

CALTECH FAINT GALAXY REDSHIFT SURVEY. XI. THE MERGER RATE
TO REDSHIFT 1 FROM KINEMATIC PAIRSR. G. CARLBERG,^{1,2} JUDITH G. COHEN,³ D. R. PATTON,^{1,2} ROGER BLANDFORD,⁴ DAVID W. HOGG,^{5,6} H. K. C. YEE,^{1,2}
S. L. MORRIS,^{1,7} H. LIN,^{1,6,8} PATRICK B. HALL,^{1,2} M. SAWICKI,^{1,9} GREGORY D. WIRTH,^{1,10} LENNOX L. COWIE,^{11,12}
ESTHER HU,^{11,12} AND ANTOINETTE SONGAILA^{11,12}*Received 1999 December 7; accepted 2000 February 1; published 2000 March 3*

ABSTRACT

The rate of mass accumulation due to galaxy merging depends on the mass, density, and velocity distribution of galaxies in the near neighborhood of a host galaxy. The fractional luminosity in kinematic pairs combines all of these effects in a single estimator that is relatively insensitive to population evolution. Here we use a k -corrected and evolution-compensated volume-limited sample having an R -band absolute magnitude of $M_r^{k,e} \leq -19.8 + 5 \log h$ mag drawing about 300 redshifts from the Caltech Faint Galaxy Redshift Survey and 3000 from the Canadian Network for Observational Cosmology field galaxy survey to measure the rate and redshift evolution of merging. The combined sample has an approximately constant comoving number and luminosity density from redshift 0.1 to 1.1 ($\Omega_M = 0.2$, $\Omega_\Lambda = 0.8$); hence, any merger evolution will be dominated by correlation and velocity evolution, not density evolution. We identify kinematic pairs with projected separations less than either 50 or 100 h^{-1} kpc and rest-frame velocity differences of less than 1000 km s^{-1} . The fractional luminosity in pairs is modeled as $f_L(\Delta v, r_p, M_r^{k,e})(1+z)^{m_L}$, where $[f_L, m_L]$ are $[0.14 \pm 0.07, 0 \pm 1.4]$ and $[0.37 \pm 0.7, 0.1 \pm 0.5]$ for $r_p \leq 50$ and 100 h^{-1} kpc, respectively ($\Omega_M = 0.2$, $\Omega_\Lambda = 0.8$). The value of m_L is about 0.6 larger if $\Lambda = 0$. To convert these redshift-space statistics to a merger rate, we use the data to derive a conversion factor to a physical space pair density, a merger probability, and a mean in-spiral time. The resulting mass accretion rate per galaxy ($M_1, M_2 \geq 0.2M_*$) is $0.02 \pm 0.01(1+z)^{0.1 \pm 0.5} M_* \text{ Gyr}^{-1}$. Present-day high-luminosity galaxies therefore have accreted approximately $0.15M_*$ of their mass over the approximately 7 Gyr to redshift 1. Since merging is likely only weakly dependent on the host mass, the fractional effect, $\delta M/M \approx 0.15M_*/M$, is dramatic for lower mass galaxies but is, on the average, effectively perturbative for galaxies above $1M_*$.

Subject headings: galaxies: evolution — large-scale structure of universe

1. INTRODUCTION

Merging is a fundamental mode of the stellar mass addition to galaxies. Moreover, merging brings in new gas and creates gravitational disturbances that enhance star formation or that fuel nuclear black holes. The general process of the substructure infall may be the rate-fixing process for the buildup of a galaxy's stars, and consequently this process may largely regulate its luminosity history. Gravitational forces on relatively large scales dominate merger dynamics, and this allows the direct observation of the mechanism, although with the considerable complication that dark matter dominates the mass. N -body simulations (Toomre & Toomre 1972; Barnes & Hernquist 1992) illustrate the detailed orbital evolution, the morphological dis-

turbances, and the eventual outcomes of the encounters of pairs of galaxies.

The purpose of this Letter is to estimate the rate of mass gain per galaxy due to mergers over the redshift 0–1 interval. Our primary statistic is the fractional luminosity in close kinematic pairs, which is readily related to N -body simulations and sidesteps a morphological interpretation. This approach provides a clear sample definition that is closely connected to the large-scale dynamics of merging. In common with all merger estimates, it requires an estimate of the fraction of the pairs that will merge and a mean time to merger.

