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INHERENT REACTOR STABILITY

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Abstract

This paper concerns the natural stability of thermal nuclear reactors. Of primary interest is the reactivity feedback effected by changes in xenon concentration and coolant temperature. A mathematical study is presented, based on the frequency response method of stability analysis, and is supplemented by results of analog computer tests of a bare hypothetical reactor.

It is shown that even with a negative temperature coefficient of reactivity a reactor may be unstable, resulting in continuous oscillations of power about some average value. The minimum value of the negative temperature coefficient required for stability is determined as a function of design parameters and the flux level.

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Introduction

A power producing reactor with a negative temperature coefficient of reactivity has many advantages from a control and safety point of view. The idea of designing nuclear power plants to adjust naturally to changes in load is becoming quite popular. In fact, there is considerable interest in complete elimination of control systems, except for start-up and shut-down requirements. Hence, a thorough understanding of factors affecting inherent stability is more important now than in the days of fast automatic reactor control systems.

The problem of analyzing a reactor and plant for stability may be divided into sub-problems quite naturally on a time scale, or from a stability analyst's point of view, on a frequency spectrum. A very long range view (low frequency) shows that a reactor tends gradually to shut itself off as its fuel is being depleted. Reactivity control, e.g. through burnable poison or rod motion, is required to compensate for depletion. Transients in xenon concentration suggest another frequency range of interest, determined primarily by the iodine and xenon decay constants. The immediate response of a plant to changes in steam load concerns still higher frequencies (shorter time intervals). In conventional two-loop plants the primary loop recirculation time determines the upper cut-off frequency for power control. Where fast reactivity accidents are possible still higher frequencies are of interest. This spectrum is shown in Fig. 1.

One usually thinks of natural stability in terms of the immediate response to changes in load. A negative temperature coefficient may result in what appears to be satisfactory performance, in the short run. However, before all automatic and manual reactivity control devices are eliminated, stability throughout the complete frequency spectrum must be established.

The purpose of this paper is to present an analysis of the effect of xenon reactivity feedback on inherent stability. The frequency range of interest lies below one cycle per hour. Thermal reactors with negative temperature coefficients of reactivity are considered, resulting in absolute stability at zero frequency and at frequencies above the xenon range. A list of definitions of terms is given on page 14.

Inherent Feedback Loops[1]

Excluding all external sources of reactivity adjustment, the inherent contribution to reactivity variation is primarily a function of the state of the coolant,

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assumed to be water, and the variable concentration of neutron absorbing fission fragments (poisons). Since the state of the coolant is defined uniquely by the temperature and pressure, it is convenient to use two coefficients of reactivity:[2]

$$\text{Pressure coefficient} \equiv \left(\frac{\partial \delta k}{\partial P_c} \right)_{T_a}$$

$$\text{Temperature coefficient} \equiv \left(\frac{\partial \delta k}{\partial T_a} \right)_{P_c} .$$

Any nuclear dependence on temperature may be added to the coolant temperature coefficient. The variable reactivity of poison is assumed to depend almost entirely on xenon concentration. Proof that samarium poisoning may be neglected is given in the Appendix. Thus, the reactors considered here have three inherent reactivity feedback loops, which may be referred to as pressure feedback, temperature feedback, and xenon feedback (Fig. 2a).

The pressure feedback loop usually can be assumed to be unimportant for two reasons: (1) The "incompressibility" of water results in a very small pressure coefficient, and (2) proper pressurizer design permits essentially no pressure variations. The behavior of this loop must be examined when designing a pressurizing system. However, the dynamics of the temperature and xenon loops may be studied assuming constant pressure, as will be done here.

Stability

Any active system with feedback, such as a reactor with its inherent reactivity feedback loops, is capable of being unstable. The frequency response method is a convenient means of analyzing such a system for stability. This method is derived from general stability considerations for linear systems which require that the Laplace transformation of the output of a system have no poles with positive real parts, for all input functions which remain finite. For details of the development of the method the reader is refered to any text on feedback theory.[3] To clarify the significance of frequency response and stability a brief qualitative application to the reactor stability problem will be described, following which we will return to the details of the analysis.

The effects of both temperature and xenon feedback are degenerative; that is, any tendency for the flux level to change is opposed by the reactivity feedback of the resulting changes in temperature and xenon concentration. Thus, the overall system appears to be stable. However, if one postulates a small sinusoidal oscillation of reactivity as being inserted into the reactor, operating at a given average power level, the resulting sinusoidal reactivity feedback of temperature and xenon lags behind the disturbance. If this phase lag at some frequency is equal to 360° the loop, in effect, becomes regenerative. Oscillations at this critical frequency are reenforced by the feedback. In particular, if the magnitude of the reactivity fed back is greater than the magnitude of the hypothetical disturbance causing it, the system is capable of sustaining the oscillation. Such an unstable condition results in oscillations of reactivity and flux which build up in amplitude until limited by the nonlinearities of the system.

The ratio of the magnitude of feedback reactivity to that of input reactivity together with the loop phase shift form a complex function of frequency called the complex loop gain, the loop frequency response function, or the loop transfer function. The criterion for stability then is that the magnitude of the loop gain must be less than unity at all frequencies where the argument (phase shift) is 2π or an integral multiple thereof. We return now to the particulars of the problem.

The two reactivity feedback loops, temperature and xenon, may be reduced to one by considering the reactor, with temperature feedback, as a part of the xenon loop (Fig. 2b). Thus, the xenon loop gain is the product of two transfer functions: that which describes the response of xenon reactivity to oscillations of flux and that of the reactor with temperature feedback, identified respectively by the expressions

$$-\ \alpha_x \frac{\delta X}{\delta n/n_o} \quad \text{and} \quad \frac{\delta n/n_o}{\delta k}$$

where $-\alpha_x\,\delta X$ represents the reactivity feedback of xenon. The product of these functions is the dimensionless, complex loop gain which must satisfy the stability criterion. These transfer functions are derived below.

