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Phenomenology of infrared smooth warped extra dimensions

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Abstract: We study the effect of the infrared (IR) geometry on the phenomenology of warped extra dimensions with gauge and fermion fields in the bulk. We focus in particular on a "mass gap" metric which is AdS in the ultraviolet, but asymptotes to flat space in the IR, breaking conformal symmetry. These metrics can be dialed to approximate well the geometries arising in certain classes of warped string compactifications. We find, similar to our earlier results on the Kaluza-Klein (KK) graviton, that these metrics give rise to phenomenologically significant shifts in the separation of KK gauge modes in the mass spectrum (up to factors ~ 2) and their couplings to IR localized fields (up to factors $\sim 5-10$ increase). We find that, despite shifts in the spectra, the constraint $m_{\rm KK} \gtrsim 3$ TeV from S remains robust in the class of 5-d mass gap metrics, and that the change to T is not significant enough to remove the need for custodial symmetry.

Keywords: Field Theories in Higher Dimensions, Compactification and String Models.

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1. Introduction

Extra dimensions appear in many extensions of the Standard Model (SM), from solutions to the hierarchy problem, to efforts to unify gravity with the rest of the forces in string theory. One appealing solution to the hierarchy problem comes from warped extra dimensions, where Randall and Sundrum (RS) [1] showed that the 5-dimensional Anti-de Sitter (AdS) space appearing often in string theories can produce a naturally small Higgs boson mass through exponential suppression of the Planck scale.

Since the idea of RS appeared almost a decade ago, much work has been done in the phenomenology of warped extra dimensions. Much of this work relates to embedding the SM in the extra dimension, and the resultant implications for collider searches (e.g. [2 – 6]). While allowing the SM fields to propagate in the bulk of the extra dimension alleviates problems with Flavor Changing Neutral Currents [7, 8] and allows an understanding of the flavor hierarchy, other problems and constraints are introduced, most notably from the Peskin-Takeuchi parameters, S and T [9 – 12], and from B factories [6, 13]. With additional work (usually involving the addition of other symmetries, such as custodial symmetry in the AdS bulk [14], or an RS GIM mechanism [15]), the problems can be alleviated and constraints relaxed, making Kaluza-Klein (KK) gauge bosons with $m_{\rm KK} \gtrsim 3$ TeV consistent with current data, and within observational reach of the Large Hadron Collider (LHC).

All of these analyses, both collider studies and constraints from precision Electroweak (EW), were carried out within the context of pure AdS geometry. The AdS metric, however, is singular in the IR. Moreover, the hierarchy is not stabilized as the warp factor can be

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arbitrarily small. An attempt to stabilize the weak-Planck hierarchy in the RS scenario has been made in [16], by introducing a bulk field that generates a potential for the distance between the IR and UV branes. However, it remains to be seen if this mechanism is realized in a fully UV complete model. Furthermore, as one allows fermion and gauge fields to propagate in the bulk, the motivation for introducing the IR and UV branes becomes less clear. Interestingly, warped models can be constructed in string theory where the IR geometry is smooth. As a result, the hierarchy is stabilized by the smooth IR geometry without the need of introducing an IR brane. A particularly well studied example of such warped geometry is the Klebanov-Strassler (KS) throat [17]. The KS geometry is non-compact, but upon compactification with fluxes (see e.g. [18] and examples therein), the weak scale hierarchy is stabilized by quantized fluxes:

$$m_{\rm TeV} = M_{Pl} e^{A_{\rm min}} = M_{Pl} e^{-2\pi K/3Mg_s} \tag{1.1}$$

where K and M are 3-form flux quanta, g_s is the string coupling, and $e^{A_{\rm min}}$ is the minimum (non-vanishing) warp factor in the IR. In addition to the KS geometry (and its generalizations to the baryonic branch [19]) whose metrics are known, a wide class of string theory backgrounds [20, 21] admit similar IR modifications although explicit metrics for such smooth warped geometries are yet to be constructed. These warped throat solutions are 10-d relatives of the 5-d RS metric which, instead of continuing to become exponentially small, go to a constant in the IR. The other five dimensions are angular coordinates (whose topology is $S^2 \times S^3$ in the examples above). Upon integrating out the angular KK modes (whose masses are higher than the low lying radial KK modes, see, e.g., [22]), one obtains an effective 5-d RS-like model.

In an earlier paper [23], we showed that these 10-d relatives of RS, though their metric differs only a little from the RS metric, and only in the last decade of the 16 decade hierarchy between the Planck and TeV scales, lead to dramatically different collider signatures. In particular, we found that the spacing between the KK graviton modes with the KS metric changes by a factor of several relative to the KS case. Their couplings to TeV localized states also changes significantly: the couplings of all KK graviton modes is no longer universal, and is larger by a factor of at least ~ 30. We thus showed that the spectrum of KK graviton states can be a sensitive probe of the geometry of the warped background, so that we may learn something about the nature of warped string compactifications at the LHC (the effects of other types of geometries have also been studied in, e.g. [24, 25]).

The KS metric is complicated, and thus presents a barrier to doing detailed phenomenological model building. It has been shown, however, that the KS metric (and its generalizations mentioned above) can be well approximated by a simple "mass gap" metric [26, 23], which interpolates between AdS warping in the UV and flat space in the IR. In the AdS/CFT language, the "mass gap" metric corresponds to a breaking of the conformality in the IR, with the mass gap parameter set by the confinement scale of the field theory dual. Such modifications have been shown to have important effects in the contexts of inflation [27] or phase transitions [28]. In our earlier paper [23], we also numerically solved for the masses and wavefunctions of the KK gravitons with the mass gap metric in 5-d and 10-d and again found large differences in the masses and wavefunction overlaps

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with the IR brane. If more than one KK graviton can be observed at the LHC, the ratio of the masses and ratio of couplings to SM states measures the background metric, and may give insight into the string compactification which generates the warping.

Here we extend our previous work to studies of gauge bosons propagating in the bulk. We restrict ourselves to mass gap metrics-they may be handled numerically much more easily than KS metrics, and can be dialed to reproduce the desired IR behavior. We solve the spectrum with both the 5-d and 10-d mass gap metrics, but restrict ourselves to five dimensions in the precision EW analysis on S and T in order to compare our results with that of the well-studied RS models. Furthermore, the Higgs mechanism cannot be embedded in the way that it is in the extended RS scenario into higher-dimension mass gap metrics without placing restrictions on the angular space. While much progress has been made toward constructing realistic D-brane models [29 – 31], explicit realistic warped string models with features of the extended RS scenario have yet to be found. The present work is thus a first step in exploring the electroweak constraints of such warped string models. Our results could be adopted to specific compactifications once this scenario finds its embedding in string theory.

We are interested in two primary questions. First, how do the spectrum and couplings of KK gauge bosons change with the new metric? Can we learn about how conformality is broken in the IR from the spectrum of KK states? This is a natural follow-up to the earlier study on KK gravitons, which showed strong sensitivity to the IR behavior of the metric. Second, very detailed model building and precision EW constraints have been derived in the context of the AdS background. Do these constraints change much when a new metric is introduced? If the KK spectrum changes significantly with each new metric, the constraints on the lowest KK gauge boson mass from S (currently sitting at 3 TeV) may be modified (this question was also considered in [32]). Even a modest shift in the constraint from 3 TeV to 1 TeV, for example, will change LHC physics reach dramatically.

The outline of this paper is as follows. In the next section we set up the equations for gauge and fermions propagating in the bulk, with an arbitrary number of extra dimensions with arbitrary modifications and of the metric in the IR, numerically solving the equations to obtain the spectrum of states and couplings to IR localized fields. We apply the formalism to the case where the Higgs vev on the IR brane modifies the profiles of the gauge boson zero modes in the extra dimension. We then turn to analyzing the Peskin-Takeuchi parameters with the new metrics in 5-d and again compare the results and constraints to RS. We consider the effect of the new metrics on the fermion profiles, including constraints on $Z \to b\bar{b}$. We then conclude.

2. Warped geometries with IR smooth behavior

We consider a class of warped $D = (5 + \delta)$-dimensional backgrounds with line element

$$ds^2 = G_{MN} dx^M dx^N = f^{-1/2}(r)\, \eta_{\mu\nu} dx^\mu dx^\nu - f^{1/2}(r) \left(dr^2 + r^2 ds^2_{X^\delta} \right) \tag{2.1}$$

where μ, ν run over $0, 1, 2, 3$ and M, N run over all dimensions. The line element for the

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compact angular space X^δ is

$$r^2 ds^2_{X^\delta} = r^2 \tilde{g}_{\xi\zeta} dy^\xi dy^\zeta \tag{2.2}$$

where $\tilde{g}$ is independent of x^μ but not necessarily of r. The determinant of the full metric tensor is then $G = f^{(\delta-3)/2} r^{2\delta} \tilde{g}$. In the Randall-Sundrum scenario, the warp factor is given in terms of the AdS curvature $k = 1/R$ by

$$f(r) = \frac{R^4}{r^4}. \tag{2.3}$$

The identification with the RS metric may be made by a substitution $r = e^{-ky}$. The location of the UV brane in the extra dimension is taken to be $r = R$ where the warp factor becomes 1 and the warped throat ends. In RS, an IR brane is placed at $r = r_{\text{tip}}$ where $r_{\text{tip}}/R = \epsilon = e^{-11.27\pi} = 4.2 \times 10^{-16}$ to address the hierarchy problem.

Backgrounds with smooth behavior in the infrared can be parametrized using the mass gap ansatz [26, 23]:

$$f(r) = \frac{R^4}{r_{\text{tip}}^4 + f_2 R^2 r^2 + r^4}. \tag{2.4}$$

Note that unlike the RS metric, this warp factor remains finite as $r \to 0$. This form of the warp factor is chosen so that a brane placed at $r = 0$ will produce the same hierarchy as the RS background while for $r \sim R \gg \sqrt{f_2} R$ the space is approximately AdS. This is the scenario that we consider. As discussed above, the IR brane is introduced to localize the Higgs[1] but is not necessary to cut off the space or stabilize the hierarchy.

