}\cdot\underline{r}}$, is replaced by $\underline{\varepsilon}$, its polarization vector. Thus we have to evaluate the matrix element

$$ME = \langle\Psi^{I_f}_{M_f}|\, \frac{e}{mc}\ \underline{p}\cdot\underline{\varepsilon} - \frac{e}{c} f(r)\underline{\sigma}\cdot(\underline{r} \times \underline{\varepsilon})\,|\Psi^{I_i}_{M_i}\rangle = -\frac{im}{\hbar}\langle\Psi^{I_f}_{M_f}|\,[\underline{r},H]\cdot\varepsilon\,|\Psi^{I_i}_{M_i}\rangle$$

$$= -im\omega\langle\Psi^{I_f}_{M_f}|\underline{r}\cdot\underline{\varepsilon}|\Psi^{I_i}_{M_i}\rangle$$

Therefore when matrix elements of the usual E1 operator, Eq (14) are computed this effect has already been taken into account[21].

Consequently it would appear that the simple mechanisms for getting such a large isospin admixture do not work and the theorist is faced with finding an adequate explanation for these results. Moreover, the explanation, when it is found, much be such that it does not lead to large isospin forbidden E1's in nuclei other than the 4n- nuclei listed in Table 4.

(b) Beta Decay

As stated in the previous section, the beta-gamma-circular-polarization experiments are consistent with small isospin admixtures (small values of $\langle H \rangle$). It is, of course, important to know whether these small estimates based on polarization are consistent with other methods of extracting isospin impurities and we shall now discuss this question.

An attempt to measure isospin mixing has recently been made by Garvey et al.[22] who look for a beta branch from the I=0 T=1 ${}^{42}_{21}Sc_{21}$ ground state to the 1.84 MeV excited I=0 T=1 state in ${}^{42}_{20}Ca_{22}$. They estimate that due to Coulomb effects one would expect

$$\frac{{}^{42}_{21}Sc_{21}\ \text{(ground state)} \rightarrow {}^{42}_{20}Ca_{22}(0^+;\ 1.84\ \text{MeV})}{{}^{42}_{21}Sc_{21}\ \text{(ground state)} \rightarrow {}^{42}_{20}Ca_{22}(0^+;\ \text{ground state})} = 0.6 \times 10^{-3}$$

Experimentally they find this branching ratio to be less than 1.2×10^{-3} and hence no evidence for any anomalously large mixing.

Another alternative is to deduce these admixtures from the data on isospin forbidden $0^+ \rightarrow 0^+$ transitions [23]. In general the wave functions of the nuclear states involved in the decay can be written as

$$\Psi = \phi^{IT}_{MT_z} + \sum_{T'} \alpha_{T'}(I)\, \phi^{IT'}_{MT_z}$$

where $\alpha^2_{T'}(I)$ is the probability that a state with isospin T' (T'>T) will be mixed into the state which is mainly isospin T. For the case that $I_i = I_f = 0$, $T_i \neq T_f$ it follows from Eq (11) that isospin admixtures alone contribute to the beta decay. If we denote the larger value of (T_i, T_f) by T and concentrate on decays for which $T_{z_i} = T_i$, $T_{z_f} = T_f$ it follows that for $0^+ \rightarrow 0^+$ isospin forbidden decays

$$ft = \frac{6250}{2T\alpha^2_T} \qquad (16)$$

Table 5

Isospin mixing matrix elements deduced from beta decay. The first three entries give the value of <H> deduced from the isospin forbidden $0^+ \rightarrow 0^+$ transitions. The last three give an upper limit for <H> based on the assumption that the Gamow-Teller matrix element is zero. The notation 2^{+*} indicates that the decay goes to the second 2^+ state in Fe.

