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MAGNETICALLY-DRIVEN METAL LINERS FOR PLASMA COMPRESSION

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MAGNETICALLY-DRIVEN METAL LINERS FOR PLASMA COMPRESSION*

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I. INTRODUCTION

A well-known fusion reactor proposal is the compression of deuterium-tritium plasma to thermonuclear temperatures by means of an imploding metallic liner.[1,2,3] Today the largest liner study programs are located in the USSR[4], and at the Naval Research Laboratory, USA.[5]

In this paper we shall formulate an approximate analytical model of liner compression in cylindrical geometry, and apply the results to reactor applications. The emphasis will be on the imploding metal liner itself as a means of energy compression for fusion.

Figures 1 and 2 show the essential features of the cylindrical liner system under consideration. A large energy outer magnetic field implodes the metal liner, which surrounds a DT plasma, which is insulated from the liner by a magnetic field. The outer magnetic field energy is converted to kinetic energy of the liner, and is then converted again to plasma and field energy in the interior. A lithium blanket surrounds the reactor for two purposes: to capture the neutron reaction energy, and to regenerate tritium. The overall objective of the system is to heat the plasma to fusion temperatures and contain it long enough for sufficient reactions to occur to provide a net energy gain.

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In our version of the scheme, the 14 meV neutrons from the reaction penetrate the liner and the container to reach the lithium blanket where their energy is absorbed. This is in contrast to other proposals which assume that the liner itself is lithium which is thick enough to stop the neutrons before they reach the wall.[2] When the thick lithium liner is used, it protects the container wall from damage by the 14 meV neutron flux. Unfortunately, the density of lithium is so low that it provides poor inertial containment for the reacting plasma. It may be possible to overcome this objection by alloying lithium with a heavier metal.[6]

The model developed here is more suitable for our thin heavy liner concept, although it may be applicable to some composite lithium-heavy metal liners.

II. NON-NUCLEAR ENERGIES AT TURNAROUND

At turnaround, where the liner radius reaches its minimum radius r_o (see Figure 2), the energy has three major components: the plasma energy, the axial magnetic field energy, and the compressional energy of the liner. The kinetic energy remaining in the liner is independent of radius r. These should be good approximations because the sound velocity (or Alfven velocity) of the hot plasma is much greater than the sound velocity of the cold dense liner. In addition, we shall neglect the diffusion of the magnetic field through both the liner and the plasma; this will be justified in a later section.

The total pressure P_o inside the liner can be written (see Figure 2):

$$P_o = \frac{B_M^2}{8\pi} = 2nkT + \frac{B_p^2}{8\pi} \tag{1}$$

where n is the ion or electron density of the DT plasma, and T is its temperature. Define the pressure ratio β in terms of the external magnetic field:

$$\beta \equiv 16\pi nkT/B_M^2 \tag{2}$$

Also, define the geometrical fill factor f_f in terms of the area ratio:

$$f_f \equiv (r_p/r_o)^2 \tag{3}$$

Then the plasma energy E_p per unit length is:

$$E_p = 3nkT\ f_f\ \pi r_o^{\ 2} = (3/2)\ \beta f_f\ \pi r_o^{\ 2}\ P_o \tag{4}$$

The magnetic energy E_M per unit length is:

$$E_M = \frac{B_M^{\ 2}\ (1-f_f) + B_p^{\ 2}\ f_f}{8\pi}\ \pi r_o^{\ 2} = (1-\beta f_f)\ \pi r_o^{\ 2}\ P_o \tag{5}$$

In order to find the compressional energy of the liner, we must make a fit to published compression data, and we must also estimate the pressure and density distribution in the liner. The data for many metals[7] can be fitted by an equation of the form:

$$\frac{\rho_o}{\rho} = \frac{v}{v_o} = \sum_i a_i e^{-P/P_i} \tag{6}$$

For example, for copper two terms suffice to make a close fit over the pressure range 0-4.5 Megabars:

$$\left(\frac{v}{v_o}\right)_{cu} = 0.3e^{-P/.65} + 0.7e^{-P/15.} \tag{7}$$

Let W be defined as the energy of compression per gram. Then one can find:

$$W = -\int_0^P P\ \frac{dv}{dP}\ dP = v_o \sum_i a_i P_i \left[1 - \left(1 + \frac{P}{P_i}\right) e^{-P/P_i}\right] \tag{8}$$

To estimate the pressure and energy distribution in the liner, it is convenient to define a mass thickness parameter η in units of gm/cm^2:

$$d\eta \equiv \rho dr \tag{9}$$

At the outside of the liner [$r = r_0(1+f_L)$, see Figure 2] where the pressure is zero, we define $\eta \equiv 0$. At the inside of the liner ($r=r_0$), we define $\eta \equiv \sigma_0$ where σ_0 is called the total mass thickness. For the pressure distribution to be consistent with the impulse-momentum theorem, we write:

$$P/\eta = P_0/\sigma_0 \tag{10}$$

This approximate formula for the pressure distribution neglects the inertia of the plasma, convergence effects in the cylindrical geometry, and liner heating.

Next, we need a relation between the total mass thickness σ_0 and the thickness parameter f_L (Figure 2). This is found by integration:

$$f_L\ r_0 = \int_{r_0}^{r_0(1+f_L)} \frac{dr}{d\eta}\frac{d\eta}{dP}\,dP = \frac{v_0\sigma_0}{P_0}\sum_i a_i\ P_i\left(1-e^{-P_0/P_i}\right) \equiv v_0\sigma_0\ z\left(P_0\right) \tag{11}$$

where we have used equations (6), (9), and (10), and where we have defined the "first compression function" $z(P)$.

Now we are ready to calculate the liner compressional energy per unit length E_L:

$$E_L = \int_0^{\sigma_0} 2\pi r_0\ W d\eta \tag{12}$$

Substitute equations (8) and (10), and carry out the integration:

$$E_L = 2\pi r_0\sigma_0 v_0 P_0 \sum_i a_i \frac{P_i}{P_0}\left[e^{-P_0/P_i} + 1 + 2\frac{P_i}{P_0}\left(e^{-P_0/P_i} - 1\right)\right] \tag{13}$$

Finally, we wish to express the energy E_L in a form similar to equations (4) and (5), and as a function of f_L rather than σ_o. Substitute equation (11) into equation (13):

$$E_L = f_L \, s(P_o) \, \pi r_o^2 \, P_o \qquad (14)$$

where the "second compression function" s(P) is given by:

$$s(P) \equiv \frac{2}{z(P)} \left\{ \sum_i a_i \frac{P_i}{P} \left[e^{-P/P_i} + 1 + 2 \frac{P_i}{P} \left(e^{-P/P_i} - 1 \right) \right] \right\} \qquad (15)$$

where z(P) was defined in equation (11).

