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APAE-Memo-87

REACTORS—POWER

I

UNITED STATES ATOMIC ENERGY COMMISSION

PUMP FAILURE AND THE APPR-1

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May 1956

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PUMP FAILURE

AND THE APPR-1

ASTRA

Milford, Connecticut

May, 1956

Work performed under Contract No. AT(11-1)-318
For Alco Products, Inc.

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CONTENTS

VIII. APPENDICES

LIST OF ILLUSTRATIONS

APAE-MEMO-87

I. SUMMARY

The object of the work described in this report was to find out how soon, after failure of the primary coolant pump, the reactor needed to be scrammed. The reactor was assumed to be operating, initially, at full normal power. The results obtained indicate that the reactor need not be scrammed at all.

This applies for an indefinite period if the amount and pressure of the water on the secondary side of the heat exchanger remain essentially the same as before primary pump failure. It applies, in any case, for times on the order of minutes.

In attaining the results, primary flow momentum and free-convection effects were investigated, as well as the interrelationship of temperature, reactivity, and power.

II. INTRODUCTION AND RESULTS

A. BACKGROUND, ADMINISTRATIVE

The work represented by this report was carried out for, and at the request of, Alco Products, Inc. The work is part of the overall effort leading towards the design, construction, and operation of the Army Package Power Reactor system, also known as the APPR-1.

Although the work described herein used the original Alco report, APAE-4, as the point of departure, it could not have been successfully carried out without the co-operation of, and the data from Messrs. William Richards and Joseph G. Gallagher of Alco.

B. BACKGROUND, TECHNICAL

In designing any type of equipment, it always is well to investigate what might happen in the event of a breakdown of a portion of the system. This is especially true with reactor powerplants because there is so little experience in their operation and because of the possibly more serious consequences of a breakdown or malfunction.

One of the possibilities to consider, in any reactor system which requires forced circulation, is the result of a failure of the pump providing that circulation. With this in mind the probable results of the failure of the primary coolant pump of the APPR-1 have been investigated. (The effect of starting the other pump was not included in this study.)

Some experimental results exist which indicate that excessive difficulties will not be encountered. These include tests on the LITR with no circulation, and include operating experience on the BSR at one megawatt. Nevertheless, these results are not completely and specifically applicable. Therefore, it was considered necessary to investigate possibilities analytically prior to actual operation.

C. METHOD OF ATTACK

1. General Approach

The general approach taken in carrying out this work was to use conservative assumptions. This was done for the usual two reasons. First, as a matter of philosophy, it was done in order to make the safe assumption. Second, it was done to make computations simpler. This approach had the usual result that the system investigated is not exactly the real system, but a more conservative version of that system.

2. The Three Situations Considered

The problem was attacked in three parts. The first answered the question of what the situation would be if the system passed safely through the transient conditions and arrived at some new steady state or quasi-steady state. The second part predicted the trends in reactor power and temperatures while passing through the transient stage. The third part considered what would happen with little or no heat removal.

3. Post-Transient Situation

If the transient phase can be passed through safely, a steady state situation will be reached.

By the definition of steady state, the heat must be removed from the reactor as fast as it is generated. This requires a matching of the primary coolant flow and the power level.

Heat must be removed from the heat exchanger at the same rate. This requires a matching of the primary coolant flow and the system temperatures.

Also, for steady reactor power, the reactivity must have returned to that zero value assumed to have existed just prior to pump failure. This fixes the system temperatures.

A combined graphical and analytical method of attack was found to be both easier and more readily understood. The above approach is discussed more completely in Chapter IV.

4. Transient Phase

In analyzing the transient phase, the most important component (insofar as possible damage is concerned) is the reactor. In carrying out the analyses, the values for the interrelated variables of temperature, reactivity, and power were obtained by numerical integration.

The transient phase can be considered as occurring in steps. In fact, for numerical integration, advantage can be taken of this step-wise method of attack.

The first step occurs during the transport lag period, after the pump failure and prior to the arrival at the reactor of lower temperature water. All of the numerical integration carried out was done within this step.

The second step occurs in a period during which the reactor inlet temperature is decreasing, due to the changing situation within the steam generator. (Since the rate of flow has decreased considerably by the time of the second step, its onset will not have an abrupt effect.)

5. A Third Situation

In addition to the analyses on the transient and post-transient situations, a third set of analyses were made. In these analyses two conditions of almost vanishingly small probability were studied.

The conditions were that the reactor power would be automatically and arbitrarily held at the normal operating level, despite the partial or complete absence of moderator. These analyses, although carrying assumptions to ridiculous extremes, give an ultimate conceivable situation.

D. RESULTS

1. General

As previously noted, the results reported here (and in greater detail elsewhere) are for a somewhat more conservative version of the actual reactor.

2. Transient Phase

It was found that both the flow and the power drop very sharply within the first few seconds. The time before the onset of net steam generation in the hottest channel was found to be about 7 seconds. The maximum metal temperature reached was 658°F which occurred after about 9 seconds. At about 11 seconds after pump failure, the temperatures began to decrease.

Because of the labor involved, numerical integration was not carried much beyond this point. However, it cannot be seen how any really fast increase in reactivity could occur between this maximum and the quasi-steady state.

It is true that the reactor inlet temperature (and ultimately the average reactor temperature) will finally decrease. However, the flow rate has decreased so much by then that it cannot cause any really fast drop in water temperature and rise in reactivity.

3. Post-Transient Situation

After the initial, relatively fast (times on the order of seconds) transients die away, a steady state or quasi-steady state is reached. This state is one in which primary circulation is maintained by natural thermal convection.

Whether the primary loop situation, after the fast transients die away, is actually a steady state situation or a quasi-steady state situation depends upon what happens in the secondary loop. If the amount and pressure of the water in the secondary side of the steam generator is held constant, then it is a true steady state.

If, as might normally occur, the situation on the secondary side changes (due to continuing operation at normal throttle, or to failure of the secondary coolant pump, or to some other cause), a quasi-steady state will be reached. The quasi-steady state will approximate the previously mentioned steady state but will change slowly (compared to the previous changes) with changes in the secondary loop.

In the steady state or quasi-steady state which is reached, the power is about 43 per cent of normal operating power. The flow is down to about 11 per cent of normal, pump-driven flow. The maximum metal temperature is less than 640°F. There is no net steam generation, although there is nucleate boiling.

(Although the above data were obtained with the assumption of scale present in steam generator, they apply to the no-scale situation within a few per cent.)

4. <u>No Circulation and No Water</u>

For the highly improbable situation of blocked circulation, with the water boiling away and not returning (no chugging or percolating), it was found that the plate with the highest power density would start to melt after a lapse of about 7 seconds.

For the almost impossible condition of steady full power with no heat removal at all, the plate with the highest power density would start to melt at the end of about 3 seconds.

5. Additional Results

In arriving at the results obtained for the transient and post-transient situations, considerable intermediate results were obtained in the way of flow momentum, and free convection relationships, in addition to the relationships obtained for use in connection with the numerical integration.

III. CONVECTION CIRCULATION

A. GENERAL

The derivations for thermal siphoning will be found in Appendix C. While this analysis implicitly assumes no boiling, it should not be too inaccurate as long as there is not any net steam generation in the reactor as a whole. The reason is that a good part of the driving head, as well as most of the friction head, is found in parts of the loop outside the reactor.

B. RELATIONSHIPS

1. In turbulent flow only, that is down to

$F = 82.7$ GPM $\quad (G = F/4000 = 82.7/4000)$

$(T_2 - T_1) = 3790\ G^{1.8}$ Turbulent

$kw = 2.06 \times 10^6\ G^{2.8}$

(See derivation, Appendix C)

2. For either turbulent or laminar flow

$(T_2 - T_1) = 164.5\ H$

and

$kw = 22.3\ FH$

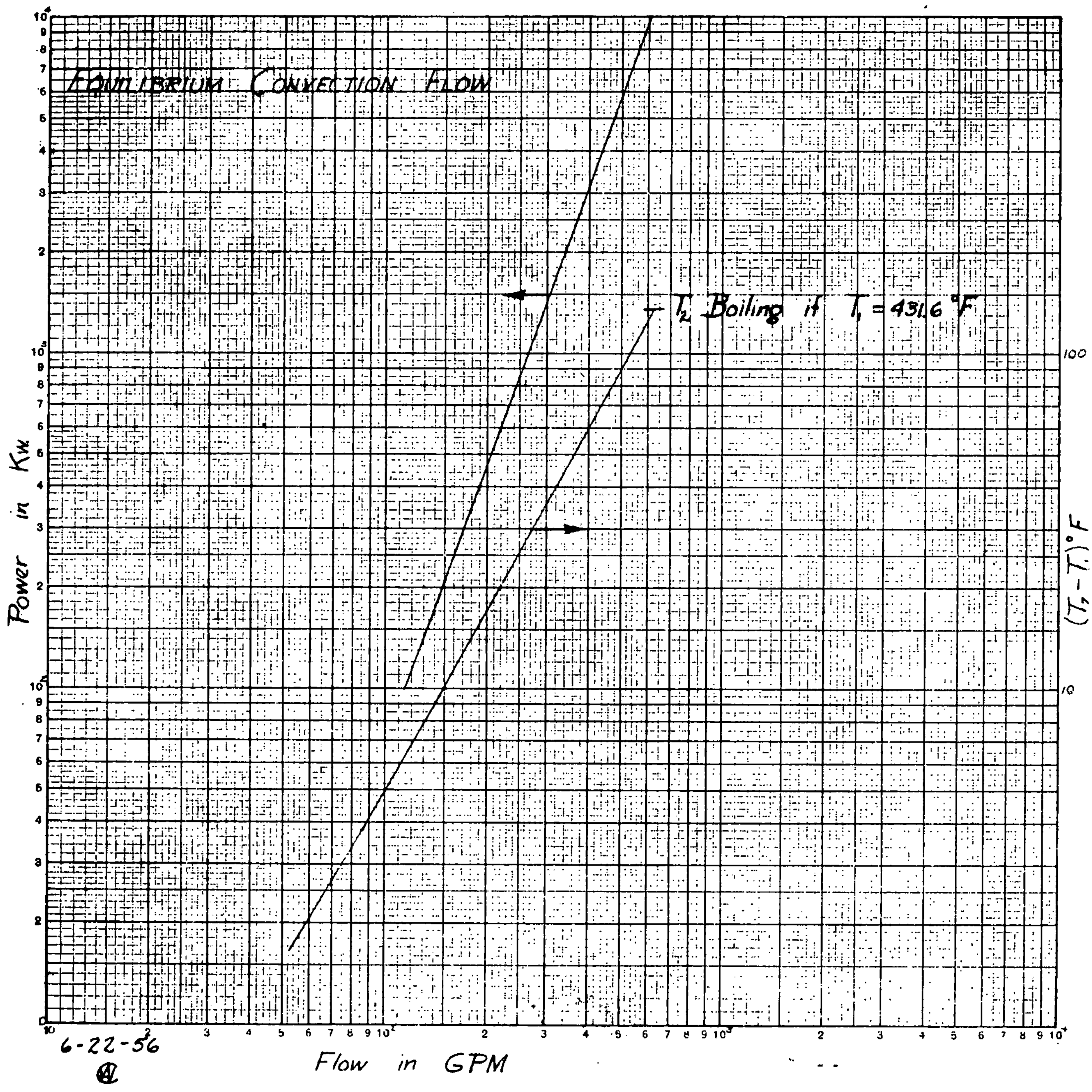
C. CALCULATIONS

1. From the relations of item B-1 above, we calculate power level (kw) and loop $\Delta T = (T_2 - T_1)$ as functions of primary loop F (GPM).

2. Tabulated Results of ΔT and kw found as above are tabulated below:

F gpm	C(=F/4000)	$G^{1.8}$	ΔT (^oF) $3790\,G^{1.8}$	$G^{2.8}$	kw $2.06 \times 10^6 G^{2.8}$
600	.15	32.8×10^{-3}	124.4	49.2×10^{-4}	10100
400	.10	15.9×10^{-3}	60.2	15.9×10^{-4}	3270
200	.05	4.56×10^{-3}	17.3	2.28×10^{-4}	469
100	.025	1.32×10^{-3}	5.0	$.327 \times 10^{-4}$	67.2

3. The above will be found plotted on Figure 1.


EQUILIBRIUM CONVECTION FLOW
T_2 Boiling if $T_1 = 431.6\,°F$
Power in Kw
$(T_2 - T_1)\,°F$
Flow in GPM
6-22-56

SHEET No. 1

IV. QUASI-STEADY STATE

A. REQUIREMENTS

1. After the pump fails (and if the system passes through the transient stage without damage), it will eventually settle into a new steady state.

2. Actually we are not concerned with the final steady state. We are interested in a quasi-steady state, one in which the rates of change of the variables are slow compared to the rapid changes which the system underwent immediately after the pump failure. Thus we are talking about a state which does not vary greatly over a period of perhaps a minute. We can ask, "What are the characteristics of such a state?"

3. Reactivity. If the control rods have not been moved, the reactor must have returned to zero reactivity by itself. This essentially means that the effective density must be the same as at the normal operating state. To a first approximation this means that the mean water temperature must be the same in the reactor as before.

4. Flow and Heat Removal. Assuming that the reactor is running at some power (and that the heat is being removed elsewhere), the flow must be adequate to remove the heat.

This is not difficult. If the temperatures reached are not destructive, the temperature difference (T_2-T_1) will automatically increase until the thereby-induced thermal convection flow is adequate to prevent further increase. Flow and (T_2-T_1) are in turn determined by the kw of output power.

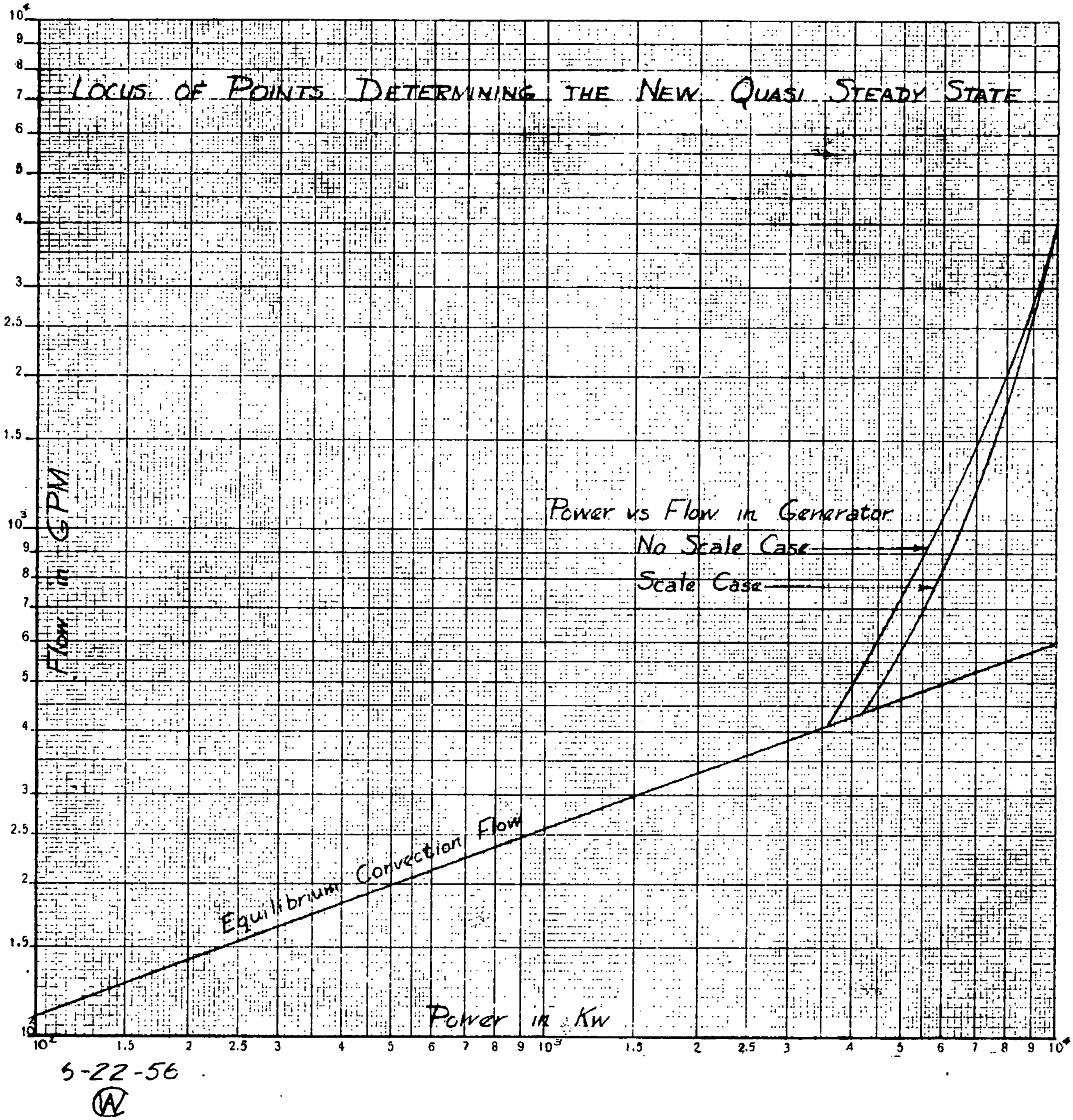
This does not fix the steady state, however. As can be seen by the previously mentioned curve in Chapter III (Curve Sheet 1), there is a whole infinity of points at which power and flow are in equilibrium. And all of them could have the same mean temperature. There must be one additional requirement on the flow.

5. Steam Generator. The additional requirement comes from the steam generator. The way this comes about requires some explanation. First the mean temperature (primary) of the steam generator (based on inlet and outlet temperatures) must be the same as for the reactor. (This is readily apparent since the inlet of one is the outlet of the other.)

Furthermore, the mean primary temperature must be at the same value as for normal operating conditions, due to reactivity considerations. We also assume, for our quasi-steady state, a constant temperature on the secondary side of the steam generator.

If we make the above assumptions, this fixes power vs. flow relationship. That is, if we assume a fixed pair of values for primary water mean temperature and secondary water saturation temperature, then for each value of power there is a particular value of flow required to maintain equilibrium.

6. <u>All Requirements</u>. Fortunately the curves of reactor flow requirements vs. power, and steam generator flow requirements vs. power, intersect. By so doing, they give the characteristics of a state which satisfies all requirements. See curves on next page.



LOCUS OF POINTS DETERMINING THE NEW QUASI STEADY STATE
Power vs Flow in Generator
No Scale Case
Scale Case
Equilibrium Convection Flow
Flow in GPM
Power in Kw
5-22-56

SHEET NO. 2

B. CALCULATION

1. As far as the reactor is concerned, the calculations of thermal convection flow vs power have already been done. (See Chapter III.)

2. For the steam generator, what is desired is Flow or G vs kw at constant ΔT where ΔT represents the difference between the saturation temperature on the secondary side and the arithmetic mean of entrance and exit temperatures on the primary side. If effective temperature equalled arithmetic mean, the situation would be fairly simple since $h \sim G^{.8}$, k for the metal is a constant and h for boiling is a known function of heat flow.

3. However, the effective mean is more nearly equal to log mean ΔT. Since the method of carrying out these calculations is not of particular importance here, the method and its derivation are included in Appendix D.

C. RESULTS

1. It will be noted that two curves are shown, one with scale and one without scale. The difference in initial conditions assumed is that with no scale the T_s on the secondary side of the steam generator is higher. All primary-side conditions are the same.

2. Knowing the flow, a value for (T_2-T_1) can be obtained from the convection flow curve in Chapter III.

3. Characteristics of the new quasi-steady state which results are tabulated below.

	SCALE		NO SCALE	
VARIABLE	OPERATING	QUASI-STEADY STATE	OPERATING	QUASI-STEADY STATE
G -	1.00	0.110	1.00	0.104
F gpm	4000	440	4000	416
P -	1.00	0.425	1.00	0.363
P' Kw	10,000	4250	10,000	3630
T_s °F	382	382	393	393
P_s psia	200	200	230	230
T °F	441	441	441	441
(T_2-T_1)	18	70	18	62
T_1 °F	432	406	432	410
T_2 °F	450	476	450	472

V. FLOW AFTER PUMP STOPS

A. GENERAL AND ASSUMPTIONS

1. In analyzing the situation after pump failure, one of the most important variables is the change in coolant flow with time.

2. The parameters which determine the rate of decrease of coolant flow are the kinetic energies stored in the coolant and in the pump (and drive mechanism) and the friction forces operating to absorb such kinetic energies.

3. A rough, quick calculation based on data supplied by Alco indicated that in the event of electrical power failure the kinetic energy stored in the pump and drive mechanism might contribute appreciably toward maintaining coolant flow. Therefore, it seemed more conservative to assume a type of failure (shaft breakage or seizure) in which the pump could <u>not</u> contribute toward maintaining flow. This was done.

4. It was felt that the friction head loss due to a "frozen" or "floating" impeller would be small compared to the friction head loss due to the steam generator piping, so that the former loss was ignored.

B. RELATIONSHIPS

With the above simplifying assumptions, a very straightforward derivation was made of flow vs. time, as is shown in Appendix E. The result was

$$G = \left(\frac{a}{a + t}\right)^{1.25}$$

Where G = normalized flow or flow/initial flow (in this case F/4000)

t = time in seconds

a = a constant depending upon friction, velocity, and length characteristic of the primary loop (in this case a = 1.02 sec.)

(It is of interest to note that "a" is in the nature of a film drop "half life" since "h", film heat-transfer coefficient, depends upon $G^{.8}$ and $G^{.8} = 1/2$ when t = a.)

C. RESULTS

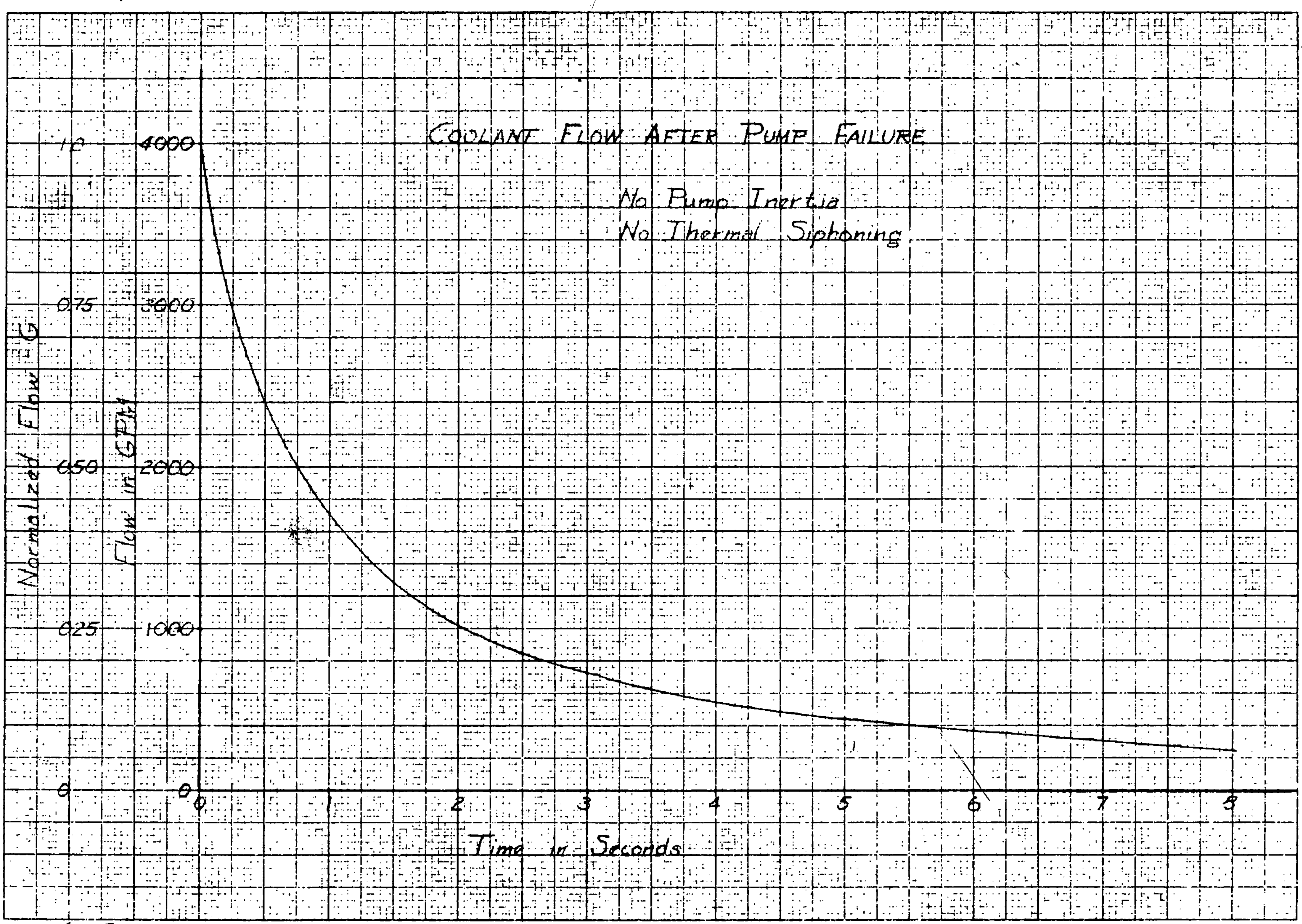
1. Interrelationships of these variables for a few representative times are tabulated below, and are shown as a curve on the next page. For the pump failure problem, convection flow becomes important at flows greater than some of the latter values in the table. For such values of flow, the curve can be interpreted as being one in which the reactor has been partially or wholly shut down.