Xenon Transfer Function

The following is a derivation of the xenon transfer function, describing the response of xenon concentration to oscillations of flux. Xenon 135 is formed directly from fission and from the decay of iodine 135, which is not a poison. The applicable equations are[2]:

$$\frac{dx}{dt} = \gamma_x\,\Sigma_f\,n + \lambda_i\,i - \sigma_x\,nx - \lambda_x\,x$$

and

$$\frac{di}{dt} = \gamma_i\,\Sigma_f\,n - \lambda_i\,i\ .$$

The steady state solutions at any flux level n_o are

$$x_o = \frac{\Sigma f(\gamma_x + \gamma_i)n_o}{\lambda_x + \sigma_x n_o}$$

and

$$i_o = \frac{\Sigma f\gamma_i}{\lambda_i}\,n_o\ .$$

Since deviations from average values are of interest, the steady state solutions are subtracted, and the equations become:

$$\frac{d\Delta x}{dt} = \Sigma f\,\gamma_x\,\Delta n + \lambda_i\,\Delta i - \sigma_x(n_o\Delta x + x_o\Delta n + \Delta x\Delta n) - \lambda_x\,\Delta x\ ,$$

and $$\frac{d}{dt}\Delta i = \Sigma_f \gamma_i \Delta n - \lambda_i \Delta i \quad .$$

The assumption necessary to obtain a transfer function is that the cross-product $\Delta x \; \Delta n$ is small compared with $n_o \Delta x$ and $x_o \; \Delta n$. That this assumption is justified becomes apparent later. For sinusoidal deviations, expressed in complex form, the time derivative operations are replaced by $j\omega$, and the xenon transfer function is obtained:

$$\frac{\delta X}{\delta n/n_o} = \frac{\left[\frac{\lambda_x}{\sigma_x n_o}(\gamma_x + \gamma_i) + \left(\frac{\lambda_x}{\sigma_x n_o}\gamma_x - \gamma_i\right)\frac{j\omega}{\lambda_i}\right]}{\left(1 + \frac{\lambda_x}{\sigma_x n_o}\right)\left[\left(1 + \frac{\lambda_x}{\sigma_x n_o}\right) + \frac{j\omega}{\sigma_x n_o}\right]\left[1 + \frac{j\omega}{\lambda_i}\right]} \quad .$$

The magnitude and phase of this complex function of frequency are plotted in Figs. 3a and 3b for various values of n_o.

The reactivity feedback of xenon is the above function multiplied by $-\alpha_x$. The negative sign introduces 180^o phase shift, since

$$-\alpha_x = \alpha_x e^{j\pi} \quad .$$

Thus, the reactivity feedback of xenon is in-phase with the oscillation of flux at a frequency where the phase lag of the xenon transfer function above is 180^o, in the neighborhood of one cycle per day (Fig. 3b). Whether or not instability will occur cannot be determined until the reactor transfer function is included, completing the xenon loop gain.

Reactor Transfer Function

The transfer function of the reactor, with temperature feedback only, is simply a positive, real constant for frequencies sufficiently close to zero. The plausibility of this statement is explained before the detailed treatment is presented.

Consider a typical plant operating at a given power level. The plant consists of a primary coolant loop and a secondary steam plant. Now if one should increase the reactivity, e.g. through manipulation of control rods, the average coolant temperature in the reactor would increase to a new steady state value, such that the reactivity of the temperature increase just offsets the reactivity disturbance, not including xenon. The increase in coolant temperature increases the steam temperature and pressure in the secondary loop. Increased pressure results in increased steam flow, for a fixed throttle setting. Therefore, the reactor is returned to the critical condition, but at a higher power level, for the load on the plant has increased. These changes in the steady state values of the plant variables are approximately directly proportional to the amount of the reactivity disturbance. Thus, the change of flux level is directly proportional to the disturbance, in the steady state; that is, the transfer function of the reactor with temperature feedback is a positive real constant, at zero frequency.

It follows that if the frequency is low enough the plant variables will follow directly an oscillating reactivity disturbance, as they would in the steady state. Therefore, the transfer function of the reactor with temperature feedback remains a positive real constant up to a frequency where the time lags in the plant begin to have an effect. We return now to a mathematical justification of the above statements, and the derivation of an expression for the constant.

The overall frequency response of a system with feedback is of the form[3]:

$$\frac{G}{1-GH},$$

where G represents the transfer function without feedback and H is the feedback function. The product GH is the dimensionless complex loop gain. Note that if

$$GH \gg 1,$$

the overall transfer function becomes simply

$$\frac{G}{1-GH} = -\frac{1}{H}.$$

For this derivation of reactor response with temperature feedback the reactor transfer function (without feedback) is G, and the temperature reactivity feedback function is H. Based on certain justifiable assumptions, it will be shown that the loop gain is much greater than unity throughout the frequency range of interest, such that the overall transfer function is simply the negative reciprocal of H. It will be shown further that H is constant throughout the frequency range of interest. An approximate method of evaluating this constant is derived.

The familiar bare reactor transfer function G is derived from a linear approximation of the elementary reactor kinetic equations.[4,5] Curves of the magnitude and phase of G are given in Figs. 4a and 4b for various values of ℓ^*, the mean neutron lifetime.

The feedback function H depends on the frequency response of average coolant temperature in the core:

$$H = \frac{\delta k}{\delta n/n_o} = -\alpha_T \frac{\delta T_a}{\delta n/n_o}.$$

The value of this function at zero frequency (steady state gain) is readily computed. If the reactor inlet temperature T_c is constant the temperature gain is simply

$$\frac{\delta T_a}{\delta n/n_o} = n_o \left(\frac{\partial T_a}{\partial n} \right)_{T_c} \cong \frac{1}{2} \Delta T_R$$

assuming $$T_a \cong \frac{1}{2}(T_h + T_c) .$$

However, with recirculating coolant, T_c in general will change with power level. The recirculation introduces regenerative feedback which affects the overall response of average temperature:

$$\frac{\delta T_a}{\delta n/n_o} = n_o \left(\frac{\partial T_a}{\partial n} \right)_{T_c} \frac{1+K_c}{1-K_c}$$

where K_c is the loop gain (zero frequency) of the primary coolant loop. The value of K_c is equal to the boiler gain dT_c/dT_h assuming the boiler to be the only recipient of reactor power. The value of the boiler gain (and K_c) is always less than unity, and may be approximated using the following proportionalities for the boiler:

$$\text{Power} \propto (T_h - T_c)$$

$$\text{Power} \propto (T_a - T_s)$$

at a constant coolant flow rate, and

$$\text{Power} \propto P$$

at a constant steam throttle setting. This last proportionality is based on the assumption that the power delivered is porportional to the steam flow rate, which is true if the enthalpy rise through the steam generator is independent of load. Assuming that the steam generator furnishes dry and saturated steam and the average coolant temperature is approximately equal to

$$T_a \cong \frac{1}{2} (T_h + T_c) ,$$

the proportionalities combine to give

$$K_c = \frac{dT_c}{dT_h} = \frac{\left(\frac{dT_s}{dP} P_o + \Delta T_f\right) - \frac{\Delta T_R}{2}}{\left(\frac{dT_s}{dP} P_o + \Delta T_f\right) + \frac{\Delta T_R}{2}} .$$

The constants ΔT_R , ΔT_f and P_o are design parameters and dT_s/dP is determined from saturated steam tables.

The feedback function is a function of frequency, just as the reactor transfer function depends on frequency. However, it may be assumed that the feedback function is constant from zero frequency to beyond the highest frequency of interest, one cycle per hour. This is reasonable because the time delays which affect the temperature response are usually small compared with one hour.