As an example, for copper (equation 7), one obtains the compression functions shown in Figure 3.

To obtain the total non-nuclear energy per unit length E_T, add equations (4), (5), and (14):

$$E_T = E_p + E_M + E_L = [1 + (1/2)\, \beta f_f + f_L s(P_o)] \, \pi r_o^2 \, P_o \qquad (16)$$

Most of this energy must be provided by the external energy supply that implodes the liner; this aspect of the problem will be considered in a later section.

III. TURNAROUND DYNAMICS AND NUCLEAR ENERGY OUTPUT

Consider the time-dependance of the liner motion up to the time of turnaround (see Figure 2). We ignore the subsequent expansion, because Rayleigh-Taylor instabilities on the inner surface of the liner would probably destroy the symmetry at later times.[8] For the thin liner approximation we have:

$$\frac{d^2r}{dt^2} = P/\sigma = P_o\left[\left(\frac{r_o}{r}\right)^2\right]^2 \frac{1}{\sigma_o}\ \frac{r}{r_o} = \frac{P_o}{\sigma_o}\left(\frac{r_o}{r}\right)^{2\gamma-1} \tag{17}$$

where we have approximated the dynamics of the plasma/field combination by adiabatic compression of a simple gamma-law gas.

The first integral of equation (17) is obtained by introducing the velocity $u = dr/dt$:

$$\int\frac{d^2r}{dt^2}\,dt = \int_u^0 u\,du = \frac{1}{\tau_o^2}\,r_o^{2\gamma}\int_r^{r_o}\frac{dr}{r^{2\gamma-1}} \tag{18}$$

where the hydrodynamic time constant τ_o is defined by:

$$\tau_o \equiv (\sigma_o\ r_o/P_o)^{1/2} \tag{19}$$

Integration of equation (18) leads to the following equation for the velocity u:

$$u = \frac{dr}{dt} = -\frac{1}{\tau_o\sqrt{\gamma-1}}\left[1 - \left(\frac{r_o}{r}\right)^{2(\gamma-1)}\right]^{1/2} \tag{20}$$

For most values of γ (such as 3/2 or 4/3), the integration of equation (20) leads to transcendental equations for r which are analytically awkward. However, for the value $\gamma = 2$, one finds a simpler result:

$$\left(\frac{r}{r_0}\right)^2 = 1 + \left(\frac{t}{\tau_0}\right)^2 \tag{21}$$

For our approximate analytic model, we will use this simple result, although most plasma-field systems would be expected to be "softer" (lower γ). Systems with large magnetic energies would be closest to having an effective gamma of 2. Plasma-dominated systems would be closer to a gamma of 5/3; an additional heating source, however, such as a laser or electron beam, could be used to raise the effective gamma during the liner compression.

Now consider the nuclear energy output Y from the DT reaction in the plasma near turnaround (see Figure 4):

$$Y = E_{DT}\, \pi r_0^{\,2}\, f_f \int_{-\infty}^{0} \frac{n^2}{4}\, \overline{\sigma v}\,(T)\, dt \tag{22}$$

where E_{DT} is the useful energy release per reaction, where $\overline{\sigma v}$ (T) is the nuclear cross-section averaged over the Maxwellian velocity distribution at temperature T, and where we have assumed a 1:1 mixture of deuterium and tritium ions.

The reaction cross-section[9] $\overline{\sigma v}$ (T) will be approximated by a quadratic fit which is accurate to about 20% in the temperature range 7 keV < T < 20 keV:

$$\overline{\sigma v}\,(T) \stackrel{\sim}{=} 0.4\,(kT)^2 \tag{23}$$

Combining equations (1), (2), (17), (21), (22), and (23) we obtain:

$$Y = \frac{1}{40} E_{DT} \pi r_o^2 f_f \beta^2 P_o^2 \int_{-\infty}^{0} \frac{dt}{\left[1 + \left(\frac{t}{\tau_o}\right)^2\right]^4} \qquad (24)$$

Using the substitution $u = (t/\tau_o)^2$ the integral can be rewritten in the following form:

$$\tau \equiv \frac{\tau_o}{2} \int_0^{\infty} \frac{u^{-1/2}\,du}{(1+u)^4} = \frac{\tau_o}{2} B\left(\frac{1}{2}, \frac{7}{2}\right) = .491\ \tau_o \qquad (25)$$

where $B\left(\frac{1}{2}, \frac{7}{2}\right)$ is a "Beta Function".[10] τ is the effective nuclear reaction time constant; it is smaller than the hydrodynamic time constant τ_o because the nuclear reaction rate is a steep function of both plasma density and plasma temperature.

Combining equations (24) and (25), one obtains:

$$Y = (.0123\ E_{DT}\ \beta^2 f_f P_o \tau_o)\ \pi r_o^2 P_o \qquad (26)$$

where the nuclear energy output Y is written in the same form as the total non-nuclear energy E_T (see equation 16). This result for the output energy Y will be an underestimate for those cases where the effective γ of the plasma-field mixture is less than 2. In those cases the total pressure will not fall as rapidly when the radius is increased; consequently, there will be a few more reactions at large radii then have been calculated here. However, this conservatism is offset by the fact that we have not taken radiative losses into account.