2. In case it becomes desirable to verify power and flow magnitudes in the interval $7 < t < 11$ sec, where both coasting and convection are important, the following relationship will apply:

$$168\dot{G} + 206G^{1.8} - P/G = 0$$

3. For the situation as in C-1. a tabulation of G vs t follows:

t (sec)	$G^{0.8}$	G	F(gpm)
0.00	1.000	1.000	4000
0.10	0.911	0.888	3552
.20	.837	.799	3196
.30	.773	.724	2896
.50	.671	.607	2428
0.75	.577	.503	2012
1.0	.505	.426	1704
1.5	.405	.321	1284
2.0	.338	.258	1032
3.0	.254	.181	724
4.0	.203	.136	544
5.0	.169	.109	436
6.0	.145	.091	364
7.0	0.127	0.076	304



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SHEET No. 3

VI. TRANSIENT CONDITIONS

A. GENERAL REMARKS ON METHOD

1. The essence of numerical methods such as are used here is that certain variables are held constant over a time interval which is sufficiently short that serious inaccuracies do not occur. This does not mean that all relationships must be linearized, nor that the values of variables which must be held constant over the time interval are always the initial values.

2. As an example of the former (linearization of all relationships), consider the reactor kinetics equations. If we linearize and use finite differences, we "build in" a mathematical (but not a physical) oscillation.

3. a. To show this we take the first few steps, hold the delayed neutron contribution constant at the initial value, and use the following relationship:

$$P_{i+1} = P_i + \left[\frac{P_i}{\ell^*}(\Delta K_i - \beta) + \frac{P_o\,\beta}{\ell^*}\right]\Delta t_i \text{ , where}$$

$$\frac{P_o\,\beta}{\ell^*} = D_o\lambda \quad \text{which becomes:}$$

$$P_{i+1} = P_i + \left[(P_i)(\Delta K_i) + (P_o - P_i)\,\beta\right]\frac{\Delta t_i}{\ell^*}$$

b. If we take the following values,

P	=	Power level, where P_o	=	1
ℓ^*	=	Generation Time	=	0.2×10^{-4} sec
β	=	Fraction of Neutrons Produced Delayed	=	75×10^{-4}
Δt	=	Time Interval	=	.01 sec
ΔK	=	Change effective K	=	$.1\times10^{-4}$

c. Then we obtain:

$$P_1 = 1 + \left[-1\times10^{-4} + 0\right] 5\times10^2 \qquad = \quad .95$$

and

$$P_2 = .95 + \left[-.95\times10^{-4} + .05\times75\times10^{-4}\right] 5\times10^2 \qquad = 1.09$$

4. Thus it can be seen that, even with a continuously negative ΔK, we obtain an <u>increase</u> in P after two intervals only .01 second in duration. Hence this method is inapplicable (at least for Δt's of the order of .01 sec.) because it produces a fictitious power oscillation.

5. As an example of using other than initial values, we can cite relationships (such as temperature relationships) in which coolant flow is involved. Since we know the flow at the beginning and end of the time interval, it is preferable to use an average value rather than an initial value.

6. Where there are two interrelated variables, such as wall temperature and coolant temperature, the final value is first computed for that variable which is expected to vary more across the interval. This having been computed, an "Average" value can be used for computing the more slowly varying value.

7. The approaches set forth above were used in setting up the numerical calculations which are described more fully below and in Appendix F.

B. METHOD OF CALCULATION

1. Sequence of Cyclic Numerical Calculation

a. For each time interval, the variables are calculated in the following order:

i. G's - (Normalized flows to the .8 power, normalized flows, and average of both)

ii. T_f - (Effective metal temperature)

iii. T - (Effective water temperature)

iv. ΔK - (Change in effective K)

v. P - (Normalized powers)

vi. D - (Contribution from delayed neutron precursors)

b. The reasoning for using the above order is as follows (numbers refer to sequence in #1 above):

i. The G's are pretty much independent of T's initially, and remain so until thermal siphoning becomes important.

ii. Metal temperature changes faster than water temperature.

iii. With the G's known and after the average T_f has been calculated, the T can be calculated.

iv. With T calculated, the ΔK can be calculated for this interval. The final T for interval is used, since analysis indicates that P closely follows the T-induced ΔK (as modified by the delayed neutron contributions) for the interval used.

v. With the ΔK known and with the previously calculated (or $t = 0$ value) of D available, the P can be calculated.

vi. With P calculated, a new D can be calculated.

2. Relationships Used

The following are the relationships used. Values are those at end of interval unless otherwise stated. Derivations will be found in Appendix F.

a. $$G = \left(\frac{a}{a+t}\right)^{1.25}$$

b. $$T_{f_{i+1}} = Y - (Y - T_{f_i})\, e^{-\frac{\Delta t_i}{\tau_{f_i}}} \; ; \; Y = \left(c_7 \frac{P_i}{U_i} + T_i\right)$$

c. $$T_{i+1} = T_i + \left(\frac{T_{f_i avg} - T_{1_i}}{\tau_{1_i}}\right) - \frac{(T_i - T_1)}{\tau_{2_i}} \Delta t_i$$

$$\frac{1}{\tau_1} = \frac{AU}{wc_p}$$

$$\frac{1}{\tau_2} = \frac{2GM_o}{w}$$

d. $$\Delta K = -c_3(T - 440.8)$$

e. $$P_{i+1} = \frac{D_i \lambda \ell^*}{\Delta K_{i+1} - \beta}$$ plus a negligible small exponential expression

f. $$D_{i+1} = D_i + \left(\frac{\beta}{\ell^*} P_{i_{avg}} - D_i \lambda\right) \Delta t_i$$

3. Symbols Used

a = A constant defining change in flow rate after pump stops. Has dimensions of seconds.

c = All c's are constants. See next section, on constants.

D = Density of delayed neutron precursions put on a power basis

G = Normalized flow. G = 1 at time 0. (G = GPM/4000)

h = Film Coefficient = $cG^{.8}$ (for turbulent flow)

K = Effective neutron multiplication
K = neutrons produced per neutrons lost = 1 for critical reactor.

ℓ^* = Generation time, average lifetime of neutrons from birth to death in seconds. Takes leakage into account.

P = Normalized power
= Power $/34.1\times10^6$ Btu/hr
= 1 at t = 0

T = Mean water temperature. Assumed to be equal to effective water temperature for both reactivity and heat transfer. $^{\circ}$F.

T_1 = Inlet water temperature to reactor $^{\circ}$F.

T_f = Effective metal temperature for heat transfer to water. $^{\circ}$F.

t = Time in seconds.

U = Heat transfer coefficient, metal to water, Btu/hr$^{\circ}$F ft^2

$$= \frac{1}{\frac{1}{h} + c_8}$$

Y = A variable used for simplifying relationships

= $(c_7 \frac{P_i}{U_i} + T_i)$

β = Proportion of neutrons produced delayed

Δ = Signifies changes in a variable over an interval

λ = Decay constant for delayed neutron precursions sec^{-1}

τ = Time constant, has dimensions of seconds.

4. <u>Constants</u>

a = 1.02 sec

C_1 = is not really a constant but is used for density of delayed-neutron precursors.

c_2 = $\frac{P}{n}$ = Ratio or power to neutron density

c_3 = $-2 \times 10^{-4}\ (^{o}F)^{-1}$

c_4 = $\frac{c_3 \Delta T - B}{\ell^*}$ negative

c_5 = $D\lambda$

c_6 = 9472 Btu/sec

c_7 = 15.5 Btu/ft^2-sec

c_8 = $\frac{1}{U_s} = 3.00\ (\frac{sec\text{-}ft^2\text{-}F}{Btu})$

ℓ^* = $.2 \times 10^{-4}$ sec

β = 75×10^{-4}

λ = $.08\ sec^{-1}$ (avg)

5. Initial Values

The following are initial values for the principal variables:

a. $G_o = 1$

b. $t_o = 0$

c. $T_{f_o} = 509^oF$

d. $\tau_{f_o} = 0.3025$ sec.

e. $(T_1)_o = 431.6^oF$

f. $(T_2)_o = 450.0^oF$

g. $T_o = 440.8^oF$

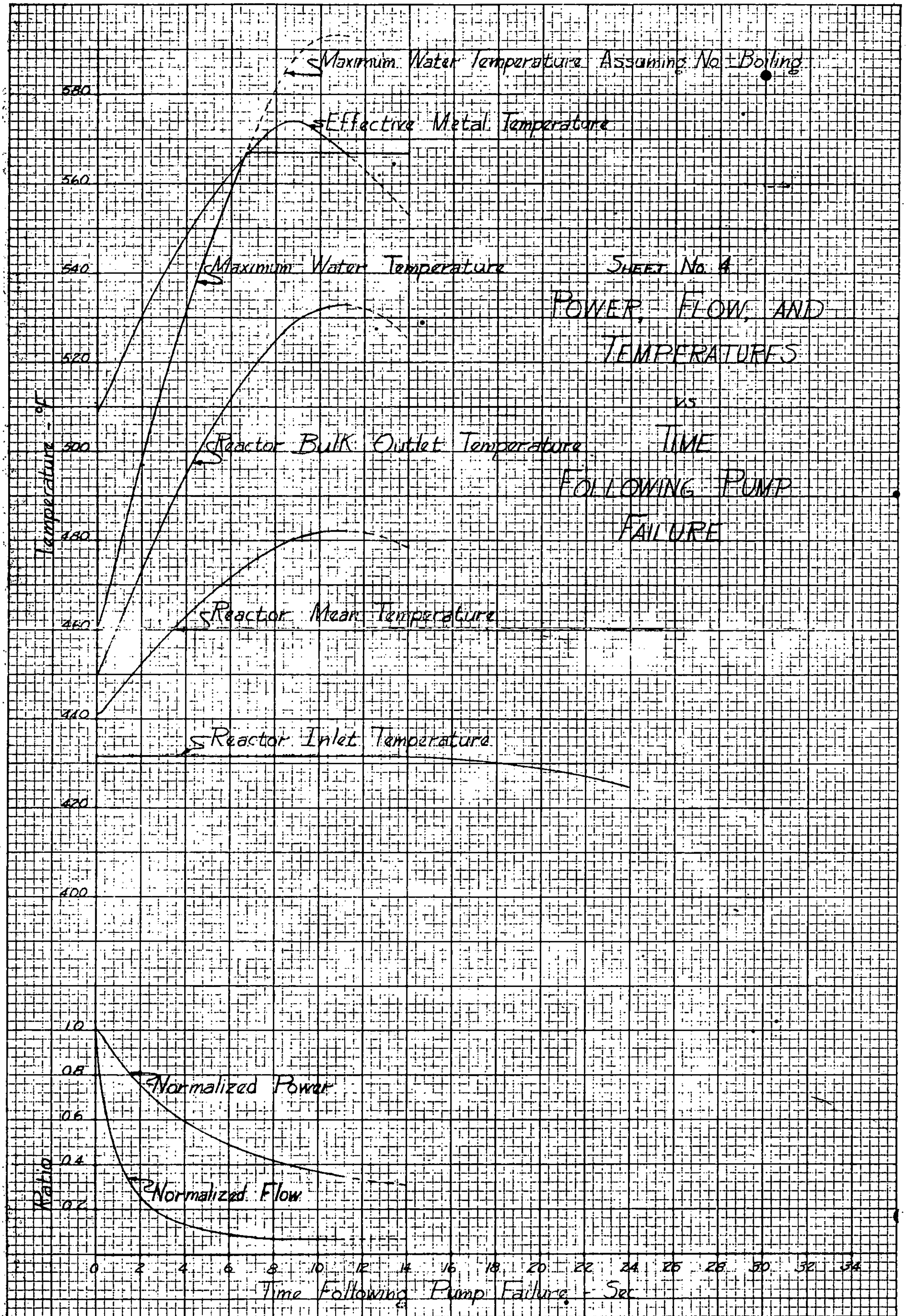
h. $1/\tau_{1_o} = 0.6565\ \text{sec}^{-1}$

i. $1/\tau_{2_o} = 4.869\ \text{sec}^{-1}$

j. $\Delta K_o = 0$

k. $P_o = 1$

l. $D_o = 4687.5$


Maximum Water Temperature Assuming No Boiling
Effective Metal Temperature
Maximum Water Temperature
Sheet No. 4
POWER, FLOW, AND TEMPERATURES vs TIME FOLLOWING PUMP FAILURE
Reactor Bulk Outlet Temperature
Reactor Mean Temperature
Reactor Inlet Temperature
Temperature - °F
580
560
540
520
500
480
460
440
420
400
Normalized Power
Normalized Flow
Ratio
1.0
0.8
0.6
0.4
0.2
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
Time Following Pump Failure - Sec

VII. INSTANTANEOUS ZERO FLOW AT CONSTANT POWER

A. VIEWPOINTS AND SUMMARY

1. One of the limiting cases is the time it would take to melt fuel plates if the flow were stopped instantaneously and the reactor power generation rate remained constant.

2. The reactor would first boil away all water and then melt the plates.

3. An even more stringent (and impossible) set of conditions is one in which the plates have no water to transfer heat to, but the heat generation rate remains constant. The latter case was investigated first.

4. The result obtained when it was assumed that no heat could be removed was that the plates melted in about 3 seconds.

5. It might be expected that the temperature rise would be appreciably slower with water present. Calculations indicate that this is not so initially. The reason is that the heat transfer coefficient with no flow is taken as only about 5% of that existing with normal flow.

6. With this and similar assumptions, it turns out that

with no return of the water boiled away the plates would melt after about 7 seconds.

7. Of course this is extremely pessimistic, as has been illustrated experimentally with an only slightly lower powered reactor. In any case, it gives a rock bottom lower limit.

B. NO HEAT OUT

1. $\frac{\Delta T_f}{\Delta t} = \frac{2.8P}{w_f c_f}$ (see derivation, Appendix J)

or

$$\Delta t = \frac{w_f c_f}{2.8P} \Delta T_f$$

2. First obtain $w_f c_f$ using fuel plates only.

a. Fuel

Weight U^{235} = 22.5 kg

Weight U $= \sim \frac{22.5}{.9} = 25$ kg

Weight UO_2 $= 25 \left(\frac{235+32}{235}\right) = 28.4$ kg

$= 28.4 \times 2.2$ lb/kg ~ 62.5 lb

(While fuel is burned fission products remain and we can assume a rough equivalency in heat capacity.)

Take c_f for UO_2 as = .08 Btu/lb $^{\circ}$F

$w_f c_f = (62.5)(.08) = 5$

b. Stainless Steel

Weight stainless matrix 98 kg*

Weight stainless clad 42 kg

Weight plates 140 kg
308 lb

c_f stainless $\sim$ .12 Btu/lb $^{\circ}F$

$w_f c_f$ stainless (308) (.12) = 37

c. Total $w_f c_f$ = 37 + 5 = 42 Btu/$^{\circ}F$

3. $\underline{\Delta T}$

a. ΔT is difference between internal temperature at point of maximum heat generation and the melting temperature.

b. Take internal temperature as 650$^{\circ}F$ (based on new max/av flux ratio of 2.8 given by Alco); max metal temperature with scale present.

c. Take melting temperature at 1420$^{\circ}C$ or 2590$^{\circ}F$

d. ΔT = 2590 - 650 = 1940$^{\circ}F$.

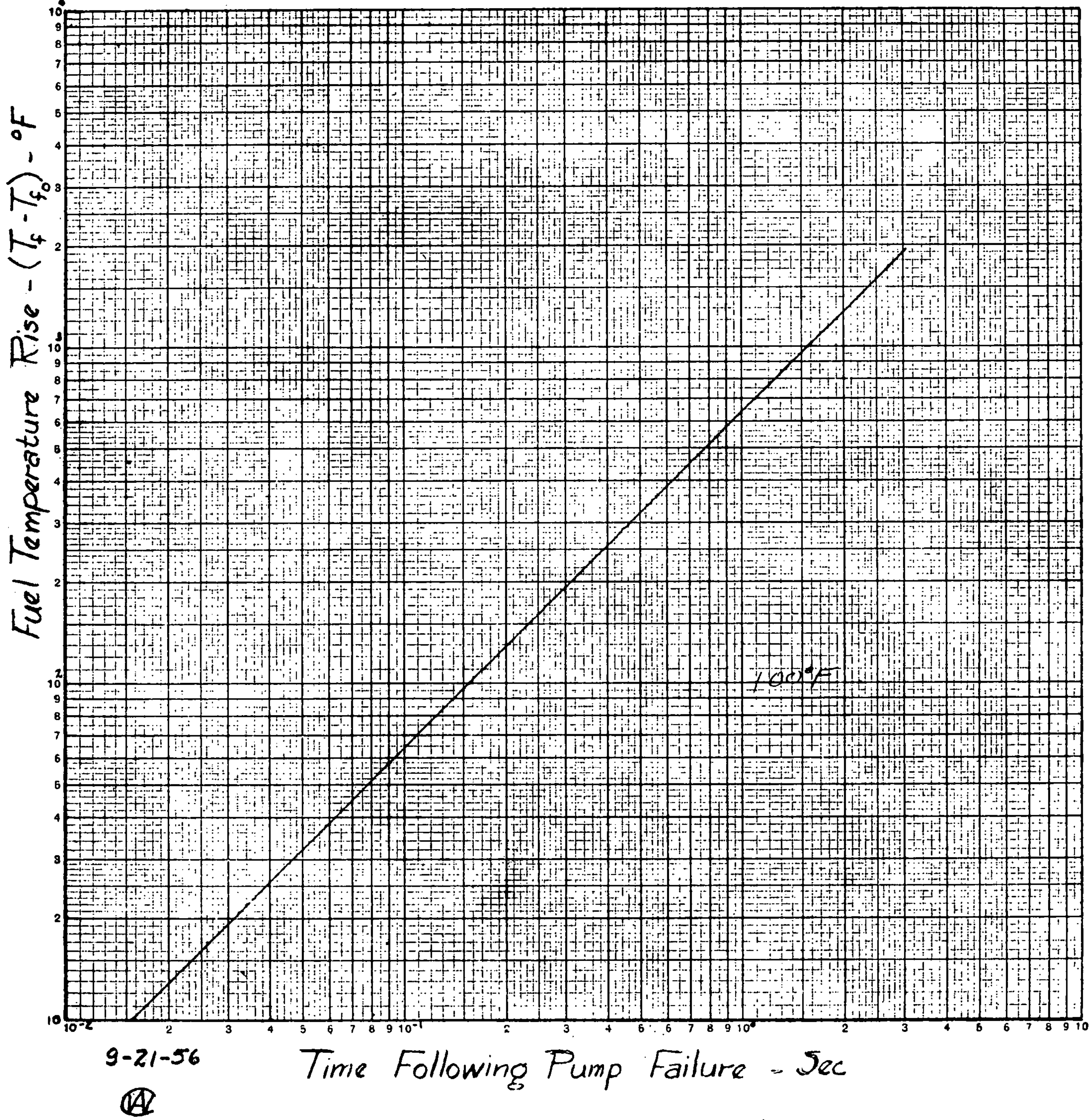
4. $P = 34.1 \times 10^6$ Btu/hr

5. $$\frac{\Delta T}{\Delta t} = \frac{2.845 \times 34.1 \times 10^6}{42 \times 3600} = 642 \ ^{\circ}F/\text{sec}$$

6. For ΔT of 1940$^{\circ}F$

$$t = \frac{1940}{642} = 3.02 \text{ sec}$$

Fuel Temperature vs Time for No Heat Removal at Steady State Power



Sheet No. 5

C. WATER EXPELLED AND NOT RETURNED

1. This calculation was very similar to that in section B except that part of the energy is now removed by boiling away a portion of the water in the core. It was assumed that this heat could be removed until approximately three-fourths of the water had been expelled. After this point had been reached the plate temperature rise was then calculated as in section B on the basis of no heat out.

2. The weight of water expelled from the core was determined from a set of differential equations derived in the first part of Appendix J. The calculation, itself, involved four separate time intervals. The first interval covered that period during which both the plate surface temperature and the bulk coolant temperature remained below the saturation temperature of the water. The second period encompassed that time during which the bulk water temperature remained below saturation while the plate surface temperature exceeded the saturation temperature. The third interval covered that period during which net boiling was taking place, terminating when three-fourths of the water had been expelled. The final interval covered that period during which no heat was removed until finally the melting

point of the fuel element had been reached.

3. The melting point of the plates was taken at 2590°F and the plate attained this value 7.4 seconds following the pump failure, as compared with the figure of 3.0 seconds with no water present.

APPENDIX A - RECONCILIATION OF ALCO-SPECIFIED INITIAL CONDITIONS

Contents Page

AA. GENERAL

Data on pressure drops, etc. for normal operating conditions were obtained from Alco. The relationships used in obtaining such data were not obtained from Alco. Such relationships are well known. However it was highly desirable that conditions existing just prior to pump failure jibe with steady state data supplied by Alco. (Otherwise we might find ourselves analyzing a different system.) Therefore it was necessary to reconcile results obtained by our relationships with given data.

AB. HEAD LOSSES

1. Alco Data on Head Losses:

Given below are Alco data on head losses, due to friction, in feet of hot water.

Reactor		
Through Cone	.6 ft	
Exit and Entrance	.7 ft	
Orifice	.4 ft	
Total, Reactor		1.7 ft
Piping		4.6 ft
Steam Generator		16.7 ft
Total, Loop		23.0 ft

2. Analysis of Reactor

a. $v_o = 4.3^* \text{f.p.s.}$

$= (4.3)(3.6)(10^3) = (15.5)(10^3)$ ft/hr

b. ρ out $= 51.75^*$ lb/ft^3 @ 450°F

ρ in $= 52.6^*$ lb/ft^3 @ 431.6°F

take $\rho = 52.0$ @ 445°F

c. $\mu = .295^*$ lb/ft hr @ 445°F.

d. Flow Channel

i. Width = size of box - 2x side plate thickness

$= 2.837^* - .100^*$

$= 2.737$ in.

ii. Thickness = Plate Pitch - Plate Thickness

$= .163 - .030$

$= .133$ in.

iii. D_e = (4)(Channel flow area)/Channel Circumference

$$= \frac{(4)(2.737)(.133)}{(2)(.737 + .133)}$$

$$= \frac{(2.737)(.266)}{(2.87)}$$

$= .254$ in.

$= .0211$ ft.

*Given by Alco

e. Re $= Dv\,\rho/\mu$

$$= \frac{(.0211)(15.5\times10^3)(52.0)}{(.295)}$$

$= 57{,}700$ (Alco gives 58,400)

f. $L_f = 23/12 = 1.92$ ft (friction length)

$L_h = 22/12 = 1.83$ ft (heat transfer length)

g. $$H = \frac{4f\ v^2L}{2gD}$$

h. From graph, McAdams ρ 156, for commercial pipes, etc.

$f = .006$ @ Re = 57,700

or

$f = .054\ Re^{-.2}$

$= .054/(57{,}700)^{.2}$

$= .054/8.95 = .00604$ say .006

i. $$H = \frac{(4)(.00604)(4.3)^2(3600)^2(1.92)}{(2)(32.2)(3600)^2(.0211)}$$

$= .631$ ft

$= .631/1.92 = .329$ ft/ft

While it is realized that core plates have smooth finish, the $f = .054\ Re^{-.2}$ relationship will be used since it checks Alco data (.6 ft) better.

j. For all exit and entrance losses at reactor we can assume loss of 2x velocity head.

$$H = \frac{2v^2}{2g}$$

$$= \frac{(2)(4.3)^2(3600)^2}{(2)(32.2)(3600)^2}$$

$$= .574 \text{ ft}$$

k. Total head loss for reactor

= .629 + .574 = 1.203 ft, which checks Alco data of .6 + .7 = 1.3

l. To take care of the above core, exit and entrance losses, and also the orifice loss, we will use a fictitious, equivalent length of 1.7 ft head (Alco) ÷ .329 ft/ft = 5.17 ft equivalent length. This only has meaning for friction and <u>not</u> for heat transfer.