Over most of the 16 orders of magnitude of the Planck-Higgs hierarchy, the warping is very well approximated by AdS, differing from it only in the last order of magnitude before reaching the tip of the throat at $r = 0$ where the IR brane is located. We plot the warp factor for several different choices of f_2 in figure 1.

2.1 Gauge fields with 5-d and 10-d mass gaps

The physics for a gauge field propagating in the bulk of AdS was considered in [34, 35]. We generalize the analysis here for an arbitrary warp factor. The action for a canonically normalized gauge field A_M propagating in the bulk of a geometry described by eq. 2.1 is

$$S = -\frac{1}{4} \int d^{5+\delta}x \sqrt{G}\, G^{MS} G^{NT} F_{MN} F_{ST}$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strength tensor and the integral runs from the IR brane to the UV brane and over all of X^δ. The action is

$$S = \int d^{5+\delta}x f^{(\delta-3)/4} r^\delta \sqrt{\tilde{g}} \left(-\frac{f}{4} \eta^{\mu\sigma}\eta^{\nu\tau} F_{\mu\nu} F_{\sigma\tau} + \frac{1}{2}\eta^{\mu\sigma} F_{\mu r} F_{\sigma r} + \frac{1}{2}\eta^{\mu\sigma} \frac{\tilde{g}^{\xi\zeta}}{r} F_{\mu\xi} F_{\sigma\zeta} \right) + \ldots, \tag{2.5}$$

where the ellipses denote terms which involve only the fields from the six extra dimensions. The mixing terms between our four dimensions and the six extra dimensions can be removed

[1] One can also localize the Higgs using instantons as in [33], in which case the IR brane can be eliminated.

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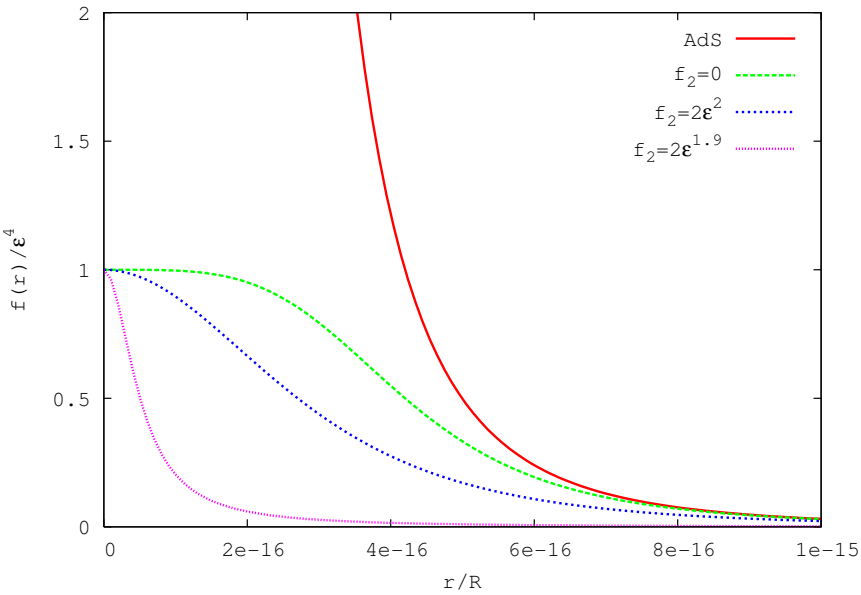

Figure 1: The warp factor for different geometries. The warp factor for AdS grows without bound while those for the mass gap geometries approach a finite value. Thus the AdS space, unlike the mass gap metrics which are the subject of this paper, must be truncated at finite r. The distinct shapes manifest themselves in many ways discussed here.

by an appropriate gauge choice. The Kaluza-Klein decomposition of A_μ is then written

$$A_\mu\left(x,r,y\right)=\sum_{n=0}^{\infty}\sum_{\{\ell\}}A_\mu^{(n,\{\ell\})}\left(x\right)a_{n,\{\ell\}}\left(r\right)\Theta_{\{\ell\}}\left(y\right) \tag{2.6}$$

where the set $\{\ell\}$ labels the angular mode. If we limit ourselves to the lighter s-wave modes (that is, where $\Theta_{\{\ell\}}\left(y\right)$ is a constant) then the KK decomposition simplifies to

$$A_\mu\left(x,r\right)=\sum_{n=0}^{\infty}A_\mu^{(n)}\left(x\right)a_n\left(r\right). \tag{2.7}$$

After integrating over r, the KK fields $A_\mu^{(n)}$ will be canonically normalized in the 4-d language if we impose the orthonormality condition

$$\int dr\int d^\delta y\, f^{(\delta+1)/4}r^\delta\sqrt{\tilde{g}}\,a_n a_m=\delta_{nm}. \tag{2.8}$$

The equation of motion will turn out to be an eigenvalue equation so that we can impose this orthonormality condition so long as a_n and a_m satisfy the same set of boundary conditions.

The $F_{\mu r}F_{\nu r}$ term in the $(5+\delta)$-d action corresponds to mass terms for the KK modes in the dimensionally reduced theory. That is, after integrating by parts, we can write the action as (again, neglecting the effects of heavier angular modes)

$$S=\int d^4x\sum_{n=0}^{\infty}\left\{-\frac{1}{4}\eta^{\mu\sigma}\eta^{\nu\tau}F_{\mu\nu}^{(n)}F_{\sigma\tau}^{(n)}+\frac{1}{2}m_n^2\eta^{\mu\sigma}A_\mu^{(n)}A_\sigma^{(n)}\right\}, \tag{2.9}$$

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where the equation of motion for the radial wavefunction is

$$\partial_r \left(f^{(\delta-3)/4} r^\delta \sqrt{\tilde{g}}\, \partial_r a_n \right) + f^{(\delta+1)/4} r^\delta \sqrt{\tilde{g}}\, m_n^2 a_n = 0. \tag{2.10}$$

In order to write the action in this form, the boundary term resulting from integration by parts must be made to vanish. That is,

$$\left[V_{X^\delta} f^{(\delta-3)/4} a_m \partial_r a_n \right]_{r=r_{\rm IR}}^{r=r_{\rm UV}} = 0, \tag{2.11}$$

where

$$V_{X^\delta} = r^\delta \int d^\delta y \sqrt{\tilde{g}} \tag{2.12}$$

is the (unwarped) volume of the angular space X^δ. The UV contribution to the surface term can be made to vanish by imposing either the Neumann boundary condition

$$\partial_r a_n \big|_{r=r_{\rm UV}} = 0 \tag{2.13}$$

or the Dirichlet boundary condition

$$a_n \left(r = r_{\rm UV} \right) = 0. \tag{2.14}$$

The Dirichlet condition breaks 4-d gauge invariance so the Neumann condition will be applied on the UV brane below.

If $\delta = 0$ (i.e. 5-d spacetime) then a similar argument can be applied to the boundary conditions on the IR. However, if $\delta \neq 0$, then the boundary conditions that can be consistently applied depend on the shape of X^δ. If X^δ is such that the volume shrinks to zero at the IR, then the requirement that the boundary term vanishes is not enough to determine the boundary condition. For example, in the simplest case, X^δ is a δ-sphere, S^δ, and $\tilde{g}$ is independent of r. In the near tip region, the warp function approaches a constant $(R/r_{\rm tip})^4$ and the equation of motion eq. 2.10 simplifies to

$$\partial_r \left(r^\delta \partial_r a_n \right) + \rho^2 r^\delta a_n = 0, \tag{2.15}$$

where $\rho^2 = m_n^2 \left(R/r_{\rm tip} \right)^4$. In 5-d, the solutions are sines and cosines. In higher dimensions, the solutions in the near-tip region can be written in terms of Bessel functions

$$a_n \left(r \right) \to \frac{1}{r^{(\delta-1)/2}} \left(a J_{(\delta-1)/2}\big(\rho r\big) + b Y_{(\delta-1)/2}\big(\rho r\big) \right). \tag{2.16}$$

The first term satisfies the Neumann condition $a_n' \left(r = 0 \right) = 0$ while the second term diverges as $r \to 0$. This means that in this case the only boundary condition that be consistently applied is the Neumann condition. However, if the volume does not shrink to zero, then other boundary conditions can be applied.

When the background is AdS, the arguments change. If the angular space is S^δ, the effect of the δ-dimensions drops out of the equation of motion since the additional factor of r^δ is cancelled out by the additional power of the warp factor. This means that, up to normalization, the radial wave function is independent of the existence of additional

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dimensions (though there is still interesting physics when extra dimensions are added, as in [36]). Because of this independence, a divergent solution exists in RS for any $\delta \geq 0$. However, the space is cut off in the Randall-Sundrum scenario by the IR brane before this divergence occurs.

As in the original RS scenario, we consider the Higgs to be localized on an IR 3-brane located at at $r = r_{\rm IR}$ where $r_{\rm IR} = 0$ for the mass gap geometries while $r_{\rm IR} = r_{\rm tip}$ in RS. In $5+\delta$ dimensions, the angular coordinates are taken to be $y^\xi = 0$. If a gauge field propagating in the bulk couples to this Higgs with coupling constant g_D, then after the Higgs obtains a vev $\tilde{v}$, the action for the gauge field can be written in $(5+\delta)$-d language as

$$S = \int d^{5+\delta}x\sqrt{G}\left\{-\frac{1}{4}G^{MS}G^{NT}F_{MN}F_{ST} + \frac{\tilde{v}^2 g_D^2}{8}\frac{\delta\left(r-r_{\rm IR}\right)\delta^\delta\left(y\right)}{\sqrt{G_{rr}}r^\delta\sqrt{\tilde{g}}}G^{MS}A_M A_S\right\} \quad (2.17)$$

where the determinant is from the Jacobian used in transforming the delta function from Cartesian coordinates to warped coordinates. The equation of motion for the internal wavefunctions does not change in the presence of the Higgs vev, but the boundary condition on the IR brane becomes modified [37]. For the geometries considered here, the boundary condition at the IR brane becomes

$$\partial_r a_n\big|_{r=r_{\rm IR}} = \frac{\tilde{v}^2 g_D^2}{4}\frac{\left[f\left(r=r_{\rm IR}\right)\right]^{(1-\delta)/4}}{V_{X^\delta}}a_n. \quad (2.18)$$

For $\delta > 0$, if the X^δ vanishes at the tip, then the boundary condition is not well-defined.