IV. ENERGY MULTIPLICATION AND ENERGY PER UNIT LENGTH

We define the energy multiplication α for the total system involving plasma, field, and liner:

$$\alpha \equiv \frac{Y}{E_T} = .0123\ E_{DT} \frac{\beta^2 f_f P_0 \tau_0}{1 + (1/2)\ \beta f_f + f_L\ s(P_0)} \qquad (27)$$

where we have used equations (16) and (26). If we set E_{DT} = 17.6 meV and solve for the product $P_0 \tau_0$, we find:

$$P_0 \tau_0 = 2.89 \times 10^6 \frac{\alpha}{\beta^2 f_f} [1 + (1/2)\ \beta f_f + f_L\ s(P_0)] \qquad (28)$$

Equation (28) can be compared with Lawson's $n\tau$ criterion at T = 10 keV, by means of equations (1), (2), and (25):

$$n\tau = \left(\frac{\beta P_0}{2kT}\right)\left(.491\tau_0\right) = 4.44 \times 10^{13} \frac{\alpha}{\beta f_f} [1 + (1/2)\ \beta f_f + f_L\ s(P_0)] \qquad (29)$$

As a specific example, choose "breakeven" (α = 1) for a field-free (β = 1) plasma contained inside a rigid nonconductin wall (f_f = 1, $s(P_0)$ = 0). Then we find

$$n\tau = 6.66 \times 10^{13} \qquad (30)$$

This value is consistent with the Lawson criterion.[11]

In order to consider specific models, it is interesting to find the final system radius r_0 for a given choice of plasma and liner parameters (α, β, f_f, f_L, and P_0). r_0 is obtained from equations (11) and (19):

$$r_o^2 = \left(\frac{1}{\rho_o f_L}\right)\left[\frac{z(P_o)}{P_o}\right]\left(P_o \tau_o\right)^2 \qquad (31)$$

where we can substitute for $P_o \tau_o$ from equation (28). The total non-nuclear energy E_T (see equation 16), can then be rewritten in terms of this solution:

$$E_T = \frac{\pi}{\rho_o f_L}\left[z(P_o)\right]\left(P_o \tau_o\right)^2\left[1 + (1/2)\ \beta f + f_L\ s(P_o)\right] \qquad (32)$$

Combining equations (28) and (32), we obtain:

$$E_T = 2.624 \times 10^{13} \frac{z(P_o)}{\rho_o f_L} \frac{\left[1 + (1/2)\ \beta\ f_f + f_L\ s(P_o)\right]^3}{\beta^4 f_f^2} \alpha^2 \qquad (33)$$

Equation (33) is an important result of this approximate analysis; it is the energy per unit length that the liner system must have as a function of the plasma model β, f_f; of the desired energy multiplication α; of the final pressure P_o; of the liner parameters ρ_o, f_L; and of the liner compressibility functions $z(P)$, $s(P)$.

Consider again the idealized rigid wall example of equation (30) $[\alpha = \beta = f_f = z(P_o) = 1,\ s(P_o) = 0]$. Then equation (33) becomes:

$$E_T = (885/\rho_o f_L) \quad \frac{\text{megajoules}}{\text{meter}} \qquad (34)$$

Note that this result is independant of the final pressure P_o. The coefficient is a large amount of energy; a dense liner (high ρ_c) is desirable to reduce E_T. A thick liner (high f_L) would also seem desirable; however, it must be remembered that this approximate theory only holds for comparatively thin liners. Choosing a maximum value of $f_L = 1$, and a maximum practical liner density of 10 - 20, we find that the ideal minimum possible value of the non-nuclear energy E_T approaches 50 - 100 megajoules/meter.

Next, consider a somewhat more realistic breakeven ($\alpha = 1$) case. Choose $\beta = f_f = 0.707$, and consider copper liners for which $\rho_0 = 8.93$ and $f_L = 1$. Then we find:

$$E_T \approx (460)\ z(P_0)\ [1 + 0.8\ s(P_0)]^3 \quad \frac{\text{megajoules}}{\text{meter}} \tag{35}$$

In this case the more realistic plasma model causes the minimum value of the non-nuclear energy to be about 5 - 10 times higher than the previous example. The compressibility function $z(P_0)$ tends to lower this value by increasing the effective liner density, but the corresponding energy function $s(P_0)$ tends to cancel the effect. The overall result is a gentle variation in the value of E_T as a function of P_0, as shown in Figure 4. In addition to the total energy E_T, Figure 4 also shows the compressional energy in the liner for this case.

Other important parameters of the liner compression are the liner mass m_0 and initial velocity u_L. The mass m_0 per unit length is most easily determined from equations (19) and (28):

$$m_0 = 2\pi r_0\ \sigma_0 = 2\pi \frac{(P_0\ \tau_0)^2}{P_0} \tag{36}$$

The approximate initial liner velocity u_L is found by neglecting the initial energy of the plasma and field:

$$u_L{}^2 = \frac{2E_T}{m_o} = \frac{P_o\, z(P_o)}{\rho_o f_L} \left[1 + (1/2)\, \beta f_{\bar{T}} + f_L\, s(P_o)\right] \qquad (37)$$

where we have substituted equations (32) and (36). Note that the required initial velocity u_L is an increasing function of the final pressure P_o. This method of finding the initial velocity u_L is more exact than differentiation of equation (21), because it does not assume $\gamma = 2$.

The final radius r_o and the initial liner velocity u_L are plotted in Figure 5 for the plasma breakeven case previously considered in equation (35) and Figure 4. From this figure it appears that the most practical breakeven copper liner experiment would have a final pressure P_o near one megabar. Lower pressures imply large radii, and higher pressures imply large liner velocities.

It should be remarked that an energy E_T of 450 megajoules/meter is comparable to the energy release from 90 kilograms of TNT per meter. Such large energies imply very large systems; furthermore $E_T \sim \alpha^2$, so it is of considerable interest to investigate methods of keeping practical liner devices small. Possible ways of doing this will be considered in a later section of this paper.

V. MAGNETIC DIFFUSION IN THE LINER

As mentioned above, we have thus far neglected the diffusion of the magnetic field into the metallic liner. Now we shall estimate the size of this effect. To do this we make use of a previously published shin depth δ approximation:[12]

$$\frac{d}{dt}\left(\delta^2\right) \cong 0.6\,\frac{\eta_0}{4\pi}\,\frac{B^2}{8\pi\,\rho_0\,c_p\,T_0} \tag{38}$$

where η_0 is the metal liner resistivity at temperature T_0, ρ_0 is its density, and c_p is its specific heat. For the copper liner of our previous example, we obtain:

$$\frac{d}{dt}\left(\delta^2\right) = 2.62 \times 10^{-10}\, B^2 \tag{39}$$

when B is in gauss.