3. <u>Analysis of Connecting Piping</u>

a. Alco specified:

27 ft of 12 in pipe with an "equivalent" length (including fittings, etc.) of L = 85 ft, ID = 11.376 in = .948 ft.

b. Take avg T = 445°F

c. Take avg ρ = 52.0 lb/ft^3

d. Take avg μ = .295 lbs/ft hr @ 445^oF

e. $W = 1.66x10^6$ lb/hr

f. $Re = \dfrac{4W}{\mu \, \pi \, D}$

$$= \frac{(4)(1.66x10^6)}{(.3)(3.14)(.948)}$$

$$= 7.44x10^6$$

g. Also $W = \dfrac{\pi \, D^2}{4} \dot{v} \rho$

$$v_o = \frac{4W}{\pi \, D^2 \rho}$$

$$= \frac{(166x10^4)(4)}{(3.14)(.948)^2(52.0)}$$

= 45,300 ft/hr

= 12.58 ft/sec, say 12.6

h. From graph, McAdams p 156

$f = .0026$ @ $Re = 7.44x10^6$

also try

$f = .054 \, Re^{-.2}$

$$= \frac{.054}{(74.4)^{.2}(10^5)^{.2}}$$

$$= \frac{.054}{(2.37)\ (10)}$$

$$= .00228$$

Since, if possible, we wish to use an analytic expression for future transient calculations, we try f = .00228.

i. $(H/L) = 4f \dfrac{v^2}{2gD}$

$$= \frac{4(2.28)10^{-3}(12.6)^2(3600)^2}{(2)\ (32.2)\ (3600)^2(.948)}$$

$$= .0237 \text{ ft/ft}$$

j. For 85 ft

H = (.0237) (85)

H = 2.015 ft

as compared to Alco figure of:

H = 4.6

Ratio 4.6/2.015 = 2.28

k. Since we do not know source of discrepancy and since H for pipe is small fraction of total, we will use value of H = 4.6 per Alco

4. <u>Analysis of Steam Generator</u>

a. Primary Loop is through tube side.

b. $v = 11.5^{*}$ ft/sec

$= 41{,}400$ ft/hr

c. OD tube	=	.75 in*
Wall thickness	=	.065 in*
ID	=	(.75 - .13)
	=	.62 in.
	=	.0516 ft

d. Assume avg temperature is 445^{o}. This is high but doesn't affect accuracy much and corresponds to convenient values of ρ and μ.

e. $\rho = 52.0\ \text{lb/ft}^3$ @ $T = 445^{o}F$

f. $\mu = .295$ lb/ft-hr @ $T = 445^{o}F$

g. $$\frac{Dv\rho}{\mu} = \frac{(.516\text{x}10^{-1})(4.14\text{x}10^{4})(.52\text{x}10^{2})}{(.295)}$$

$= 3.76\text{x}10^{5}$ Friction Reynolds number

* From Alco

h. Alco gives 9.33×10^5; however it is noted that if Alco figure for Re: fuel plate is multiplied by ratios of diameters and velocity

$$(58{,}000)(.0516/.0211)(11.5/4.3) = 3.8 \times 10^5$$

So it is assumed that 933,000 is a typographical error, probably a transposition of 393,000.

i. From McAdams, p 156, f = .0041 @ Re = 3.8×10^5 **for commercial** pipe.

From relationship, $f = .054\ Re^{-.2}$

$$= \frac{(.054)}{(3.8)^{0.2}(10)^{0.2 \times 5}}$$

$$= \frac{.0054}{1.306} = .0041$$

j. $H/L = \dfrac{4fv^2}{2gD}$

$$= \frac{(.0164)(11.5)^2(3600)^2}{(2)(32.2)(3600)^2(.0516)}$$

$$= \frac{(1.6 \times 10^{-2})(1.322 \times 10^{+6})}{(.64 \times 10^2)(.5 \times 10^{-1})}$$

$$= .654 \text{ ft/ft}$$

k. If we take the figure given by Alco

$H = 16.7$ ft

The equivalent length would be

$$L = \frac{16.7}{.654} = 25.5 \text{ ft}$$

l. Vaporizer and superheater of steam generator are in parallel, insofar as primary loop is concerned.

m. The asterisked columns of data tabulated below were given. The others follow in an obvious way.

Portion	OD in.	Outside Heat Transfer Area ft^2/ft	No. Tubes	Total Area ft^2	Avg Area Per Tube ft^2	Avg Length Per Tube	Plus .916* ft in Header
Vap.	.75*	.1965	326*	836*	2.55	13.0	13.9
Sup.	.75*	.1965	44*	194*	4.41	22.4	23.3
Total			370	1030			

n. If we assume one velocity head as the total of the exit, entrance, and bend losses,

$$\frac{v^2}{2g} = \frac{(11.5)^2}{64.4}$$

$$= 2.05 \text{ ft}$$

o. $H_{vap} = \left[(13.9)(.654) + 2.1 \right] \times \left(\frac{11.65}{11.5}\right)^2$

$= \left[9.1 + 2.1 \right] \times 1.028$

$= 11.5$ ft head

$H_{sup} = \left[(23.3)(.654) + 2.1 \right] \times \left(\frac{9.95}{11.5}\right)^2$

$= \left[15.2 + 2.1 \right] \times .748$

$= 12.9$ ft head

Alco gives 16.7 ft head.

We will use 16.7 and fictitious equivalent L = 25.5

5. Loop Parameters

Given below is a recapitulation of flow constants and parameters for initial conditions.

Portion of Loop	L_h ft	L_f ft	D ft	v_o ft/sec	T_{avg} ^oF	ρ lb/ft^3	μ lb/ft hr	Re	H_f ft
Core	1.83	5.1	.0211	4.3	445	52.0	.295	5.8×10^4	1.7
Steam Gen.		25.5	.0516	11.5	445	52.0	.295	3.8×10^5	16.7
Piping		85.0	.948	12.6	445	52.0	.295	7.4×10^6	4.6

AC. HEAT TRANSFER

1. Steam Generator

a. Steam Generator Characteristics and Proportion of Heat to Vaporizer.

The following are roughly the characteristics of the steam generator.*

Part	Heat Removal Rate Btu/hr	No. Tubes	OD in.	Thick. in.	Length ft.	Area ft^2	Velocity ft/sec
Vap.	3.35×10^7*	326	.75	.065	13	836	11.65*
Sup.	0.06×10^7	44	.75	.065	22.4	194	9.95*
Total	3.41×10^7*	370				1030	

b. Ratios of Flows

i. If exit and entrance temperatures were the same for both vaporizer and superheater, the ratio of heat flow rates for each would be proportional to the velocity and the number of tubes in each.

ii. Therefore:
$$\frac{\text{Heat to Vap}}{\text{Heat to (super) + (vap)}} =$$

$$\frac{\text{(No. tubes x v)}}{\text{(No. tubes x v) + (No. tubes x v)}}$$

Where v = velocity

* From Alco

$$= \frac{(326)(11.65)}{(326)(11.65) + (44)(9.95)}$$

$$= \frac{3800}{3800+438} = \frac{3800}{4238} = .90$$

iii. This is the correct fraction of the flow that goes through vaporizer tubes, but as shown in the next paragraph, the vaporizer must supply 98.1% of the total heat transfer.

c. Temperature Drops Through Vaporizer and Superheater

Taking 450°F as inlet temperature to both sets of tubes and T_1 and T_2 as respective outlet temperatures of vaporizer and superheater, we require the vaporizer to supply 979.9 Btu/lb (250°F feedwater temperature up to 99.7% quality) and the superheater to supply 16.3 Btu/lb, to 25°F superheat (407°F, 200 psia); respectively 98.1% and 1.9% of the total heat transfer. These figures are confirmed by

$$.981 \times 3.41 \times 10^7 = 3.35 \times 10^7 \text{ Btu/hr}$$

given by Alco as vaporizer heat removal rate. On this basis we may solve for the outlet temperatures. The mixed mean of T_1 and T_2 must be the reactor inlet temperature, 431.6°F.

$$\frac{9T_1 + T_2}{10} = 431.6$$

Also the ratio of total heat removal in vaporizer to that in superheater is

$$\frac{98.1}{1.9} = \frac{9(450-T_1)}{450-T_2}$$

The temperatures come out $\begin{cases} T_1 = 429.9^\circ F \\ T_2 = 446.6^\circ F \end{cases}$

d. Temperature Drops vs Heat Flow Rates

Total ΔT relationship can be obtained from p 701 of Glasstone, which disregards scale relationship for ΔT, is

$$\Delta T = \frac{b}{ah}\frac{q}{A} + \frac{b \ln b/a}{k}\frac{q}{A} + \frac{1}{c^{.413}}\left(\frac{q}{A}\right)^{.413}$$

where $\frac{q}{A}$ = Heat flow rate per unit area, with area based on OD.

b = Outside Radius, ft.

a = Inside Radius, ft.

h = Heat transfer coefficient of coolant, Btu/hr ft^2 $^\circ$F

k = Conductivity of metal of tube, $\frac{\text{Btu ft}}{\text{hr ft}^{2\circ}\text{F}}$

e. Film Drop

i. From p 678 of Glasstone, film drop "h" can be computed from relationship

$$h = 170\ (1+10^{-2}T - 10^{-5}\ T^2)\ \frac{v^{.8}}{D^{.2}}$$

where

$$h = \text{Btu/hr ft}^2\ {}^{o}F$$

$$T = 440.8^{o}F \text{ mean bulk water temperature*}$$

$$v = 11.5 \text{ ft/sec}$$

$$D = .62 \text{ in, ID } (.75\text{-}2\text{x}.065)$$

ii. $$h = 170\ (1{+}4.4.{-}1.945)\left(\frac{7.06}{.908}\right)$$

$$= (170)(3.46)(7.79)$$

$$= 4580 \text{ Btu/hr ft}^2\ {}^{o}F$$

iii. $$\Delta T = \frac{q}{A}\left(\frac{b}{a}\right)h$$

$$b = \left(\frac{.75}{2}\right)\left(\frac{1}{12}\right) = .0313 \text{ ft}$$

$$a = \left(\frac{.62}{2}\right)\left(\frac{1}{12}\right) = .0258 \text{ ft}$$

$$\Delta T = \frac{q}{A}\ \frac{.0313}{.0258}\ \frac{1}{4580} = \frac{1.21}{.458\text{x}10^4}\ \frac{q}{A}$$

$$= 2.64\text{x}10^{-4}\ \left(\frac{q}{A}\right){}^{o}F$$

f. Drop Through Metal

i. $$\Delta T = \frac{q}{A}\ \frac{b \ln \frac{b}{a}}{k}$$

where

$$k = 9.4\ \frac{\text{Btu ft}}{\text{hr ft}^2\ {}^{o}F} \text{ for stainless 304}$$

---

*Using inlet or outlet temperature changes h by less than 1/3 of 1%.

ii. $\Delta T = \frac{q}{A} \frac{.0313 \ln 1.21}{9.4}$

$= \frac{q}{A} \frac{(.0313).191)}{9.4}$

$\Delta T = .000636(\frac{q}{A})\ {}^{o}F$

g. <u>Drop Through Boiling Region</u>

$$\Delta T = \frac{1}{c^{.413}} \left(\frac{q}{A}\right)^{.413}$$

$c = 81$ @ 200 psia (p 698, Glasstone)

$1/c^{.413} = \frac{1}{6.12} = 1.635\text{x}10^{-1}$

h. <u>Total Drop</u>

i. $\Delta T = 2.64\text{x}10^{-4} \frac{q}{A} + 6.36\text{x}10^{-4} \frac{q}{A} + 1.635\text{x}10^{-1} \left(\frac{q}{A}\right)^{.413}$

$= 9.00\text{x}10^{-4} \frac{q}{A} + 1.634\text{x}10^{-1} \left(\frac{q}{A}\right)^{.413}$

ii. <u>Computation of T vs $\frac{q}{A}$ (See graph)</u>

(1)	(2)	(3)	(4)	(5)	(6)
$(q/A)\text{x}10^{-4}$	$(q/A)^{.413}$	ΔT film and wall	ΔT from boiling	ΔT	$R_{boil} = \Delta T_B/q/A$
Assume	$(1)^{.413}$x45.0	(1) 9.0	(2) $1.635\text{x}10^{-1}$	(3)+(4)	(4)/(1)
1	45.0	9.0	7.35	16.4	.000735
4.0	79.8	36.0	13.06	49.1	.000326
5.0	87.5	45.0	14.30	59.3	.000286
7.0	100.4	63.0	16.43	79.4	.000235

i. <u>Log Mean Temperature Difference</u>

$$\text{Log Mean } \Delta T = \frac{\Delta T_3 - \Delta T_4}{\ln \frac{\Delta T_3}{\Delta T_4}}$$

$$\Delta T_3 = 450.0 - 381.8$$

$$= 68.2$$

$$\Delta T_4 = 49.8$$

$$\text{Arith Mean } \Delta T = 59.0^{\circ}$$

$$\text{Log Mean } \Delta T = \frac{68.2-49.8}{\ln \frac{68.2}{49.8}}$$

$$= \frac{18.4}{\ln 1.37}$$

$$= \frac{18.4}{.3145} = 58.4$$

Log Mean ΔT = $58.4^{\circ}F$. This can be compared with arithmetic mean of $59.0^{\circ}F$. (However at other than initial conditions, such as reduced flow in primary system, the discrepancy becomes important.)

j. <u>Comparison of ΔT's and Thermal Resistances</u>

i. Mean (q/A) = 4.00×10^4 Btu/ft^2hr in vaporizer.

Log Mean ΔT = $58.4^{\circ}F$

ii. At mean q/A a comparison is given of "resistances", both as computed and as supplied by Alco, and ΔT's both as computed directly and as computed from thermal resistances supplied by Alco. Using the Alco figure, 33.5×10^6 Btu/hr for vaporizer, mean $q/A = \frac{33.5 \times 10^6}{836} = 4.00 \times 10^4$ Btu/ft^2/hr.

k. Comparison of Computed and Alco Supplied Data

Quantity or Location	Used in Computing Both ΔT s	Computed		Supplied by Alco		
		Resist.	ΔT	Conduct	Resist.	ΔT
q/A	4.00×10^4					
Film		.000264**	10.6	3810*	.000263	10.5
Metal		.000636**	25.4		.000620*	24.8
Scale					.000300*	12.0
Boiling		.000326	13.1**	5200*	.000192	7.7
Total		.001245	49.8		.001375	55.0
Log Mean ΔT			58.4			57.2*
Overall U 1/U				702*	.001425	

* Primary Information from Alco
** Primary Computation

2. Reactor

a. Heat Transfer Coefficient

Using the expression for the heat transfer coefficient, h, as found on page 678 in Glasstone's Principles of Nuclear Reactor Engineering, for turbulent flow:

$$h = 170(1+10^{-2}T-10^{-5}T^2)v^{0.8}/D_e^{0.2}$$

where

T = average water temperature - $^{\circ}F$

v = coolant velocity - ft/sec

D_e = equivalent diameter - inches.

$$T = \frac{T_1+T_2}{2} = \frac{431.6+450}{2} = 441^{\circ}F$$

$$v = 4.3G \text{ ft/sec}$$

$$D_e = (0.0211)(12) = 0.254 \text{ inches.}$$

$$(1+10^{-2}T-10^{-5}T^2) = 3.47$$

$$h = 170(3.47)v^{0.8}/D_e^{0.2} = 590v^{0.8}/D_e^{0.2}$$

$$v^{0.8} = 3.21G^{0.8}$$

$$D_e^{0.2} = 0.76$$

$$h = 2490G^{0.8} \text{ Btu/hr-}^{\circ}\text{F-ft}^2$$

The above figure compares very favorably with the value of 2570 listed in the Hazard Report. To keep problem consistent with other Alco data, the value of 2570 Btu/hr-°F-ft^2 will be used in subsequent calculations for the heat transfer coefficient.

b. <u>Check of Maximum Wetted-Surface Temperature.</u>

Using the value for the maximum to average power distribution of 4:1, the maximum wetted surface temperature was calculated on the basis of a chopped cosine axial distribution having a maximum to average of 1.31:1.

c. <u>Calculations</u>

$Q_{max} = 4\ Q_{av}$

$(P_{max}/\ P_{av})_{\ axial} = 1.31$

$$\frac{1}{a}\int_{o}^{a} \cos \frac{\pi x}{2L}\, dx = \frac{1}{1.31}$$

$$\frac{2L}{\pi a} \sin \frac{\pi x}{2L} \Big|_{o}^{a} = \frac{2L}{\pi a} \sin \frac{\pi a}{2L}$$

$$\sin \frac{\pi a}{2L} = \frac{2\pi a}{2(1.31)\ L} = 1.2\ \frac{a}{L}$$

$$\sin 1.57\ \frac{2}{L} = 1.2\ \frac{a}{L}$$

$\frac{\pi a}{2L} = 1.235$ $\qquad\qquad$ a= 0.9165 ft.

$\frac{\pi}{2L} = 1.346$

$Q = Q_o \cos 1.346x$

$$T_f - T_o = \frac{A}{Wc_p}\int_{-a}^{x} Q\,dx$$

$$T_f - T_o = \frac{Q_{o_{Max}} A}{Wc_p} \int_{-a}^{x} \cos\frac{(\pi x)}{2L}\,dx$$

$$\frac{2Q_{o_{max}} AL}{\pi Wc_p}\left[\sin\frac{\pi x}{2L} + \sin\frac{\pi a}{2L}\right] = T_f - T_o$$

$$T_s - T_f = \frac{Q_{o_{max}} V}{hA_h} \cos\frac{\pi x}{2L}$$

$$T_s - T_o = \frac{Q_{o_{max}} VL}{\pi Wc_p\, a}\left[\sin\frac{\pi x}{2L} + \sin\frac{\pi a}{2L}\right] + \frac{Q_{o_{max}} V}{hA_h} \cos\frac{\pi x}{2L}$$

Differentiating to locate the maximum temperature,

$$\frac{Q_{o_{max}} V}{2Wc_p a} \cos \frac{\pi x}{2L} = \frac{Q_{o_{max}} V\pi}{2LhA_h} \sin \frac{\pi x}{2L}$$

$$\tan \frac{\pi x}{2L} = \frac{LhA_h}{\pi a Wc_p}$$

$$T_{s_{max}} - T_o = \frac{Q_{o_{max}} VL}{\pi \; Wc_p a} \left[\sin \frac{\pi x_{max}}{2L} + \sin \frac{\pi a}{2L} + \cos \frac{\pi x_{max}}{2L} \operatorname{ctn} \frac{\pi x_{max}}{2L} \right]$$

$$= \frac{Q_{o_{max}} VL}{\pi Wc_p a} \left[\sin \frac{\pi a}{2L} + \csc \frac{\pi x_{max}}{2L} \right]$$

Let $\delta = \sin \frac{\pi a}{2L} + \csc \frac{\pi x_{max}}{2L}$

$$\frac{Q_{av} V}{Wc_p} = 18.4F^o \qquad\qquad Q_{o_{max}} = 4 \; Q_{av}$$

$$\frac{Q_{o_{max}} V}{Wc_p \pi} = 18.4 \; (4/\pi) = 23.4F^o, \qquad \frac{L}{a} = 1.27$$

$$\sin \frac{\pi a}{2L} = \sin 70.8^o = 0.9442$$

$$\operatorname{ctn} \frac{\pi x_{max}}{2L} = \frac{\pi \; aWc_p}{LhA_h} = 2.92$$

$$\text{ctn}^2 \left(\frac{\pi x_{max}}{2L}\right) = 8.5$$

$$\csc^2 \left(\frac{\pi x_{max}}{2L}\right) = 9.5$$

$$\csc \frac{\pi x_{max}}{2L} = 3.085$$

$$= 3.085 + 0.944 = 4.029$$

$$T_{s_{max}} - T_o = 4.03(1.27)(23.4) = 120 \text{ F}^{o}$$

$$T_{s_{max}} = 431.6+120 = 552^{o}\text{F}$$

The value of 552°F checks very well with the figure of 554°F listed for the maximum surface temperature in the Hazards Report.

APPENDIX B - FRICTION HEAD LOSS vs FLOW

Contents

1. Velocities and Flow Rates

   a. For densities which can be taken as constant, v depends only upon F.

   b. In fact:

Reactor	$v = (4.3/4000)F$
Steam Gen.	$v = (11.5/4000)F$
Piping	$v = (12.6/4000)F$

2. Reynolds Number vs. Flow

   a. By using same line of reasoning as for "H" in turbulent flow:

$$Re = C'F$$

   with C' such that it gives same Re for F = 4000 as is found in Appendix A.

   b. Therefore:

$$\text{Reactor } Re = 58{,}000\ (F/4000) = 14.5F$$

$$\text{Steam Gen } Re = 380{,}000\ (F/4000) = 95\ F$$

$$\text{Piping } Re = 7{,}440{,}000\ (F/4000) = 1860F$$

3. Transition, Turbulent to Laminar Flow

   a. Laminar flow runs to Re as high as 2×10^3. Actual curve for f dips below curves for f for laminar and turbulent.

b. We use two expressions:

$$f = 16/Re \qquad \text{Laminar}$$

$$f_t = .054/(Re)^{.2} \qquad \text{turbulent}$$

c. At intersection $f = f_t$

$$\frac{Re}{16} = \frac{Re^{.2}}{.054}$$

$$(Re)^{.8} = \frac{16}{.054}$$

$$Re = \left(\frac{16}{.054}\right)^{1.0/.8}$$

$$Re = (296)^{1.25} = 1230$$

If we use this as transition, we are conservative.

4. Turbulent Flow

a. For turbulent flow, friction head is

$$H = \frac{4fv^2L}{2gD}$$

b. For a system whose dimensions are fixed, H depends only upon f and v^2.

c. From Appendix A,

$$f = .054\ Re^{-.2}$$

or if ρ and μ are taken constant, f is proportional to $F^{-.2}$

d. Therefore:

$$H = CF^2\ F^{-.2}$$

$= CF^{1.8} = C'' \, G^{1.8}$ (since $G = \frac{F}{4000}$)

e. C'' must give same value at F = 4000 as are found in Appendix A. So for turbulent flow,

Reactor	$H = 1.7 \, G^{1.8}$
Steam Gen.	$H = 16.7 \, G^{1.8}$
Piping	$H = 4.6 \, G^{1.8}$
Total	$H = 23.0 \, G^{1.8}$

5. <u>Value of F at Transition</u>

a. Re = 1230 at transition

Re = C'F

F = 1230/C' at transition

also H = $C''G^{1.8}$

b. Therefore:

Component	① C'	② 1230/C' = F trans 1230/①	③ 4000/F 4000/②	④ ③$^{1.8}$	⑤ C'' from App A	⑥ H ⑤/④
Reactor	14.5	84.8	47.1	1.032×10^{3}	1.7	1.645×10^{-3}
Steam Gen.	95.0	12.95	309.	3.033×10^{4}	16.7	5.51×10^{-4}
Piping	1,860.	0.661	6050.	6.398×10^{6}	4.6	7.19×10^{-7}

6. Laminar Flow

a. $$H = \frac{64}{\rho} \quad \frac{\mu}{Dv\ \rho} \quad \frac{v^2 L}{2g}$$

$$H = C'''F$$

where C''' gives proper H at transition

b. C''' = (H/F) @ transition

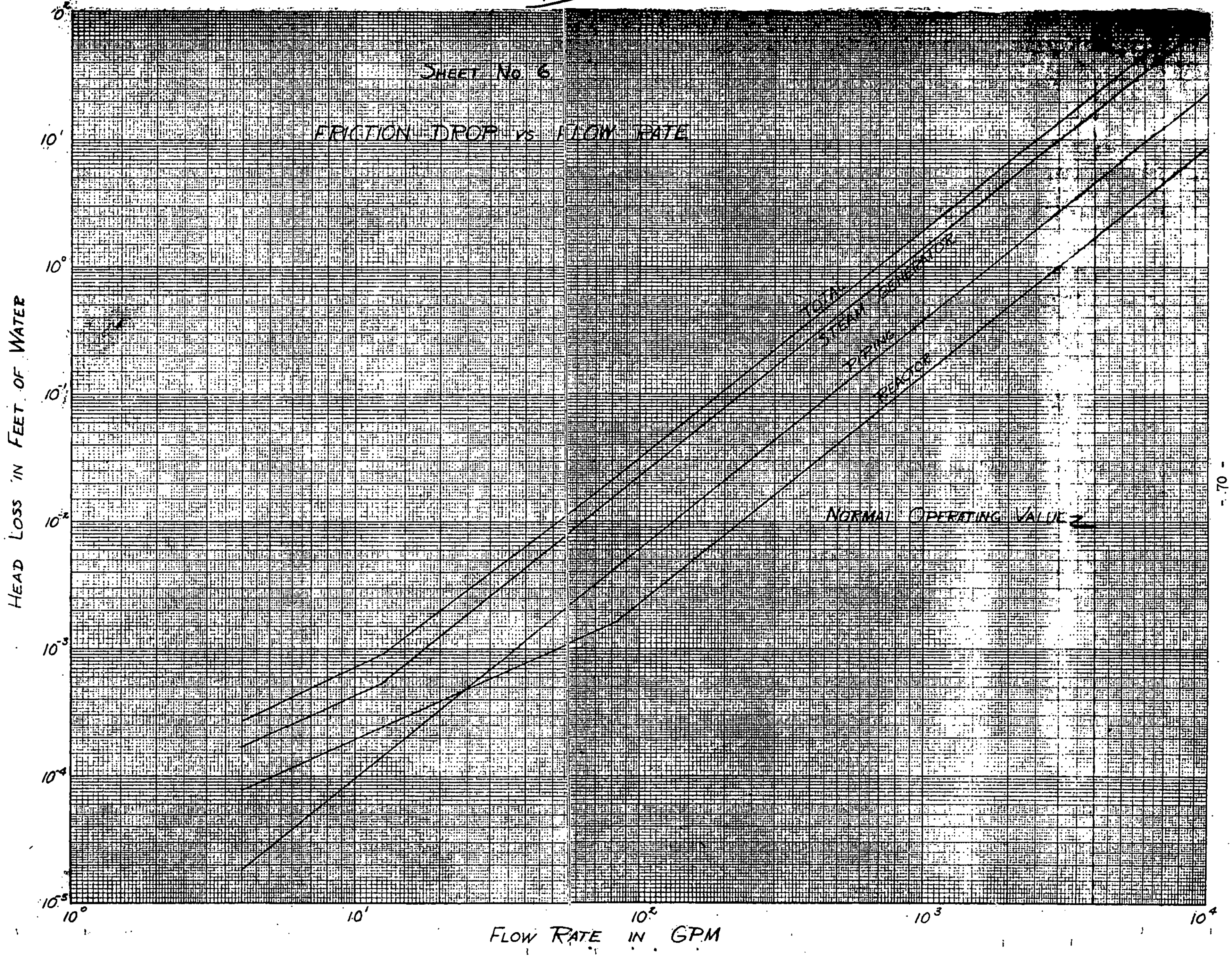
c.