The loop gain GH of the reactor with temperature feedback becomes unbounded as $f \to 0$, because the reactor transfer function G is unbounded and H is constant. Therefore, there exists a frequency f_o below which $GH > 10$. Then at frequencies below f_o the overall gain of the reactor with temperature feedback is approximately equal to $-1/H$, a constant. It is assumed that $f_o > 1$ cycle per hour; that is, $GH > 10$ and the overall gain is constant throughout the frequency range of interest. Substitution of typical numbers shows that this assumption usually is satisfied. Summarizing,

$$\frac{\delta n/n_o}{\delta k} = -\frac{1}{H},$$

$$H = -\alpha_T \frac{\Delta T_R}{2} \frac{1+K_c}{1-K_c},$$

$$K_c = \frac{\left(\frac{dT_s}{dP} P_o + \Delta T_f\right) - \frac{\Delta T_R}{2}}{\left(\frac{dT_s}{dP} P_o + \Delta T_f\right) + \frac{\Delta T_R}{2}}.$$

Xenon Loop Stability

Since the reactor transfer function is constant, the curves of Fig. 3a multiplied by

$$\alpha_x \frac{\delta n/n_o}{\delta k}$$

represent the magnitude of the xenon loop gain. The curves of Fig. 3b, minus 180°, show that the total phase lag is 360° at a frequency in the neighborhood of one to two cycles per day, depending on the flux level. Oscillation occurs if the magnitude of the loop gain exceeds unity at this critical frequency. The magnitude of the xenon transfer function at the critical frequency is plotted in Fig. 5 as a function of flux level. Note that instability cannot exist below a flux level of 4×10^{11} cm^{-2}sec^{-1}. The loop gain requirement places an upper limit on the constant

$$\alpha_x \frac{\delta n/n_o}{\delta k}.$$

Thus, the reciprocal of this constant must exceed the value given by the curve of Fig. 5 for stability. A method of evaluating the constant has been given in the preceeding section.

Sample Calculation

The order of magnitude of the temperature coefficient required for stability may be determined by considering a hypothetical plant with the following characteristics:

$$n_o = 10^{14}\ \text{cm}^{-2}\ \text{sec}^{-1}$$

$$\Delta T_R = 50^{o}F$$

$$\Delta T_f = 100^{o}F$$

$$P_o = 500\ \text{psi}$$

$$\alpha_x = 0.6$$

Substituting in the above equations:

$$K_c = 0.78$$

$$n_o \frac{dT_a}{dn} = 202^{o}F.$$

From Fig. 5 the "critical gain" is

$$\frac{\alpha_T\, n_o \frac{dT_a}{dn}}{\alpha_x} = 6.32 \times 10^{-2}$$

and the minimum negative temperature coefficient is

$$\alpha_T = 1.9 \times 10^{-4} \text{ per } {}^{o}F\ .$$

Simulation

To substantiate the analysis, the xenon feedback problem has been simulated on an analog computer. The nonlinear term (nx), which is linearized in the analysis, was accurately simulated to determine the effect on stability.

A typical analog response curve is given in Fig. 6. In this example the parameter values are:

$$n_o = 10^{14}\ \text{cm}^{-2}\ \text{sec}^{-1}$$

critical frequency = 1.3 cpd (from Fig. 3b) critical gain = 15.8 $\left(\left(\frac{\delta X}{\delta n/n_0} \right)^{-1} \text{ from Fig. 5} \right)$

actual gain used = 17.4 .

Note that following the initial disturbance the oscillation builds up rapidly in amplitude, even though the gain exceeds the critical gain by only 10%. The frequency observed is approximately 1.2 cycles per day.

The results of the analog tests confirm the critical gains and frequencies of oscillation within reasonable limits of accuracy. In addition they show that the nonlinear term, neglected in the analysis, does not limit the amplitude.

Conclusion

The analysis indicates that with no external control a reactor may be unstable because of xenon feedback if the negative temperature coefficient is not large enough. Such a condition results in an oscillation of flux which builds up in amplitude until limited by plant saturation effects, such as the boiling of the coolant.

Fortunately, the frequency of such oscillations is very low, in the neighborhood of one to two cycles per day. Therefore, a very slow control system, manual or automatic, would be sufficient to offset the instability.

Although instability in this low frequency range may appear not to be serious, it must be concluded that such an unstable plant could not be left unattended without an external control system. Periodic reactivity adjustments, manual or automatic, are required unless the magnitude of the negative temperature coefficient exceeds the critical value.

Appendix

The samarium transfer function is derived from the differential equations of samarium and promethium:[2]

$$\frac{ds}{dt} = \lambda_p\, p - \sigma_s\, ns$$

$$\frac{dp}{dt} = \gamma_p\, \Sigma_f n - \lambda_p\, p \ .$$

The steady state solutions at any flux level n_o are:

$$s_o = \frac{\gamma_p \Sigma_f}{\sigma_s}, \quad \text{independent of the flux level,}$$

and

$$p_o = \frac{\gamma_p}{\lambda_p} \Sigma_f\, n_o \ .$$

Subtracting the steady state solutions, and neglecting the cross-product $\Delta n \Delta s$, the transfer function is obtained:

$$\frac{\delta s}{\delta n / n_o} = \frac{-\gamma_p \dfrac{j\omega}{\lambda_p}}{\left(1 + \dfrac{j\omega}{\sigma_s n_o}\right)\left(1 + \dfrac{j\omega}{\lambda_p}\right)} \ .$$

Substitution of the proper values for the parameters yields the following results: The magnitude of this function cannot exceed γ_p, which equals 0.014. For frequencies above 0.5 cycles per day,

$$\frac{\delta s}{\delta n / n_o} = \frac{-\gamma_p}{1 + \dfrac{j\omega}{\sigma_s n_o}} \ .$$

For any flux level

$$\left|\frac{\delta X}{\delta n / n_o}\right| \gg \left|\frac{\delta s}{\delta n / n_o}\right|$$

at the critical frequency where the xenon loop phase lag is 360°, and at all higher frequencies. Therefore, samarium has a negligible effect on the instability of the xenon loop. At lower frequencies the magnitudes are comparable. However, it can be shown that the phase lag of the sum of the samarium and xenon reactivity transfer functions never reaches 180°, and that instability cannot exist below the critical frequency of the xenon loop. Furthermore, other causes of reactivity feedback, such as fuel depletion, would have to be included in a stability study at such low frequencies.

Nomenclature and Parameter Values[2]

n = average flux level.

δk = reactivity.

T_h = hot leg coolant temperature, reactor outlet to boiler inlet.

T_c = cold leg coolant temperature, boiler outlet to reactor inlet.

n_o = steady state average flux level.