As a first approximation we substitute the flux-conserved value of B (no diffusion), and integrate over time:

$$\delta^2 = 2.62 \times 10^{-10}\, B_0{}^2 \int_{-\infty}^{0} \frac{dt}{\left[1 + \left(\frac{t}{\tau_0}\right)^2\right]^2} \tag{40}$$

where we have substituted equation (21). The integral is $\pi/4$, and the skin depth δ becomes:

$$\delta = .0143\sqrt{\tau_0}\, B_0 = .0719\,\left(P_0\,\tau_0\right)^{1/2} \tag{41}$$

where we have used equation (1). By comparison with equations (28)-(30) one finds that the skin depth δ is almost independent of the pressure P_0, or the size r_0. For the example plotted in Figures 4 and 5, the skin depth is computed to be about 0.25 cm, which is small compared to the final inner liner radius r_0 at all pressures except $P \gtrsim 10$ MB where the compressional energy is also rising.

This skin depth estimate neglects the decrease in metallic resistivity due to compression, and so it may be an overestimate. On the other hand, the metal vapor cloud observed at high fields[13] may blow across the void gap and contaminate the DT plasma. Such effects require further investigation.

Overall, we find that the comparatively small value of the skin depth in the liner justifies our neglect of magnetic diffusion in these large liner systems. Magnetic diffusion in the plasma is also neglected; its effects can be roughly taken into account by adjustment of the plasma parameters β and f_f.

VI. NUMERICAL COMPUTATIONS OF LINER SYSTEMS

An independent evaluation of this analytical model can be made by comparing it to numerical computations of cylindrical liner compression. Several such calculations have been done in cylindrical geometry[14] using the multizone, two-temperature hydrodynamic code LASNEX.[15]

To the basic code, an axial magnetic field (B_z) has been added, similar to the MAGPIE code.[16] The equation of state of the copper liner that was used is more elaborate than the approximation of equation (6). If $\rho > \rho_0$, it uses a Gruniesen formulation.[7] If $\rho < \rho_0$, a virial expansion is made which is matched to the estimated critical point parameters.[17] In addition, the energy of the alpha particles produced in the DT reactions is redeposited in the plasma, using approximate formuli for the range and time delay of the alpha particle.

Most of the code problems cannot be compared with this model because they used thick liners; however, one problem had a thin liner for which $f_L = 1.16$, when the liner kinetic energy was minimum. At this time the inner liner radius $r_0 = 1.04$ cm, the total pressure $P_0 = 1.65$ MB, the fill factor $f_f = .55$, and the plasma $\beta = .95$.

In Table I we show a comparision of the energies computed by the code and by the approximate model. The agreement is seen to be fairly good except for the nuclear yield parameters α and Y. The principal reason for this discrepency is believed to be the fact that diffusion of the magnetic field into the plasma is neglected. Consequently, most of the plasma has a lower effective β than the value $\beta \approx .95$, which was computed at the plasma center. A lower value of β would bring both the plasma energy E_p and the nuclear parameters α and Y into better agreement.

Another important result that was shown by the computer runs[14] is that the nuclear yield near breakeven does not increase for liners thicker than $f_L \cong 1.0$, if one holds the total problem energy constant. The significance of this result is that our approximate thin liner model is a good way of estimating the total energy requirements for breakeven given by the more exact computations, even for thicker liners beyond the range of validity of the initial thin liner assumptions.

Thus, it is concluded that the computer runs support the results of this approximate model for liner compression.

VII. LENGTH AND TOTAL ENERGY

In addition to the radial compression, a complete system analysis must also consider axial flow of the plasma out of the ends of the cylindrical liner. A complete two-dimensional computation of this problem has not yet been undertaken; therefore, we shall adopt a simple approximate criterion for the length of the liner system which should illustrate the magnitudes of the quantities involved. The criterion is that the reaction time τ must be at least as short as the time it takes the plasma to escape from the ends:

$$L \gtrsim 2v_a \tau \cong 2 \times 10^8 \tau \tag{42}$$

where L is the length of the liner-plasma system, and v_a is the acoustic velocity in a 10 keV DT plasma.

For the "ideal" plasma of equation (30), we find:

$$L \gtrsim \frac{1.33 \times 10^{22}}{n} \tag{43}$$

For a dense theta pinch ($n = 10^{17}$), one obtains a length of 1.33 kilometers, which is within a factor of two of other estimates.[18,19,20] For the dense liner systems considered here, the length is considerably shorter.

A more general result for L is found by combining equations (25), (28), and (42):

$$L = \frac{2.838 \times 10^{14}}{P_o} \frac{\alpha}{\beta^2 f_f} [1 + (1/2) \beta f_f + f_L s(P_o)] \qquad (44)$$

The total non-nuclear energy E_{min} for the whole length of the liner is then found from equations (33) and (44):

$$E_{min} = L E_T = 7.447 \times 10^{27} \frac{z(P_o)}{P_o \rho_o f_L} \frac{[1 + (1/2) \beta f + f_L s(P_o)]^4}{\beta^6 f_f^3} \alpha^3 \qquad (45)$$

Inspection of equations (44) and (45) shows that to first approxmate, the required total energy is inversely proportional to the final pressure P_o. Detailed calculations of L and L E_T for the more realistic plasma model are plotted in Figure 6, which confirm the $1/P_o$ relationship except at the highest pressures where the liner compressibility becomes important.

Since $P_o \sim n_f$, we find $E_{min} \sim \alpha^3/n_f$ as stated in the abstract. Note, however, the importance of the parameters β and f_f. Low values of β and f_f require much larger values of E_{min} for breakeven.

It is interesting to compare equation (45) with the expression for the minimum energy $E_{L.P.}$ required for ignition of a spherical laser-heated DT pellet:[21]

$$E_{L.P.} \simeq 10^{15} \frac{\alpha^3}{\epsilon^4 \eta^2} \qquad (46)$$

where ε is the efficiency of laser light absorption, and η is the compression ratio $\rho/P_0(DT)$ for solid deuterium-tritium. One sees the same power of α for both inertial systems. The density factor in equation (46) is replaced by the product $n_f\ \rho_0$ in equation (45), where n_f is the plasma density and ρ_0 is the liner density. The efficiency factor ε^4 is related to the reciprocal of the fourth power of the bracketed term in equation (45). Thus, inertial containment follows similar laws in either cylindrical or spherical cases.