The number of kinematic pairs is proportional to the volume integral at small scales of the product of the two-point correlation function ξ and the luminosity function (LF). The high-luminosity galaxies appear to be evolving purely in luminosity (Lilly et al. 1995; Lin et al. 1999), which can be easily compensated. The measured evolution of ξ suggests that the density of physical pairs should not vary much with redshift, $(1+z)^{0 \pm 1}$ (LeFèvre et al. 1996; Carlberg et al. 1997, 2000). This inference is in notable contrast to the pair counts or morphological-typing approaches to merger estimation (Zepf & Koo 1989; Carlberg, Pritchet, & Infante 1994; Yee & Ellingson 1995; Patton et al. 1997; LeFèvre et al. 2000), which suggests that the merging rate by number varies as $(1+z)^{3 \pm 1}$. The *Hubble Space Telescope* photometric pairs with no redshift information lead us to a dependence $(1+z)^{1.2 \pm 0.4}$ (Neuschaefer et al. 1997).

In the next section, we combine the Caltech Faint Galaxy Redshift Survey (CFGRS) with the Canadian Network for Observational Cosmology field galaxy survey (CNOC2), from which we construct evolution-compensated, volume-limited

¹ Visiting Astronomer, Canada-France-Hawaii Telescope, which is operated by the National Research Council of Canada, le Centre National de Recherche Scientifique, and the University of Hawaii.

² Department of Astronomy, University of Toronto, Toronto, ONT, M5S 3H8, Canada.

³ Department of Astronomy, Caltech, MS 105-24, Pasadena, CA 91125.

⁴ Theoretical Astrophysics, Caltech, MS 130-33, Pasadena, CA 91125.

⁵ Institute for Advanced Study, Olden Lane, Princeton, NJ 08540.

⁶ Hubble Fellow.

⁷ Dominion Astrophysical Observatory, Herzberg Institute of Astrophysics, National Research Council of Canada, 5071 West Saanich Road, Victoria, BC, V8X 4M6, Canada.

⁸ Steward Observatory, University of Arizona, Tucson, AZ 85721.

⁹ Caltech, MS 320-47, Pasadena, CA 91125.

¹⁰ W. H. Keck Observatory, 65-1120 Mamalahoa Highway, Kamuela, HI 96743.

¹¹ Visiting Astronomer, W. M. Keck Observatory, jointly operated by the California Institute of Technology and the University of California.

¹² Institute for Astronomy, University of Hawaii, 2680 Woodlawn Drive, Honolulu, HI 97822.

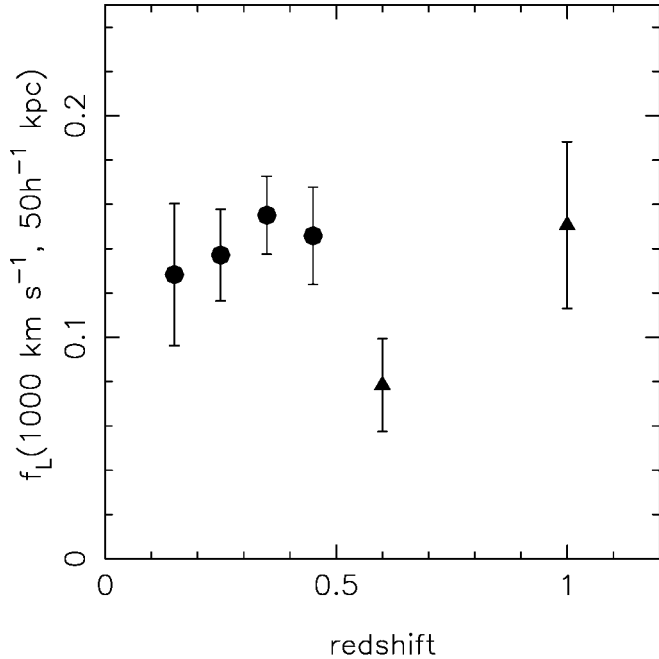


FIG. 1.—Fraction of the sample luminosity in pairs with $\Delta v \leq 1000 \text{ km s}^{-1}$ and $5 \leq r_p \leq 50 h^{-1} \text{ kpc}$ as a function of redshift. The octagons are from the CNOC2 sample, and the triangles are from the CFGRS.

subsamples. In § 3, we measure the fractional luminosity in 50 and $100 h^{-1} \text{ kpc}$ companions as a function of redshift. The CNOC2 sample is used in § 4 in order to relate this wide-pair sample to a close-pair sample that is more securely converted into a mass merger rate. In § 5, we discuss our conclusions. We use $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_M = 0.2$ in open and flat cosmologies.