As shown below and in [23], increasing the number of dimensions in an infrared smooth geometry has significant effects on the couplings and spectra. However, consistent localization of the Higgs on a TeV brane depends on the angular geometry. For this reason, we will consider only 5-d backgrounds when doing the precision electroweak analysis below and leave the effects of higher-dimensional backgrounds on S and T for future work.

In 5-d, the IR boundary condition becomes

$$\partial_r a_n = \frac{\tilde{v}^2 g_5^2}{4}\left[f\left(r=r_{\rm IR}\right)\right]^{1/4}a_n. \quad (2.19)$$

With these boundary conditions, a non-vanishing constant solution (zero mode) is not present. Instead, the flat solution is replaced by an "almost" zero mode where the profile is flat everywhere except near the IR (figure 10). This is also forces the 4-d mass to be non-vanishing for the lowest state.

2.2 Fermions with 5-d mass gaps

Next we turn to generalizing bulk fermions for arbitrary warp factors. This was done for the RS scenario in the context of neutrino masses in [38] though our conventions follow more closely those in [39]. We will specialize to 5-d because of the difficulties with embedding the Higgs discussed above. The action for a 5-d Dirac spinor Ψ propagating in the bulk is

$$S = \int d^5x\,\sqrt{G}\left\{i\bar{\Psi}\Gamma^A E^M{}_A D_M\Psi - M_f\bar{\Psi}\Psi\right\}, \quad (2.20)$$

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where $E^M{}_A$ is the inverse of the fünfbein which is given in this background by

$$E_M{}^A = \begin{pmatrix} f^{-1/4}\,\delta^a_\mu & 0 \\ 0 & f^{1/4} \end{pmatrix}, \tag{2.21}$$

and A, B, a, b are tangent space indices with A, B ranging from 0 to 4 and a, b ranging from 0 to 3. We also denote by $\hat{M}$ the tangent space index corresponding to the base space index M. The covariant derivative is

$$D_M = \partial_M + \frac{1}{4}\omega_M{}^{AB}\Gamma_{AB}, \tag{2.22}$$

where $\omega_M{}^{AB}$ is the spin connection and $\Gamma_{AB} = \frac{1}{2}\left[\Gamma_A, \Gamma_B\right]$ generate Lorentz transformations. We use the representation $\Gamma_{\hat{\mu}} = \gamma_\mu$ and $\Gamma_{\hat{4}} = i\gamma_5$ where $\{\gamma_\mu\}$ is a representation of the Dirac algebra in four-dimensions and $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ is the chirality operator.

In terms of the torsion

$$T^C{}_{AB} = \left(E^M{}_A E^N{}_B - E^M{}_B E^N{}_A\right)\partial_N E_M{}^C, \tag{2.23}$$

the spin connection is

$$\omega_M{}^{AB} = \frac{1}{2}E_M{}^C\left(T_C{}^{AB} - T^{AB}{}_C - T^B{}_C{}^A\right). \tag{2.24}$$

The non-vanishing components of the spin connection are then

$$\omega_\mu{}^{\hat{4}b} = \frac{f'}{4f^{3/2}}\delta^b_\mu \tag{2.25}$$

so that the covariant derivative is

$$\begin{aligned} D_\mu &= \partial_\mu - \frac{if'}{8f^{3/2}}\gamma_\mu\gamma_5 \\ D_r &= \partial_r. \end{aligned} \tag{2.26}$$

Defining $\hat{\Psi} = f^{-1/2}\Psi$ simplifies the action to

$$S = \int d^5x\, f^{1/4}\left\{if^{1/4}\hat{\bar{\Psi}}\eta^{\mu\sigma}\gamma_\mu\partial_\sigma\hat{\Psi} - f^{-1/4}\hat{\bar{\Psi}}\gamma_5\partial_r\hat{\Psi} - M_f\hat{\bar{\Psi}}\hat{\Psi}\right\}. \tag{2.27}$$

We can write

$$\hat{\Psi} = \hat{\Psi}_L + \hat{\Psi}_R, \tag{2.28}$$

where $\gamma_5\hat{\Psi}_{L,R} = \mp\hat{\Psi}_{L,R}$, and perform the KK decomposition

$$\hat{\Psi}_{L,R} = \sum_{n=0}^{\infty}\psi^{(n)}_{L,R}\left(x^\mu\right)\chi^{L,R}_n\left(r\right). \tag{2.29}$$

Then, after integrating over r, the four-dimensional action becomes (where all indices are now contracted with $\eta_{\mu\nu}$)

$$S = \int d^4x\sum_{n=0}^{\infty}\left\{i\bar{\psi}^{(n)}_L\gamma^\mu\partial_\mu\psi^{(n)}_L + i\bar{\psi}^{(n)}_R\gamma^\mu\partial_\mu\psi^{(n)}_R - m_n\left(\bar{\psi}^{(n)}_L\psi^{(n)}_R + \bar{\psi}^{(n)}_R\psi^{(n)}_L\right)\right\}, \tag{2.30}$$

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where we have imposed the orthonormality condition

$$\int dr\, f^{1/2}\chi_n^{L,R}\chi_m^{L,R} = \delta_{nm}, \tag{2.31}$$

and the internal wavefunctions satisfy the coupled equations

$$\pm\partial_r\chi_n^{L,R} + f^{1/4}M_f\chi_n^{L,R} = f^{1/2}m_n\chi_n^{R,L}. \tag{2.32}$$

Unlike the radial wave functions for the gauge bosons, these are first-order equations. We return to these equations in section 4 for the modified gauge-fermion couplings with 5-d mass gaps.

2.3 Spectra and couplings

In the AdS background, the solutions to eq. 2.10 can be written in terms of Bessel functions [34, 35]

$$a_n = \frac{A_n}{r}J_1\left(\frac{m_nR^2}{r}\right) + \frac{B_n}{r}Y_1\left(\frac{m_nR^2}{r}\right). \tag{2.33}$$

For the IR smooth backgrounds considered here, the solutions are more complex. For the special case in which $f_2 = 2\epsilon^2$, the solutions can be expressed in terms of (analytically continued) associated Legendre functions

$$a_n = \frac{1}{\left(r^2 + r_{\rm tip}^2\right)^{1/4}}\left(\alpha_\nu P_{1/2}^{\nu}\left(\frac{ir}{r_{\rm tip}}\right) + \beta_\nu Q_{1/2}^{\nu}\left(\frac{ir}{r_{\rm tip}}\right)\right), \tag{2.34}$$

where $\nu = \sqrt{\frac{1}{4} + \frac{m_n^2R^4}{r_{\rm tip}^2}}$. Even though an analytic solution exists, the dependence on the KK mass is highly nonlinear, entering through the order of the Legendre function. When the warp factor cannot be expressed as a perfect square, $f(r) = (R^2/(r_{\rm tip}^2 + r^2))^2$, analytic solutions are not found for general m_n. We instead solve the equation numerically.

We will consider scenarios in which the gauge symmetries are broken by a Higgs vev and not by boundary conditions. Since a Dirichlet boundary condition explicitly breaks gauge invariance, we apply Neumann conditions at each brane. As for RS, a flat zero mode solution with $m_0 = 0$ is admitted with these boundary conditions.

As expected, changing the geometry alters the spectrum. When R and the hierarchy are left fixed, as f_2 increases, the masses of the KK modes increase while the spacing between KK modes becomes tighter. As was found for gravitons [23] the geometries give a distinguishable pattern from RS (tables 1-4, figure 2). Like the KK graviton, the 10-d mass gap gives larger effects than the 5-d mass gap, with $\mathcal{O}(10)$ changes in the couplings to IR localized fields and $\mathcal{O}(1)$ changes in the ratio between the masses of the KK states (which depends only on the geometry and not on the overall scale). Thus the KK gauge spectra, like the KK graviton spectra, can be a probe of the geometry of the warped compactification. Changing the IR boundary condition to a Higgs boundary condition does not significantly change the KK masses for the values of $\tilde{v}R$ considered here.

Profiles for the first and second KK modes for the internal wavefunctions of a gauge field are shown in figures 3-5. As f_2 increases, the wave function becomes increasingly

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Mode	m_n (TeV)					
	$f_2 = 0$	$f_2 = \epsilon^4$	$f_2 = 2\epsilon^{2.1}$	$f_2 = 2\epsilon^2$	$f_2 = 2\epsilon^{1.9}$	RS
1	1.00	1.00	1.00	1.00	1.00	1.00
2	2.53	2.53	2.53	2.42	2.05	2.27
3	3.99	3.98	3.98	3.81	3.20	3.54
4	5.44	5.43	5.42	5.20	4.33	4.82

Table 1: 4-d masses for the first few modes in different 5-d geometries for $r_{\rm tip}/R = \epsilon = 4.2 \times 10^{-16}$ with the first mode normalized to the (unrealistic) value 1 TeV.

Mode	g_n/g_0					
	$f_2 = 0$	$f_2 = \epsilon^4$	$f_2 = 2\epsilon^{2.1}$	$f_2 = 2\epsilon^2$	$f_2 = 2\epsilon^{1.9}$	RS
0	1	1	1	1	1	1
1	6.95	6.96	6.97	7.34	9.85	8.39
2	6.24	6.24	6.24	6.88	10.7	8.41
3	6.27	6.27	6.27	6.82	10.4	8.43
4	6.27	6.27	6.27	6.80	10.3	8.43

Table 2: Relative couplings of 5-d KK excitations to IR localized fields for different geometries. These are given by the ratios of wave function values at the IR when the wavefunctions are normalized in the sense of eq. 2.8.