	H transition	F transition	C'''
Reactor	1.645×10^{-3}	84.8	$1.94 \times 10^{-5}F$
Steam Gen.	5.51×10^{-4}	12.95	$4.26 \times 10^{-5}F$
Piping	7.19×10^{-7}	0.661	$1.086 \times 10^{-6}F$

7. Curves

Curves of H vs F will be found on the next curve sheet.

70


Sheet No 6
Friction Drop vs Flow Rate
Total
Steam Generator
Piping
Reactor
Normal Operating Value
Head Loss in Feet of Water
Flow Rate in GPM
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^0
10^1
10^2
10^3
10^4

APPENDIX C - CONVECTION FLOW (THERMAL SIPHONING) RELATIONSHIPS

Contents | Page

1. Basic Relationships

a. Σ (Heads causing flow) = Σ (Friction heads)

$$\frac{\Delta p \text{ causing flow}}{\rho_{av} \text{ around loop}} = H_o \left[\frac{F}{4000}\right]^{1.8} = H_o G^{1.8} \qquad \text{(turbulent flow)}$$

where $G. = \frac{F}{4000}$

H_o = friction head at full flow = 23.0 ft.

b. Use simplified diagram on next page to visualize loop and to obtain vertical dimensions of loop.

2. Pressure Differentials

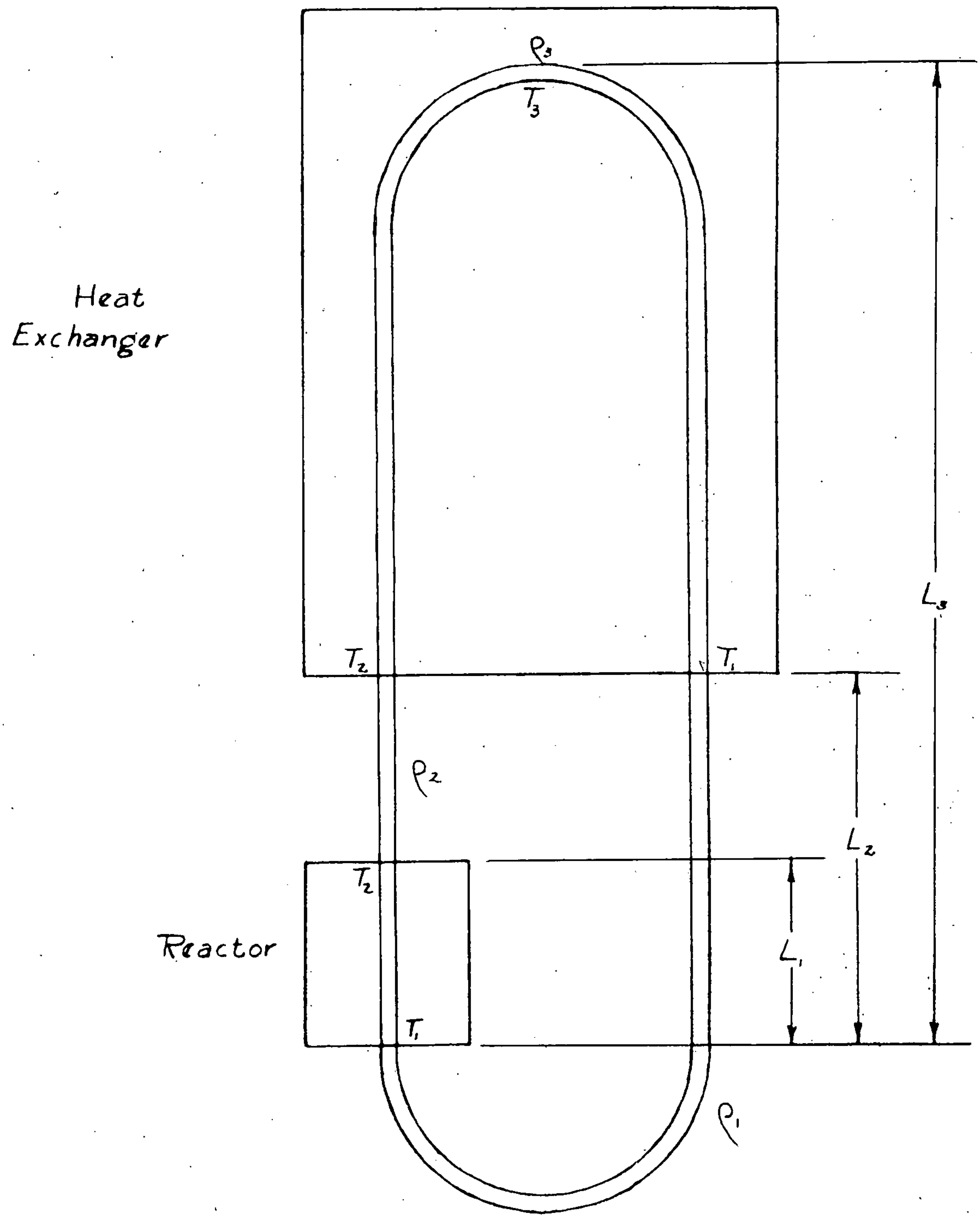
a. Pressure Differential Produced by Parts of Loop Other Than Steam Generator

i. Δp = Δp from reactor = Δp from riser above reactor and below steam generator + Δp from downward-going portion of pipe below steam generator

ii. or (see loop diagram)

$$\Delta p = -L_1 \frac{(\rho_1 + \rho_2)}{2} - (L_2 - L_1) \quad \rho_2 + L_2 \rho_1$$

$$= \frac{\rho_1 - \rho_2}{2} (2L_2 - L_1)$$



FROM ALCO DRAWINGS:

$L_1 = 1.83$ ft
$L_2 = 4.25$ ft
$L_3 = 11.33$ ft

SHEET 7. SIMPLIFIED PIPING DIAGRAM

Pressure Differential Produced by Steam Generator

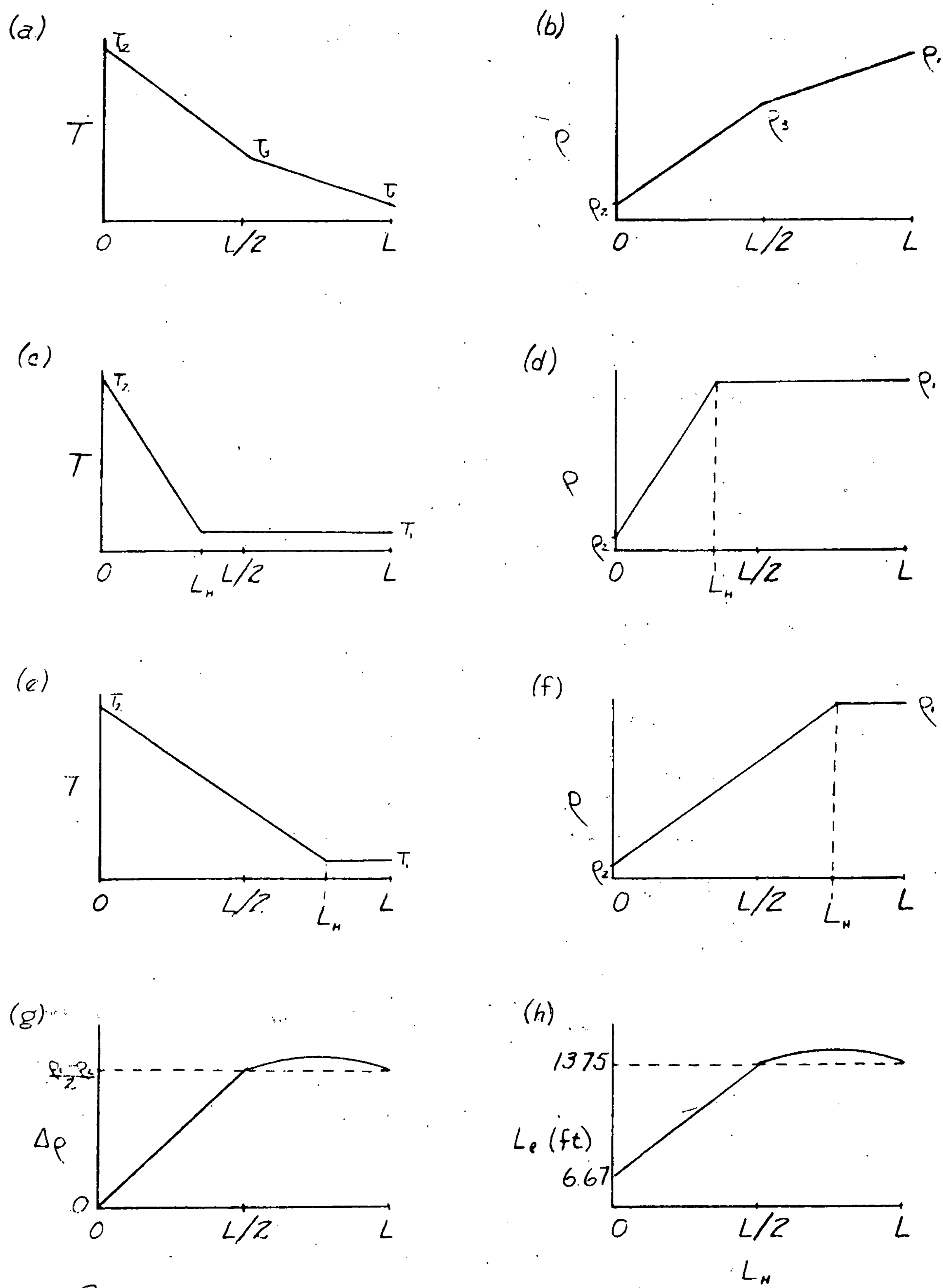
i. Actually there are several possible results which can be obtained for Δp (due to thermal convection) through the steam generator. The one obtained depends upon the assumptions made for rate of heat flow from the water as a function of distance through the steam generator. Several of these will be discussed, but the one discussed immediately below can be used in most situations.

ii. Assume that $\frac{d\rho}{dT}$ is constant over the temperature range of the loop, and that $\frac{dT}{dL}$ = one constant for up-leg of heat exchanger and $\frac{dT}{dL}$ = another constant for down-leg of heat exchanger. Then the temperature and density variations through the heat exchanger will be as shown on Figures (a) and (b) of curve sheet 8.

iii. The difference between mean densities in the two legs will be

$$\Delta\rho = \frac{\rho_1 + \rho_3}{2} - \frac{\rho_2 + \rho_3}{2} = \frac{\rho_1 - \rho_2}{2}$$

which is independent of the density (and temperature) at the top of the loop. Therefore the contribution to thermal siphoning Δp by the heat exchanger is independent of the



SHEET 8. TEMPERATURE AND DENSITY VARIATION

fraction of the total heat removal which occurs in the up-leg. This is nearly the normal situation, as demonstrated later.

iv. Based on the above,

$$\Delta p = \text{(leg length)} \quad \text{(density difference)}$$
$$= (L_3 - L_2) \frac{(\rho_1 - \rho_2)}{2}$$

c. <u>Total Pressure Differential for Loop</u>

i. $\Delta p = \Delta p$ reactor and piping + Δp steam generator

$$= \frac{(\rho_1 - \rho_2)}{2} (2L_2 - L_1) + (L_3 - L_2) \frac{(\rho_1 - \rho_2)}{2}$$

ii. $\Delta p = \dfrac{\rho_1 - \rho_2}{2} \left[L_3 - L_2 + 2L_2 - L_1\right] = \dfrac{\rho_1 - \rho_2}{2} \left[L_3 + L_2 - L_1\right]$

for heat removal not concentrated in a length $L_H < L/2$.

iii. For this case, we can define an "equivalent loop length"

$$L_e = L_3 + L_2 - L_1$$
$$= 11.33 + 4.25 - 1.88 \qquad \text{(see loop diagram)}$$
$$= 13.75 \text{ ft. for this system}$$

iv. $\Delta p = \dfrac{\rho_1 - \rho_2}{2} L_e$

$$= \frac{\rho_1 - \rho_2}{2} \; 13.75 \text{ lb/ft}^2 \quad \text{when } \rho\text{'s are lb/ft}^3$$

3. <u>Derivation of Temperature Drop, Flow and Power Relationships</u>.

a. <u>Flow and Temperature Drop</u>

i. $$1/2 \frac{(\rho_1 - \rho_2)}{\rho_{avg}} L_e = H_o G^{1.8}$$

ii. But from Alco $\frac{d\rho}{dT} = -.046$ in temperature range of interest, so that $(\rho_1 - \rho_2) = .046 (T_2 - T_1)$

iii. $$\frac{.046(T_2 - T_1)L_e}{2 \rho_{avg}} = H_o G^{1.8}$$

$$(T_2 - T_1) = \frac{\rho_{avg}}{L_e} \frac{H_o G^{1.8}}{.023}$$ for turbulent flow

$$(T_2 - T_1) = \frac{\rho_{avg}}{L_e} \frac{H}{.023}$$ with H from curve of H vs F for laminar flow

b. <u>Flow and Power</u>

i. Power Removal Rate = Weight Flow Rate x Heat Removed Per Unit Weight

ii. $Q(\text{Btu hr}) = (vA\rho_{avg}) (c_p) (T_2 - T_1)$

$= Wc_p(T_2 - T_1)$, where W = Weight flow rate, lb/hr

iii. $$W = W_o G$$

where W_o = Weight flow rate (normal operating)

$G = F/4000$

iv. $$Q = Wc_p\,(T_2 - T_1)$$

$$= W_o G c_p \frac{\rho_{avg}}{L_e} \frac{H_o G^{1.8}}{.023} \quad \text{(for turbulent flow)}$$

$$Q = W_o c_p \frac{\rho_{avg}}{L_e} \frac{H_o G^{2.8}}{.023}$$

v. $$kw = \frac{Q}{3413} = \frac{W_o H_o c_p \rho_{avg}}{78.5 L_e} G^{2.8}$$

vi. Normalized power: $$P = \frac{W_o H_o c_p \rho_{avg}}{78.5 \times 10^4 L_e} G^{2.8}$$

($P = 1$ for 10^4 kw, rated Output)

c. <u>Power vs Temperature Drop</u>

i. From flow vs temperature drop

$$G^{1.8} = \frac{(T_2 - T_1)\, L_e}{\rho_{avg}\, H_o} \times .023$$

ii. From flow vs power

$$G^{2.8} = \frac{P(.3413)(.023)L_e}{W_o H_o c_p \;\rho_{avg}}$$

iii. Also note that

$$G^{2.8} = \left[G^{1.8}\right]^{\frac{2.8}{1.8}} = (G^{1.8})^{1.556}$$

iv. Therefore

$$G^{2.8} = \left[\frac{(T_2-T_1)\ L_e\ .023}{\rho_{avg}\ H_o}\right]^{1.556} = \left[\frac{.023\ L_e}{\rho_{avg} H_o}\right]\left[\frac{.3413P}{W_o c_p}\right]$$

$$P = (T_2-T_1)^{1.556} \left[\frac{L_e .023}{\rho_{av} H_o}\right]^{.556} \left[\frac{W_o c_p}{.3413}\right]$$

4. <u>Numerical results</u>

a. $\rho_{av} = 52.17\ \text{lb/ft}^3$

$L_e = 13.75$ ft, equivalent length

$H_o = 23.0$ ft, head

$W_o = 1.66\text{x}10^6$ lb/hr

$c_p = 1.115\ \text{Btu/lb-}^{\circ}\text{F}$

b. $$\frac{\rho_{avg} H_o}{L_e \text{x}.023} = \frac{52.17\text{x}\ 23.0}{13.75\ \text{x}\ .023} = 3790$$

so that

$$(T_2-T_1) = 3790\ G^{1.8} \quad \text{for turbulent flow only.}$$

c. Also note that

$$\frac{\rho_{avg}}{L_e \times .023} = 164.5$$

so that

$$(T_2 - T_1) = 164.5 \text{ H for laminar or turbulent flow}$$

d. $$\frac{W_o c_p \ \rho_{av} H_o}{78.5 \times 10^4 L_e} = \frac{1.66 \times 10^6 \times 1.115 \times 52.17 \times 23.0}{78.5 \times 10^4 \times 13.75}$$

$$= 205.8$$

Hence

$$P = 206. \ G^{2.8}$$

for turbulent flow

$$\text{and kw} = 2.06 \times 10^6 \ G^{2.8}$$

e. Although the laminar flow range turns out to have little bearing on this problem, the corresponding relations are given:

$$\frac{W_o c_p \ \rho_{avg}}{78.5 \ L_e} = \frac{1.66 \times 10^6 \times 1.115 \times 52.17}{78.5 \times 13.75} = 8.94 \times 10^4$$

Hence $\text{kw} = 8.94 \times 10^4 \ GH$

$= 22.3$ FH (H from laminar flow curve)

$P = 8.94$ GH

5. Other Assumptions on Steam Generator

a. Effect of Assumptions

i. The above relationships break down if the temperature drop and density rise occur substantially in some distance L_H, well before reaching the top or the bottom of the loop. For example, the temperature and density variations through the heat exchanger might be represented as in Figures (c) and (d) of curve sheet 8.

ii. In this case, the effective density increment between the two legs would be

$$\Delta \rho = \rho_1 - \left[\frac{\rho_1 + \rho_2}{2} \, 2L_H/L + \rho_1 \frac{L-2L_H}{L} \right]$$

$$= (\rho_1 - \rho_2) \quad L_H/L \text{ for } L_H < L/2$$

iii. If, on the other hand, (Figures e and f), the effective length should terminate somewhere on the down-leg of the steam generator (i.e. $L/2 < L_H < L$) the effective density increment will be higher than if the entire loop were used. If, for example, the temperature and density variations in the generator were as shown in Figures e and f, the effective density increment between the two legs would become

$$\Delta \rho = (P_1 - P_2)(2 - L_H/L - L/2L_H), \text{ for } (L/2 < L_H < L)$$

This effect is represented graphically by Figure (g).

iv. The pressure differential produced by steam generator (see loop diagram) for the case of rapid heat removal ($L_H < L/2$)

$$\Delta P_H = (L_3 - L_2)\ (\rho_1 - \rho_2)\ (L_H/L)$$

(see item 5, a, ii)

b. <u>Axial Temperature Distribution in Steam Generator</u> (Normal Operation)

i. Because of the dependence of the thermal-siphoning driving head upon the difference in mean densities of the water between the heat-exchanger legs, it is important to investigate the axial temperature distribution through the primary tubes of the steam generator.

ii. Assuming constant thermal resistance between primary and secondary water, for the entire length L of the vaporizer tubes, and defining c as a constant to be determined by end conditions,

iii. $$-\frac{dT}{dx} = c\ (T - T_s) \qquad \begin{array}{l} x = 0,\ T = T_3 \\ x = L,\ T = T_4 \end{array}$$

$$\int_{T_3}^{T} \frac{dT}{T-T_s} = -c\int_{o}^{x} dx$$

$$\ln (T-T_s)\Big|_{T_3}^{T} = -cx\Big|_{o}^{x}$$

$$\ln \frac{T-T_s}{T_3-T_s} = -cx$$

relations giving T for any position x along the tubes

$$\frac{T-T_s}{T_3-T_s} = e^{-cx}$$

To evaluate c:

$$\frac{T_4-T_s}{T_3-T_s} = e^{-cL}$$

$$cL = \ln \frac{T_3-T_s}{T_4-T_s}$$

iv. Let $T_3 = 450^{o}F$, $T_4 = 432^{o}F$, $T_s = 382^{o}F$ (for full flow condition).

$$cL = \ln \frac{T_3-T_s}{T_4-T_s} = \ln \frac{68}{50} = \ln 1.36 = 0.307$$

$$\frac{T-T_s}{68} = e^{-.307\ x/L}$$

x/L	$.307\frac{x}{L}$	$e^{.307\frac{x}{L}}$	T-382	T(^{o}F)
0.00	0.000	1.000	68.0	450.0
.25	.077	1.080	63.0	445.0
.50	.154	1.166	58.3	440.3
0.75	.230	1.259	54.1	436.1
1.00	0.307	1.360	50.0	432.0

v. For $T_3 = 476^{o}F$, $T_4 = 406^{o}F$, $T_s = 382^{o}F$ (quasi-steady state),

$$cL = \ln \frac{T_3 - T_s}{T_4 - T_s} = \ln \frac{94}{24} \ln 3.92 = 1.366$$

$$\frac{T - T_s}{94} = e^{-1.366 \quad x/L}$$

x/L	1.366 x/L	$e^{1.366 \quad x/L}$	T-382	T(^{O}F)
0.00	0.000	1.000	94.0	476.0
.25	.342	1.407	66.8	448.8
.50	0.683	1.98	47.5	429.5
0.75	1.024	2.78	33.9	415.9
1.00	1.366	3.92	24.0	406.0

The results of Sections 5-b iv and v, for the full-flow and quasi-steady axial temperature distributions through the steam generator, are plotted on the next page. It will be seen that for both cases the representation by straight lines over spans L/2 is a very good approximation. Hence the density increment $\frac{\rho_1 - \rho_2}{2}$ (Section 2 - b) is applicable to our problem, and the variations discussed in Section 5-a are not needed.

c. Total Pressure Differential for Loop; Case of Zero Contribution from Steam Generator

i. It should be noted that for heat removal in an extremely short length $L_H \rightarrow 0$, the steam generator would make zero pressure contribution to thermal siphoning.

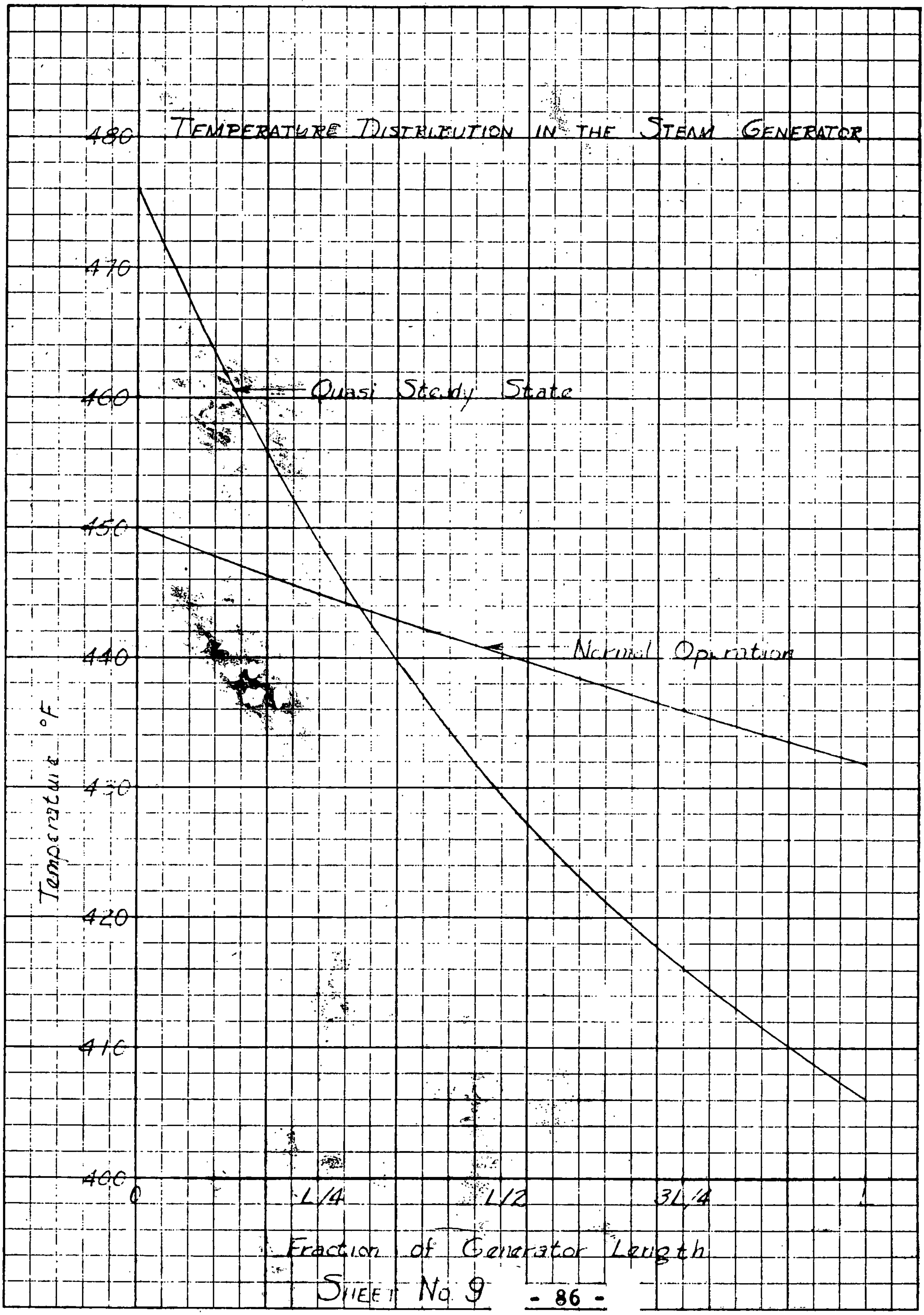
ii. Therefore

$$L_e = 2L_2 - L_1 = 8.50-1.83 = 6.67 \text{ ft.} \quad (L_H \rightarrow 0)$$

whereas

$$L_e = 13.75 \text{ ft} \quad (L_H = L/2 \text{ or } L) \qquad \text{(see Figure h)}$$

We have shown that the "full" value of equivalent length, 13.75 ft, is applicable to our problem.