2. THE CFGRS AND CNOC2 VOLUME-LIMITED SAMPLES

The CFGRS sample of the Hubble Deep Field plus flanking fields is discussed in detail elsewhere (Hogg et al. 2000; Cohen et al. 2000). We use the high-coverage subsample lying within a $240''$ radius circle, with a center located at $12^{\text{h}}36^{\text{m}}50^{\text{s}}$, $62^{\circ}12'55''$ (J2000). The computed magnitude selection function $s(m_R)$ (in Cousins R) is accurately approximated as a constant 90% spectroscopic completeness for $m_R < 22.8 \text{ mag}$ with a linear decline to 19% at $m_R < 23.4 \text{ mag}$, which is our sample limit. The magnitude weight is $1/s(m_R)$. The CFGRS k -corrections and evolution compensation are approximated here as $k(z) = Kz \text{ mag}$ from the tables of Poggianti (1997). For galaxies that Cohen et al. (2000) classify as “E” (emission), $K = 1.0$; “A” types have $K = 2.0$, and all other types have $K = 1.7$.

The CNOC2 selection weights and k -corrections are discussed in Yee et al. (2000). The evolution of the luminosity function is approximated as a uniform $M_*(z) = M_* - Qz$, with $Q \approx 1$ (Lin et al. 1999), which we use over the entire CNOC2–CFGRS redshift range.

The kinematic pair fraction is directly proportional to the mean density of the sample and is therefore sensitively dependent on correcting for incompleteness (Patton et al. 2000b). The most straightforward approach is to impose a strict volume limit. For our primary sample, we will limit the CFGRS and the CNOC2 samples to $M_R^{k,e} = -19.8 + 5 \log h \text{ mag}$, which

yields volume-limited samples of about 300 CFGRS galaxies between redshifts 0.3 and 1.1 and 3000 CNOC2 galaxies between redshifts 0.1 and 0.5. The volume density of the sample is approximately constant at $1.2 \times 10^{-2} h^3 \text{ Mpc}^{-3}$ over the entire redshift range for $\Omega_\Lambda = 0.8$ but rises roughly as $(1+z)^{0.8}$ for $\Omega_\Lambda = 0$. Both the CFGRS and the CNOC2 surveys are multiply masked, which minimizes the effects of slit crowding; however, there is still a measurable pair selection function. The CNOC2 catalog has about a 20% deficiency of close angular pairs. We model the measured angular pair selection weight as $w(\theta) = [1 + a_s \tanh(\theta/\theta_s)]^{-1}$, where $[a_s, \theta_s]$ is $[0.5, 5'']$ for the CFGRS sample and $[-0.3, 10'']$ for the CNOC2 sample, with typical pair corrections being 10%.

3. THE PAIR FRACTIONAL LUMINOSITY FRACTION

The preferred choice of pair statistic depends on the application (Patton et al. 2000b). Here we are primarily interested in the impact of merging on the increase of galaxy mass, for which the k -corrected, evolution-compensated R luminosity is a stand-in. The rate of merging per galaxy depends on the density of galaxies in the near neighborhood and their velocity distribution. As a practical redshift-space estimator, we compute the fractional luminosity in close kinematic pairs,

$$f_L(z|\Delta v^{\text{max}}, r_p^{\text{max}}, M_R^{k,e}) = \frac{\sum_{ij} \sum_{i \neq j, < \Delta v^{\text{max}}, < r_p^{\text{max}}} w_i w_j w(\theta_{ij}) L_i}{\sum_j w_j L_j}, \quad (1)$$

where the weights w_i allow for the magnitude selection function. Note that the ij and ji pairs are both counted. The ratio has the benefit of being fairly stable for different luminosity limits, self-normalizing for luminosity evolution, and identical to a mass ratio for a fixed M/L population. For an unperturbed pair luminosity function, it is mathematically identical to the N_c of Patton et al. (2000b), although constructed out of somewhat different quantities. The two parameters Δv^{max} and r_p^{max} are chosen on the basis of merger dynamics and the characteristics of the sample. The rate of mass increase per galaxy is calculated from this statistical estimator using a knowledge of merger dynamics and the measured correlations and kinematics of galaxy pairs in the sample.