Mode	m_n (TeV)					
	$f_2 = 0$	$f_2 = \epsilon^4$	$f_2 = 2\epsilon^{2.1}$	$f_2 = 2\epsilon^2$	$f_2 = 2\epsilon^{1.9}$	RS
1	1.00	1.00	1.00	1.00	1.00	1.00
2	1.67	1.67	1.74	1.64	1.31	2.27
3	2.29	2.29	2.36	2.24	1.62	3.54
4	2.90	2.90	3.02	2.83	1.94	4.82

Table 3: 4-d masses for the first few modes in different 10-d geometries (with S^5 angular space) for $r_{\rm tip}/R = \epsilon = 4.2 \times 10^{-16}$ with the first mode normalized to 1 TeV.

Mode	g_n/g_0					
	$f_2 = 0$	$f_2 = \epsilon^4$	$f_2 = 2\epsilon^{2.1}$	$f_2 = 2\epsilon^2$	$f_2 = 2\epsilon^{1.9}$	RS
0	1	1	1	1	1	1
1	11.3	11.3	13.7	28.7	49.2	8.39
2	46.8	46.8	47.7	86.7	119	8.41
3	106	106	110	183	249	8.43
4	193	193	197	323	461	8.43

Table 4: Relative couplings of 10-d KK excitations to IR localized fields for different geometries when the angular space is S^5.

localized towards the IR when the wavefunctions are normalized in the sense of eq. 2.8. This effect is enhanced in 10-d, leading to strong coupling between the TeV fields and the KK modes. Although the strength of the coupling is such that the physics can no longer

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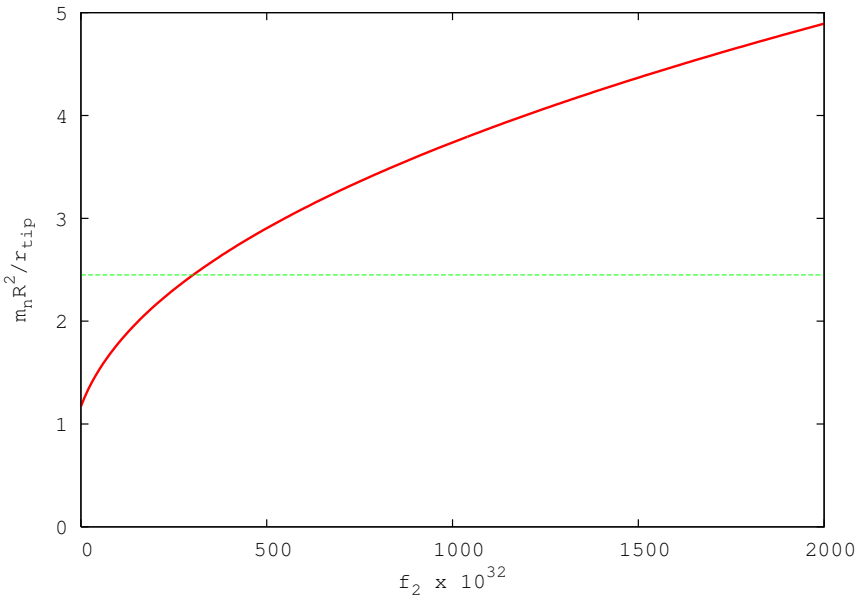

Figure 2: Lowest KK mass in different geometries. The value for RS is 2.45, which is shown in green.

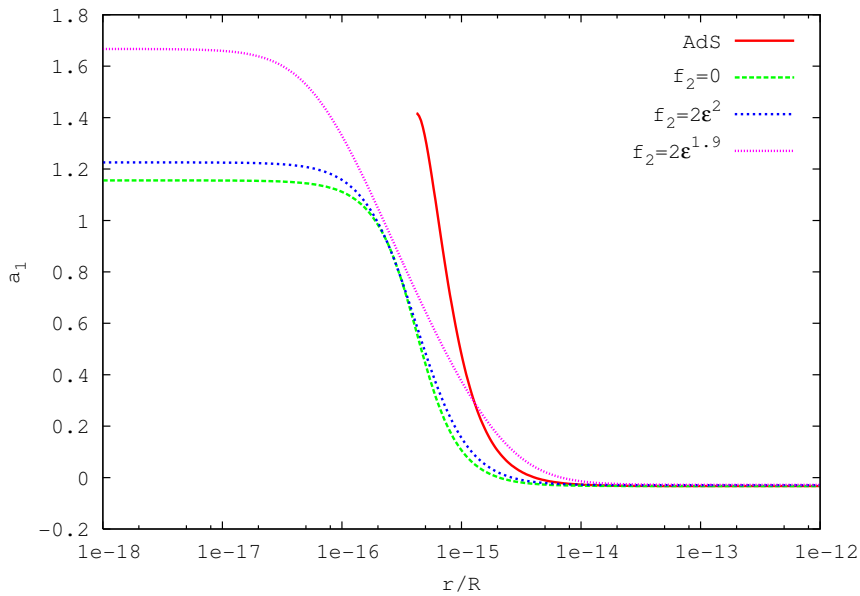

Figure 3: Internal wavefunctions for the first KK in 5-d mode in units of $1/\sqrt{R}$.

be treated perturbatively for larger KK modes, it would guarantee strong collider signals.

The standard model fermions are associated with the lowest modes of Ψ_L while the zero mode of Ψ_R is assumed to be projected out by (for example) boundary conditions. The bulk mass of Ψ is written $M_f = \nu/R$. For $\nu \lesssim -\frac{1}{2}$, the profile χ_0^L is localized toward the IR brane, while for $\nu \gtrsim -\frac{1}{2}$, the profile is localized toward the UV brane. For the case in which a Higgs field that obtains a vev is localized on the IR brane, the 4-d mass of the

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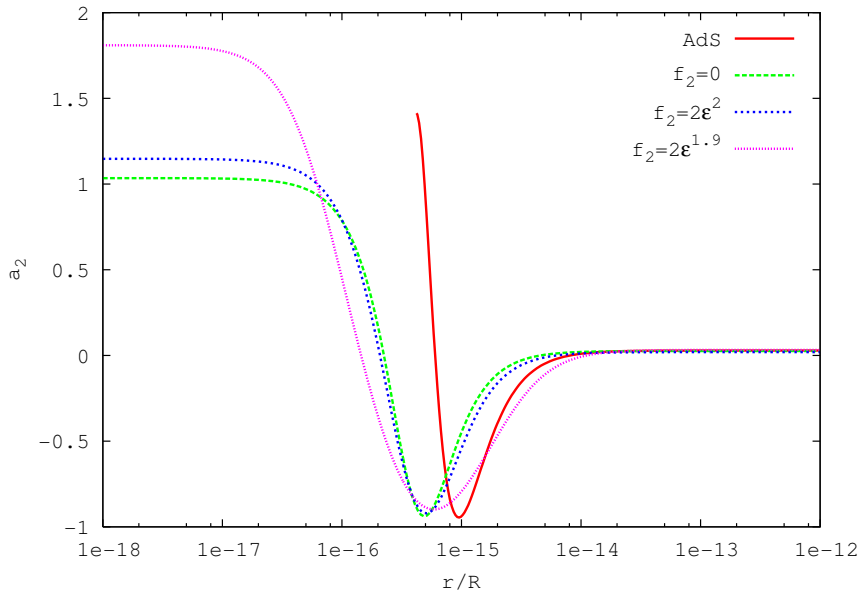

Figure 4: Internal wavefunctions for the second KK mode in 5-d in units of $1/\sqrt{R}$.

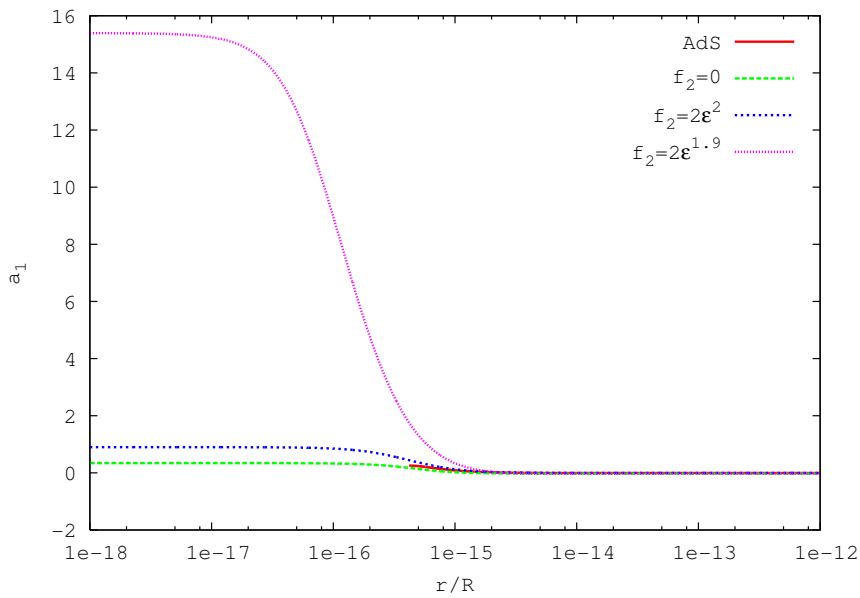

Figure 5: Internal wavefunctions for the first KK mode in 10-d in units of $1/\sqrt{R}$. The modes are more strongly localized to the IR than they are in 5-d, leading to stronger couplings to IR localized fields.

standard model fermion is not zero, but is related to the overlap of the zero mode with the Higgs. More precisely, if $\Psi^{(\mathrm{d})}$ transforms as a part of a doublet under SU(2) and $\Psi^{(\mathrm{s})}$ is the corresponding singlet, then the coupling to the Higgs vev is

$$S = -\int d^5x\sqrt{G}\,(\lambda_5 R)\,\frac{\tilde{v}}{2}\left(\bar{\Psi}^{(\mathrm{d})}\Psi^{(\mathrm{s})} + \mathrm{h.c.}\right)\frac{\delta\left(r - r_{\mathrm{IR}}\right)}{\sqrt{G_{55}}} \tag{2.35}$$