TEMPERATURE DISTRIBUTION IN THE STEAM GENERATOR
480
470
460
450
440
430
420
410
400
Temperature °F
Quasi Steady State
Normal Operation
0
L/4
L/2
3L/4
L
Fraction of Generator Length
SHEET No 9

APPENDIX D - STEAM GENERATOR RELATIONSHIPS FOR QUASI-STEADY STATE

Contents | Page

1. Assumptions on Steam Generator

On the basis of the comparisons in Appendix A, and considering that the steam generator was presumably designed according to conservative practices, we are justified in assuming conditions which seem reasonable and which are conservative for the particular computation involved. With this in mind, the following are assumed:

a. For the new quasi-steady situation (existing just after the fast major transients die away), there is constant pressure and temperature on the secondary side.

b. Scale exists. This is apparently more conservative than no-scale (and also more consistent with reactor assumptions). However, the no-scale situation will also be examined.

c. The vaporizer dominates the steam generator and the inlet, outlet, and mean temperatures of the steam generator are those of the vaporizer.

d. The effective primary fluid temperature is the mean temperature (440.8°F) during the early part of the transient phase. However, to obtain the new quasi-steady state the log mean ΔT will be used.

The following relationships exist:

e. The film ΔT (primary fluid) varies directly with P and inversely with $G^{.8}$.

f. Metal and scale ΔT's vary directly with power.

g. Boiled ΔT varies directly with generated power to and exponent of 0.413.

2. Thermal Resistance Components

Looking at the comparisons in Appendix A, we can build up a table of percentages of total temperature drop, and taking the total at $59^{o}F$ (with scale) we can obtain a quantity to be assigned as the contribution from each resistance to heat flow.

	Percentages			ΔT's
	Computed	Alco	Assigned	Assigned
Film	19.3%	19.1%	19%	$11.2^{o}F$
Metal	46.4	45.1	43	25.4
Scale	- - -	21.9	19	11.2
Boil	25.0	13.9	19	11.2
Total	90.7%	100.0%	100%	$59.0^{o}F$

3. Temperature Drops for Decreasing Power and Flow

a. We determine ΔT's (with scale) as follows:

ΔT = Total = Film + Metal & Scale + Boiling

Effective ΔT at Start $= 59 = 11.2 + 36.6 + 11.2$

and for new quasi-steady state, with scale,

$$\text{Log-mean } \Delta T = (\Delta T \leqq 59) = \frac{11.2P}{G^{0.8}} + 36.6 + 11.2P^{4.13}$$

b. For the case without scale, we will assume that the primary loop situation is the same as that with scale, and that the pressure in secondary loop is kept higher (by throttle) thus giving the same heat flow at a lower ΔT. (Raising the pressure in secondary loop raises T_{sat}.)

c. The above assumption results in the following relationship:

$$\text{Total } \Delta T = \text{Film} + \text{Metal} + \text{Boiling}$$

$$\text{LM } \Delta T = \frac{11.2P}{G^{.8}} + 25.4\ P + 11.2\ P^{.413}$$

where log-mean $\Delta T = 47.8^{\circ}F$, initial ΔT.

d. Numerically, the no-scale assumption has the following effect. The Alco-assumed scale resistance for steam generator tubes was 3×10^{-4} (equivalent of .0036 inches with $k = 1$ Btu/hr-ft-$^{\circ}$F). This amounted to 19% of the total thermal resistance in the evaporator ($11.2^{o}F$ out of total $\Delta T = 59^{o}$). The steam will thus be generated at $393^{o}F$ before scale has formed, instead of at $382^{o}F$. Therefore the start-up powerplant will produce 230 psia steam which will be expected to drop gradually to 200 psia on formation of scale with the assumed resistance.

4. Power-Flow Relationships to Satisfy Steam Generator Conditions

An indirect calculation is necessary to determine the relationship between P and G imposed by the heat exchanger. The complication is introduced by the necessity of considering the log-mean temp difference in place of an arithmetic mean.

a. For the scale condition:

$$\text{log-mean } \Delta T = \frac{\Delta T_1 - \Delta T_2}{\ln \dfrac{\Delta T_1}{\Delta T_2}} = \frac{T_1 - T_2}{\ln \dfrac{T_1 - 382}{T_2 - 382}}$$

(since $\Delta T_1 = T_1 - 382$, $\Delta T_2 = T_2 - 382$)

Also, $\text{log-mean } \Delta T = \dfrac{11.2P}{G^{.8}} + 36.6P + 11.2P^{.413}$

b. Other relationships permitting elimination of the unknown T's are

$$\frac{T_1 + T_2}{2} = 441^{o}F,$$

$$\frac{\Delta T_1 + \Delta T_2}{2} = 59^{o}F,$$

$$P = \left(\frac{T_1 - T_2}{18}\right)G$$

c. Eliminating T's by this means, one obtains (letting $P/G = x$)

$$\frac{18x}{\ln \frac{6.55 + x}{6.55 - x}} = 11.2P^{.2} \quad x^{.8} + 36.6P + 11.2P^{.413}$$

which, by trial solution, yeilds the values in the first half of the following table, for the scale case.

d. The corresponding relationships for the no scale case are:

$$\text{log-mean } \Delta T = \frac{T_1 - T_2}{\ln \frac{T_1 - 393.2}{T_2 - 393.2}}$$

since $\Delta T_1 = T_1 - 393.2$, $\Delta T_2 = T_2 - 393.2$

$$\text{log-mean } \Delta T = \frac{11.2P}{G^{.8}} + 25.4P + 11.2P^{.413}$$

$$\frac{\Delta T_1 + \Delta T_2}{2} = 47.8^{o}F$$

$P = \left(\frac{T_1 - T_2}{18}\right)$ G, as before.

The equation yielding P vs G (letting $P/G = x$) is now

$$\frac{18x}{\ln \frac{5.31 + x}{5.31 - x}} = 11.2P^{.2} \; x^{.8} + 25.4P + 11.2P^{.413}$$

whose results are tabulated in the second half of the table below.

e. Heat Exchanger Power-vs-Flow Relationships

Scale		No Scale	
P	G	P	G
1.000	1.000	1.000	1.000
.775	.388	.690	.345
.580	.193	.440	.147
.405	.101	.363	.104

APPENDIX E - DECREASE IN FLOW AFTER PUMP FAILURE

Contents

Page

1. <u>Assumptions</u>

a. Ignore thermal siphoning (until about 8 sec. after pump failure).

b. Also ignore KE of pump (more conservative, particularly initially, and certainly easier).

c. After pump fails, the flow decreases while the kinetic energy of the water is being converted into work of friction

2. <u>Conversion of Kinetic Energy to Friction Work</u>

a. For each part of the hydraulic circuit

$KE = Mv^2/2 = \frac{AL\rho v^2}{2g}$ ft lb where v is in ft/sec and

A = Flow cross-sectional area of pipe or component

b. Rate at which KE is being turned into friction work

$\frac{dKE}{dt} = \frac{2Mv}{2} \frac{dv}{dt} = \frac{AL\rho v}{g} \frac{dv}{dt}$ ft-lb/sec

c. Similarly the rate of work loss due to friction,

$\frac{dKE}{dt} = -H\rho Av$ ft-lb/sec

$\therefore \Sigma L \left(\frac{vA\rho}{g}\right) \frac{dv}{dt} = -\Sigma H (A\rho v)$

d. However $(A\rho v)$ is constant around circuit, at any one time, water being considered incompressible, and ignoring small local boiling for the nonce.

$$\therefore \quad \Sigma \; \frac{L(A\rho v)}{g} \; \frac{dv}{dt} = -\Sigma \; H$$

e. However for each component,

$v = v_o G$ and $H = BG^{1.8}$**

(or $\frac{dv}{dt} = v_o \frac{dG}{dt}$)

Where v_o and B are constants peculiar to each component (see Appendices A and B) and G is normalized flow or F/4000.

f. $$\therefore \quad \frac{1}{g} \; \frac{dG}{dt} \, \Sigma \, L \, v_o = -G^{1.8} \quad \Sigma B$$

and

$$\frac{\Sigma L v_o}{g \Sigma B} \quad \frac{dG}{G^{1.8}} = -dt$$

** For turbulent flow in all components or down to G=.0207 (F= 82.7). Thereafter, for component affected, $H=CG^{1.0}$.

3. Integration of Decreasing-Flow Relationships

a. $$\frac{\Sigma L v_o}{g \Sigma B} \int_1^G \frac{dG}{G^{1.8}} = - \int_o^t dt$$

$$\frac{\Sigma L v_o}{g \Sigma B} \left[\left(\frac{-1}{.8}\right) \left(\frac{1}{G^{.8}} - 1\right) \right] = -t$$

or

$$a \left[\frac{1}{G^{.8}} - 1 \right] = t, \text{ where } a = \frac{\Sigma L v_o}{.8 g \Sigma B}$$

b. Transposing,

$$\frac{1}{G^{.8}} = \frac{t}{a} + 1$$

or

$$G^{.8} = \frac{a}{t + a}$$

where

$$a = \frac{\Sigma L v_o}{.8 g \Sigma B} \text{ sec.}$$

and B's are constants which give proper H at G = 1

L's are effective lengths of components for K. E.

v_o's are component velocities at G = 1.

4. Evaluation of Constants

a. Ignoring KE in reactor, other than that due to flow in core, we tabulate:

Component	L(ft) (effective for KE)	v (ft/sec) ($=v_oG$)	Lv_o	H(ft) ($=BG^{1.8}$)
Reactor	2.0	4.3 G	8.6	$1.7\ G^{1.8}$
Steam Gen.	14.7	11.5 G	169.0	$16.7\ G^{1.8}$
Piping	33.7	12.6 G	425.	$4.6\ G^{1.8}$
			603	$23.0\ G^{1.8}$

b. $$a = \frac{\Sigma L v_o}{.8g\Sigma B}$$

$$= \frac{603}{(.8)(32.2)(23.0)} = 1.016$$

Use a= 1.02.

$$G = \left(\frac{1.02}{t+1.02}\right)^{1.25} \qquad (0 \leq t \leq 8 \text{ sec})$$

c. The free-convection flow predominates for $t > 8$ sec; see Appendix C.

5. Combination of Coasting and Thermal Siphoning Effects

For $7 < t < 11$ sec, a more exact relationship connecting flow and power is obtained by combining item 2f (above) with the equations of Appendix C (sections 3 and 4):

$$\text{Coasting:} \quad -\frac{\Sigma L v_o}{g} G = H_o G^{1.8}$$

$$\text{Convection:} \quad \frac{.023 \times 3.41 \times 10^7}{\rho_{av} W_o c_p} L_e P G^{-1} = H_o G^{1.8}$$

Adding these effects and inserting constants,

$$168G + 206G^{1.8} - P/G = 0$$

APPENDIX F - DERIVATIONS OF TRANSIENT APPROXIMATIONS

Contents

1. Fuel Plate Temperatures

a. The derivation can be handled more easily using actual power P', in Btu/sec rather than the normalized power, P.

b. Rate heat delivered = Rate heat transferred + Heat going to raise the temperature of the fuel element.

c. $P' = c_6 P = AU\,(T_f - T) + w_f c_f \dot{T}_f$

d. Relative to changes in T_f, P and T can be considered constant and h can be obtained using an average G.

e. $$\left(\frac{P'}{w_f c_f} + \frac{AUT}{w_f c_f}\right) - \frac{AU}{w_f c_f} T_f = \dot{T}_f$$

where the first two terms are constant.

f. The above is of the form:

$$A - BT_f = \dot{T}_f$$

and has the solution of form:

$$T_f = \frac{A}{B} - ce^{-Bt}$$

where c must satisfy boundary condition that

$$T_f = T_{f_o} \quad @\ t = 0$$

so $c = (\frac{A}{B} - T_{f_o})$, and $-c = (T_{f_o} - \frac{A}{B})$

g. Therefore:

$$T_f = (T_{f_o} - \frac{A}{B})\, e^{-Bt} + \frac{A}{B}$$

$$\frac{A}{B} = (\frac{P'}{AU} + T)$$

h. $T_f - \frac{A}{B} = T_{f_o} - \frac{P'}{AU} - T$

i. Changing coefficients for convenience of computing sheet

$$T_f = (T_{f_o} - \frac{A}{B})\, e^{-Bt} + \frac{A}{B}$$

becomes

$$T_{f_{i+1}} = (T_{f_i} - \frac{P_i}{AU_i} - T_i)\quad e^{-\Delta t_i/\tau_{f_i}} + \frac{P_i}{AU_i} + T_i$$

j. Becomes

$$T_{f_{i+1}} = (\frac{P_i}{AU_i} + T_i) - \left[\frac{P_i{}'}{AU_i} - (T_{f_i} - T_i)\right] e^{\Delta t_i/\tau_{f_i}}$$

$$= Y_i - \left[Y_i - T_{f_i}\right] e^{-\Delta t_i/\tau_{f_i}}$$

where $Y_i = (\frac{P_i{}'}{AU_i} + T_i)$

k. However $\frac{P'}{P_o{}'} = P$

so $P' = PP'_o = 9472\ P$

l. So $Y_i = \left(\frac{9472}{A} \frac{P_i}{U_i} + T_i \right)$

or calling $\frac{9472}{A} = \frac{9472}{611.1} = c_7$

$$Y_i = (c_7 \frac{P_i}{U_i} + T_i)$$

where $c_7 = 15.5$

m. The effective T_{f_o} is obtained as follows:

$$T_{f_o} = T_o + \Delta T_o$$

Total ΔT = ΔT through film + ΔT through scale.

and $\frac{Q_o}{A} = h\Delta T_1 = U_s \Delta T_2$

so $\Delta T_o = \frac{Q_o}{Ah} + \frac{Q_o}{AU_s}$

and $T_{f_o} = T_o + \frac{Q_o}{Ah} + \frac{Q_o}{AU_s}$

$$T_{f_o} = T_o + \frac{Q_o}{AU}$$

where $U = \frac{1}{\frac{1}{h} + c_8}$ $\qquad c_8 = \frac{1}{U_s}$

$$= \frac{1}{\frac{1}{h_o G_i} 0.8 + c_8}$$

n. This gives as a final result:

$$T_{f_{i+1}} = Y_i - \left[Y_i - T_{f_i} \right] e^{-\Delta t_i / \tau_{f_i}}$$

where $Y_i = (c_7 \dfrac{P_i}{U_i} + T_i)$

and $U_i = \dfrac{1}{\dfrac{1}{h_o G_i^{0.8}} + c_8}$

o. Numerically, values are as follows:

$T_o = 440.8^oF$

$P_o = 1$

$G_o = 1$

$h_o = \dfrac{2570}{3600} = 0.714$ Btu/sec - ^oF - ft^2

$U_s = \dfrac{1200}{3600} = \dfrac{1}{3} = \dfrac{1}{c_8}$ Btu/sec-^oF - ft^2

$c_7 = 15.5$

This permits us to obtain

$$U_i = \frac{1}{\dfrac{1}{0.714\ G_i^{0.8}} + 3}$$

$U_o = \dfrac{1}{4.4007} = 0.2272$ Btu/sec -ft^2 -^oF

$$Y_i = (15.5 \frac{P_i}{U_i} + T_i)$$

$$T_{f_o} = 440.8 + \frac{34.1x10^6}{(611.1)(2570)} + \frac{34.1x10^6}{(611.1)(1200)}$$

$$= 440.8 + 21.7 + 46.4$$

$$T_{f_o} = 508.9^oF$$

$$T_{f_i} = \frac{w_f c_f}{AU_i} = \frac{42}{AU_i}$$

$$\frac{1}{f_i} = \frac{AU_i}{w_f c_f} = \frac{611.1}{42} U_i = 14.55U_i$$

2. <u>Water Temperatures</u>

a. Rate of Heat to water = Rate of heat Carried away + Rate of heat going to raise average water temp.

b. $AU\ (T_f - T) = c_p\ GM_o\ (T_2 - T_1) + wc_p\dot{T}$

c. $(T_2 - T_1) = 2(T - T_1)$ (approximately)

d. $\frac{AU}{wc_p}\ (T_f - T) - \frac{2GM_o}{w}\ (T - T_1) = \dot{T}$

e. Or $\frac{1}{\tau_1}\ (T_f - T) - \frac{1}{\tau_2}\ (T - T_1) = \dot{T}$

where $\frac{1}{\tau_1} = \frac{AU}{wc_p}$, $\frac{1}{\tau_2} = \frac{2GM_o}{w}$

f. T_i is constant for the first 11 seconds

T_f average can be used

We assume that T varies slowly enough so that the exponential need not be used.

$$T_{i+1} = T_i + \frac{\Delta t_i}{\tau_{1_i}} (T_{f_{avg.}} - T_i) - \frac{t_i}{\Delta \tau_{2_i}} (T_i - T_1)$$

g. $\frac{1}{\tau_2} = \frac{2GM_o}{w}$

and since $G_o = 1$

$$\left(\frac{1}{\tau_2}\right)_i = \left(\frac{2M_o}{w}\right) G_i = \left(\frac{1}{\tau_2}\right)_o G_i$$

h. $\left(\frac{1}{\tau_1}\right)_i = \left(\frac{A}{wc_p}\right) U_i$

3. <u>Power and Reactivity Relationships</u>

a. For Prompt Neutrons:

i. Rate Prod. Neutrons − Rate Loss Neutrons = Rate Change Neutron Density

ii. But:

$$\text{Rate Loss Neutrons} = \frac{n}{\ell^*}$$

$n =$ Neutron Density

$\ell^* =$ Mean Lifetime

iii. And:

Rate Prod. = Rate Prod. Prompt + Rate Prod. from Delayed Neutron Precursors

(We neglect a source term. Any sources if present are negligible.)

iv. Where:

$$\text{Rate Prod. Prompt} = \left(\begin{matrix}\text{Rate}\\ \text{Loss}\end{matrix}\right)\left(\text{Ratio } \frac{\text{Neutrons Prod.}}{\text{Neutrons Loss}}\right)\left(\text{Ratio } \frac{\text{Neutrons Prod. Prompt}}{\text{Neutrons Prod.}}\right)$$

$$= \frac{n}{\ell^*}\,(K)\,\frac{1-\beta}{1},\quad K = K \text{ effective}$$

Rate Prod. Delayed $= \lambda C_1$ — C_1 = Density of Delayed Neutron Precursors; λ = Decay Constant

(We assume one effective group of delayed neutrons since incremental changes in multiplication are small)

v. Combining:

$$\frac{n}{\ell^*} K(1-\beta) + C_1\lambda - \frac{n}{\ell^*} = \dot{n}$$

vi. Rearranging

$$\frac{n}{\ell^*} \quad K(1-\beta)-1 + C_1\lambda = \dot{n}$$

vii. For first 1/10th second following a step change in K, a useful approximation is to take delayed neutron contribution constant. 1/10th second is a small fraction of half life of shortest half life delayed neutrons. Also, since change in K here is expected to be more of a ramp function, this assumption should be sufficiently accurate for even longer intervals. If we make this approximation, then $C_1\lambda$ is constant over interval and is equal to $\frac{n_o}{\ell^*}$ over first interval.

viii. A usual approximation is to take $K = 1 + \Delta K$, as is usually done in kinetic analyses.

then

$$K(1-\beta) = (1-\beta)(1+\Delta K)$$

$$= 1 + \Delta K - \beta - \beta\Delta K$$

and if ΔK is small, we can ignore $\beta\Delta K$, as is usually done in kinetic analyses.

ix. Then

$$\frac{n}{\ell^*} \quad (\Delta K - \beta) \quad + C_1\lambda = \quad \dot{n}$$

x. If $P = c_2 n$

and $D = c_2 \; C_1$

and if we multiply through by c_2,

we obtain,

$$\frac{c_2 n}{\ell^*} \quad (\Delta K \quad - \beta) + c_2 C_1 \lambda = \quad (c_2 \dot{n})$$

$$\frac{P}{\ell^*} \quad (\Delta K \quad - \beta) \quad + \quad D\lambda = \quad \dot{P}$$

Where, as before, $\Delta K \;\; = c_3 \;\; \Delta T$

xi. Over any sufficiently small interval of time, ΔT and D can be taken as constant and equations solved for P.

xii. Letting $\dfrac{c_3 \Delta T - \beta}{\ell^*} = c_4$ (A negative number)

and $D\lambda = c_5$

$$c_4 P + c_5 = \dot{P}$$

xiii. Solution is

$$P = \left(P_o + \frac{c_5}{c_4}\right) e^{c_4 t} - \frac{c_5}{c_4}$$

where the P_o is the value of P at the start of the interval.

xiv. Or as set up for time intervals,

$$P_{i+1} = \left[P_i + \left(\frac{c_5}{c_4}\right)i\right] e^{c_4 \Delta t_i} - \left(\frac{c_5}{c_4}\right)_i$$

xv. Now $\dfrac{c_5}{c_4}$ is negative and of the same general order of magnitude as P, so that $P_i + \left(\dfrac{c_5}{c_4}\right)_i$ is certainly smaller than P_i.

xvi. Since, for this problem, ΔT is always positive, c_4 must be less than $-\dfrac{\beta}{\ell^*}$ and

$$\frac{\beta}{\ell^*} = \frac{-75}{2} = -37.5$$

so that for Δt's as large as .1 sec (or $c_4\ \Delta t$ as large as 37.5)

$$\left[P_i + \left(\frac{c_5}{c_4}\right)_i\right] e^{c_4 \Delta t_1}$$ is negligibly small.

xvii. Actually, even for $\Delta t = .01$, the expression turns out to be very small, thus indicating that P follows ΔT very closely and in determining P at the end of the time interval, indicating in turn that it is value of ΔT at end of time interval which should be used. While it is really the average D over the period which should be used, D changes an order of magnitude slower than P, initially, so that this type of iteration is not needed.

$$P_{i+1} = -\frac{c_5}{c_4}$$

$$= +\frac{D_i \lambda \ell^*}{c_3 \Delta T_i + 1 - \beta}$$

xviii. Another way of looking at it is to consider that the power at the end of the period is equal to the delayed neutron contribution at beginning of period multiplied by two conversion ratios, one of which is the ratio.

$$\frac{\text{Precursors which become neutrons}}{\text{Neutrons which become precursors plus neutron imbalance}}$$

The other of which is ratio of lifetimes of neutrons and precursors or $\left(\frac{\ell^*}{1/\lambda}\right)$. This gives

$$D_i \left[\frac{1}{1+\Delta K-\beta}\right]\left[\frac{\ell^*}{(1/\lambda)}\right]$$

b. Delayed Neutrons and Precursors

i. Rate of Precursors Formed - Rate of Precursors Lost = Rate Change in No. of Precursors

or

$$\frac{n}{\ell^*}\beta K - C_1\lambda = \dot{C}_1$$

ii. Again if we set $K = 1 + \Delta K$

$$\frac{n}{\ell^*}\beta(1 + \Delta K) - C_1\lambda = \dot{C}_1$$

and if ΔK is small, $\beta\Delta K$ can be ignored, so

$$\frac{n}{\ell^*}\beta - C_1\lambda = \dot{C}_1$$

iii. Again multiplying through by c_2, where $P=c_2n$ and $D=c_2C_1$

$$\frac{P}{\ell^*}\beta - D\lambda = \dot{D}$$

iv. Variation in D is very slow and is controlled by P so that it is possible and easier to linearize.

v. $\therefore\ \Delta D = \left[P\left(\frac{\beta}{\ell^*}\right) - D\lambda\right]\Delta t$ (where $P = \frac{P_i+P_{i+1}}{2}$)

and $D_{i+1} = D_i + P(\beta/\ell^*) - D\lambda\ \ \Delta t_i$

4. Variation of Steam Generator Outlet Temperature

a. The inlet temperature (T_3) of the steam generator does not depart appreciably from 450°F until t=1.6 sec, at which time it has risen 0.16°F. This is the "slug" which left the reactor at t= 0.1 sec, and which will leave the steam generator at t= 11.7 sec. Therefore, we may use $T_3 = 450^\circ F$ for the portion of the transient flow period which is of primary interest.

b. The time-variation of outlet temperature (T_4) of the steam generator is determined as follows: (cf App C:5b)

$$\frac{1}{U} \quad \frac{dT}{T-T_s} = \frac{Adx}{Wc_pL}$$

c. Ignoring the variation of boiling resistance: (cf App. D, sec's 4,5)

$$\frac{10^4}{U} = 2.71G^{-.8} + 11.55$$

$$= 2.66t + 14.26$$

$$A = 836 \text{ ft}^2, \; c_p = 1.115 \frac{\text{Btu}}{\text{lb-}^\circ F}, \; L = 13.0 \text{ ft.}$$

$$W = W_oG = 1.66 \times 10^6 \; G$$

d. Letting $dx = v \, dt = v_oGdt$
(where v_o = 11.65 ft/sec in evaporator tubes)

e. $(2.66t + 14.26) \frac{dT}{T-T_s} = \frac{-836 \times 11.65\ G}{1.115 \times 13.0 \times 1.66 \times 10^6 G}\ dt.$

f. First we integrate over a time range during which the "slug" leaving the steam generator had entered before the time of pump failure, and had a temperature T_o at $t = 0$ (from curve of App.C).

g. $$\int_{T_o}^{T_4} \frac{dT}{T-T_s} = -1.525 \int_o^{t_4} \frac{dt}{t + 5.36} (0<t_4<2.64 \text{ sec})$$

$$\ln \frac{T_o - T_s}{T_4 - T_s} = 1.525 \ln \left(\frac{t_4 + 5.36}{5.36}\right)$$

$$\frac{T_o - T_s}{T_4 - T_s} = \left(\frac{t_4 + 5.36}{5.36}\right)^{1.525} \quad (0<t_4<2.64 \text{ sec})$$

h. For "slugs" entering the steam generator after pump failure (at times $0<t_3<1.6$ sec), the initial temperature to be used in evaluating the definite integral will be $T_3 = 450^oF$.