The mean fractional luminosity, based on 18 CFGRS pairs and 91 CNOC2 pairs, with $\Delta v \leq 1000 \text{ km s}^{-1}$ and $5 \leq r_p \leq 50 h^{-1} \text{ kpc}$ pairs is displayed in Figure 1. These kinematic separation parameters are larger than is suitable for reliably identifying “soon-to-merge” pairs. However, they provide a statistically robust connection to those pairs and take into account the lower velocity precision and sample size of the CFGRS relative to the CNOC2. The errors are computed from the pair counts, $n_p^{-1/2}$. The measurements of $f_L(z)$ in Figure 1 are fitted to $f_L(\Delta v, r_p, M_R^{k,e})(1+z)^{m_L}$, and we find $[f_L, m_L]$ of $[0.14 \pm 0.07, 0 \pm 1.4]$ for $r_p \leq 50 h^{-1} \text{ kpc}$ pairs and $[0.37 \pm 0.7, 0.1 \pm 0.5]$ for $r_p \leq 100 h^{-1} \text{ kpc}$ pairs, both for $\Omega_M = 0.2$ and $\Omega_\Lambda = 0.8$. The increase with r_p of f_L is consistent with a $\gamma = 1.8$ two-point correlation function. If $\Omega_M = 0.3$ and $\Omega_\Lambda = 0.7$, then m_L at $100 h^{-1} \text{ kpc}$ rises by about 0.05, whereas if $\Omega_M = 0.2$ and $\Omega_\Lambda = 0.0$, then $m_L = 0.50$. The increase is largely the result of the rise in the implied comoving sample density over this redshift range.

The merger probability of a kinematic pair depends sensitively on the pairwise velocity dispersion σ_{12} of galaxies. The model pairwise velocity distribution is computed as the convolution of the correlation function with the distribution of

random velocities. The infall velocities are negligible at these small separations, and we will assume that the peculiar velocities are drawn from a Gaussian distribution. The measured fraction of the CNOC2 pair sample with velocities smaller than some Δv , normalized to the value at 1000 km s^{-1} , is displayed in Figure 2. The $50 h^{-1}$ kpc-wide pairs limited at -19.5 mag are plotted as open squares, the $20 h^{-1}$ kpc pairs limited at -18.5 and -19.5 mag are plotted as filled octagons and filled diamonds, respectively. The upper curve assumes that σ_{12} is 200 km s^{-1} , and the lower one assumes that σ_{12} is 300 km s^{-1} ; these values approximately span the data.

4. MERGER RATE ESTIMATION

The merger rate is best estimated from very close kinematic pairs, $20 h^{-1}$ kpc or less, about half of which are physically close and have significant morphological disturbance (Patton et al. 2000b). However, the fraction of galaxies in such close pairs is about 1%, giving poor statistics. Since the number of pairs increases smoothly as $r^{3-\gamma}$, where γ is the slope of the small-scale correlation function, we can use pairs at somewhat larger separations as statistically representative of the close pairs; however, we prefer to stay within the radius of virialized material around a galaxy over our redshift range, which is no larger than about $100 h^{-1}$ kpc. The mass accretion rate from major mergers is therefore estimated as

$$\mathcal{R}_M = \frac{1}{2} f_L(\Delta v, r_p, z) C_{zs}(\Delta v, \gamma) F(v < v_{\text{mg}}) \langle M \rangle T_{\text{mg}}^{-1}(z, r_p), \quad (2)$$

where the factor of $\frac{1}{2}$ allows for the double counting of pairs, $C_{zs}(\Delta v, \gamma)$ converts from redshift-space to real-space pairs, F gives the fraction of the pairs that will merge in the next T_{mg} (the “last orbit” in-spiral time from r_p), and $\langle M \rangle$ is the mean incoming mass as estimated by assuming a constant M/L . For the relatively massive galaxies considered here, the dynamical friction is so strong that it has a more violent relaxation with little timescale dependence on the masses. The measured ratio of the numbers of 50 and $100 h^{-1}$ kpc pairs to that of $20 h^{-1}$ kpc pairs in the CNOC2 sample is 3.8 ± 1.0 and 9.4 ± 3.0 , respectively, in accord with the expectation of a growth as $r_p^{3-\gamma}$ with an inner cutoff of $5 h^{-1}$ kpc.