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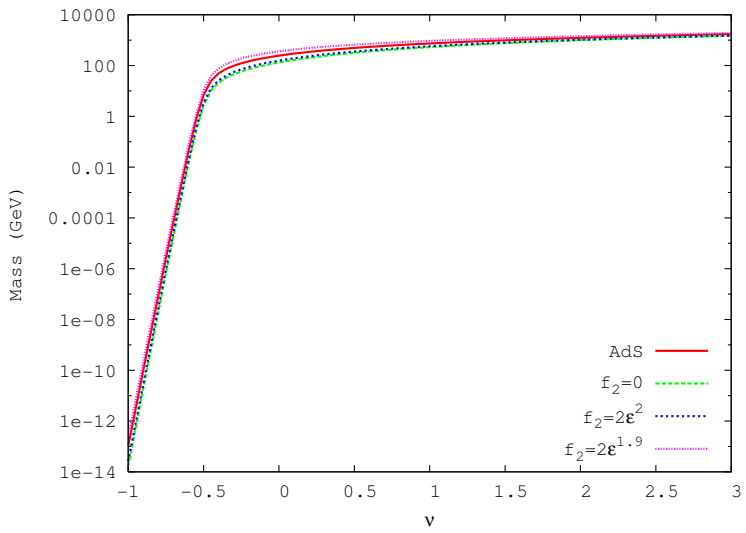

Figure 6: 4-d masses for the lowest KK mode of Ψ_L due to the presence of a Higgs field localized on the IR brane assuming $\lambda_5 = 1$ and that ν is the same for both the singlet and the doublet.

where R is introduced to make λ_5 dimensionless (fermions are dimension 2 in 5-d, while the Higgs vev is still dimension 1 since the Higgs is restricted to 4-d). Using the delta function, this gives a 4-d coupling to the warped Higgs vev $v = \tilde{v}\frac{r_{\rm tip}}{R} \approx 246$ GeV

$$\lambda_4 = \lambda_5 \frac{R^2}{r_{\rm tip}} \xi_0^{L(d)}\left(r = r_{\rm IR}\right) \xi_0^{L(s)}\left(r = r_{\rm IR}\right) \tag{2.36}$$

Since the equation of motion is first-order, the value of the normalized radial wave function on the IR (and hence the Yukawa coupling and the 4-d mass) is determined by the values of M_f for the singlet and the doublet. This relation allows the hierarchy in Yukawa couplings to be explained by small changes in the parameters ν. For simplicity, we take the five dimensional Yukawa coupling $\lambda_5 = 1$ for every fermion. Standard model fermions with small 4-d mass have a weak coupling to the Higgs and are thus taken to be IR localized ($\nu \lesssim -\frac{1}{2}$) while heavier fermions are localized toward the UV brane (figure 6). Because the geometries considered here differ from the RS scenario in only the IR, the physics for light fermions is not significantly altered from RS in this scenario. Since the top has a large 4-d Yukawa coupling, either the left-handed or the right-handed (singlet) top must be heavily localized towards the IR so we take ν for the right-handed top to be equal to 1. The value of ν for the left-handed top is chosen such that the overlap with the Higgs gives the correct 4-d top mass. Since the left-handed top and the left-handed bottom are in the same $SU(2)$ doublet, they must have the same profile in the internal space (figure 7).

3. Precision electroweak constraints with 5-d mass gaps

While allowing standard model fields, most notably the fermions, to propagate in the bulk reduces some serious problems, such as flavor changing neutral currents, it gives rise to other potential problems, such as corrections to the precision electroweak observables S

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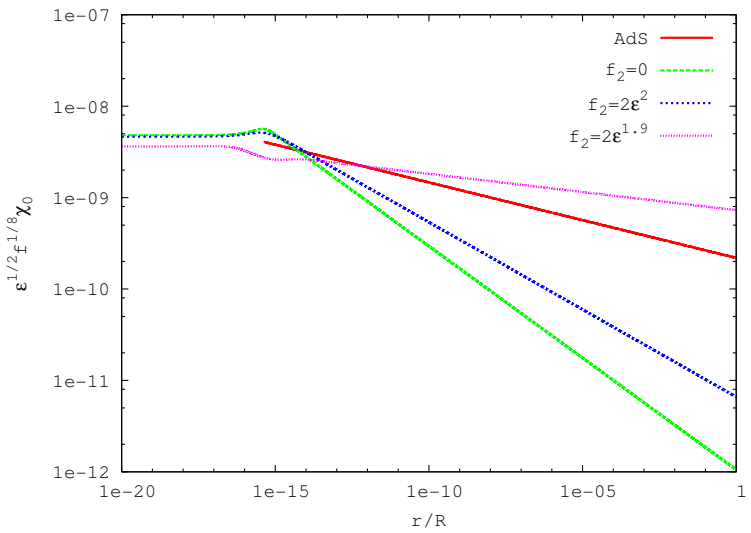

Figure 7: Internal wavefunction for the left-handed bottom quark in units of $1/\sqrt{R}$, assuming $\lambda_5 = 1$ and that ν for the right-handed top is 1. As f_2 decreases, the profile becomes increasing localized in the IR.

and T (U remains small for these models). In the absence of a Higgs vev on the TeV brane, Neumann b.c. may be consistently applied to both the UV and IR branes, and the wavefunctions of the zero modes of the gauge bosons are completely flat. The presence of the Higgs vev on the IR brane forces the Neumann b.c. to be modified so that the derivative is non-vanishing there. This modifies the gauge boson profiles from flatness in the extra dimension. Since the W and Z gauge bosons have different couplings to the Higgs, their profiles in the extra dimension are modified in different ways. The overlap of these profiles with the fermion profiles gives the gauge coupling of the gauge bosons to the fermions, which are now non-standard. For fermions localized in the same place in the extra dimension, the non-standard gauge couplings can be absorbed into a gauge boson wavefunction renormalization, and hence into the Peskin-Takeuchi parameters S and T. In this class of models, the Peskin-Takeuchi parameters arise at tree-level, which we consider here. For completeness, we describe this procedure in detail, following Csaki et al. [9] though our normalization scheme differs from theirs.

We begin by writing the KK decomposition for the $SU(2) \times U(1)$ gauge fields propagating in the bulk as

$$W_\mu^\pm = \sum_{n=0}^{\infty} W_\mu^{\pm(n)}(x^\mu)\, w_n(r)$$
$$Z_\mu = \sum_{n=0}^{\infty} Z_\mu^{(n)}(x^\mu)\, z_n(r)$$
$$A_\mu = \sum_{n=0}^{\infty} A_\mu^{(n)}(x^\mu)\, a_n(r)\,.$$

The standard model fields are identified with $W^{\pm(0)}$, $Z^{(0)}$, and $A^{(0)}$. For the photon, which

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does not couple to the Higgs, the corresponding internal wavefunction is a flat true zero mode with zero 4-d mass. However, the W and Z couple to the IR-localized Higgs field, forcing the internal wavefunction to be an almost zero mode and the 4-d mass to be non-vanishing. In this section, we adopt a normalization for the internal wavefunctions different from eq. 2.8 as explained below.

3.1 Formalism

The 5D action for the electroweak gauge fields (in a non-canonical normalization) is

$$S = \int d^5x\sqrt{G}\left\{-\frac{1}{4g_5^2}G^{MS}G^{MT}W^a_{MN}W^a_{ST} - \frac{1}{4g_5'^2}G^{MS}G^{NT}B_{MN}B_{ST}\right.$$
$$\left. + \frac{\tilde{v}^2}{8}\frac{\delta\left(r-r_{\rm IR}\right)}{\sqrt{G_{55}}}G^{MS}\left[W^1_MW^1_S + W^2_MW^2_S + \left(W^3_M - B_M\right)\left(W^3_S - B_S\right)\right]\right\} \tag{3.1}$$

where W^a are the $SU(2)$ gauge fields and B is the $U(1)$ hypercharge gauge field. In this non-canonical normalization, the Z and photon are written

$$Z_M = W^3_M - B_M \tag{3.2}$$
$$A_M = s^2W^3_M + c^2B_m \tag{3.3}$$

where the weak mixing angle is defined by the usual relation

$$\sin\left(\theta_{\rm W}\right) = s = \frac{g_5'}{\sqrt{g_5^2+g_5'^2}}\,, \tag{3.4}$$

and $c = \sqrt{1-s^2}$.

After integrating over the extra-dimension and keeping only the first member of the KK towers, the action becomes

$$S = \int d^4x\left\{-\frac{1}{2g^2}Z_WW^+_{\mu\nu}W^{-\mu\nu} - \frac{1}{4\left(g^2+g'^2\right)}Z_ZZ_{\mu\nu}Z^{\mu\nu} - \frac{1}{4e^2}Z_\gamma F_{\mu\nu}F^{\mu\nu}\right.$$
$$\left. + \left(\frac{v^2}{4} + \frac{1}{g^2}\Pi_{WW}\left(0\right)\right)W^+_\mu W^{-\mu} + \frac{1}{2}\left(\frac{v^2}{4} + \frac{1}{\left(g^2+g'^2\right)}\Pi_{ZZ}\left(0\right)\right)Z_\mu Z^\mu\right\}, \tag{3.5}$$

where all indices have been contracted with $\eta_{\mu\nu}$ and

$$\frac{1}{g^2}Z_W = \frac{1}{g_5^2}\int dr\, f^{1/4}w_0w_0 \tag{3.6}$$
$$\frac{1}{g^2+g'^2}Z_Z = \frac{1}{g_5^2+g_5'^2}\int dr\, f^{1/4}z_0z_0$$
$$\frac{1}{e^2}Z_\gamma = \left(\frac{1}{g_5^2}+\frac{1}{g_5'^2}\right)\int dr\, f^{1/4}a_0a_0$$

and

$$\frac{1}{g^2}\Pi_{WW}\left(0\right) = \frac{1}{g_5^2}\int dr\, f^{-3/4}\left(\partial_rw_0\right)^2 + \frac{v^2}{4}\left(w_0\left(r_{\rm IR}\right)^2 - 1\right) \tag{3.7}$$
$$\frac{1}{g^2+g'^2}\Pi_{ZZ}\left(0\right) = \frac{1}{g_5^2+g_5'^2}\int dr\, f^{-3/4}\left(\partial_rz_0\right)^2 + \frac{v^2}{4}\left(z_0\left(r_{\rm IR}\right)^2 - 1\right),$$

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while v is simply the warped Higgs vev

$$v = \tilde{v} f^{-1/4} \approx 246 \text{ GeV}. \tag{3.8}$$

Here, z_0, w_0 are the almost zero modes for the Z and $W^{\pm}$ fields while a_0 is the true zero mode for the photon.