i. $$\int_{T_3}^{T_4} \frac{dT}{T-T_s} = -1.525 \int_{t_3}^{t_4} \frac{dt}{t+5.36} \quad (0<t_3<1.6)$$

$$\frac{T_3 - T_s}{T_4 - T_s} = \left[\frac{t_4 + 5.36}{t_3 + 5.36}\right]^{1.525} \quad (0<t_3<1.6)$$

where t_4, the exit time, is determined (cf App G) for each entry time t_3 by

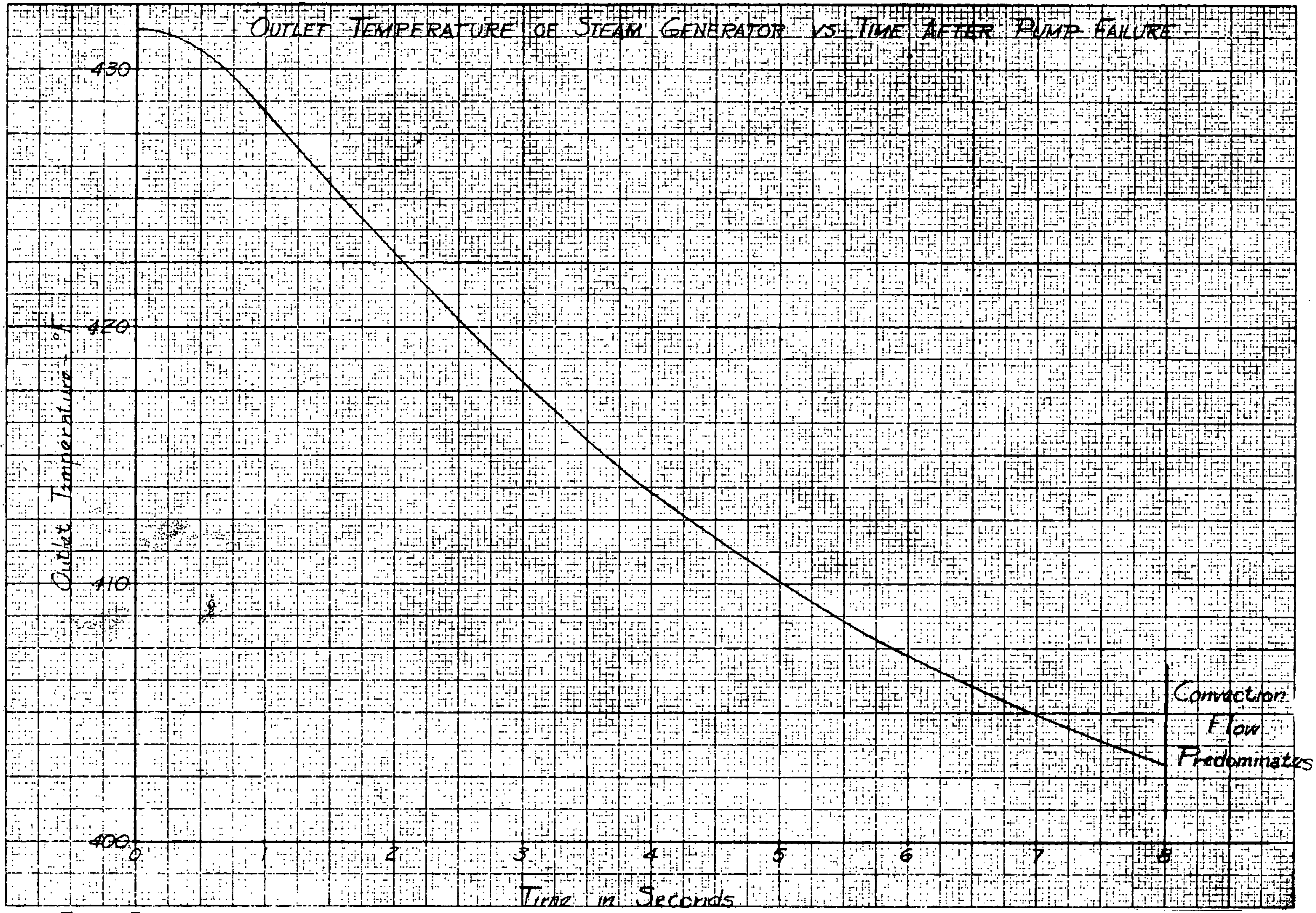
$$t_4 = \left[\frac{1.02 + t_3}{1 - .272\,(1.02 + t_3)^{.25}}\right]^4 - 1.02$$

The following table lists some of the values for the outlet steam generator temperature as a function of the time following the pump failure. The results are plotted on Curve Sheet 10.

Enter t_3	Leave t_4	Initial T_o	T_o-T_s	T_4-T_s	T_4
---	0.00	431.6	49.8	49.8	431.6
---	0.80	439.9	58.1	47.0	428.8
---	1.60	445.1	63.3	42.5	424.3
0.00	2.64	450.0	68.2	37.0	418.8
0.20	3.65	450.0	68.2	32.7	414.5
0.40	4.76	450.0	68.2	28.9	410.7
0.60	6.01	450.0	68.2	25.5	407.3
0.80	7.27	450.0	68.2	22.8	404.6

5. Inlet Water Temperature to the Reactor

Since the water flow rate is dropping initially, the steam generator outlet temperatures will be spread over a much longer time interval than is used in the above calculation. At the end of eight seconds,



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SHEET No. 10.

convection becomes the driving force and flow becomes almost constant (G = 0.0666).

b. Distance from steam generator to reactor = 24.2 ft.

c. $$s = \frac{12.6\ (1.0261}{0.25} \left[\frac{1}{(a+t_1)^{0.25}} - \frac{1}{(a+t_2)^{0.25}} \right]$$

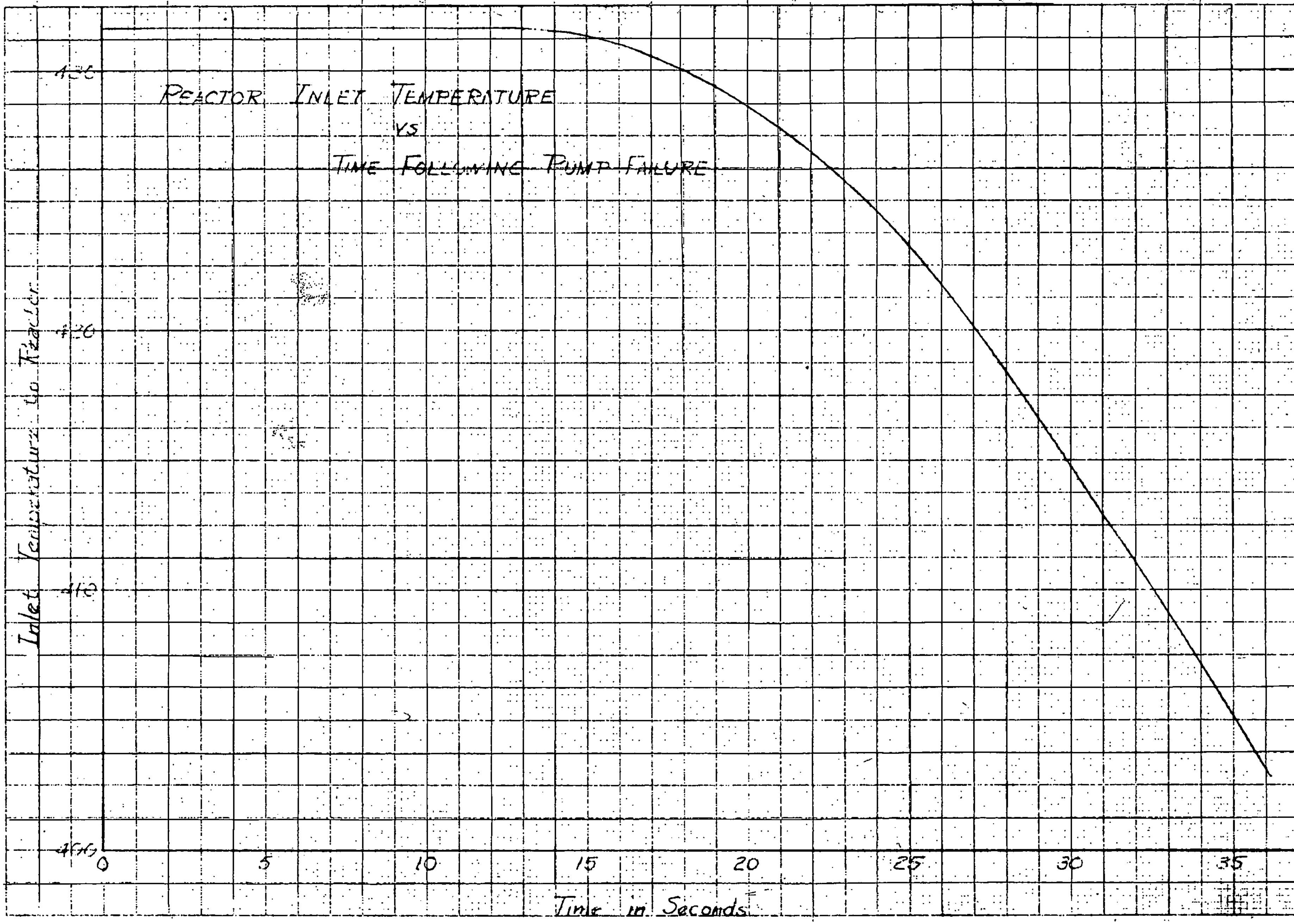
$$= 51.6 \left[\frac{1}{(a+t_1)^{0.25}} - 0.577 \right] = 51.6 \left[\frac{1}{(a+t)^{0.25}} - 0.577 \right]$$

d. After 8 sec, v = 0.846 ft/sec

e. Total time to reactor $= 8 + \frac{24.2-s}{0.846}$

t_1	$a+t_1$	$\frac{1}{(a+t_1)^{0.25}}$	$\frac{1}{(a+t_1)^{0.25}}-0.577$	s	24.2-s	t_c	t_T
0	1.02	0.995	0.418	21.55	2.65	3.09	11.1
0.4	1.42	0.916	0.339	17.48	6.72	7.84	15.8
0.8	1.82	0.861	0.284	14.65	9.55	11.16	19.2
1.2	2.22	0.819	0.242	12.48	11.72	13.68	21.7
1.6	2.62	0.786	0.209	10.78	13.42	15.67	23.4
2.0	3.02	0.759	0.182	9.39	14.81	17.29	25.3
2.4	3.42	0.735	0.158	8.15	16.05	18.73	26.7
2.6	3.62	0.725	0.148	7.63	16.57	19.34	27.4
3.14	4.16	0.700	0.123	6.34	17.86	20.85	28.9
3.65	4.67	0.680	0.103	5.31	18.89	22.05	30.1
4.20	5.22	0.661	0.084	4.33	19.87	23.18	31.2
4.76	5.76	0.645	0.068	3.51	20.69	24.14	32.2
5.39	6.41	0.628	0.051	2.63	21.57	25.2	33.2
6.01	7.03	0.610	0.033	1.70	22.50	26.3	34.3
6.65	7.67	0.601	0.024	1.24	22.96	26.8	34.8
7.27	8.29	0.589	0.012	0.62	23.58	27.5	35.5
7.93	8.95	0.578	0.001	0.05	24.14	28.2	36.2

The results of this table are plotted on Curve Sheet 11.



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SHEET NO. 11

6. Constants for Numerical Integration

CORE:

$A = 611.1\ ft^2$

$c_p = 1.115$ Btu/lb - ^{o}F

$D_o = 4688$

$F_o = 4000$ GPM

$G_o = \frac{F_o}{4000} = 1$

$M_o = 4.611 \times 10^2$ lb/sec

$\ell^* = 0.2 \times 10^{-4}$ sec

$P_o = 1$

$P' = 9472$ Btu/sec $= c_6 P_o$

$T_o = 440\ 8^{o}F$

$T_{o_1} = 431.6^{o}F$

$T_{o_2} = 450.^{o}F$

$\Delta T_o = \frac{P'}{AU} = \frac{9472}{611.1\ (0.2272)} = 68.2^{o}F$

$T_{f_o} = T_o + \Delta T_o = 509^{o}F$

$U_o = 0.2272$ Btu/sec - ft^2-^{o}F

$w = 189.8$ lb

$w_f c_f = 42$ Btu/^{o}F

$$\beta = 75 \times 10^{-4}$$

$$\lambda = 0.8 \ \text{sec}^{-1}$$

$$c_3 = -2 \times 10^{-4} \ (^{o}F)^{-1}$$

$$c_4 = \frac{c_3 \Delta T - \beta}{\ell^*}$$

$$(c_4)_o = -375 \ \text{sec}^{-1}$$

$$c_5 = D \lambda$$

$$(c_5)_o = 375 \ \text{sec}^{-1}$$

$$c_6 = 9472 \ \text{Btu/sec}$$

$$c_7 = 15.5 \ \text{Btu/sec - ft}^2$$

$$A/wc_p = 2.89 \ \frac{\text{ft}^2 - {}^{o}F}{\text{Btu}}$$

$$A/w_f c_f = 14.56 \ \frac{\text{ft}^2 - {}^{o}F}{\text{Btu}}$$

$$(1/\tau_f)_o = \frac{AU}{w_f c_f} = 3.306 \ \text{sec}^{-1}$$

$$(1/\tau_1)_o = \frac{AU_o}{wc_p} = 0.6565 \ \text{sec}^{-1}$$

$$(1/\tau_2)_o = \frac{2G_o M_o}{w} = 4.869 \ \text{sec}^{-1}$$

APPENDIX G - TRANSPORT LAGS

Contents | Page

1. Pipe Lengths

Listed below are approximations of actual line lengths from Alco drawing AEL - 16. Data were taken from the plan view with corrections for changes in the elevation of the lines.

a. Steam Generator to Y Joint

i. For pump closer to reactor:

From steam generator to C. L. of pump	25.6	in.
From C. L. of pump to Y joint	46.5	
	72.1	in=6.01ft

ii. For pump farther from reactor:

From steam generator outlet to C. L. of pump	32	in.
From C. L. of pump to Y joint	96.3	
	128.3	in=10.69ft

iii. Elevation corrections:

One 45^{o} bend	15.5	in.
One riser to pump	38.0	
	53.5	in=4.46ft

b. Y Joint to Reactor Inlet

From Y joint around bend	29.7	in
Along straight section to next bend	92.0	
Approximate distance around bend	28.3	
Straight section to reactor inlet	14.4	
	164.4	in=13.70ft

c. Reactor Outlet to Steam Generator

From reactor outlet to bend	20.5 in
Around bend	28.3
From bend to steam generator inlet	65.6
	114.4 in =9.53 ft.

Minimum total pipe length = 6.01 + 4.46 + 13.70 + 9.53 = 33.70 ft.

Maximum total pipe length = 38.4 ft.

2. Flow vs. Time

a. Use the equation derived in appendix E:

$$t = 1.02 \left[(4000/F)^{.8} - 1 \right] \text{ sec}$$

Eliminate F in this equation by using the relationship

$v = 12.6 \ (F/4000)$ ft/sec (for piping)

and

$v = 11.65 \ (F/4000)$ ft/sec (within evaporator tubes)

For the former case, we obtain

$$v(t) = \frac{12.6\ (1.02)^{1.25}}{(1.02 + t)^{1.25}}$$

$$v(t) = \frac{12.9}{(1.02 + t)^{1.25}}$$

$$v(t) = \frac{A}{(B + t)^{k}}$$

where A = 12.9 for piping; 11.94 for steam gen;

4.4 for reactor

B = 1.02

k = 1.25

b. The distance traveled by a "slug" of fluid between any times t_1 and t_2 (measured from instant of pump failure) will be

$$s_{t_1,t_2} = \int_{t_1}^{t_2} \frac{A}{(B+t)^k}\,dt = -\frac{A}{k-1}\left[\frac{1}{(B+t_2)^{k-1}} - \frac{1}{(B+t_1)^{k-1}}\right]$$

$$s_{t_1,t_2} = \frac{A}{k-1}\left[\frac{1}{(B+t_1)^{k-1}} - \frac{1}{(B+t_2)^{k-1}}\right]$$

$$= \frac{12.9}{0.25}\left[\frac{1}{(1.02+t_1)^{.25}} - \frac{1}{(1.02+t_2)^{.25}}\right]$$

$(t_2 < 8 \text{ sec})$

c. Starting at a time t_1, the time t_2 by which an additional distance, s, will have been traveled is

$$t_2 = \frac{B + t_1}{\left[1 - \frac{s(k-1)}{A}(B+t_1)^{k-1}\right]^{1/(k-1)}} - B$$

$$\frac{1.02 + t_1}{\left[1 - \frac{.25s}{12.9}(1.02 + t_1)^{.25}\right]^4} - 1.02$$

These expressions should be used only up to $t_2 = 8$ sec.;

3. <u>Transport Lags for Particular Conditions</u>

a. The total pipe length from the steam generator outlet to the reactor inlet (through the nearer pump) is 24.2 ft. Taking $t_1 = 0$ and $t_2 = 8$ sec, the distance a slug would travel in 8 sec is

$$s_{0,8} = \frac{12.9}{0.25}\left[\frac{1}{(1.02)^{.25}} - \frac{1}{(9.02)^{.25}}\right]$$

$$= 51.6\ (.995 - .577) = \underline{21.55}\ \text{ft.}$$

Considering $v = 12.6 \times 0.067 = 0.846$/sec for the remaining 2.65 ft., the total time for this slug to travel from the steam generator to the reactor will be

$$t = 8.0 + \frac{2.65}{.846} = \underline{\underline{11.14}}\ \text{sec.}$$

b. A similar calculation for a slug leaving the steam generator at $t_1 = 2.6$ sec (temp. $t_4 = 419^\circ$F) gives

$$s_{2.6,8.0} = \frac{12.9}{0.25}\left[\frac{1}{(3.62)^{.25}} - \frac{1}{(9.02)^{.25}}\right]$$

$$= 51.6\ (.725 - .576) = 7.68\ \text{ft.}$$

Assuming free-convection flow at 0.846 ft/sec for the remaining 16.5 ft., the total time from the instant of pump failure to reach the reactor is $t_2 = 8.0 + \frac{16.5}{0.846}$ $= 27.5$ sec

c. Flow distance from reactor outlet to steam generator inlet is 9.53 ft. A slug leaving the reactor at $t_1 = 0$ would leave the steam generator at

$$t_2 = \frac{1.02}{\left[1 - \frac{.25 \times 9.53}{12.9}(1.02)^{.25}\right]^4} - 1.02$$

$$= \frac{1.02}{(1 - .186)^4} - 1.02$$

$$t_2 = \underline{\underline{1.30 \text{ sec}}}$$

d. Taking $t_1 = 0.2$ sec (by which time reactor outlet temperature has increased $1.2^\circ F$), the time t_2 (from pump failure) for this hotter "slug" to reach the steam generator will be

$$t_2 = \frac{1.02 + 0.20}{\left[1 - \frac{.25 \times 9.53}{12.9}(1.22)^{.25}\right]^4} - 1.02$$

$$= \frac{1.22}{(1 - 0.194)^4} - 1.02$$

APPENDIX H - MAXIMUM TEMPERATURES

Contents | Page

1. <u>General</u>

In setting up the constants for the analysis for the transient conditions of the pump failure problem, the figure for the axial maximum-to-average heat flux was taken directly from the Hazards Summary Report of October 1955 to be 1.31:1. This value would indicate that the axial flux distribution is that of a chopped cosine. Consequently, all the constants for the transient relationships were set up using the entire fuel core length for the heated length of each plate. Recent information now indicates that the value listed in the Hazards Report was much too low. Instead of a chopped cosine flux distribution, as was assumed, the flux distribution is instead closer to a cosine function plus a zero function on the upper end. It thus appears that an error was introduced into the numerical integration as something less than the actual fuel length should have been used. However, in view of the time and effort that would be required to make this correction, it does not appear feasible to do so, especially since the corrections are probably of second order.

In calculating maximum plate temperatures, however, the value for maximum-to-average flux becomes extremely important. It was decided, therefore, to use the following assumptions to be consistent with the transient calculations:

a. The axial power (flux) distribution will be taken as a pure cosine function (maximum/average = 1.57:1)

b. The radial maximum-to-average power distribution will be taken to be about 1.8:1.

These assumptions should be conservative as they should yield higher temperatures than taking a higher value for the axial distribution. By raising the radial maximum-to-average ratio, one assumes that more power is generated in the hottest or central channel than is the case. Also, since there is some net steam generation near the outlet of some of the central fuel elements, the calculated temperatures will be high in this region since we have assumed that there is power production in that area where actually a zero power function will exist.

On the other hand, these calculations do not take into consideration the possible reduction in flow area due to a combination of poor fuel-element tolerances, nor does it consider the volume heating effect in the water film; however, these phenomena should not appreciably alter the results. Also there will be some decrease in flow through the central fuel elements due to the formation of steam and the corresponding increase in friction accompanying two-phase flow. This effect will be somewhat compensated by an increase in convection flow through these channels and in any case, should introduce relatively little error into the calculations, as the mixture leaving the hottest channel has a steam quality of less than ten per cent. Moreover, the introduction of steam voids in this portion of the reactor will inject a considerable amount of negative reactivity, which will cause a sharp decrease in power and alleviate the situation.

In no case is it conceivable in the analysis of the pump failure, that any cooling surface will enter the region of film boiling where burnout is likely to occur. Since the reactor will operate in the region of nucleate boiling following pump failure, the temperature at the wetted surface is not expected to exceed 600^{o}F.

2. Maximum Metal Temperatures

The problem of determining the location and magnitude of the maximum metal temperatures during the transient period before the new quasi steady state defies straightforward solution because there are too many variables. Rather than attempt an exact analytical solution of the problem it was decided to use a different method of attack which should give conservative values.

The method is to take some time of exit, t_2, from the reactor, calculate the time of entry, t_1, from the inertia and convective flow relationships; average the normalized flow, G_{av}, over the period along with a linear average of the normalized power, P_{av}; and assuming steady-state flow conditions obtain the temperature profile of the hottest channel. The temperatures thus obtained should be somewhat higher than the actual case because the linear average of the power is higher than the true average over the same time interval. The average outlet coolant temperature was checked at each interval and every case was found to be higher than that obtained from the numerical integration.

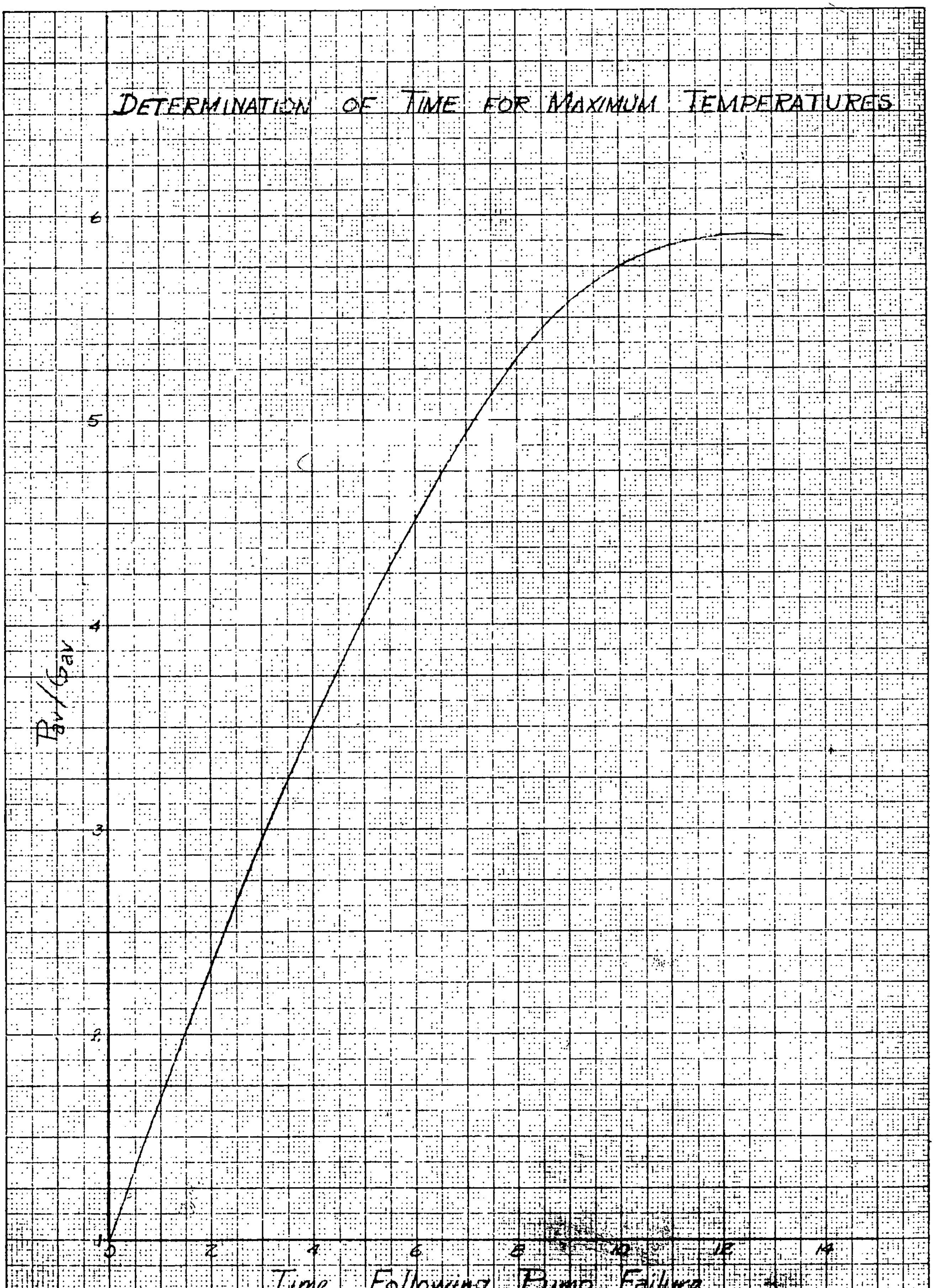
The next problem was to determine at what time and under what conditions during the transient period was the maximum temperature obtained. Since the worst conditions will prevail approximately during that period where the ratio of power to coolant flow is a maximum, this ratio was calculated and is plotted in figure 12 on the following page. As can be seen from the figure, the maximum value of 5.9 for P/G was reached at an exit time of 12 seconds following the pump failure. The normalized flow and power conditions at this point were

$$P_{av} = 0.4135$$

$$G_{av} = 0.0701$$

and these values were used in obtaining the temperature profiles of the central coolant channel. The data used for this graph are tabulated in the table below.

