Fermilab

$\bar{p}$ Note #251

Deflection of Curved Beams;
Holding Curved Magnets Together

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Coordinates

s along beam

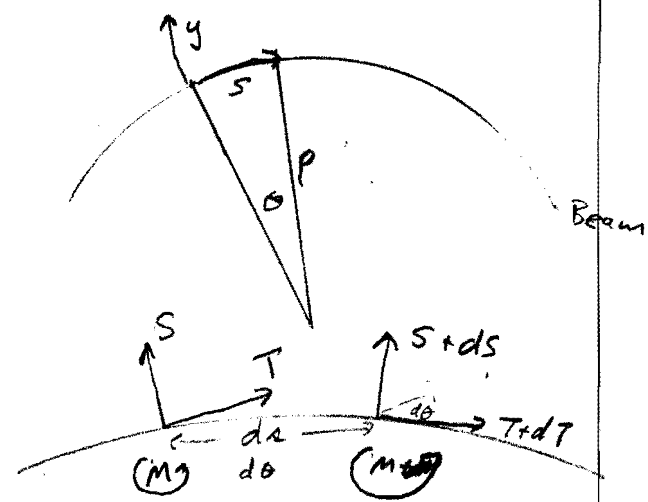
y $\perp$ to beam

ρ = radius of curvature

S - shear Force

T - Tension Force

M - Bending Moment.



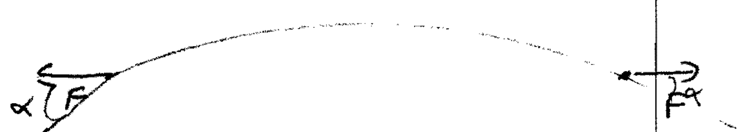
Equilibrium

a) $-T + (T+dT)\cos d\theta + (S+dS)\sin d\theta = 0 \Rightarrow S = -\frac{dT}{d\theta}$

b) $-S + (S+dS)\cos d\theta - (T+dT)\sin d\theta = 0 \Rightarrow T = \frac{dS}{d\theta}$

c) $0 = -M + M + dM + (S+dS)\cos d\theta\, ds - (T+dT)\sin d\theta\, ds \Rightarrow \frac{dM}{ds} = -S$

For a beam pulled at the ends



$T = A\cos\theta + B\sin\theta$

$S = A\sin\theta - B\cos\theta$

$S = 0$ at $\theta = 0 \Rightarrow B = 0$

at the end of the beam $T = A\cos\alpha = F\cos\alpha$
$S = A\sin\alpha = F\sin\alpha$ $\Rightarrow A = F$

$S = F\sin\theta$

$M = -\int S\,ds = -\rho\int F\sin\theta\,d\theta = \rho F[\cos\theta - \cos\alpha]$

since $M = 0$ at $\theta = \alpha$

Now let y be the deflection away from the unsheared position

$y'' = \frac{M}{K}$ $K = EI$ $I = \frac{t w^3}{12}$

where w is the width and t the thickness of the beam. E = Youngs Modulus

$$y'' = -\frac{\rho F}{K}\left[\cos\theta - \cos\alpha\right]$$

$$y' = \frac{\rho F}{K}\int \rho\, d\theta \left[\cos\theta - \cos\alpha\right] = \frac{\rho^2 F}{K}\left[\sin\theta - \theta\cos\alpha\right] + \text{const}$$

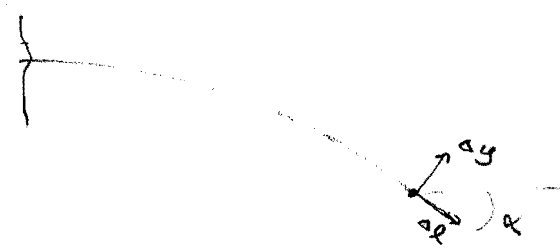
const $= 0$, since $y' = 0$ at $\theta = 0$

Similarly $y = +\frac{\rho^3 F}{K}\left[-\cos\theta - \frac{\theta^2}{2}\cos\alpha + 1\right]$ where the constant of integration is chosen to make $y = 0$ at $\theta = 0$

Similarly the beam is stretched, so that its length increases

$$\ell(\theta) = \int ds\left(1 + \frac{T(\theta)}{E w t}\right) \quad \text{or} \quad \Delta\ell = \rho\int_0^\theta d\theta\, \frac{T(\theta)}{E w t} = \frac{\rho F}{E w t}(1 - \cos\theta)$$

Then at each end of the beam there are parallel and perpendicular deflections

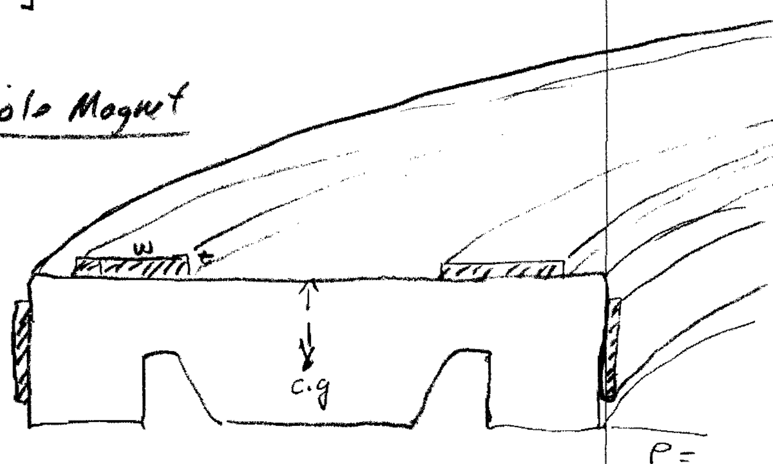


$$\Delta\ell = \frac{\rho F}{E w t}(1 - \cos\alpha) \quad \text{and}$$

$$\Delta y = \frac{\rho^3 F}{I E}\left[1 - \cos\alpha - \frac{\alpha^2}{2}\cos\alpha\right]$$

$$\left[1 - \cos\alpha\left(1 + \tfrac{\alpha^2}{2}\right)\right]$$

Application to (Large) Dipole Magnet



Curved Bars on the top and sides are meant to restrain the laminations when clamping pressure is released. The side bars have negligible moment of inertia and will be neglected.

For the above problem, assume 150 PSI clamping pressure, $E = 3\times10^7$ PSI

$w = 8''$, $t = 2''$ (two plates), $\rho = 687$ in, $\alpha = 7.5°$, $A = 796$ in^2

Then $\Delta y = .92''$ and

$\Delta\ell = 1.5\times10^{-3}\,''$

Of course, the magnet ends will not move this far, since the lamination "spring" is very non linear. It just means that these curved bars are unable to restrain the laminations, and they will relax to a lower pressure.

If instead the bars were straight, they would stretch by an amount $\Delta l = l \frac{F}{wtE} = .022''$ at each end. $(l = 90'')$

Out of plane warping

Since the side bars cannot effectively exert any tension, the magnet can only be restrained from "fanning out" by a moment exerted by the side bars. The center of gravity of the lamination is about 7.5" from the top, so the moment which must be exerted is $M = 7.5'' \times 150\,psi \times 796\,in^2 = 9 \times 10^5$ in #

The side bars (and the magnet) must have a radius of curvature

$$\rho = \frac{EI}{M} = 2860''$$

, not so different from the curvature in the other direction. The sagitta, the amount it pulls up at each end, is 1.42".

The most reasonable way out is to reduce the clamping pressure, use straight top bars and heavier or higher (better) side bars.