The physical meaning of these choices can be understood easily. In the absence of the Higgs vev, the wavefunctions for the gauge bosons are all constants and equal. When this is the case, we may take $Z_X = 1$ and then eqs. 3.6 are a dependent set of equations that simply relate the 5-d coupling constants to the 4-d coupling constants,

$$\frac{1}{g_x} = \frac{1}{g_{x5}^2} a^2 \int dr\, f^{1/4} \tag{3.9}$$

where $a = a_0(r) = w_0(r) = z_0(r)$, g_x is any of the 4-d couplings and g_{x5} is the corresponding 5-d coupling. However, when the Higgs has a non-zero vev, the wave functions for W and Z become modified in the IR and eqs. 3.6 can be satisfied only if some of the Z_X are different from unity, which results in corrections to electroweak precision observables.

This can also be understood in the canonical normalization where the coupling appears in the fermion terms. As discussed below, the 4-d effective coupling of the gauge fields to fermions can be written

$$\begin{aligned} g &= g_5 \int dr\, f^{1/2} w_0 \chi^0 \chi^0 \\ \sqrt{g^2 + g'^2} &= \sqrt{g_5^2 + g_5'^2} \int dr\, f^{1/2} z_0 \chi^0 \chi^0 \\ e = \frac{g g'}{\sqrt{g^2 + g'^2}} &= \frac{g_5 g_5'}{\sqrt{g_5^2 + g_5'^2}} \int dr\, f^{1/2} a_0 \chi^0 \chi^0 \end{aligned} \tag{3.10}$$

When the wavefunctions are flat, these equations are again a redundant set of equations. However, when the wavefunctions for the Z and W are modified by the Higgs vev, these equations can only be satisfied if the normalization of the wavefunctions for the heavy gauge bosons are modified, again giving rise to corrections to the well measured properties of the W and Z.

We normalize the wavefunctions so that $a_0 = w_0 = z_0 = 1$ on the UV brane. Since light fermions must have a small overlap with the Higgs vev, they are highly peaked at the UV brane and this choice ensures that the coupling of these fermions to the gauge bosons are as in the standard model. This choice of normalization then redefines the corrections in precision EW observables into a wavefunction and mass renormalizations, eqs. 3.6, appearing in S, T and U. The zero mode of the photon, which does not couple to the Higgs and is thus a true zero mode takes the constant value $a_0 = 1$.

Given the discussion above, it is natural to choose $Z_\gamma = 1$ since the photon remains a true zero mode, in which case

$$\frac{1}{e^2} = \left(\frac{1}{g_5^2} + \frac{1}{g_5'^2} \right) \int dr\, f^{1/4}. \tag{3.11}$$

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Then in order to preserve the relations between the coupling constants and the weak mixing angles, we must take

$$\frac{1}{g^2} = \frac{1}{g_5^2} \int dr\, f^{1/4} \tag{3.12}$$

$$\frac{1}{g^2 + g'^2} = \frac{1}{g_5^2 + g_5'^2} \int dr\, f^{1/4}. \tag{3.13}$$

Then the wavefunction renormalizations and mass corrections become

$$Z_W = \frac{\int dr\, f^{1/4} w_0 w_0}{\int dr\, f^{1/4}} \tag{3.14}$$

$$Z_Z = \frac{\int dr\, f^{1/4} z_0 z_0}{\int dr\, f^{1/4}} \tag{3.15}$$

$$\Pi_{WW}(0) = \frac{\int dr\, f^{-3/4} (\partial_r w_0)^2}{\int dr\, f^{1/4}} + \frac{g^2 v^2}{4} \left(w_0 (r_{\rm IR})^2 - 1 \right) \tag{3.16}$$

$$\Pi_{ZZ}(0) = \frac{\int dr\, f^{-3/4} (\partial_r z_0)^2}{\int dr\, f^{1/4}} + \frac{\left(g^2 + g'^2\right) v^2}{4} \left(z_0 (r_{\rm IR})^2 - 1 \right) \tag{3.17}$$

We can relate these quantities to the oblique parameters $S, T,$ and U. We identify

$$Z_W = 1 - g^2 \Pi_{11}' \tag{3.18}$$

$$Z_Z = 1 - \left(g^2 + g'^2\right) \Pi_{33}' \tag{3.19}$$

$$\Pi_{WW}(0) = g^2 \Pi_{11}(0) \tag{3.20}$$

$$\Pi_{ZZ}(0) = \left(g^2 + g'^2\right) \Pi_{33}(0) \tag{3.21}$$

from which the Peskin-Takeuchi parameters [40] are:

$$S = 16\pi \Pi_{33}' \tag{3.22}$$

$$T = \frac{4\pi}{s^2 c^2 M_Z^2} \left(\Pi_{11}(0) - \Pi_{33}(0) \right) \tag{3.23}$$

$$U = 16\pi \left(\Pi_{11}' - \Pi_{33}' \right) \tag{3.24}$$

Note that the expression for S has simplified because of the lack of Z–γ mixing at the classical level.

In this context, the Peskin-Takeuchi parameters are a measure of how much the wavefunctions of the W and Z deviate from a true zero mode in the extra dimension. When the Higgs is turned off, the full electroweak symmetry is unbroken, the boundary conditions on the IR are restored to Neumann boundary conditions and the Peskin-Takeuchi parameters vanish at tree-level.

In the Randall-Sundrum scenarios, this deviation from the zero flat mode leads to a large T parameter unless the size of the extra dimension is made small. This translates into a bound on the KK mass of order 10 TeV [9]. It is known however, that by introducing a custodial $SU(2)$ symmetry into the bulk which is broken partially by boundary conditions that T can be made small. The primary constraint on $m_{\rm KK}$ ($\gtrsim$ 3 TeV) is then derived

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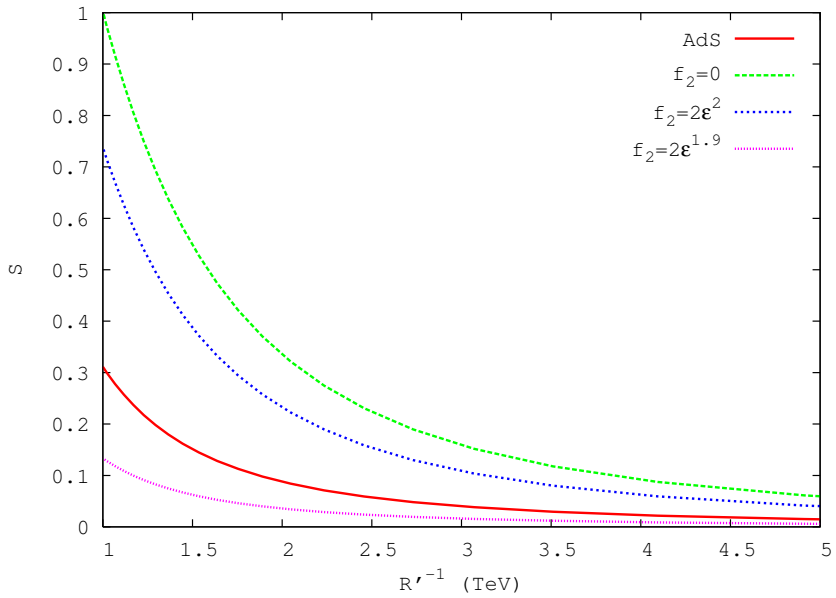

Figure 8: The Peskin-Takeuchi parameter S for different geometries where the hierarchy is fixed to $r_{\rm tip}/R = 4.2 \times 10^{-16}$.

from S [14]. Since the geometries give rise to significant shifts in the spectrum, one might suspect that constraints from S and T on the KK scale might be alleviated within these modified classes of metrics. It turns out that this is not the case with the 5-d mass gap metrics, as we discuss next.

3.2 Results

The effect of the size (given in terms of $R' = R/\epsilon$) and shape of the internal manifold on S and T when no custodial symmetry is implemented is shown in figures 8 and 9 (U remains negligibly small in this setup). For every value of f_2, S and T are always positive and increase monotonically as R increases.

Measurements on the Z pole provide bounds on S and T and so restrict the size of the extra dimensions and the mass of the first KK excitation ($m_{\rm KK}$). Since the functional dependence of the Peskin-Takeuchi parameters on R depends on the geometry (see figure 8), this might a priori suggest that the bound on $m_{\rm KK}$ might change from one geometry to another. However, $m_{\rm KK}R^2/r_{\rm tip}$ also changes with the geometry (figure 2) in such a way that the two effects cancel for S (figure 11) while T changes from one geometry to another though the changes remain small for 5d mass gap metrics (figure 12). From figure 10, this insensitivity to the geometry for UV localized e and μ might have been expected. In the UV, the W, Z wavefunctions change very little from geometry to geometry as long as the mass of the first KK mode is normalized to the same value for each geometry. For fields localized near the UV brane, the effects of the KK modes can be summarized by their masses and their corresponding wavefunctions at the position of the Planck brane. With a fixed $m_{\rm KK}$, the couplings of W and Z to e and μ are not significantly affected by the IR modifications. It is these couplings to W and Z in eq. 3.10 , which are redefined into

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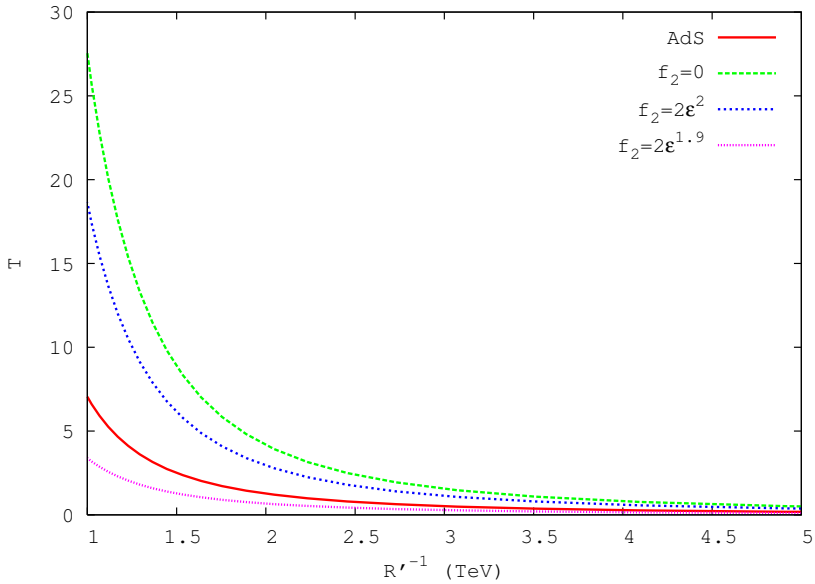

Figure 9: The Peskin-Takeuchi parameter T for different geometries where the hierarchy is fixed to $r_{\rm tip}/R = 4.2 \times 10^{-16}$.