Exit Time	Entry Time	P_{av}	G_{av}	P_{av}/G_{av}
2	0.78	0.821	0.361	2.28
3	1.30	.7425	.251	2.96
4	1.80	.6795	.1938	3.445
6	2.70	.586	.1293	4.53
8	3.62	.515	.0973	5.29
9	4.08	.4845	.0866	5.59
10	4.62	.458	.0796	5.75
11	5.25	.4345	.0742	5.86
12	5.92	.4135	.0701	5.90
13	6.69	.398	.0676	5.89



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SHEET No. 12

3. Outlet Conditions From The Hottest Channel

For the highest surface temperatures, outlet time = 12 seconds.

$P_{av} = 0.4135 \qquad G_{av} = 0.0701$

$$q_c = \int_0^L Q_{o_{max}} P_{av} A_c \sin \frac{\pi x}{L} dx = \frac{2}{\pi} Q_{o_{max}} P_{av} V_c$$

$$Q_{o_{max}} = \frac{34.13 \times 10^6 (1728)}{(22)(2.5)(0.02)(800)} 2.845 = 1.907 \times 10^8 \text{ Btu/hr-ft}^3$$

$$q_c = \frac{2}{\pi} (1.907 \times 10^8) (0.4135) (\frac{1.1}{1728}) = 31{,}940 \text{ Btu/hr}$$

$$W_c = \frac{W_o G_{av}}{800} = \frac{1.66 \times 10^6 (0.0701)}{800} = 145 \text{ lb/hr}$$

$$c_p = 1.184 \text{ Btu/lb-}^oF$$

$$q_c = W_c c_p \Delta T + W_c \Delta H$$

$$c_p \Delta T + \Delta H = \frac{31940}{145} = 220.5 \text{ Btu/lb}$$

$$c_p \Delta T = 1.184 (567.2 - 431.6) = 160.5 \text{ Btu/lb}$$

$$\Delta H = 220.5 - 160.5 = 60 \text{ Btu/lb}$$

$$\text{STEAM QUALITY} = \frac{60}{611.7} = 9.8\%$$

$$q/A = \left(\frac{q}{A}\right)_o \sin \frac{\pi x}{L}$$

$$\left(\frac{q}{A}\right)_o = \frac{3.195 \times 10^4 (1.57)}{5\ (22)} 144 = 6.56 \times 10^4 \text{ Btu/hr-ft}^2$$

Because of the very low quality of the steam generated in the channel there appears to be no danger of going into film boiling. The reactor Handbook, Volume II, states that burnout experiments demonstrated that "burnout does not occur until very high steam qualities have been obtained."

4. Temperature Analysis of Central Flow Channel at Time of Maximum Fuel Plate Temperatures (12 Seconds)

$$T_4 - T_o = \frac{Q_o V}{\pi W c_p} \left[1 - \cos \frac{\pi x}{L} \right]$$

$$T_s - T_o = \frac{Q_o V}{\pi W c_p} \left[1 - \cos \frac{\pi x}{L} \right] + \frac{Q_o V}{h\ A_h} \sin \frac{\pi x}{L}$$

$$T_{sat} - T_o = 567.2 - 431.6 = 135.6$$

$$G_{av} = 0.701, \quad P_{av} = 0.4135$$

$$h = 2570\ (0.0701)^{0.8} = 2570\ (0.1195) = 307\ \text{Btu/hr-}^{o}\text{F-ft}^2$$

$$\frac{Q_o V}{\pi W c_p} = \frac{1.907 \times 10^8\ (0.509)\ (0.4135)}{\pi\ (1.66 \times 10^6)\ (0.0701)\ (1.115)} = 98.4$$

$$\frac{Q_o V}{h A_h} = \frac{1.907 \times 10^8\ (0.509)\ (0.4135)}{307\ (611.1)} = 214$$

x	$180\frac{x}{L}$	$\sin\frac{\pi x}{L}$	$\cos\frac{\pi x}{L}$	$214\sin\frac{\pi x}{L}$	$(1-\cos\frac{\pi x}{L})$	$98.4(1-\cos\frac{\pi x}{L})$	T_s-T_o	T_s
0.5	4.09	0.0713	0.99744	15.3	0.00256	0.25	15.6	447.2
1.0	8.18	0.142	0.9898	30.4	0.0102	1.0	31.4	463.0
1.5	12.27	0.2125	0.9772	45.5	0.0228	2.2	47.7	479.3
2.0	16.36	0.282	0.9595	60.4	0.0405	4.0	64.4	496.0
2.5	20.45	0.3495	0.9371	74.8	0.0629	6.2	81.0	512.6
3.0	24.54	0.4154	0.9097	88.9	0.0903	8.9	97.8	529.4
3.5	28.63	0.4792	0.8777	102.6	0.1223	12.0	114.6	546.2
4.0	32.7	0.5402	0.8416	115.7	0.1584	15.6	131.3	562.9
4.13	33.8	0.5562	0.8310	119.0	0.1690	16.6	135.6	567.2
							T_f	
6.0	49.1		0.6545		0.3455	34.0	465.6	
8.0	65.5		0.4148		0.5852	57.6	489.2	
10.0	81.8		0.1423		0.8577	34.4	516.0	
12.0	98.2		-0.1423		1.1423	112.5	544.1	
13.7	112.1		-0.3765		1.3765	135.3	566.9	

From the curve for boiling heat transfer coefficients from vertical tubes on page 699 of Glasstone's <u>Principles of Nuclear Reactor Engineering</u>.

$$h_b = c(\Delta T_b)^n \qquad \begin{array}{llll} h & = 50 & \Delta T_b & = 4.6 \\ h & = 5000, & \Delta T_b & = 42.0 \\ h & = 10, & \Delta T_b & = 2.1 \end{array}$$

$$\frac{5000}{50} = \frac{c(42)^n}{c(4.6)^n} = 100 = (9.13)^n$$

$$n = 2.08$$

$$c = 50/(4.6)^{2.08} = 50/23.9 = 2.09$$

$$h_b = 2.09\ (\Delta T_b)^{2.08}$$

check:

$$\Delta T = 2.1$$

$$h = 2.09\ (4.68) = 9.78 (\text{OK})$$

$$q/A = (\frac{q}{A})_o \sin\frac{\pi x}{L} = 65600 \sin\frac{\pi x}{L} = h_b \Delta T_b = 2.09(\Delta T_b)^{3.08}$$

$$\Delta T_b = (\frac{65600}{2.09} \sin\frac{\pi x}{L})^{1/3.08} = (31400 \sin\frac{\pi x}{L})^{0.325}$$

x	$180 \frac{x}{L}$	$\sin \frac{\pi x}{L}$	$31400 \sin \frac{\pi x}{L}$	$(31400 \sin \frac{\pi x}{L})^{0.325}$	T_s
14	180-65.5	0.91	28600	27.6	594.8
16	180-49.1	0.756	23740	26.4	593.6
17	180-40.8	0.632	19850	24.8	592.0
18	180-32.7	0.5403	16970	23.6	590.8
19	180-24.5	0.4148	13030	21.7	588.9
20	180-16.3	0.281	8820	19.1	586.3
21	180-8.1	0.1408	4420	15.3	582.5
21.5	180-4.1	0.0715	2250	12.3	579.5
21.8	180-1.7	0.0299	939	9.2	576.4
21.9	180-0.8	0.0138	343.5	6.6	573.6

A plot of the temperature profiles in the central or hottest channel at the time of maximum temperature is shown on Curve Sheet 13.

5. Maximum Center Plate Temperatures

For the calculation of the center plate temperatures, the following data and assumptions were used:

(1) All conductive heat flow is perpendicular to the plate surface.

(2) Thermal conductivity of the stainless steel (type 304) will be taken as 8 Btu/hr ft^{o}F.

(3) The thermal conductivity of the fertile material will be taken as 90% of the value for stainless steel since it is composed of a mixture of finely divided UO_2 particles suspended in a stainless steel matrix.

(4) There is no film drop at the clad-meat interface.

(5) It will be assumed that the maximum plate temperature occurs at the same position of the maximum surface temperature.

The maximum center plate temperature of a flat fuel plate is discussed and the various equations are derived for this temperature in Chapter 11 of Glasstone's Principles of Nuclear Reactor Engineering. For our case, the following equation can be used:

$$T_o - T_{s_{max}} = Qa \left[\frac{a}{2k_f} + \frac{b-a}{k_{ss}} + \frac{1}{h_s} \right]$$

where

a = One-half the thickness of the fuel portion of the plate = 0.01" = 0.000833'

b = One-half the thickness of the plate = 0.015" = 0.00125'

h_s = Scale heat transfer coefficient = 1200 Btu/hr-^oF-ft^2

k_f = Thermal conductivity of fuel = 7.2 Btu/hr-^oF-ft

k_{ss} = Thermal conductivity of clad = 8 Btu/hr-^oF-ft

Q = Volume heating - Btu/hr-ft^3

T_o = Center temperature

$T_{s_{max}}$ = Maximum surface temperature

a. No-Scale Case (h_s = o)

Maximum surface temperature $\cong$ 596 ^oF = $T_{s_{max}}$

$$\left(\frac{q}{A}\right)_{max} = 6.56 \times 10^4 \quad \text{Btu/hr-ft}^2$$

$$q = QAa, \qquad Q = \frac{6.56 \times 10^4}{0.0008333} = 7.87 \times 10^7 \text{Btu/hr-ft}^3$$

$$T_o - T_{s_{max}} = Qa \left[\frac{a}{2k_f} + \frac{b-a}{k_{ss}}\right]$$

$$T_o - T_{s_{max}} = 7.87 \times 10^7 \left(\frac{10^{-2}}{12}\right)\left[\frac{10^{-2}}{14.4\ (12)} + \frac{10^{-2}}{24(8)}\right]$$

$$= 6.56 \times 10^4 \left[6.13 \times 10^{-5} + 5.21 \times 10^{-5}\right] = 6.56 \times$$

$$10^4 \left[11.34 \times 10^{-5}\right] = 7.44^{\circ}F$$

$$T_o = 7.44 + 596 = 603^{\circ}F$$

b. With Scale

$$T_o - T_{s_{max}} = Qa \left[\frac{a}{2k_f} + \frac{b-a}{k_{ss}} + \frac{1}{h_s}\right]$$

$$= 6.56 \times 10^4 \quad 11.34 \times 10^{-5} + 83.33 \times 10^{-5}$$

$$= 62F^{\circ}$$

$$T_o = 596 + 62 = 658^{\circ}F$$

APPENDIX I - THE EFFECT OF TEMPERATURE ON PRESSURE DROP IN THE CORE

1. Under the assumed operating conditions, water at 431.6°F enters the core and is heated to a bulk temperature of 450°F before leaving the core. Because the power distribution is not uniform, the water exit temperature will vary between different fuel elements. These temperature differences will introduce small differences in flow between elements but will not appreciably affect overall pressure drop values.

2. The average total pressure drop through the core is the sum of individual pressure drops tabulated below:

a. Orifice, $$\Delta P_0 = K_o \frac{W_1^2}{\rho_o}$$

b. Entrance, $$\Delta P_E = K_E \frac{W_1^2}{\rho_E}$$

c. Plate Friction, $$\Delta P_P = \frac{W_2^{1.8} \mu^{0.2}}{\rho_P}$$

d. Exit, $$\Delta P_x = K_x \frac{W_1^2}{\rho_x}$$

In the above equations ΔP may be either psi or psf depending on K; W_1 is the mass flow through one fuel element, W_2 is the mass flow through one channel in a fuel element, ρ is the average density of the coolant, and K is a lumped constant.

3. Initial pressure drops (tabulated below) are from APPR-1 data supplied by Alco. It was assumed the exit loss was 1/2 the entrance loss.

		T ^{o}F	ρ lb/ft^3	h ft	ΔP lb/ft^2
a.	Orifice	431.6	52.6	0.4	21.04
b.	Entrance	431.6	52.6	2/3 x 0.7	24.55
c.	Plate	440.8	52.2	0.6	31.32
d.	Exit	450.	51.75	1/3 x 0.7	12.08
					88.99

4. For the case of constant flow and constant inlet temperature an increased rate of heat production will increase the exit and average temperatures. Difference factors for increased heat production appear in plate and exit terms only. Assuming average and exit temperatures to rise $10^{o}F$ and $20^{o}F$, respectively.

		T	ΔP_o	$f(\rho)$	$f(\mu)$	ΔP_1
a.	Orifice	431.6	21.04	1		21.04
b.	Entrance	431.6	24.55	1		24.55
c.	Plate	450.8	31.32	$\left(\frac{52.20}{51.72}\right)$	$\left(\frac{.290}{.297}\right)^{0.2}$	31.48

		T	ΔP_o	$f(\rho)$	$f(\mu)$	ΔP_1
d.	Exit	470.	12.08	$(\frac{51.75}{50.81})$	$(\frac{.276}{.291})^{0.2}$	12.18
	Total					89.25

The difference per degree change in average reactor temperature is then:

$$\frac{89.25 - 88.99}{88.99 \times 10} = 2.9 \times 10^{-4}/^{o}F$$

5. For the case of constant heat production with a change in bulk coolant temperature, all factors will be affected:

		T	ΔP_o	$f(\rho)$	$f(\mu)$	ΔP_2
a.	Orifice	441.6	21.04	$(\frac{52.6}{52.17})$	$(\frac{.297}{.304})^{0.2}$	21.12
b.	Entrance	441.6	24.55	$(\frac{52.6}{52.17})$	$(\frac{.297}{.304})^{0.2}$	24.65
c.	Plate	450.8	31.32	$(\frac{52.2}{51.72})$	$(\frac{.290}{.297})^{0.2}$	31.48
d.	Exit	460.	12.08	$(\frac{51.75}{51.3})$	$(\frac{.283}{.291})^{0.2}$	12.13
	Total					89.38

The difference per degree change in average reactor temperature is then:

$$\frac{89.38 - 88.99}{88.99 \times 10} = 4.4 \times 10^{-4}/^{o}F$$

6. From the above it can be concluded that the effect of temperature on pressure drop can be neglected.

APPENDIX J - INSTANTANEOUS ZERO FLOW-STEADY POWER RELATIONSHIPS

Contents

JA. TEMPERATURE DERIVATIONS

1. No Heat Out

a. Rate of Heat Produced = Rate of Heat to Water + Rate of Heat to Raise Plate Temp.

b. For this case, rate to water = 0

c. $P = 0 + w_f c_f \left(\frac{dT_f}{dt}\right)_{avg}$

d. $\frac{P}{w_f c_f} \int_0^t dt = \int_{T_{f_o}}^{T_f} dT_{f_{avg}}$

e. However, since we are not interested in average but maximum, we need a factor of 2.8.

f. $\frac{2.8P}{w_f c_f} = \frac{\Delta T_f}{\Delta t}$

2. <u>Water Expelled and Not Returned</u>, T_f

a. Rate of Heat Produced = Rate of Heat to Water + Rate of Heat to Plate

b. For <u>Max</u> rate of change of T_f

$$2.8P = Ah\,(T_f - T) + w_f c_f \frac{dT_f}{dt}$$

where

T_f = Max T fuel ** at point of maximum power production

T = Bulk Water Temperature

h = Heat transfer coefficient (no flow)

A = Total heat transfer area

w_f = Total weight of fueled parts of fuel plates

c_f = Specific heat of fuel

t = Time

c. Also

Rate of Heat to Water = Rate of Heat to raise temp. of water or boil water

d. a. $Ah(T_f - T) = wc_p \dfrac{dT}{dt}$ $\quad T < T_{sat.}$
(w Constant)

b. $Ah_b\,(T_f - T) = c_b \dfrac{dw}{dt}$, where $T = T_{sat}$

**Temperature rise in fuel element is small compared to film drop. Decrease in h from operating conditions will flatten distribution in element still more. Therefore average T_f was taken equal to max T_f.

h_b = Boiling heat transfer coefficient, no forced circulation

h = Heat transfer coefficient no forced circulation, differing if

$T_f > T_{sat}$ and

$T < T_{sat}$

c_b = Heat of vaporization

dw/dt = Steam rate

e. Rearranging da slightly it becomes

$$(T_f - T) = \frac{wc_p}{Ah} \frac{dT}{dt}$$

$$= \tau_2 \frac{dT}{dt}, \text{ where } \frac{wc_p}{Ah} = \tau_2$$

$$T_f = T + \tau_2 \frac{dT}{dt}$$

f. db becomes, $(T_f - T_{sat}) = \dfrac{c_b}{Ah_b} \dfrac{dw}{dt}$

g. b becomes, $(T_f - T) = \dfrac{2.8P}{Ah} - \dfrac{w_f c_f}{Ah} \dfrac{dT_f}{dt}$

$$= D - \tau_1 \frac{dT_f}{dt}$$

$$T = T_f + \tau_1 \frac{dT_f}{dt} - D$$

Note: The scale resistance (1/1200) has been neglected in comparison with the static film resistance (1/50); also the distinction between T_f near the wall and actual bulk temperature (at zero flow) has been eliminated. The latter distinction will have vanishing significance as boiling proceeds.

h. Laplace-Transform Notation

F(t)		G(s)
T_f	Max Fuel Temp	G_f
T	Temp water, bulk, at max plate temp location	G
D	Constant	(D/S)
$\frac{dT}{dt}$	Derivative of water temp.	$sG-T_o$
$\frac{dT_f}{dt}$	Derivative of Fuel Temp.	$sG_f-T_{f_o}$
τ_1	Coefficient	τ_1
τ_2	Coefficient	τ_2

i. da & e when transformed (Laplace) go from

$$T_f = T + \tau_2 \frac{dT}{dt}$$

to

$$G_f = G + \tau_2 (sG-T_o)$$

$$G_f + \tau_2 T_o = G(1 + \tau_2 s)$$

$$G = \frac{G_f + \tau_2 T_o}{(1 + \tau_2 s)}$$

j. b & g go from

$$T + T_f = \tau_1 \frac{dT_f}{dt} - D$$

to

$$G = G_f + \tau_1 (sG_f - T_{f_o}) - (D/s)$$

k. Substituting i into j

$$\frac{(G_f + \tau_2 T_o)}{(1 + \tau_2 s)} = G_f + \tau_1 sG_f - \tau_1 T_{f_o} - D/s$$

$$G_f + \tau_2 T_o = G_f + \tau_2 sG_f + \tau_1\tau_2 s^2 G_f - \tau_1 T_{f_o} - \tau_1\tau_2 s\, T_{f_o} - D/s - D\tau_2$$

$$sG_f\,(\tau_2 + \tau_1 + \tau_1\tau_2 s) = (\tau_2 T_o + \tau_1 T_{f_o} + D\tau_2) + D/s + \tau_1\tau_2 s T_{f_o}$$

$$sG_f\,(\alpha + \beta\, s) = \gamma + D/s + \beta\, s\, T_{f_o}$$

$$= (s\,\gamma + D + \beta\, s^2\, T_{f_o})\ 1/s$$

where

$$\gamma = (\tau_2 T_o + \tau_1 T_{f_o} + D\tau_2)$$

$$\alpha = (\tau_1 + \tau_2)$$

$$\beta = \tau_1\tau_2$$

$$\alpha/\beta = \frac{1}{\tau_2} + \frac{1}{\tau_1} = \sigma$$

$$\frac{\gamma}{\beta} = \frac{T_o}{\tau_1} + \frac{T_{f_o}}{\tau_2} + \frac{D}{\tau_1} = a_1$$

l.
$$G_f = \frac{D + \gamma s + \beta T_{f_o}\, s^2}{s^2\,(\alpha + \beta\, s)}$$

Divide by β

$$G_f = \frac{D/\beta + (\cancel{\gamma}/\beta)\, s + T_{f_o}\, s^2}{s^2\,(\frac{\alpha}{\beta} + s)}$$

m. $$G_f = \frac{a_o + a_1 s + a_2 s^2}{s^2 (\sigma + s)}$$

where

$$a_o = D/\beta = \frac{2.8P}{Ah\ \tau_1 \tau_2} = \frac{2.8P\ Ah}{w_f c_f w c_p}$$

$$a_1 = \frac{\gamma}{\beta} = \frac{T_o}{\tau_1} + \frac{T_{f_o}}{\tau_2} + \frac{2.8P}{Ah\tau_1}$$

$$= \frac{T_o\ \tau_2 Ah + T_{f_o}\ \tau_1\ Ah + 2.8P\ \tau_2}{\tau_1 \tau_2\ Ah}$$

$$a_2 = T_{f_o}$$

$$\sigma = \alpha/\beta = \frac{1}{\tau_2} + \frac{1}{\tau_1} = \frac{\tau_1 + \tau_2}{\tau_1 \tau_2}$$

$$= \frac{Ah}{wc_p} + \frac{Ah}{w_f c_f} = \frac{Ah\ (wc_p + w_f c_f)}{w_f c_f w c_p}$$

n. $$T_f = K_1\ e^{-\sigma t} + K_2 t + K_3$$

By inspection K_2 $$= \frac{2.8P}{(w_f c_f + wc_p)}\ *$$

$$= \frac{2.8P}{Ah} \left(\frac{1}{\tau_1 + \tau_2} \right)$$

$$= D \left(\frac{1}{\tau_1 + \tau_2} \right)$$

Also K_3 $$= T_{f_o} - K_1$$

---

* From "Equilibrium" rate of temperature increase, also checks by more pedestrian methods.

o. From Tables **

$$K_1 = a_2 - \frac{a_1}{\sigma} + \frac{a_{o_2}}{\sigma}$$

i. $a_2 = T_{f_o}$

ii. $$\frac{a_1}{\sigma} = \frac{\gamma\beta}{\beta\ \alpha} = \frac{\gamma}{\alpha}$$

iii. $$\frac{\gamma}{\alpha} = \frac{(\tau_2 T_o + \tau_1 T_{f_o} + D\,\tau_2)}{(\tau_1 + \tau_2)}$$

$$= T_{f_o} - \frac{\tau_2(T_{f_o} - T_o)}{(\tau_1 + \tau_2)} + \frac{D\tau_2\tau_1}{(\tau_1 + \tau_2)^2} + \frac{D\tau_2^{\,2}}{(\tau_1+\tau_2)^2}$$

iv. $$\frac{a_o}{\sigma^2} = \frac{D}{\beta}\ \frac{\beta^2}{\alpha^2} = \frac{D\tau_1\tau_{2_2}}{(\tau_1+\tau_2)^2}$$

v. $$K_1 = T_{f_o} - T_{f_o} + \frac{\tau_2 \, (T_{f_o} - T_o)}{(\tau_1 + \tau_2)} - \frac{D\,\tau_2^{\,2}}{(\tau_1 + \tau_2)^2} - \frac{D\tau_2\tau_1}{(\tau_1 + \tau_2)^2} + \frac{D\tau_1\tau_2}{(\tau_1 + \tau_2)^2}$$

$$= -\frac{2.8P}{Ah} \left(\frac{\tau_2}{\tau_1+\tau_2}\right)^2 + (T_{f_o} - T_o)\left(\frac{\tau_2}{\tau_1 + \tau_2}\right)$$

** "Tables of Integral Transforms", Vol. I Bateman Manuscript Project, McGraw Hill 1954, p. 230, No. 8.

p. Combining Constants

$$T_f = T_{f_o} + \frac{2.8P}{Ah} \left(\frac{t}{\tau_1+\tau_2}\right) + \left[\frac{2.8P}{Ah} \left(\frac{\tau_2}{\tau_1+\tau_2}\right)^2 - (T_{f_o} - T_o) \times\right.$$

$$\left.\left(\frac{\tau_2}{\tau_1+\tau_2}\right)\right] \left[1 - e^{-(1/\tau_1 + 1/\tau_2)t}\right]$$

q. $\frac{2.8P}{Ah}$ = Temp difference to transfer all heat to water

r. K_2 = "Equilibrium rate of temp rise" for fuel and water at that (mythical) time when temperatures are rising at the same rate, or looking at it another way, it is the rate of temp rise for "average" temp.

$$K_1 = \frac{\tau_2}{\tau_1+\tau_2} \left[\left(\frac{2.8P}{Ah}\right) \left(\frac{\tau_2}{\tau_1+\tau_2}\right) - (T_{f_o} - T_o)\right]$$

s. $\frac{\tau_2}{\tau_1+\tau_2}$ = $\frac{\text{Rate of Heat Flowing into water}}{\text{Rate of Total Heat}}$ At "Equilibrium" (i.e. after a long time and assuming no change in properties)

$2.8P \frac{\tau_2}{\tau_1+\tau_2}$ = Rate of heat flowing into water at "Equilibrium"

$\frac{2.8P}{Ah} \frac{\tau_2}{\tau_1+\tau_2}$ = Temp diff between water and fuel to bring about "Equilibrium" heat flow to water

$(T_{f_o} - T_o)$ = Initial temp diff.

$$\frac{2.8P}{Ah} \frac{\tau_2}{\tau_1 + \tau_2} - (T_f - T_o) = \text{Change in temp difference re-}$$

quired to bring about "equilibrium"

etc.

t. $\frac{\tau_2}{\tau_1 + \tau_2}$ also $= \left[\begin{array}{c}\text{Ratio} \\ \text{at} \\ \text{"Equilibrium"}\end{array}\right] \left[\begin{array}{l}\text{Temp difference between} \\ \text{fuel element and average} \\ \text{temp difference between} \\ \text{fuel elements and water}\end{array}\right]$

so $\frac{\tau_2}{\tau_1 + \tau_2} \left[\frac{2.8P}{Ah} \frac{t_2}{\tau_1 + \tau_2} - (T_{f_o} - T_o)\right]$

= "Equilibrium" difference between

average temp and fuel element

temp.