ΔT_R = steady state difference between T_h and T_c at flux level n_o.

T_a = space average of coolant temperature in reactor, assumed equal to the space average of coolant temperature in boiler.

T_s = steam temperature (secondary loop saturation temperature).

ΔT_f = steady state difference between T_a and T_s (average boiler film drop), at flux level n_o.

P = steam pressure.

P_o = steady state steam pressure.

P_c = primary coolant pressure.

K_c = coolant loop gain, at constant coolant flow and constant throttle setting.

Σ_f = macroscopic fission cross section.

Σ_a = total macroscopic cross section (excluding xenon).

α_x = $\frac{\Sigma f}{\Sigma a}$.

α_T = negative temperature coefficient of reactivity.

α_p = pressure coefficient of reactivity.

x = xenon concentration.

i = iodine concentration.

s = samarium concentration.

p = promethium concentration.

γ_x = 0.0036 = fission yield of xenon.

γ_i = 0.0681 = fission yield of iodine.

γ_p = 0.014 = fission yield of promethium.

λ_x = 2.1×10^{-5} sec^{-1} = xenon decay constant.

λ_i = 2.9×10^{-5} sec^{-1} = iodine decay constant.

λ_p = 4.1×10^{-6} sec^{-1} = promethium decay constant.

σ_x = 3.5×10^{-18} cm^2 = microscopic xenon cross section.

σ_s = 5.3×10^{-20} cm^2 = microscopic samarium cross section.

$X = \frac{\Sigma x}{\Sigma f} = \frac{\sigma_x x}{\Sigma f}$.

$S = \frac{\Sigma s}{\Sigma f} = \frac{\sigma_s s}{\Sigma f}$.

f = frequency.

ω = angular frequency.

$j = \sqrt{-1}$

Δ is used as a prefix to denote a real deviation of a variable from equilibrium.

δ is used as a prefix to denote a sinusoidal deviation, in complex form, of a variable from equilibrium.

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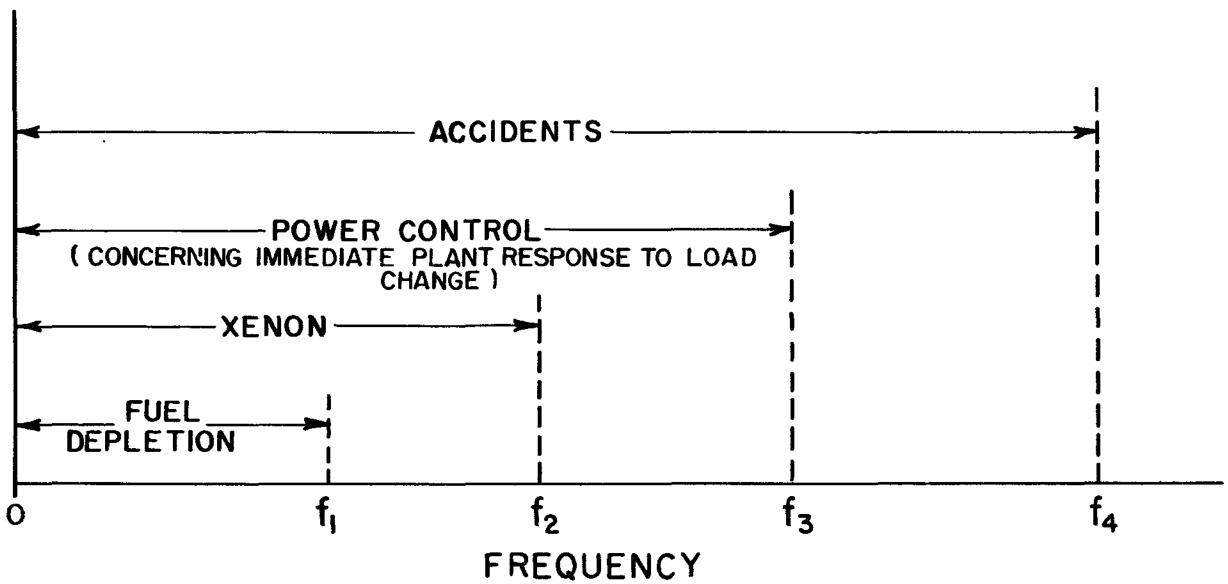


f_1 - IS A FUNCTION OF PLANT LIFETIME

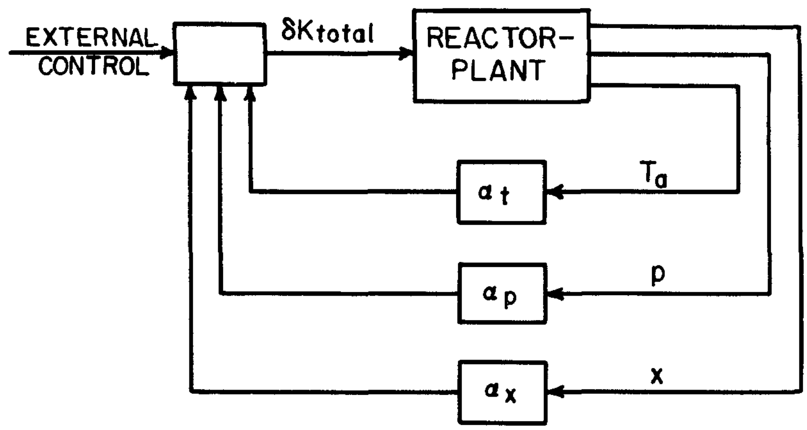
f_2 - IS A FUNCTION XENON AND IODINE DECAY CONSTANTS

f_3 - IS A FUNCTION OF PRIMARY LOOP CIRCULATION TIME

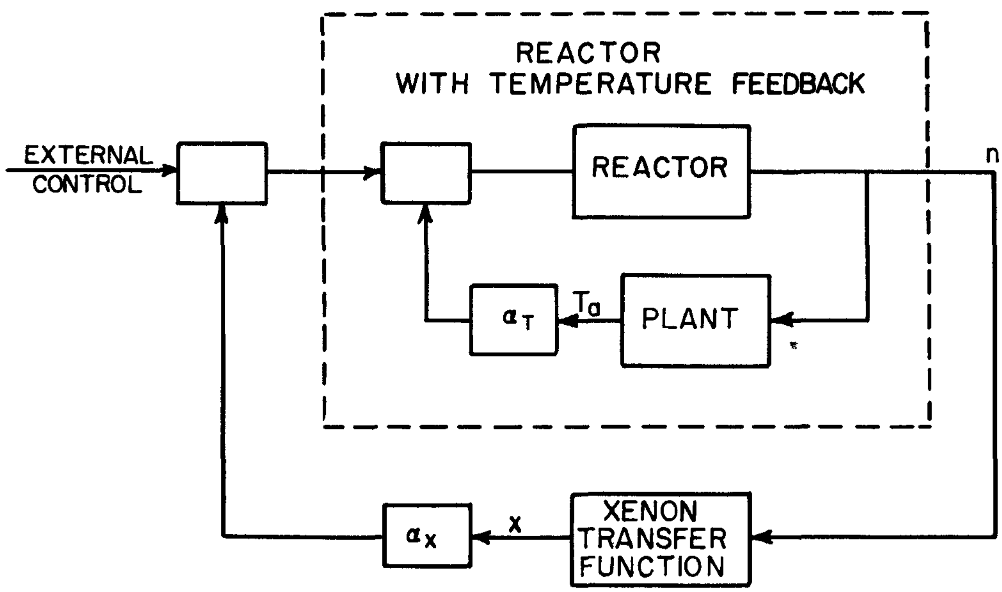
f_4 - IS A FUNCTION OF MEAN NEUTRON LIFETIME, ℓ^*