VIII. SMALLER LINER SYSTEMS

Figure 6 shows that breakeven for the DT plasma example will require a total liner energy of the order of a gigajoule. Pulses greater than this ($\alpha > 1$) will require even larger energies, in proportion to α^3, as given by equation (45). One gigajoule is approximately equivalent to the energy release from 200 kilograms of TNT explosive. The applicat on of such very large explosions to electrical power production would require extraordinarily large containers, and novel engineering solutions.

In this section we will describe some possible ways in which this large size of explosion can be reduced to more manageable size. The first of these is a hybrid system in which the lithium blanket is replaced by a composite blanket containing both lithium and fissionable material, such as uranium. Such blankets have been calculated[22] to be capable of both breeding tritium (from the lithium), and yielding an energy multiplication (from the uranium) of more than 10 times the energy release of the DT reaction.

Thus, breakeven for the overall hybrid system would require $\alpha = 0.1$ in equation (45). In that case the energy E_{min} would only be of the order of one megajoule, or 200 gm. of TNT equivalent. Containment of such an explosion is quite conceivable within current engineering practice. This radical improvement for the hybrid system is a consequence of the cubic power law for α in equation (45). Of course, all the other parameters of the system (radius, length, liner mass, etc.) will be reduced according to the various equations developed above.

Another possible approach to reducing the liner size is to form a "two-component" plasma[23] inside the liner. In this case some of the ions are non-Maxwellian, having energies of 50-200keV. As these ions slow down, they contribute additional in-flight nuclear reactions which would add an additional term to our expression for the nuclear energy output Y (see equation 26). It has been estimated[23] that such a plasma might have an effective $n\tau$ which is 2-3 times lower than for a Maxwellian plasma. Thus, one might conceive of lowering E_{min} by a factor of about ten. This would still be a rather large explosion, however.

A third approach is to lower the required liner length L by changing the design of the ends of the system, where the plasma escapes. End plugs,[18] multiple mirrors,[24,25] cusps,[26,27] and (in the USSR) toroidal plasmas[28] have been suggested for this purpose. These designs would reduce the overall length requirement, but would not affect the cylindrical calculations of section IV.

A fourth possibility, recovering some of the liner energy, is described in the next section. Further attempts to minimize the size of the liner system are desireable; the cubic power law for α offers hope that such an effort can lead to smaller systems.

IX. MAGNETIC IMPLOSION OF THE LINER

A complete system calculation should also include additional energy losses arising from the inefficiency of the method of liner implosion. In the case of the magnetic field implosion concept (see Figure 1), we must choose between the usual "θ-pinch" (B_z) driving field and a "z-pinch" (B_θ) driving field.[29,30] Figure 7 illustrates the practical geometry of the two concepts.

It has been shown that the B_θ system is inherently more efficient because the local magnetic field is largest at the smaller radius of the liner.[30] The simplest way to demonstrate this is to consider the liner kinetic energy W:

$$W = E_{m_i} - E_{m_f} \tag{47}$$

where E_{m_i} is the initial magnetic energy in the driving field, and E_{m_f} is the final energy. (All quantities are per unit length, and resistive effects are neglected.) The magnetic energy E_m is given by:

$$E_m = 1/2 \frac{\phi^2}{L} \tag{48}$$

where ϕ is the flux, and L is the inductance per unit length. But the flux ϕ is a constant; therefore, the driving efficiency η can be written:

$$\eta \equiv \frac{W}{E_{m_i}} = 1 - \frac{L_i}{L_f} \tag{49}$$

Substituting the appropriate inductance formuli for the two cases, one obtains:

$$\eta_z = \frac{r_i^2 - r_f^2}{R^2 - r_f^2} \tag{50a}$$

$$\eta_\theta = \frac{\ln (r_i/r_f)}{\ln (R/r_f)} \tag{50b}$$

where R is the radius of the container. Comparing the two efficiencies, one finds that $\eta_\theta > \eta_z$, as was to be shown.

In addition to its higher efficiency, the B_θ container geometry (Figure 7) may permit the construction of a higher pressure vessel due to the possibility of having a higher hoop stress in a cylinder which is unbroken in the azimuthal direction.

In a complete fusion system one must compensate for the inefficiency $1 - \eta$ by specifying a higher α (equation 27). However, it has been suggested [37] that if the liner maintains its integrity after the implosion, its outward motion (explosion) will pump energy back into the driving field, thus reducing the required α. It is not presently known whether such stability is possible.

The power supply for the driving magnetic field is the main energy source for the liner implosion. For a radius ratio of 30 (corresponding to an adiabatic temperature ratio of 90 and an initial plasma temperature of 200eV), the liner implosion time τ_i is roughly $30r_0/u_L$, where u_L is given by equation (37). Thus to first order τ_i varies as $1/P_0$. At P_0 = 6 MB, $\tau_i \simeq 50$ μsec, which requires a very fast system. For the larger energy systems, τ_i is longer. In that case one can consider inherently slow power supply systems, such as inductive energy storage.

X. LINER FORMATION

Consider a small fusion reactor power requirement of 10^8 watts average power. Extrapolating from Figure 6, a 1 meter breakeven system at 6 megabars pressure would have $Y = 4 \times 10^8$ joules per explosion. If a reactor system operated at the same final pressure (6 MB) with a multiplication of $\alpha = \sqrt{5}$, then the yield Y per explosion would be

$$Y = \alpha E_{min} = 10^{10} \text{ joules/explosion} \tag{51}$$

A power of 10^8 watts would require a new explosion inside the container every 100 seconds (36 explosions per hour). Within this cycle time one would have to pump out all of the debris from the previous explosion, and form the next liner-plasma system.

We can suggest two possible methods of forming the liner which might be investigated further. The first is the continuous casting of a solid cylinder in place inside the container. As a result of a brief contact with the light metals industry, we estimated that the capability to cast 36 metal liners per hour would cost about $\$1.2 \times 10^7$. If interest plus payment of principal amounts to 20% per year, then the casting cost would come to about \$275 per hour. For a 10^8 watt output, this would be \$.00275 per kw-hr.

A similar cost estimate can be made for the containment shell, and amounts to \$.0010 per kw-hr. No complete system cost estimates have been made, but the sum of these two costs is less than the market value of \$.01 per kw-hr.