Not all kinematic pairs are close in physical space. The relation between the kinematic pairs closer than r_p and Δv and the pairs with a three-dimensional physical distance r_p is readily evaluated by integrating the velocity-convolved correlation function over velocity and projected radius and by taking the ratio of the three-dimensional integral of the correlation function. We find that $C_{zs} = 0.54$ for $\Delta v = 1000 \text{ km s}^{-1}$ and $\gamma = 1.8$. There is support for this value on the basis of morphological classification, as tested in Patton et al. (2000a), where about half of the kinematic pairs exhibited strong tidal features.

A key part of the rate calculation is the fraction of physically close pairs that is at sufficiently low velocity so as to merge. It is clear that many galaxies will have close encounters that do not lead to immediate mergers, although mergers could of course occur on subsequent orbital passages. The key quantity that we need is the ratio of the critical velocity for merging (v_{mg}) to σ_{12} . The timescale for close pairs to merge is much shorter than the time over which morphological disturbances are clearly evident, by nearly an order of magnitude (Barnes & Hernquist 1992; Mihos & Hernquist 1996; Dubinski, Mihos, & Hernquist 1999). This is one of our reasons for preferring

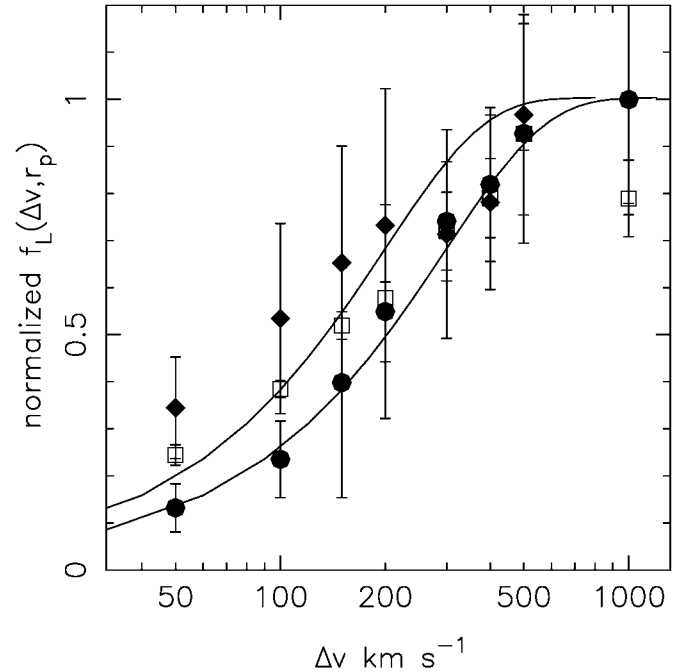


FIG. 2.—Velocity distribution function for CNOC2 galaxy pairs. The filled symbols are for $20 h^{-1}$ kpc pairs limited at M_k^e of -19.5 mag (diamonds) and -18.5 mag (octagons), and the open squares are for $50 h^{-1}$ kpc pairs limited at -19.5 mag. The lines are the distributions expected for an $r^{-1.8}$ correlation function convolved with Gaussian velocity distribution functions of 200 (upper line) and 300 (lower line) km s^{-1} .

kinematic pairs as a merger estimator. The simulation results indicate that the time to merge is, on the average, roughly that of a “half-circle” orbit, which, at an r_p of $20 h^{-1}$ kpc at a velocity of 200 km s^{-1} , is close to 0.3 Gyr. A straight-line orbit with instantaneous merging would merge in about 0.1 Gyr, although that is not likely to be representative.