FREQUENCY SPECTRUM OF REACTIVITY VARIATIONS

Fig. 1



INHERENT REACTIVITY FEEDBACK LOOPS
FIG. 2a



XENON FEEDBACK LOOP
INCLUDING
REACTOR WITH TEMPERATURE FEEDBACK

Fig. 2b

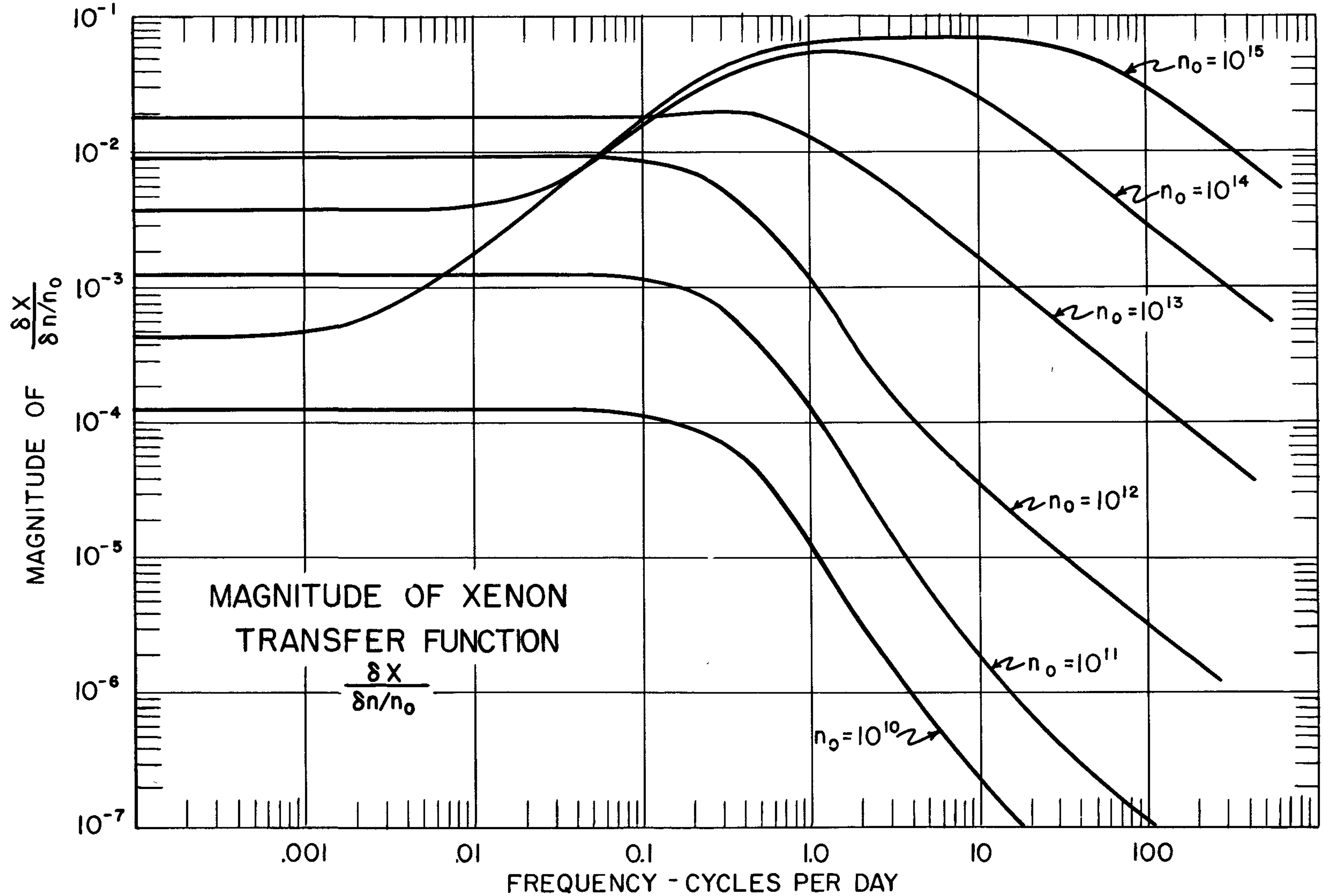
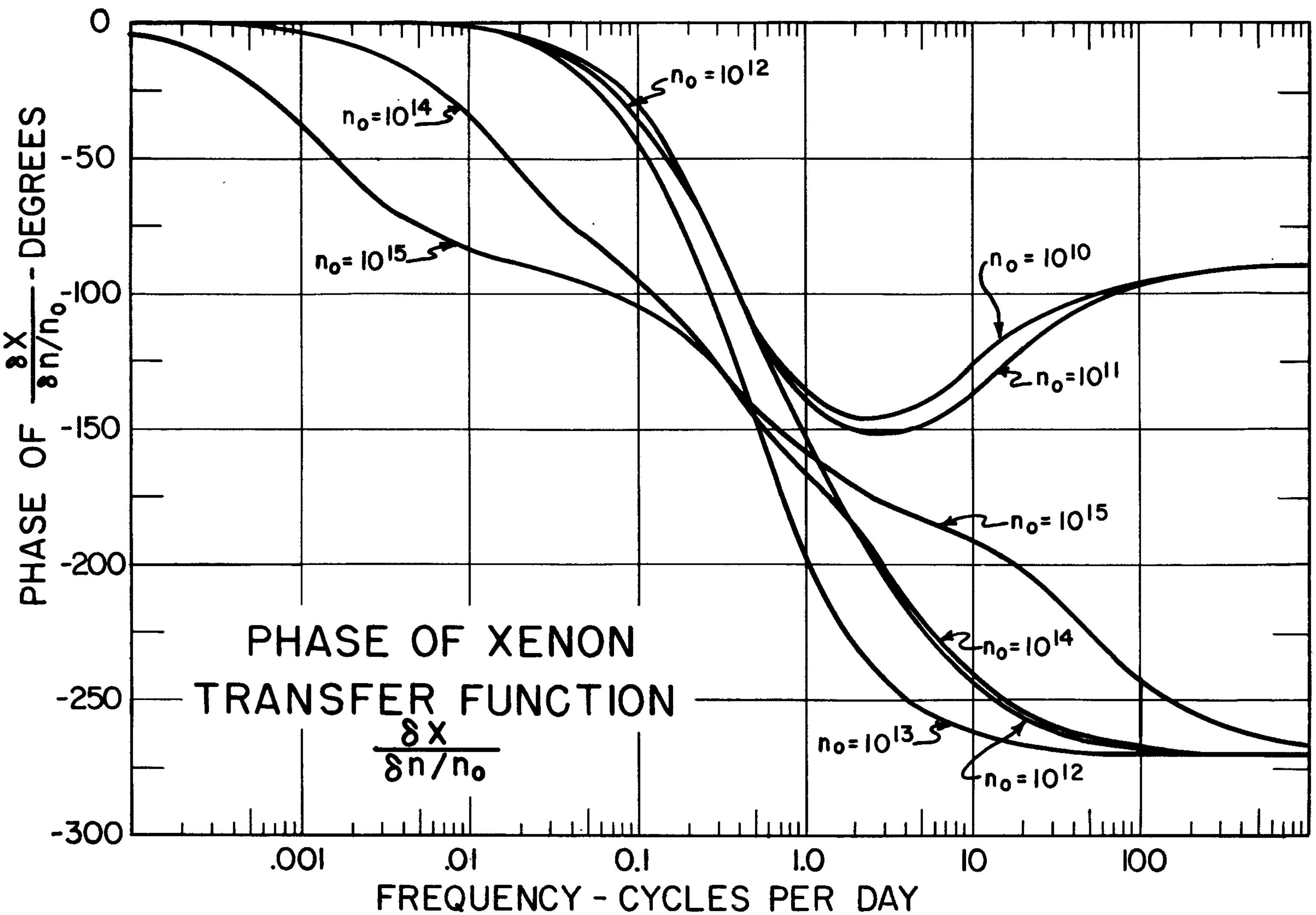