3. <u>Derivation of T</u>

a. $T = T_f + \tau_1 \frac{dT_f}{dt} - D$

b. $\tau_1 \frac{dT_f}{df} = \tau_1 \left[0 + K_2 - K_1 \sigma e^{-\sigma t}\right] = \tau_1 K_2 - \tau_1 K_1 \sigma e^{-\sigma t}$

c. $T_f - T = D - \tau_1 K_2 + \tau_1 K_1 \sigma e^{-\sigma t}$

$$= \frac{2.8P}{Ah} - \tau_1 \frac{2.8P}{Ah} \frac{1}{(\tau_1+\tau_2)} + \tau_1 K_1 \sigma e^{-\sigma t}$$

$$= \tau_2 K_2 + \tau_1 \sigma K_1 e^{-\sigma t}$$

$$= \frac{2.8P}{Ah} \left(\frac{\tau_2}{\tau_1+\tau_2}\right) - \frac{2.8P}{Ah} \left(\frac{\tau_2}{\tau_1+\tau_2}\right) \sigma\tau_1$$

$$\times \left[\left(\frac{\tau_2}{\tau_1+\tau_2}\right) - \frac{T_{f_o} - T_o}{\frac{2.8P}{Ah}}\right] e^{-\sigma t}$$

$$\sigma\tau_1 = \tau_1 \left(\frac{\tau_1+\tau_2}{\tau_1 \tau_2}\right) = \frac{\tau_1+\tau_2}{\tau_2}$$

$$T_f - T = \frac{2.8P}{Ah} \left(\frac{\tau_2}{\tau_1+\tau_2}\right) \left\{1 - \left[1 - \frac{T_{f_o} - T_o}{\frac{2.8P}{Ah}\left(\frac{\tau_2}{\tau_1+\tau_2}\right)}\right] e^{-\sigma t}\right\}$$

$$= \tau_2 K_2 - \left[\tau_2 K_2 - (T_{f_o} - T_o)\right] e^{-\sigma t}$$

4. <u>Net Boiling</u>

a. For net boiling T can be taken as constant at T_{sat}

b. Rate of Heat Produced = Rate Heat to Water + Rate Heat to Plate

$$2.8P = Ah\,(T_f - T) + w_f c_f \frac{dT_f}{dt}$$

$$\frac{2.8P}{Ah} + T = T_f + \tau_1 \frac{dT_f}{dt}$$

$$\frac{2.8P}{Ah} + T = T_f + \tau_1 \frac{dT_f}{dt}$$

$$T_f = \left(\frac{2.8P}{Ah} + T\right) - \left[\left(\frac{2.8P}{Ah} + T\right) - T_{f_o}\right] e^{-t/\tau_1}$$

5. <u>Net Boiling Weight of Water Boiled</u>

a. $$T_f - T = \frac{C_b}{Ah} \frac{dw}{dt}$$

b. $$T_f = \left(\frac{2.8P}{Ah} + T\right) - \left[\left(\frac{2.8P}{Ah} + T\right) - T_{f_o}\right] e^{-t/\tau_1}$$

$$= A - \left[A - T_{f_o}\right] e^{-t/\tau_1}, \quad A = \left(\frac{2.8P}{Ah} + T\right)$$

c. $$A - A\text{-}T_{f_o} \; e^{-t/\tau_1} - T = -B \frac{dw}{dt}$$

$$\int_o^t (A - T)\, dt - \int_o^t \left[A - T_{f_o}\right] e^{-t/\tau_1} dt - \int_{w_o}^{w} B \; dw$$

$$(A - T)\,t + (A - T_{f_o})\,(\tau_1)\,(e^{-t/\tau_1} - 1) = (w_o - w)\,B$$

$$(A-T)t - \tau_1 T_f + \tau_1 T_{f_o} = (w_o - w)\,B$$

$$\left(\frac{2.8P}{Ah} + T - T\right) t - \tau_1 (T_f - T_{f_o}) = \frac{c_b}{Ah} (w_o - w)$$

$$t = \tau_1 \left[\frac{T_f - T_{f_o}}{\frac{2.8P}{Ah}} \right] + \left[\frac{c_b\,(w_o - w)}{2.8P} \right]$$

$$\text{or } w = w_o - \frac{2.8P}{c_b}\,t + \frac{\tau_1}{\frac{c_b}{Ah}} (T_f - T_{f_o})$$

d. $$w = w_o - \frac{2.8P}{c_b} \left[t - \frac{\tau_1}{\frac{2.8P}{Ah}} (T_f - T_{f_o}) \right]$$

JB. WATER EXPELLED AND NOT RETURNED, CALCULATIONS

1. <u>T_f for Water and Surface Below Sat. Temp.</u>

a. $(T_f - T_{f_o}) = K_2 t + K_1 (e^{-\sigma t} - 1)$

b. K_2, K_1, & σ are functions of

$$\tau_1,\ \tau_2,\ \frac{2.8P}{Ah},\ T_{f_o},\ \&\ T_o$$

c. h is assumed to be 50 Btu/hr-Ft^2-^{o}F*

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* Perry, John H., Chem. Engr. Hdbk., McGraw-Hill, 1950, p 481

e. $\sigma = \frac{1}{\tau_1} + \frac{1}{\tau_2}$

$= \frac{2}{4.94} + \frac{2}{26.1} = .482 \text{ sec}^{-1}$

f. $(T_{f_o} - T_f) = 102.3t + 915\ (1-e^{0.482t})$

Where $T_{f_o}' = \text{old } T_{f_o} - 58$

and $t = \text{old } t - 0.097$

$$(A - T)\,t + (A - T_{f_o})\,(\tau_1)\,(e^{-t/\tau_1} - 1) = (w_o - w)\,B$$

$$(A-T)t - \tau_1 T_f + \tau_1 T_{f_o} = (w_o - w)\,B$$

$$\left(\frac{2.8P}{Ah} + T - T\right) t - \tau_1 (T_f - T_{f_o}) = \frac{c_b}{Ah}(w_o - w)$$

$$t = \tau_1 \left[\frac{T_f - T_{f_o}}{\frac{2.8P}{Ah}}\right] + \left[\frac{c_b\,(w_o - w)}{2.8P}\right]$$

$$\text{or } w = w_o - \frac{2.8P}{c_b}\,t + \frac{\tau_1}{\frac{c_b}{Ah}}\,(T_f - T_{f_o})$$

d. $$w = w_o - \frac{2.8P}{c_b}\left[t - \frac{\tau_1}{\frac{2.8P}{Ah}}\,(T_f - T_{f_o})\right]$$

JB. WATER EXPELLED AND NOT RETURNED, CALCULATIONS

1. T_f for Water and Surface Below Sat. Temp.

a. $(T_f - T_{f_o}) = K_2 t + K_1 (e^{-\sigma t} - 1)$

b. K_2, K_1, & σ are functions of

τ_1, τ_2, $\frac{2.8P}{Ah}$, T_{f_o}, & T_o

c. h is assumed to be 50 Btu/hr-Ft^2-^{o}F*

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* Perry, John H., Chem. Engr. Hdbk., McGraw-Hill, 1950, p 481

d. $\tau_1 = \frac{w_f c_f}{Ah} = \frac{42}{(611.1)(50)} = 1.37 \times 10^{-3}$ hr

$= 4.94$ sec

$$\tau_2 = \frac{wc_p}{Ah} = \frac{A_c L \rho c_p}{(611.1)(50)}$$

$$= \frac{(2.083)(22/12)(52)(1.115)}{(3.06)\ (10^4)} = 7.24 \times 10^{-3} \text{ hr}$$

$$= 26.1 \text{ sec}$$

$$\tau_1 + \tau_2 = 4.9 + 26.1 = 31.0 \text{ sec}$$

e. $K_2 = \frac{2.8P}{Ah} \left(\frac{1}{\tau_1 + \tau_2}\right)$

$$= \frac{3.17 \times 10^3}{31.0}$$

$$= \underline{1.023 \times 10^2}\ {}^{o}\text{F/sec}$$

$$K_1 = -\left[\frac{2.8P}{Ah} \left(\frac{\tau_2}{\tau_1 + \tau_2}\right)^2 - (T_{f_o} - T_o)\left(\frac{\tau_2}{\tau_1 + \tau_2}\right)\right]$$

$$= -\left[(3.17)(10^3)\ (.71) - (194)\ (.842)\right]$$

$$= -\left[2250 - 163\right] = -2087$$

$$\sigma = \frac{1}{\tau_1} + \frac{1}{\tau_2} = .241 \text{ sec}^{-1}$$

f. $(T_f - T_{f_o}) = 102.3t + 2087 \ (1-e^{-.241t})$

g.

(1) t Assum.	(2) .241t .241(1)	(3) $e^{-.241t}$ $e^{-(2)}$	(4) 1-(3)	(5) 2087(4)	(6) 102.3t 102.3(1)	(7) $T_f - T_{f_o}$ (5)+(6)	(8) T_f (7)+642
0.001	0.00024	0.99975	0.00025	0.522	0.1023	0.62	642.62
0.01	0.00241	0.99759	0.00241	5.02	1.023	6.04	648.04
0.02	0.00482	0.99519	0.00481	10.04	2.046	12.09	654.45
0.05	0.01205	0.98802	0.01198	25.0	5.115	30.12	672.12
0.10	0.02410	0.97618	0.02382	49.7	10.23	59.93	701.93
0.20	0.04820	0.95294	0.04706	98.2	20.46	118.66	760.66

h. The above table is assumed to be valid as long as the surface temperature is below the saturation temperature, 567.2°F. Assuming the film drop in the scale remains constant, this is

$$T_f = 567.2 + \frac{5.58 \times 10^4 \ (2.8)}{1200} = 567.2 + 132.3$$

$$= 699.5^\circ F \quad \text{Say } 700^\circ F$$

$$T_f - T_{f_o} = 700 - 642 = 58^\circ.$$

At $(T_f + T_{f_o}) = 58^\circ F$, $t = 0.097$ sec (from Curve Sheet 14)

i. For T_f above 699.5°F, assume h = 100* Btu/hr-°F-ft²

2. <u>T for Water and Surface Below Sat. Temp.</u>

a. $(T_f - T) = \tau_2 K_2 - \left[\tau_2 K_2 - (T_{f_o} - T_o)\right] e^{-\sigma t}$

b. $\tau_2 K_2 = (26.1)\ (1.023 \times 10^2) = 2680°F$

c. $(T_{f_o} - T_o) = 194°F$

d. $\tau_2 K_2 - (T_{f_o} - T_o) = 2680 - 194 = 2486°F$

(1) t Assum.	(2) $\frac{\sigma\tau}{2.3}$ .1048 (1)	(3) 3.3155-$\frac{\sigma\tau}{2.3}$ 3.3955-(2)	(4) Antilog(3)	(5) 2.680-(4)	(6) T_f Previous	(7) (6) - (5)
0.	0.	3.39550	2486	194	642	448
0.01	0.00105	3.39445	2480	200	648	448
0.02	0.00210	3.39340	2474	206	654	448
0.05	0.00524	3.39026	2456	224	672	448
0.07	0.00724	3.38826	2445	235	684	449
0.10	0.01048	3.38502	2427	253	702	449
0.20	0.02096	3.37454	2369	311	761	450
0.50	0.05240	3.34310	2203	477		
1.00	0.10480	3.29070	1953	727		

*Perry, John H., Chem. Engr. Hdbk, McGraw-Hill, 1950, p 481

3. T_f for Water Below Saturation Temp & $T_{surface}$ Above Sat. Temp.

($h=100$ Btu/hr-^oF-ft^2)

a. $(T_f - T_{f_o}) = K_2 t - K_1 (1-e^{-\sigma t})$

b. $K_2 = \frac{2.8}{Ah} \frac{1}{(\tau_1 + \tau_2)} \frac{2.8P}{Ah/Ah (w_f c_f + wc_p)}$

so changes in h do not affect K_2 and

$K_2 = 102.3\ ^oF/sec$

c. Similarly, change does not affect $(\frac{\tau_2}{\tau_1 + \tau_2})$

New $T_{f_o} = 700^oF$

New $T_o = 449^oF$

$(T_{f_o} - T_o)_{new} = 251^oF$

d. $K_1 = - \left[\frac{2.8P}{Ah} (\frac{\tau_2}{\tau_1 + \tau_2})^2 - (T_{f_o} - T_o) (\frac{\tau_2}{\tau_1 + \tau_2}) \right]$

$= - \left[\frac{(3.17(10^3))}{2} (.71) - (251) (.842) \right]$

$= - \left[1126 - 211 \right]$

$= - 915^oF$

e. $\sigma = \frac{1}{\tau_1} + \frac{1}{\tau_2}$

$$= \frac{2}{4.94} + \frac{2}{26.1} = .482 \text{ sec}^{-1}$$

f. $(T_{f_o} - T_f) = 102.3t + 915 (1-e^{0.482t})$

Where $T_{f_o}' =$ old $T_{f_o} - 58$

and t = old $t - 0.097$

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
t Assum.	.482 t .482 (1)	$e^{-(2)}$	(1) - (3)	911 (4)	102.3 t 102.3 1	$(T_f - T_{f_o})$ (5) + (6)	old $(T_f - T_f)$ (7) + 60	old T_f (8) + 642	old t (1) + 0.097
0	0	1.0	0.0	0	0	0	58	700	.097
.1	0.0482	0.95294	0.04706	42.9	10.2	53	111	753	.197
.2	0.0964	0.90810	0.0919	83.7	20.5	104	162	804	.297
.5	0.2410	0.78584	0.21416	195.1	51.2	246	304	946	.597
1.0	0.4820	0.61754	0.38246	348.0	102.3	450	508	1150	1.097
1.5	0.723	0.48480	0.51520	469.0	153.5	623	681	1323	1.597
2.0	0.964	0.38136	0.61864	564.0	204.6	769	829	1469	2.097
2.2	1.06	0.3462	0.6538	595.6	225.1	821	879	1521	2.297
2.3	1.109	0.3298	0.6702	610.6	235.3	846	904	1546	2.397

4. T For Water Below Saturation & T Surface Above Saturation

a. $(T_f - T) = \tau_2 K_2 - \left[\tau_2 K_2 - (T_{f_o} - T_o)\right] e^{-\sigma t}$

b. $\tau_2 K_2 = \frac{26.1}{2} (1.023 \times 10^2) = 1335^\circ F$

c. $(T_{f_o} - T_o)_{new} = 251^\circ F$

d. $\tau_2 K_2 - (T_{f_o} - T_o) = 1335 - 251 = 1084^\circ F$

$\log 1084 = 3.025$

e. $\sigma = .482$

f.

(1) t New Assum.	(2) $\frac{\sigma t}{2.3}$ .21(1)	(3) 3.035-(2)	(4) Antilog (3)	(5) (T_f-T) 1335-(4)	(6) old T_f	(7) T old (6) - (5)	(8) old t (1)+0.0097
.1	.021	3.014	1033	302	753	451	.197
.2	.042	2.993	984	351	804	453	.297
.5	.105	2.930	851	484	946	462	.597
.7	.147	2.888	773	552			.797
1.0	.210	2.825	668	667	1150	483	1.097
1.5	.315	2.720	525	810	1323	513	1.597
2.0	.420	2.615	412	923	1469	546	2.097
2.2	.462	2.573	374	961	1521	560	2.297
2.3	.483	2.552	356	979	1546	567	2.397

5. <u>Net Boiling</u>

a. $$T_f = \left(\frac{2.8P}{Ah}\right) + T - \left[\left(\frac{2.8P}{Ah} + T\right) - T_{f_o}\right] e^{-t/\tau_1}$$

b. $$\frac{2.8P}{Ah} = \frac{3170}{2} = 1585^\circ F$$

$$T = 567.2^\circ F$$

$$\frac{2.8P}{Ah} + T = 2152, \text{ say } 2150$$

$$\Delta T_{f_o} = 1550^oF, \quad t = 2.4 \text{ sec}$$

$$\tau_1 = \frac{4.94}{2} = 2.47 \text{ sec}$$

c. $T_f = 2150 - 600 \ e^{-.405t}$

d.

(1) New t Assume	(2) .405(1)	(3) $e^{-(2)}$	(4) 600(3)	(5) T_f 2150-(4)	(6) $(T_f - T_{f_o})$ (5) - 642	(7) old t (1) + 2.2
0	0	1	600	1550	908	2.4
.2	.081	.92219	553	1597	945	2.6
.4	.162	.850	510	1640	998	2.8
.6	.243	.784	470	1680	1038	3.0
.8	.324	.723	434	1716	1074	3.2
1.0	.405	.677	400	1750	1108	3.4
2.2	.891	.410	246	1904	1262	4.6
3.2	1.295	.273	163	1987	1345	5.6
3.6	1.458	.233	140	2010	1368	6.0
4.0	1.62	.198	119	2031	1389	6.4
4.2	1.701	.1825	110	2040	1398	6.6
4.13	1.672	.188	113	2037	1395	6.53
4.14	1.675	.1875	112.5	2037.5	1395.5	6.54

6. Net Boiling Time

a. Assume boiling can remove heat until 3/4 of liquid is gone or $w = 1/4\ w_o$

b. $$w = w_o + \frac{2.8P}{c_b}\left[\frac{\tau_1}{\frac{2.8P}{Ah}}\ (T_f - T_{f_o}) - t\right]$$

c. $w_o = 199$ lb, say 200

$$2.8P = \frac{2.8\ (34.1)\ (10^6)}{3.6 \times 10^3} = 2.7 \times 10^4 \text{ Btu/Sec}$$

$c_b = 612$ Btu/lb @ 1200 psia

$\tau_1 = 2.47$ sec

$$\frac{2.8P}{Ah} = 1589^\circ F$$

$T_{f_o} = 1550^\circ F$ For this part of the problem

$$\frac{\tau_1}{(\frac{2.8P}{Ah})} = \frac{2.47}{1589} = 1.555 \times 10^{-3}$$

$$w = 199 + 44.1\left[1.555 \times 10^{-3}\ (T_f - T_{f_o}) - t\right]$$

(1) t_{new}	(2) t_{old}	(3) T_f	(4) $T_f - T_f(o)$	(5) 1.555×10^{-3} $(T_f - T_f^{(o)})$	(6) $\alpha - t_{new}$	(7) 44.1β	(8) $199 + \gamma$
0	2.4	1550	0	0	0	0	199
0.2	2.6	1597	47	0.0731	-0.1269	-5.59	193.41
1.0	3.4	1750	200	0.311	-0.689	-30.3	168.7
3.2	5.6	1987	437	0.680	-2.52	-111.1	87.9
3.6	6.0	2010	460	0.715	-2.885	-127.3	71.7
4.0	6.4	2031	481	0.748	-3.252	-143.5	55.5
4.2	6.6	2040	490	0.762	-3.483	-151.7	47.3
4.13	6.53	2037	487	0.757	-3.373	-148.7	50.3
4.14	6.54	2037	487.5	0.758	-3.386	-149.3	49.7

7. <u>Total Time For Plates To Melt</u>

a. Since the melting point of the fuel is 1420°C or 2590°F, the time for the temperature to rise from 2040°F to 2590°F will be calculated as though there were no water present.

$$\frac{\Delta T}{\Delta t} = \frac{2.8P}{w_f c_f} \quad \text{or}$$

$$\Delta t = \frac{w_f c_f}{2.8P} \Delta T_f$$

$$T = 2590 - 2040 = 550^{o}F$$

$$w_f c_f = 42 \text{ Btu} / {}^{o}F$$

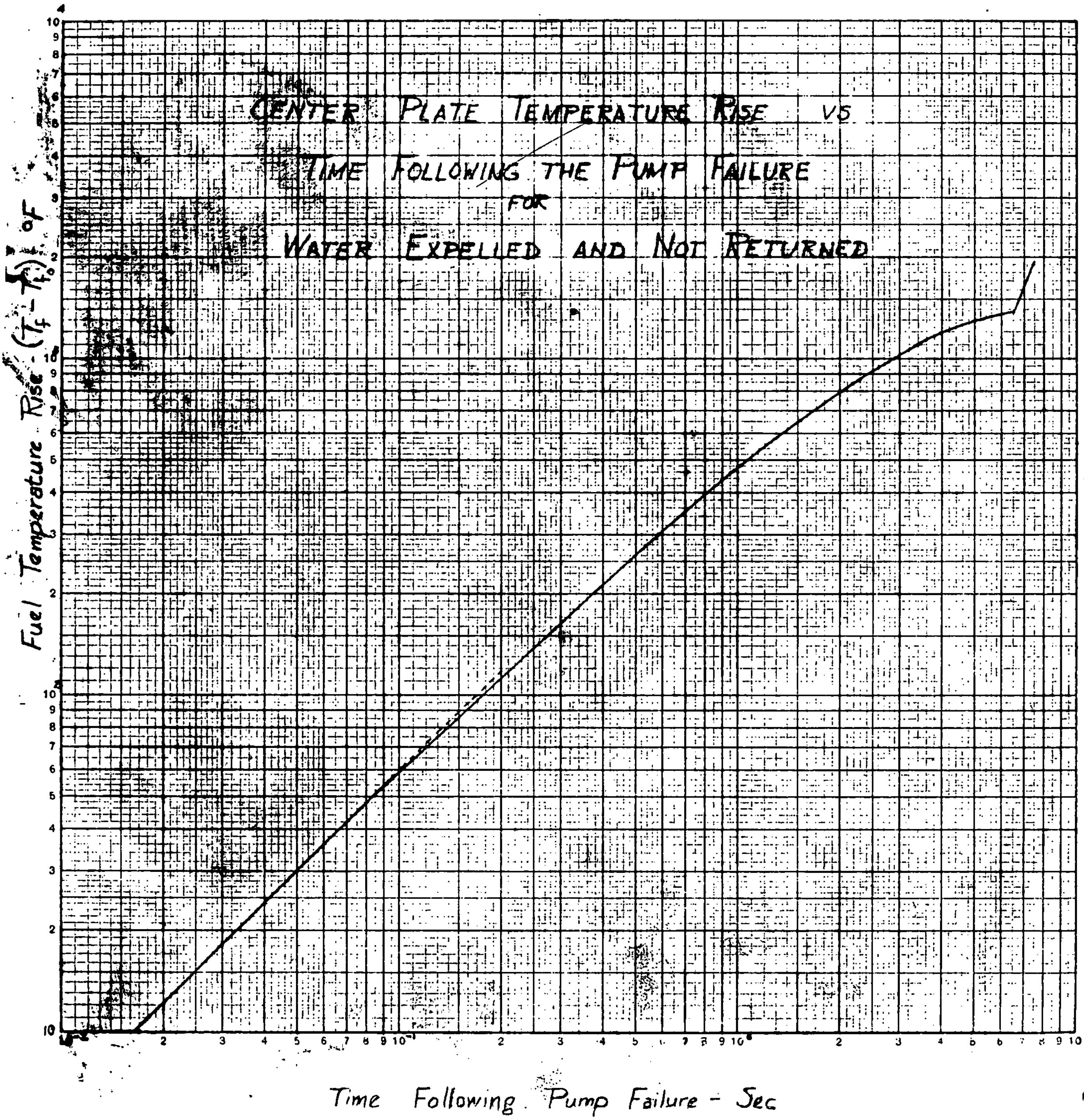
$$P = 34.1 \times 10^{6} \text{ Btu} / \text{hr}$$

$$\frac{\Delta T}{\Delta t} = \frac{(2.8)\ (3.41)\ 10^{7}}{(42)\ (3600)} = 642^{o}F / \text{sec}$$

$$\Delta t = \frac{\Delta T}{642} = \frac{550}{642} = 0.86 \text{ sec}$$

b. Total Time for Water Expelled and Not Returned

$$t_t = 6.54 + 0.86 = 7.4 \text{ sec.}$$


CENTER PLATE TEMPERATURE RISE VS
TIME FOLLOWING THE PUMP FAILURE
FOR
WATER EXPELLED AND NOT RETURNED
Fuel Temperature Rise $(T_f - T_{f_0})$ °F
Time Following Pump Failure - Sec

SHEET No 14