To compute the merger probability $F(< v_{\text{mg}})$, we need to know the maximal velocity to merge v_{mg} at a physical separation of $20 h^{-1}$ kpc for a typical M_* galaxy. A not very useful lower bound is fixed by the Keplerian escape velocity at $20 h^{-1}$ kpc, $v_c \sqrt{2} \text{ km s}^{-1}$, where the circular velocity is approximately 200 km s^{-1} . An upper bound to v_{mg} is the velocity that an object would have if it is captured in a galaxy’s extended dark halo at the virialization radius and orbits to $20 h^{-1}$ kpc with no dynamical friction. The virialization radius is approximately at the radius where the mean interior overdensity is $200\rho_c$, implying $r_{200} = v_c/(10H_0)$ or about $200 h^{-1}$ kpc for our typical galaxy. The largest possible apogalactic velocity at r_{200} is v_c , which leads to an undissipated velocity at $20 h^{-1}$ kpc of $2.37v_c$. Using $\sigma_{12} = 200$ (300) km s^{-1} at $20 h^{-1}$ kpc, we find that the fraction of all physical pairs that merge in one T_{mg} is about 0.40 (0.16). Therefore, we will normalize to a merger probability of 0.3 , noting the 50% or so uncertainty.

The absolute magnitude limit of $-19.8 + 5 \log h$ mag corresponds to $L \geq 0.5L_*$, which contains about 58% of the luminosity for the mean CNOC2 LF, $M_* = -20.4$ and $\alpha = -1.2$. To make our merger rate inclusive of major mergers, we normalize to $L \geq 0.2L_*$, which includes 85% of the luminosity. Within the current statistical accuracy, the paired and field galaxies have identical LFs. On the basis of N -body experiments (Barnes & Hernquist 1992), galaxies with masses greater than about $0.2M_*$ will merge in approximately one orbital time.

On the basis of these considerations, we find that the rate of the mass accumulation of galaxies with luminosities of $0.2M_*$ and above is

$$\mathcal{R}_M = (0.02 \pm 0.01)M_*(1+z)^{0.1 \pm 0.5} \times \frac{F(v_{\text{mg}}/\sigma_{12})}{0.3} \frac{0.3 \text{ Gyr}}{T_{\text{mg}}} \text{ Gyr}^{-1}, \quad (3)$$

where we have adopted the $100 h^{-1} \text{ kpc } m_L$ value for a flat, low-density cosmology and explicitly assumed that the velocity and timescale factors do not vary over this redshift range, as expected at these small scales in a low- Ω universe (Colin, Carlberg, & Couchman 1997). There is direct evidence that once the luminous galaxies are evolution-compensated, they exhibit no evolution in their circular velocities (Vogt et al. 1997; Mallen-Ornelas et al. 1999).

5. DISCUSSION AND CONCLUSIONS

Our main observational result is that for galaxies with $M_R^{k,e} \leq -19.8 + 5 \log h \text{ mag}$, the fraction of galaxy luminosity in $50 h^{-1} \text{ kpc}$ -wide kinematic pairs is about 14%, with no noticeable redshift dependence over the redshift range of 0–1. This implies an integrated mass accretion rate of about 2% of $L_* \text{ Gyr}^{-1} \text{ galaxy}^{-1}$ for merging galaxies that have $L \geq 0.2L_*$.

Our rate is uncertain at about the level of a factor of 2 because of the uncertainty in the dynamical details of merging for our sample definitions. This merger rate implies a 15% mass increase in an M_* galaxy since redshift 1. If the correlations of lower luminosity galaxies are only somewhat weaker than these (Carlberg et al. 1998), then the same $0.15M_*$ merged-in mass causes a 50% mass increase in a $0.3M_*$ galaxy.

There are several issues that require further investigation. First, the rate of merging of similarly selected kinematic pairs should be studied in appropriately matched N -body experiments to better determine the orbital timescales. Second, the absence of a redshift dependence of σ_{12} and v_{mg} needs to be observationally checked. Third, the connection between close kinematic pairs and morphologically disturbed galaxies needs to be better understood at high redshift; this connection does conform to the kinematic pair predictions at low redshift (Patton et al. 2000b).

This research was supported by NSERC and NRC of Canada. H. L. and D. W. H. acknowledge support provided by NASA through Hubble Fellowship grants HF-01110.01-98A and HF-01093.01-97A, respectively, awarded by the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy, Inc., for NASA under contract NAS5-26555.

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