Fig. 3a

T.F. 4-12-55 424919-a



0
-50
-100
-150
-200
-250
-300
PHASE OF $\frac{\delta X}{\delta n/n_0}$ - DEGREES
.001
.01
0.1
1.0
10
100
FREQUENCY - CYCLES PER DAY
PHASE OF XENON TRANSFER FUNCTION $\frac{\delta X}{\delta n/n_0}$
$n_0 = 10^{12}$
$n_0 = 10^{14}$
$n_0 = 10^{15}$
$n_0 = 10^{10}$
$n_0 = 10^{11}$
$n_0 = 10^{15}$
$n_0 = 10^{14}$
$n_0 = 10^{13}$
$n_0 = 10^{12}$

Fig. 3b

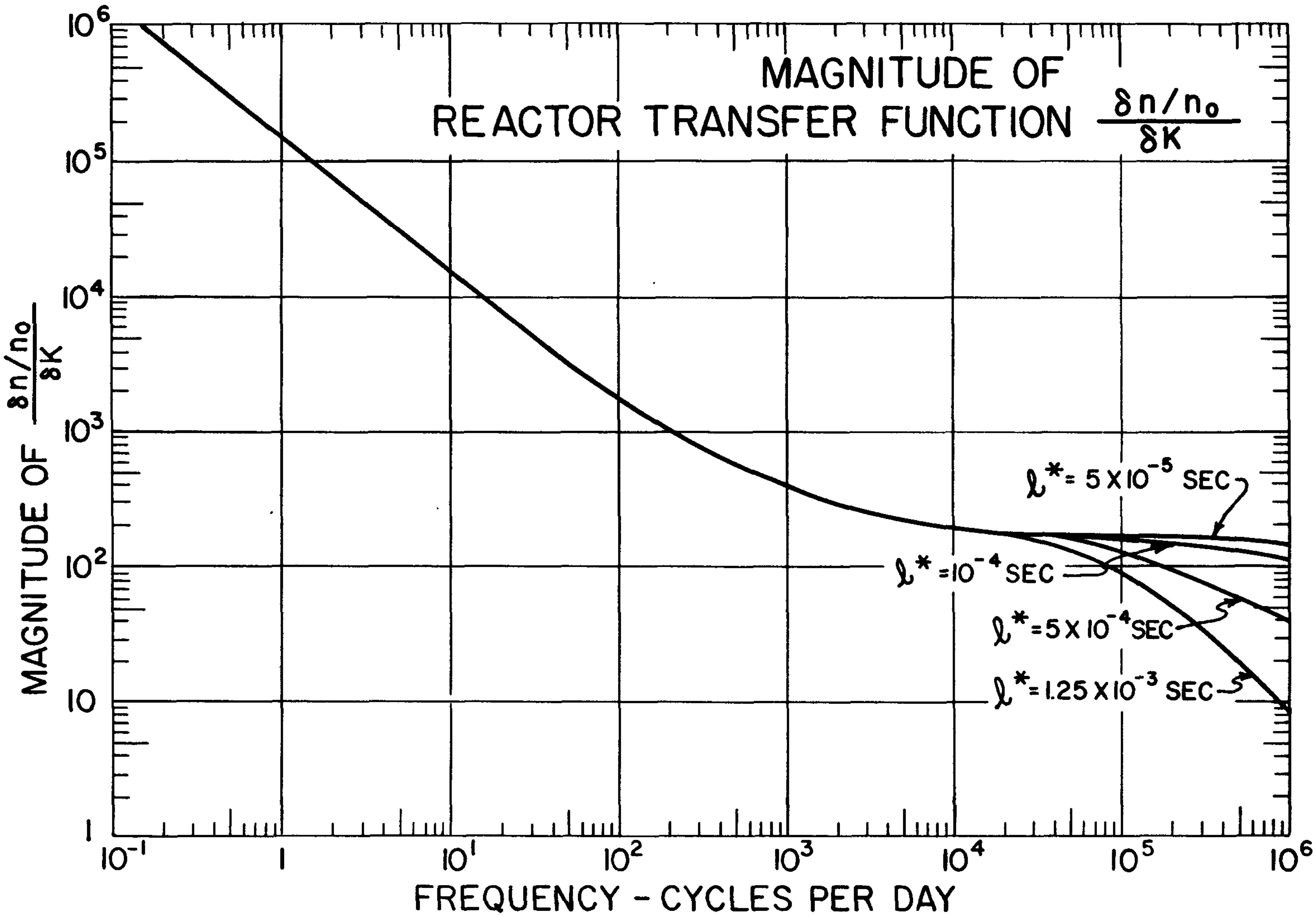

MAGNITUDE OF
REACTOR TRANSFER FUNCTION $\frac{\delta n/n_0}{\delta K}$
MAGNITUDE OF $\frac{\delta n/n_0}{\delta K}$
FREQUENCY - CYCLES PER DAY
$\ell^* = 5 \times 10^{-5}$ SEC
$\ell^* = 10^{-4}$ SEC
$\ell^* = 5 \times 10^{-4}$ SEC
$\ell^* = 1.25 \times 10^{-3}$ SEC