Transition		log ft	α_T	Energy difference, ΔE, in MeV between admixed states	Isospin Mixing matrix element, <H>, in keV
Nucleus	Spin				
${}^{64}_{31}Ga_{33} \rightarrow {}^{64}_{30}Zn_{34}$	$0^+ \rightarrow 0^+$	6.6	1.98×10^{-2}	1.7	34
${}^{66}_{31}Ga_{35} \rightarrow {}^{66}_{30}Zn_{36}$	$0^+ \rightarrow 0^+$	7.9	3.62×10^{-3}	3.6	13
${}^{66}_{32}Ge_{34} \rightarrow {}^{66}_{31}Ga_{35}$	$0^+ \rightarrow 0^+$	≥ 7.4	$\leq 7.9 \times 10^{-3}$	6.7	≤ 53
${}^{47}_{20}Ca_{27} \rightarrow {}^{47}_{21}Sc_{26}$	$7/2^- \rightarrow 7/2^-$	8.5	$\leq 1.68 \times 10^{-3}$	8.38	≤ 14
${}^{56}_{27}Co_{29} \rightarrow {}^{56}_{26}Fe_{30}$	$4^+ \rightarrow 4^+$	8.5	$\leq 2.22 \times 10^{-3}$	6.0	≤ 13
${}^{58}_{27}Co_{31} \rightarrow {}^{58}_{26}Fe_{32}$	$2^+ \rightarrow 2^{+*}$	7.6	$\leq 5.1 \times 10^{-3}$	7.35	≤ 38

In Table 5 the sparse experimental data pertaining to isospin admixtures deduced in this way are tabulated. Although Eq (16) is only rigorously true for $0^+ \rightarrow 0^+$ transitions, it can also be applied to cases where the decay is severely inhibited. In these cases if one assumes the process goes entirely through isospin mixing (i.e. one assumes that the Gamow-Teller matrix element is zero) an upper limit on α_T can be obtained. Once α_T is known <H> may be determined from the relationship

$$\alpha_T = \frac{\langle H \rangle}{\Delta E}$$

where ΔE is the energy difference between the admixed states. This latter quantity can be obtained from either a knowledge of the position of the analog state [13] or can be deduced from binding energy differences. For example, in the A=64 and 66 nuclei the position of the analog state is not known and ΔE is estimated as follows: The neutron binding energy to the N = Z = 28 core can be obtained directly from the known total binding energies [24] of $^{56}_{28}Ni_{28}$ and $^{57}_{28}Ni_{29}$

$$\varepsilon_{\nu} = BE(^{57}_{28}Ni_{29}) - BE(^{56}_{28}Ni_{28}) = -10.267 \text{ MeV}$$

Because the mass of $^{57}_{29}Cu_{28}$ is not known, one must proceed in a round-about way to find ε_{π}, the proton binding to the core. From the known mass of $^{58}_{28}Ni_{30}$ one can calculate the interaction energy, E_o, between the two neutrons outside the $^{56}_{28}Ni_{28}$ core

$$BE(^{58}_{28}Ni_{30}) - BE(^{56}_{28}Ni_{28}) = 2\varepsilon_{\nu} + E_o$$

Thus E_o = -1.936 MeV. Further, since the analog of this state in $^{58}_{29}Cu_{29}$ is known [13] to lie at 0.202 MeV it follows that the proton binding energy to the core is

$$\varepsilon_{\pi} = -0.715 \text{ MeV}.$$

The excitation energy of the analog of the (N+1, Z-1) ground state in the nucleus (N,Z) is then given by the relationship

$$\Delta E = BE(N+1, Z-1) - BE(N,Z) + (\varepsilon_{\pi} - \varepsilon_{\nu})$$

From Table 5 it is apparent that the matrix elements of <H> deduced in this way have values consistent with those given by the beta-gamma-circular-polarization experiments and hence the E1 properties of the nuclei listed in Table 4 are indeed anomalous.

In summary, it is clear that there are still many interesting problems - both theoretical and experimental - associated with conventional beta and gamma decay. The measurement of Ml's within a multiplet does much to shed light on the question of configuration purity. Studies such as those described for Niobium could be profitably carried out on $^{210}_{83}Bi_{127}$ where one deals with the $(\pi h_{9/2} \times \nu g_{9/2})$ negative parity multiplet. A re-examination of the beta decay of $^{209}_{81}Tl_{128}$ is clearly called for since if the transition to the $\frac{1}{2}^+$ state in $^{209}_{82}Pb_{127}$ is indeed severely inhibited one can unambiguously determine whether or not the proton single particle well is larger than that of the neutron. Finally, the theoretical problem of the anomalously fast El's may well be explained by the fact that there are a large number of admixed I=1 T=1 states that interfere constructively to give the "large" El matrix element. However, if this is the case one must still answer the question "Why only for 4n-nuclei?" Alternatively the fast El's may be due to an isospin non-conserving part of the nucleon-nucleon interaction. If this is the answer, the "4n-nuclei question" still remains and in addition one must then address the question "Why do the $0^+ \rightarrow 0^+$ $\Delta T \neq 0$ transitions require such small isospin admixture?

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