A second possible method is gravity flow of a liquid liner. This would require that the axis of the liner system be vertical, and that the mass flow rate through the annular orifice at the top of the container be varied in time so that the thickness of the liner would be constant (as a function of axial position z) at the time of the magnetic implosion. A simple calculation shows that a linearly decreasing mass flow rate will meet this criterion. Any low melting point metal can be used. No cost estimates have been made for this method.

Other practical problems needing further assessment are the pump out problem and the question whether the exploding liner (at late times after turnaround) will damage the container. This latter problem will be particularly severe if the liner breaks up into chunks of metal shrapnel.

Additional studies are needed to clarify these practical reactor problems.

XII. CONCLUDING REMARKS

This model is a good approximation for thin cylindrical compressible liners. Comparison with numerical calculations suggests that its usefulness can be extended to thick compressible liners. Thick incompressible liners (at lower pressures) can best be treated by the method of Robson.[32]

These calculations of cylindrical metal liner compressions for fusion purposes have shown that very large energy explosions will be needed to surpass breakeven if the usual long theta pinch plasma geometry is employed. However, the structure of the equations arouses hope that other plasma configurations may reduce the energy per explosion. If such reductions can be achieved, then metal liners should be taken seriously as a method for achieving fusion.

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Table I: Comparison of Approximate Model with a Computer Calculation, at turnaround:

$\beta = .95$, $f_f = .55$, $f_L = 1.16$, $r_o = 1.04\text{cm}$, $P_o = 1.65$ MB

	Computer Run	Approximate Model
Plasma Energy E_p (MJ/m)	37.8	43.9
Field Energy E_M (MJ/m)	26.9	26.7
Liner Compressional Energy E_L (MJ/m)	11.6	9.1
Total Non-nuclear Energy E_T (MJ/m)	76.3	79.7
Energy Multiplication α	.31	.59
Nuclear Energy Output Y (MJ/m)	23.5	47.0

FIGURE CAPTIONS

Figure 1: *Contained Liner Concept.*

Figure 2: Time & Radius Dependance of Plasma, Magnetic Field, and Liner near Turnaround.

Figure 3: Copper Compression Functions versus Pressure. (See equations (6), (7), (11), and (15).

Figure 4: Breakeven Energies for DT Plasma Example ($\alpha = f_L = 1$, $\beta = f_f = 0.707$).

Figure 5: Radius and Velocity for DT Plasma Example ($\alpha = f_L = 1$, $\beta = f_f = 0.707$).

Figure 6: *Length and Total Energy for DT Plasma Example* ($\alpha = f_L = 1$, $\beta = f_f = 0.707$).

Figure 7: Schematic Arrangements of Power Supplies for Driving the Liner.

Figure 1

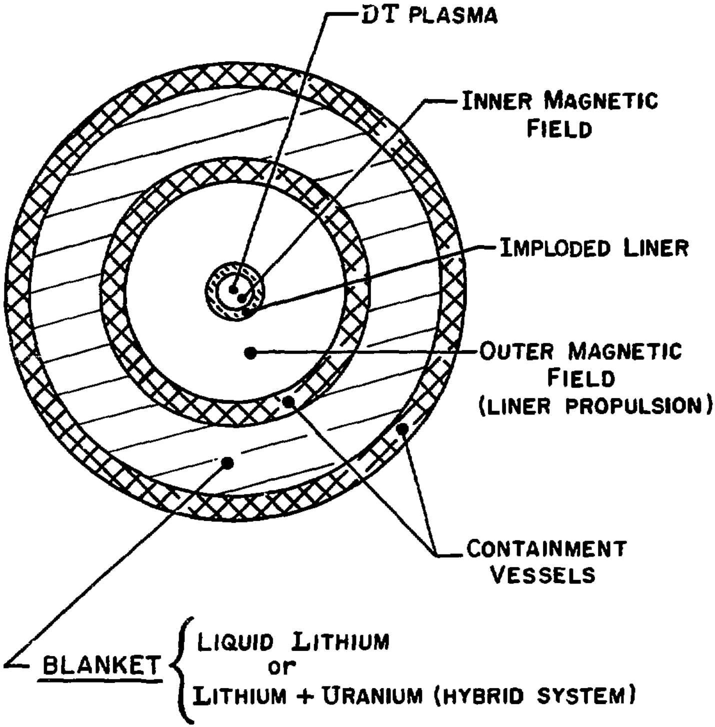

DT PLASMA
INNER MAGNETIC FIELD
IMPLODED LINER
OUTER MAGNETIC FIELD (LINER PROPULSION)
CONTAINMENT VESSELS
BLANKET
LIQUID LITHIUM
or
LITHIUM + URANIUM (HYBRID SYSTEM)

Figure 2

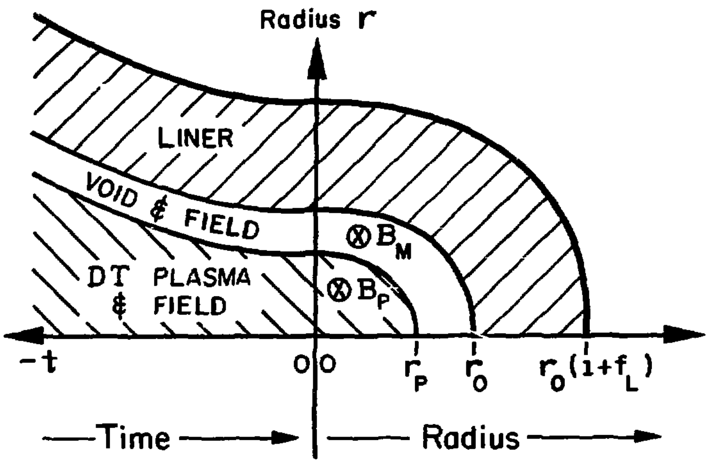

Radius r
LINER
VOID & FIELD
⊗ B_M
DT PLASMA
& FIELD
⊗ B_P
-t
0
0
r_P
r_0
$r_0(1+f_L)$
Time
Radius