Fig. 4a

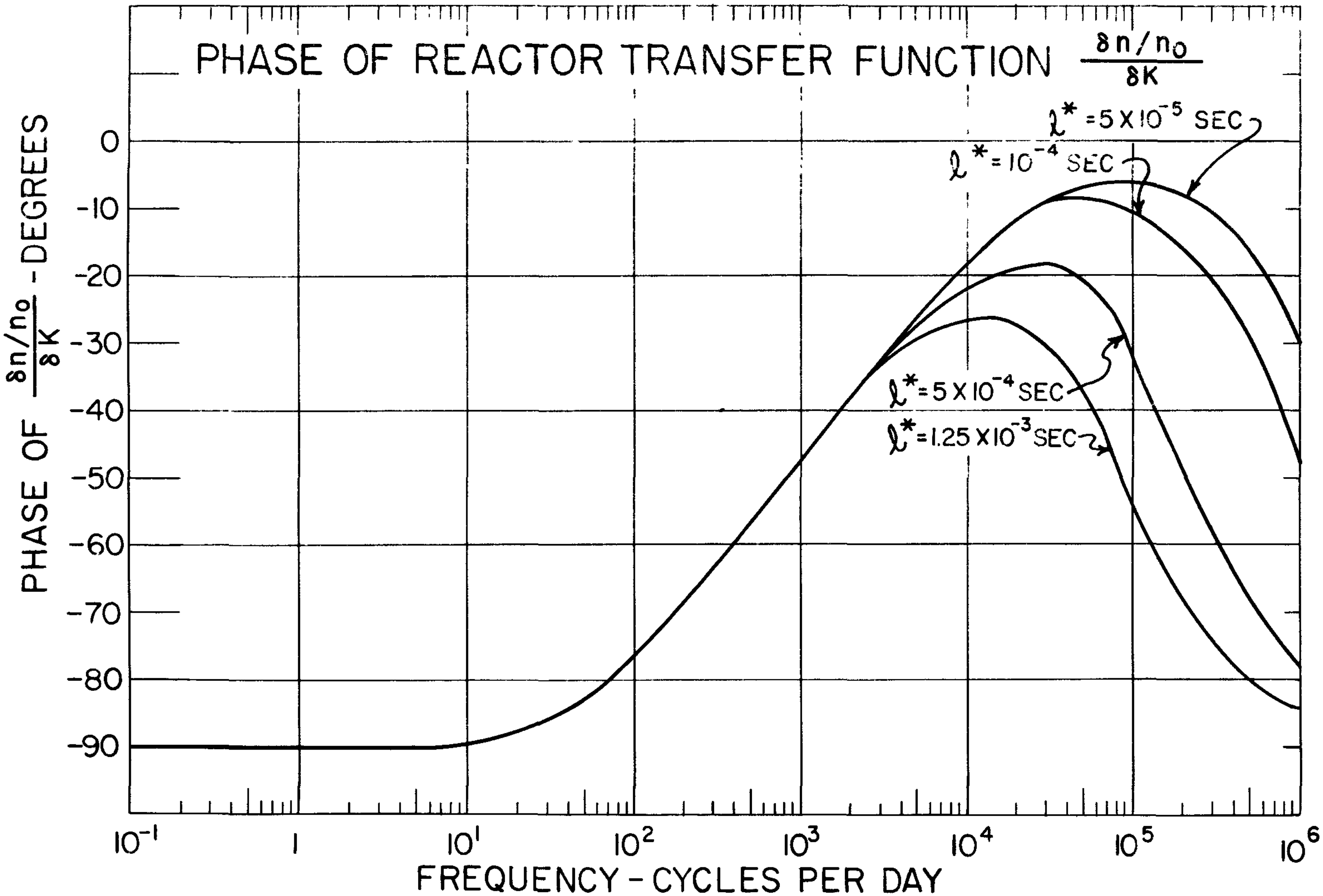

PHASE OF REACTOR TRANSFER FUNCTION $\frac{\delta n/n_0}{\delta K}$
PHASE OF $\frac{\delta n/n_0}{\delta K}$ - DEGREES
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
$l^* = 5 \times 10^{-5}$ SEC
$l^* = 10^{-4}$ SEC
$l^* = 5 \times 10^{-4}$ SEC
$l^* = 1.25 \times 10^{-3}$ SEC
10^{-1}
1
10^1
10^2
10^3
10^4
10^5
10^6
FREQUENCY - CYCLES PER DAY