S and T through eq. 3.7. If e and μ were IR localized, one would expect a much bigger effect on S. The slight variation of T with the geometry is due to the fact that T measures the derivatives of the W and Z wavefunctions, which are most significant in the IR (see figure 10) and hence the IR modifications can give rise to a shift in T. Our results are consistent with those of [32], who found that S and T depend on volume factors, which differ little in the geometries we consider here. In the future, we plan to consider the effects on S and T from 10-d mass gap geometries where the shifts on the Peskin-Takeuchi parameters (in particular, T) can be more significant.

In models that possess a custodial symmetry, the primary bound on $m_{\rm KK}$ comes from S. This implies that the bound on the $m_{\rm KK}$ is independent of the shape of the internal space for the warped geometries chosen here. For models with custodial symmetry, the bound on $m_{\rm KK}$ remains at $3-4\,$TeV [14].

4. Fermion-gauge boson couplings

The action for a 5-d Dirac fermion coupling minimally to a 5-d gauge field with coupling constant g_5 is given by

$$S = \int d^5x \sqrt{G} g_5 \bar{\Psi} \Gamma^A E^M{}_A A_M \Psi. \tag{4.1}$$

With the gauge choice $A_r = 0$, we find after performing the KK decomposition and integrating over r,

$$S = \int d^4x \sum_{n,m,q=0}^{\infty} \left\{ g^L_{nm,q} \bar{\psi}^{(n)}_L \gamma^\mu A^{(q)}_\mu \psi^{(m)}_L + g^R_{nm,q} \bar{\psi}^{(n)}_R \gamma^\mu A^{(q)}_\mu \psi^{(m)}_R \right\}, \tag{4.2}$$

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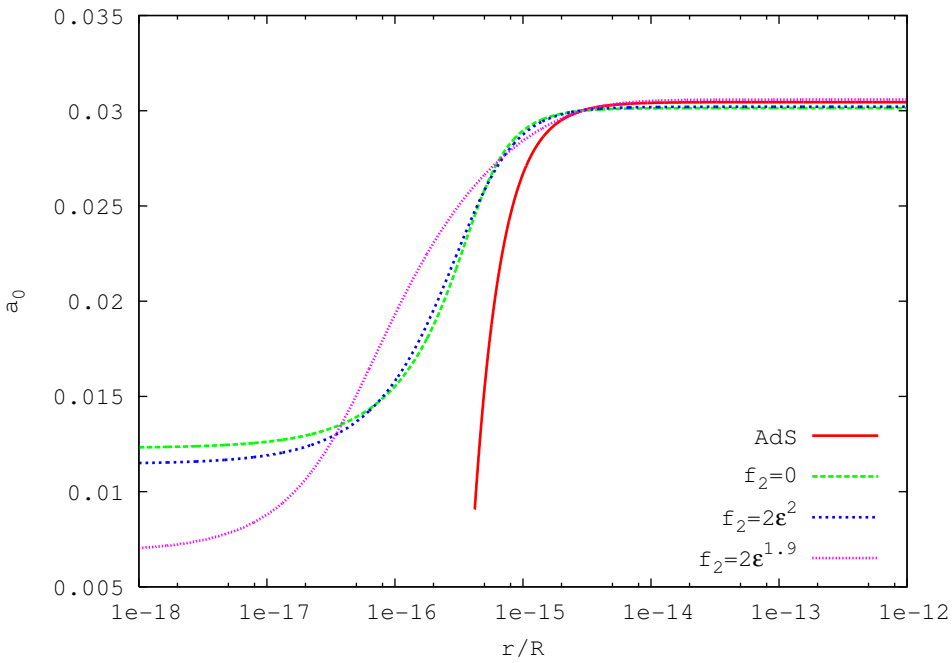

Figure 10: The "almost zero" internal wavefunctions for the Z given in units of $1/\sqrt{R}$ (normalized in the sense of eq. 2.8). The deviation from flatness is due to the presence of the Higgs vev on the IR. For each geometry, R is chosen such that $m_{\rm KK} = 3$ TeV.

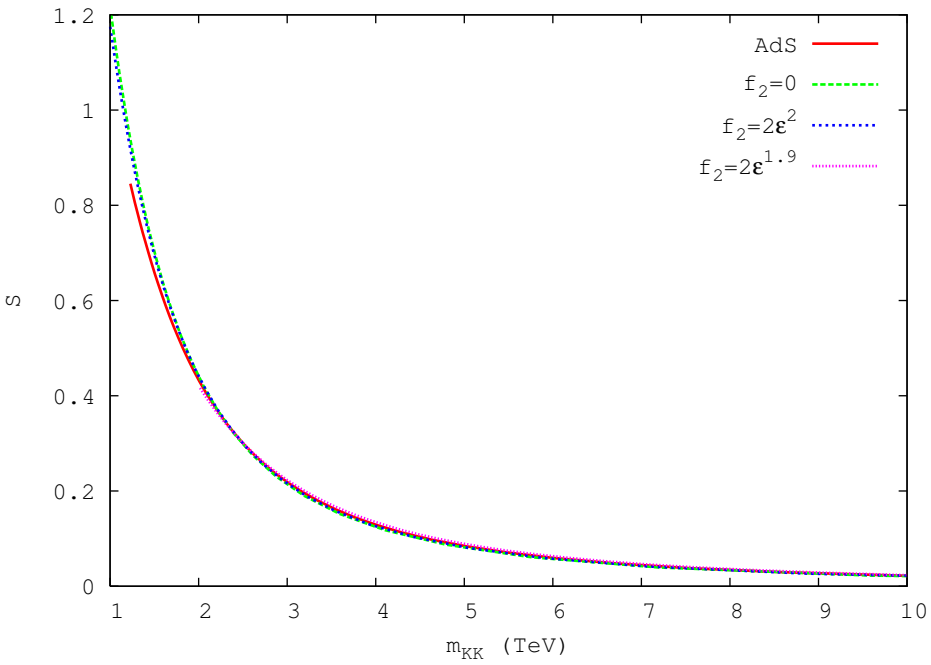

Figure 11: The Peskin-Takeuchi parameter S for different geometries and different values of $m_{\rm KK}$. Since the curves lie essentially on top of each other, the constraint on $m_{\rm KK}$ from S remains unchanged.

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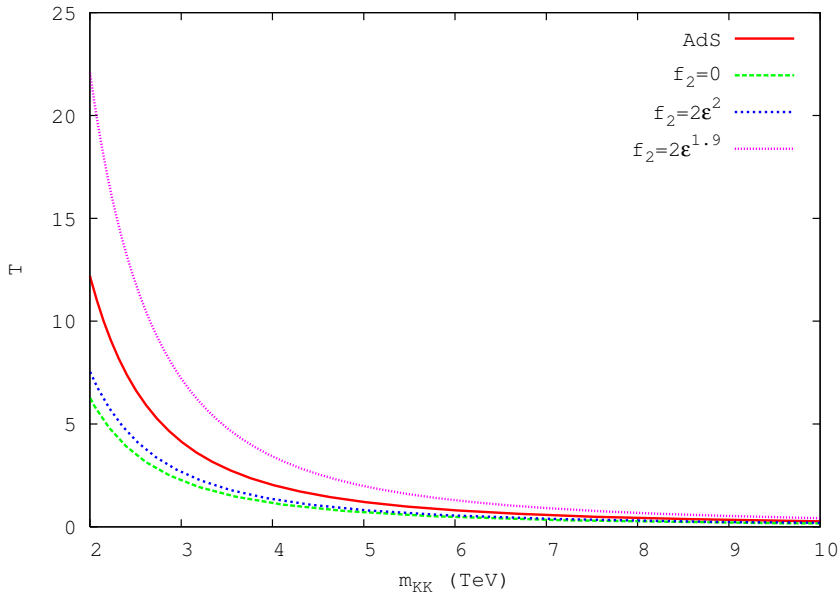

Figure 12: The Peskin-Takeuchi parameter T for different geometries and different values of $m_{\rm KK}$.

where the effective four-dimensional couplings are given by the overlap

$$g^{L,R}_{nm,q} = g_5 \int dr f^{1/2} \chi^{L,R}_n \chi^{L,R}_m a_q. \tag{4.3}$$

Electroweak precision data come mostly from the physics of lighter fermions. As discussed above and in [38, 41], lighter fermions have small coupling to the Higgs and so must be localized towards the UV. The localization is determined by the mass $M = \nu/R$ such that when $\nu \lesssim -\frac{1}{2}$, the wavefunction is localized towards the IR while for $\nu \gtrsim -\frac{1}{2}$, the wavefunction is localized towards the UV. In order to render the corrections to electroweak physics from the new physics oblique, we choose the normalization of the gauge field so that the coupling of the gauge fields to a standard model fermion localized entirely on the UV is equal to the standard model value. This requires that $a_0 = z_0 = w_0 = 1$ on the UV brane.