Figure 3

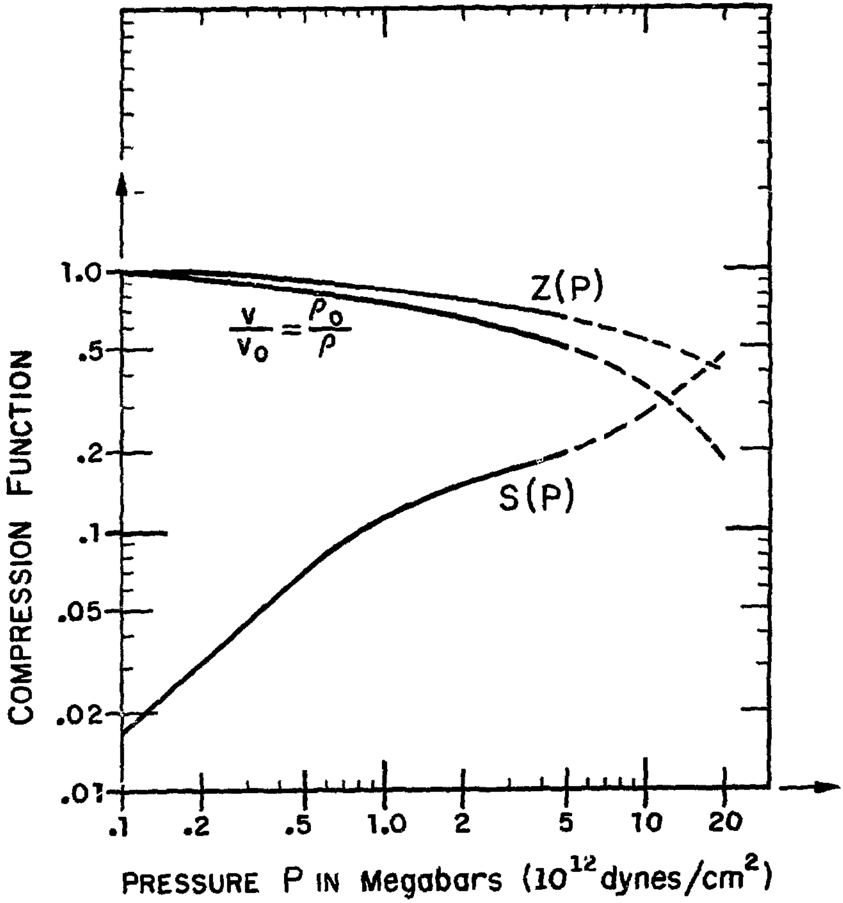
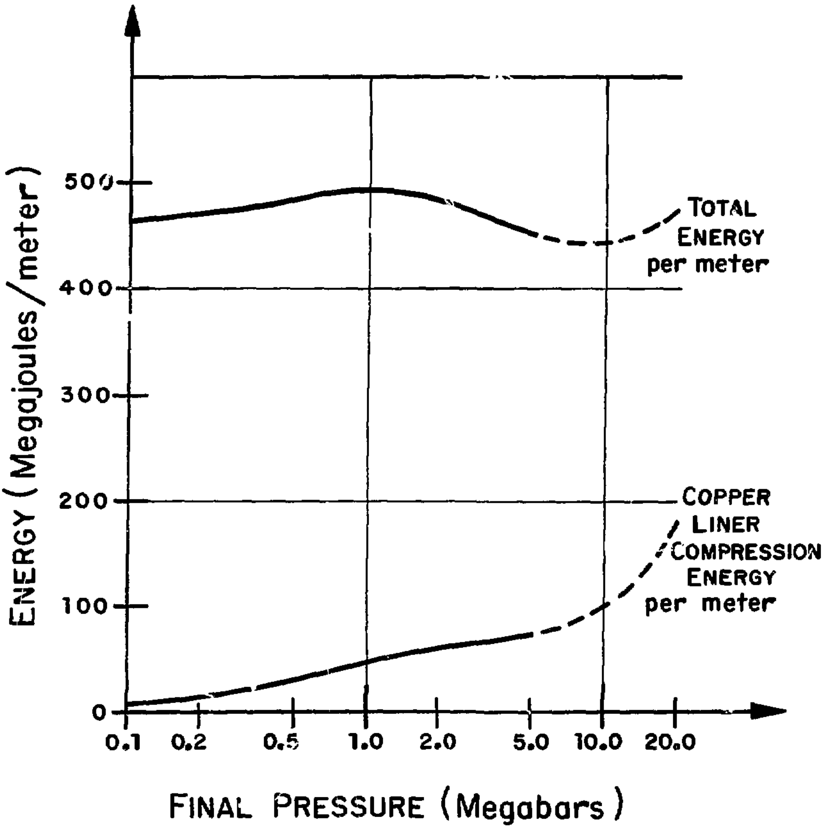

COMPRESSION FUNCTION
1.0
.5
.2
.1
.05
.02
.01
Z(P)
$\frac{v}{v_0} = \frac{\rho_0}{\rho}$
S(P)
.1
.2
.5
1.0
2
5
10
20
PRESSURE P IN Megabars (10^{12} dynes/cm^2)

Figure 4



500
400
300
200
100
0
ENERGY (Megajoules/meter)
TOTAL ENERGY per meter
COPPER LINER COMPRESSION ENERGY per meter
0.1
0.2
0.5
1.0
2.0
5.0
10.0
20.0
FINAL PRESSURE (Megabars)