Fig. 4b

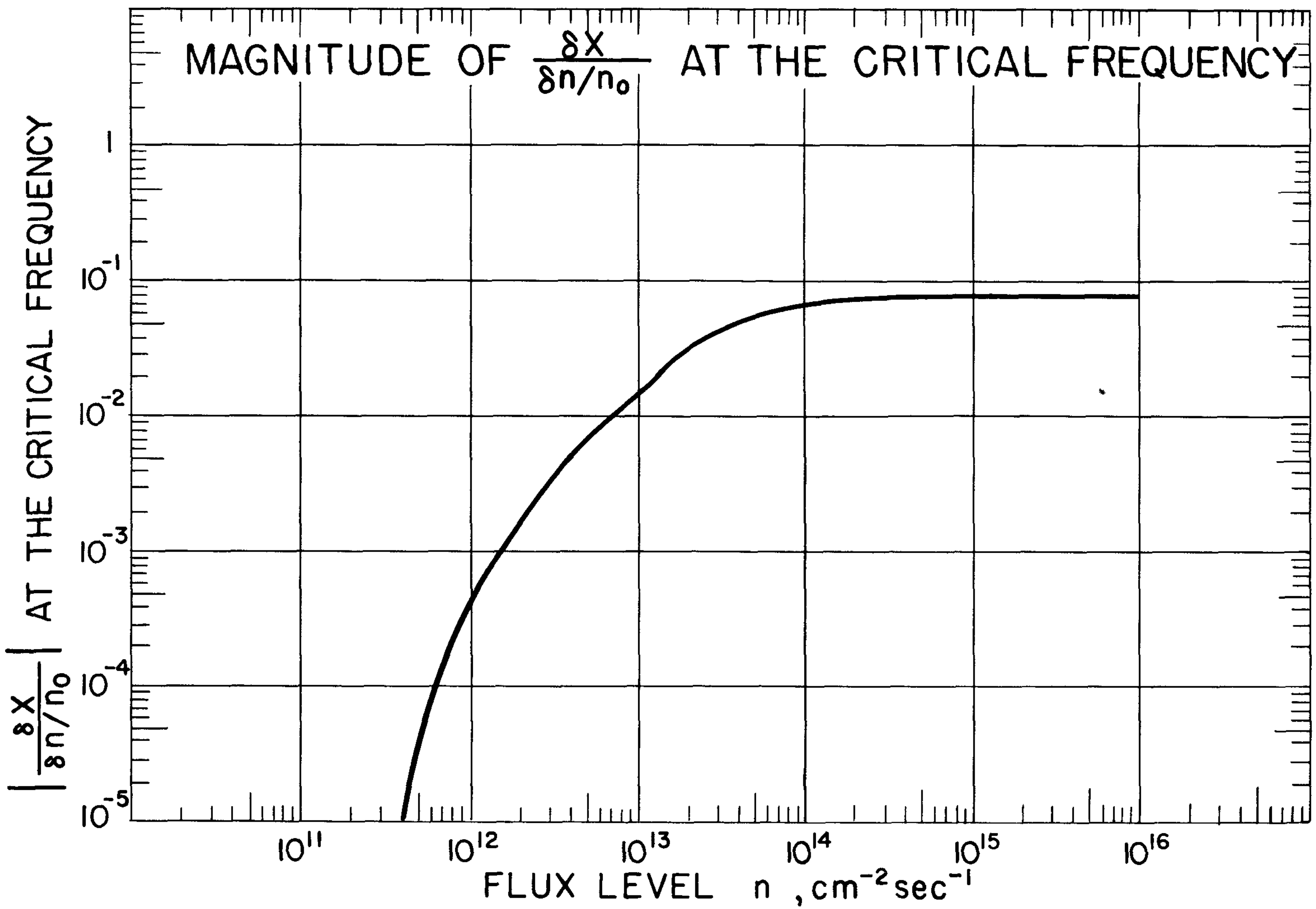

MAGNITUDE OF $\frac{\delta X}{\delta n/n_0}$ AT THE CRITICAL FREQUENCY
$\left|\frac{\delta X}{\delta n/n_0}\right|$ AT THE CRITICAL FREQUENCY
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
10^{16}
FLUX LEVEL n , $cm^{-2} sec^{-1}$

Fig. 5

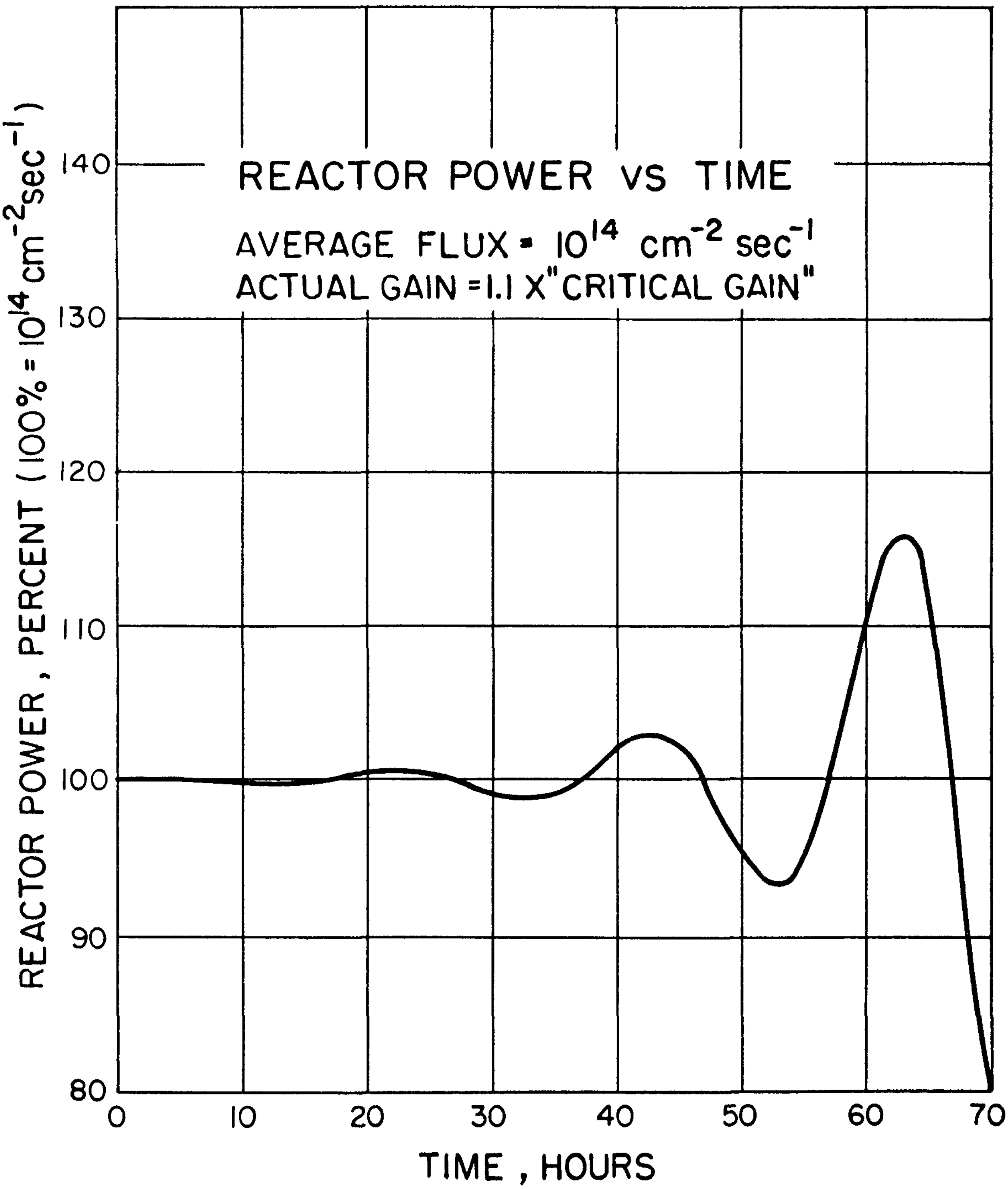

REACTOR POWER VS TIME
AVERAGE FLUX = 10^{14} cm^{-2} sec^{-1}
ACTUAL GAIN = 1.1 X "CRITICAL GAIN"
REACTOR POWER, PERCENT (100% = 10^{14} $cm^{-2}sec^{-1}$)
140
130
120
110
100
90
80
0
10
20
30
40
50
60
70
TIME, HOURS

Fig. 6

429-024