The presence of the Higgs on the TeV brane alters the boundary condition of the internal wavefunctions for the $W^\pm$ and the Z. Since the 4-d coupling constant is related to the overlap between the different KK wavefunctions, this effects a change in the coupling between the fermion and the boson. For light fermions which are localized towards the UV, this is a small effect since the modification to the gauge boson wavefunction is restricted to the tip in this normalization scheme (figure 13). For fermions localized towards the IR, the geometry can significantly affect the couplings to the gauge fields.

Precision measurements constrain the deviation of the coupling of the bottom to the Z relative to the standard model value to be less than $\sim 1\%$. Since the left-handed top must be somewhat localized toward the IR to give a large Yukawa, the left-handed bottom will also be localized toward the IR. Since this is where the Z wavefunction deviates from flat, this will give corrections to the coupling of the left-handed B to the Z. For a few

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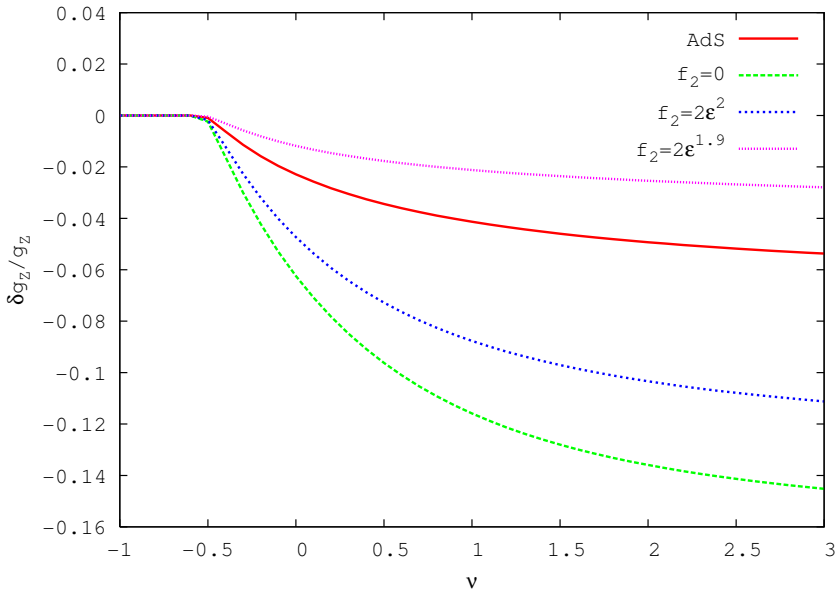

Figure 13: Relative correction of the coupling of a fermion to the Z almost zero mode due to the turning on of the Higgs vev. $\nu = M_f R$ sets the location of the fermion such that for $\nu \lesssim -\frac{1}{2}$ the fermion is localized toward the UV while for $\nu \gtrsim \frac{1}{2}$ it is localized toward the IR. Here, $R = 3.4 \times 10^{-16}$ TeV^{-1} so that for RS, $m_{\rm KK} = 3$ TeV. Smaller values of R naturally produce smaller effects.

Geometry	$1/R'$ (TeV)	$\nu_{\rm bottom}$	$\delta g_Z/G$
AdS	1.2	$-.42$	-6.7×10^{-3}
$f_2 = 0$	2.6	$-.26$	-1.2×10^{-2}
$f_2 = 2\epsilon^2$	2.1	$-.31$	-1.0×10^{-2}
$f_2 = 2\epsilon^{1.9}$	.76	$-.46$	-4.2×10^{-3}

Table 5: Relative correction of bottom coupling to the Z. Here $R' = R^2/r_{\rm tip}$ and $\nu_{\rm bottom} = M_f R$ is the bulk mass required for the left-handed top and bottom doublet to reproduce the 4-d mass for the top when the 5-d Yukawa coupling $\lambda_5 = 1$ and ν for the right-handed to p is equal to 1. The size of the geometry is such that $m_{\rm KK} = 3$ TeV.

geometries, this correction at the $m_{\rm KK} = 3$ TeV bound is shown in table 5. For any particular geometry, this bound on the coupling translates to a bound on the value for ν for the left-handed bottom. This is shown in figure 14. For only small values of f_2 does this provide a stronger bound on the geometry than S.

The higher KK modes of the gauge bosons will also couple to the standard model fermions. These couplings again depend on the localization of the fermions (figure 15).

5. Conclusions

We have studied the effects of IR modifications of the AdS metric from a mass gap. This metric, rather than becoming singular in the IR like AdS, levels off to a constant, breaking conformality. We find that in both 5-d and 10-d, the KK gauge boson spectrum is sensitive

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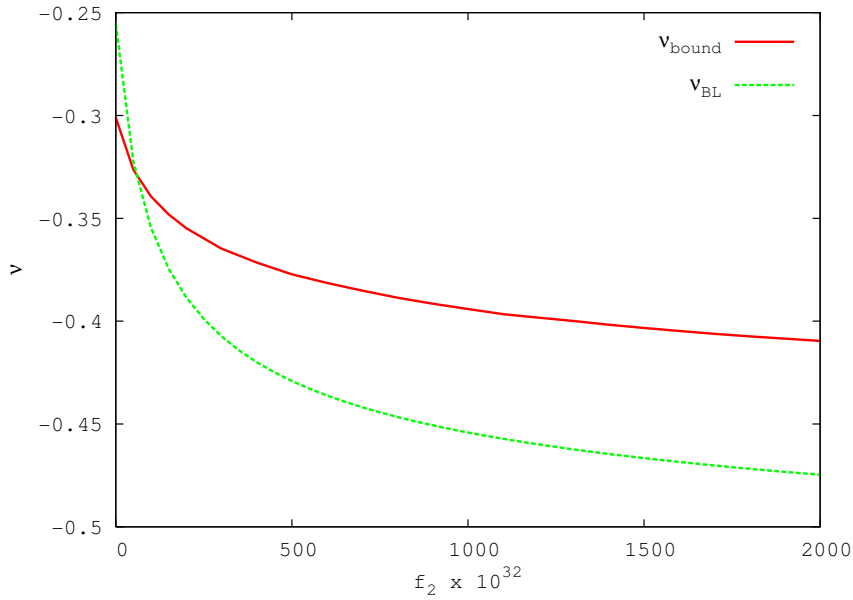

Figure 14: Upper bound on ν consistent with $\delta g_z/g_z < 1\%$ for different geometries. The value of ν for the left-handed bottom is also shown. This value is such that when $\lambda_5 = 1$ and ν for the right-handed top quark is 1, the 4-d Yukawa coupling agrees with the observed top mass.

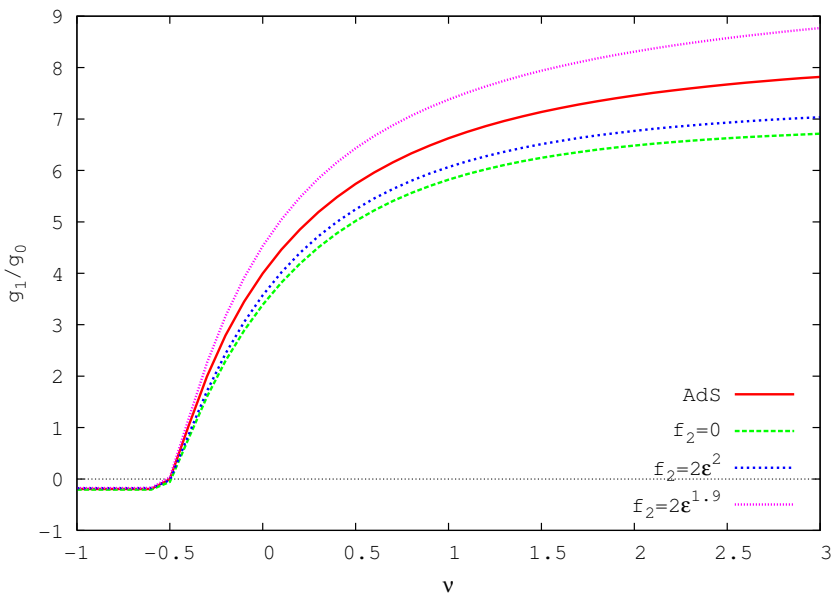

Figure 15: Coupling of a bulk fermion to the first KK mode relative to the zeroth KK mode. Because of the correction of the coupling to due a Higgs is small and the higher KK modes are almost entirely insensitive to the Higgs, this result applies whether or not the gauge boson couples to the Higgs vev. As expected, the difference in geometries are more important for heavy (IR localized) fermions than for light (UV localized) fermions.

to the form of the mass gap metric in the IR. The 10-d mass gap metric yields the most significant changes, with factor $\sim 5-10$ increase in the coupling of the gauge KK fields to

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IR localized fields, and factor ~ 2 changes in the ratio of the KK masses. With such large couplings to IR localized fields, even the lowest KK modes may be strongly coupled so that the standard perturbation theory analysis for production and decay no longer applies. While these effects are quite large, especially in the 10-d case, they are not as large as those found for the KK graviton spectrum studied in our earlier paper [23]. This is expected, since the KK graviton is more highly peaked in the IR than the KK gauge boson, so that it is more sensitive to IR modifications of the metric. The shift of the top coupling to the Z, $\delta g_Z/g_z$ in 5-d, also depends on the IR geometry by a factor $\sim 2-4$.

For the precision EW analysis, the results of the standard analysis are basically robust against 5-d deformations of the metric in the IR. There is *virtually no* change in S with the new metrics. T is modified from AdS, but not so dramatically that the need for custodial symmetry can be removed. We leave for future work an analysis of the effects of the 10-d metric on S and T, since the result with the Higgs is sensitive to the way that the five extra compact dimensions are integrated out.

Remarkable advances in string phenomenology this decade has brought us closer to the ambitious goal of connecting string theory to data. Hopefully, the LHC may allow us to probe not only the mechanism of electroweak symmetry breaking, but also the geometry of string compactifications. The present work is a modest step towards this goal.

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