Figure 5

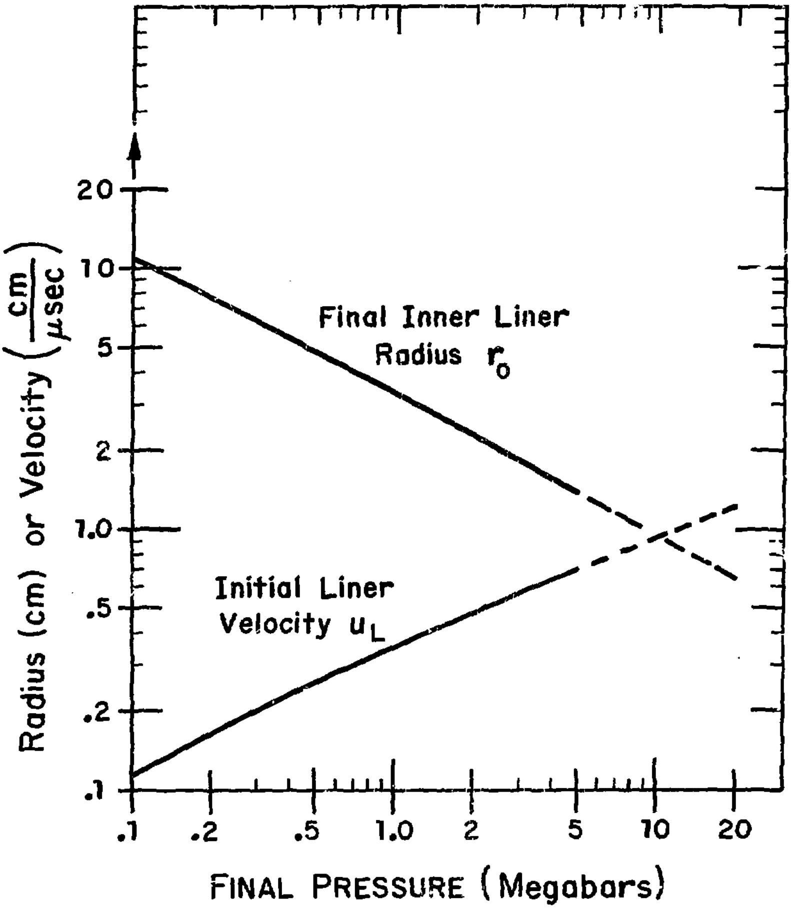

20
10
5
2
1.0
.5
.2
.1
Radius (cm) or Velocity (cm/μsec)
Final Inner Liner
Radius r_0
Initial Liner
Velocity u_L
.1
.2
.5
1.0
2
5
10
20
FINAL PRESSURE (Megabars)

Figure 6

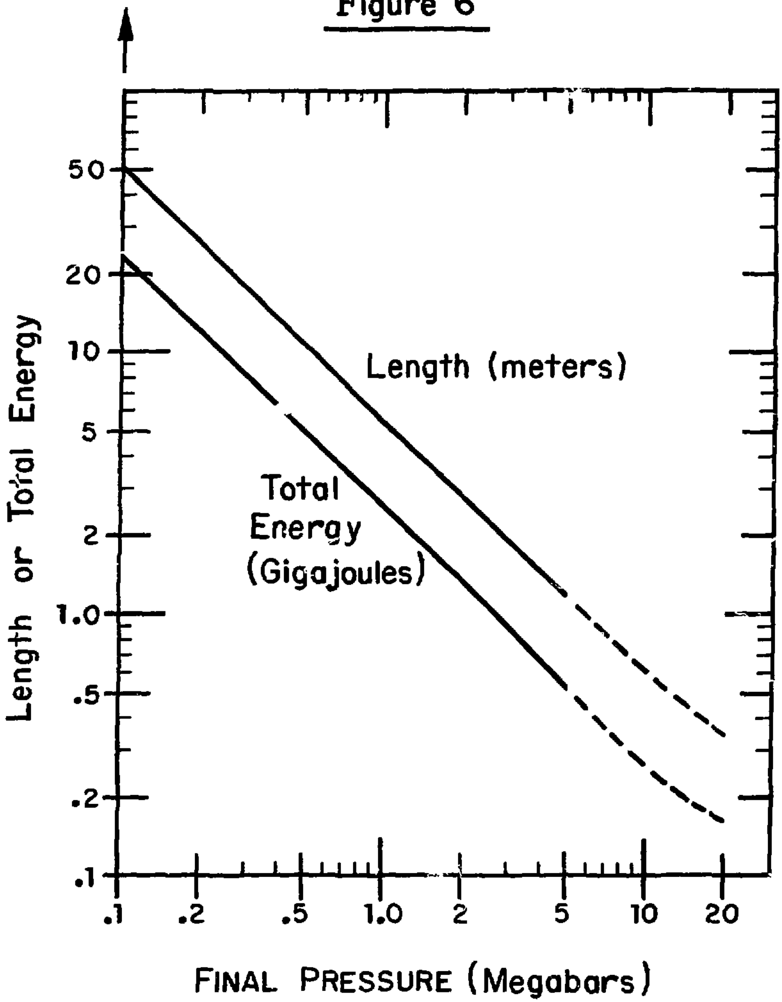
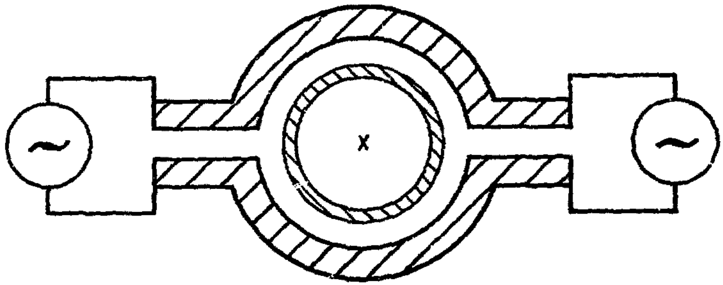
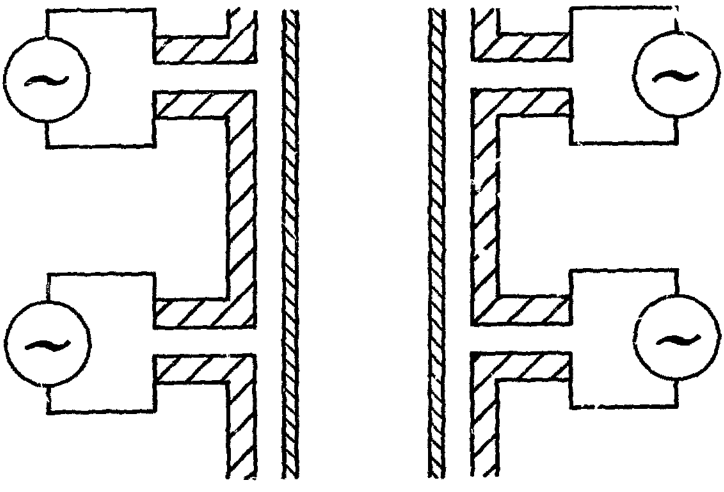

50
20
10
5
2
1.0
.5
.2
.1
Length or Total Energy
Length (meters)
Total
Energy
(Gigajoules)
.1
.2
.5
1.0
2
5
10
20
FINAL PRESSURE (Megabars)

Figure 7



(a) B_z – DRIVEN LINER



(b) B_θ – DRIVEN LINER