Steady State and Transient Electromagnetic Coupling Through Slabs

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Abstract—The problem of electromagnetic transmission through a slab where transmitting and receiving antennas are at finite distances from the slab is considered. The mathematical formulation of the problem is quite general. A detailed solution is presented for the case of a highly conducting slab exposed to sinusoidal and transient excitations. A discussion is given of the conditions under which measurements with the source and receiver at finite distances are equivalent to the same measurements with plane wave excitation.

I. INTRODUCTION

ONE OF THE simplest conceivable ways for determing the electromagnetic properties of materials is to measure the electromagnetic field transmitted through a slab of the material under test. The corresponding mathematical model consists of an infinite slab with transmitting and receiving antennas placed on opposite sides of the slab. The model provides a reasonably good approximation to the real situation of a slab of finite extent when the distance between transmitting and receiving points is small compared to the transverse slab dimensions.

Measurements can be made in the sinusoidal or the transient regime. For instance, MIL standards for evaluating the shielding effectiveness of materials [1] require that transmission measurements be made in the steady state at prescribed frequencies and then in a pulsed regime using wire and loop antennas placed at prescribed distances from the slab of shielding material. Although these standards are useful for *relative* comparisons, a fundamental question remains unanswered: does the measurement depend *only* on the electromagnetic properties of the slab (and on its thickness), or does it depend *also* on antenna type and orientation, antenna distance, and (for transient measurements) on transmitted waveform?

A crude but simple method for studying (or, at least, having an estimate of) the field coupled to the inside of an enclosure is to consider the transmission through a slab, provided the enclosure is large in terms of the incident wavelength. The slab may be perforated, inhomogeneous, or described by stochastic parameters, the last case being relevant to near-millimeter propagation through aerosols used for camouflage tactics. In electromagnetic pulse (EMP) experiments it is customary to simulate the EMP plane wave signal by using rather sophisticated antennas and guiding devices [2], [3]. An attractive alternative to this approach can result from an understanding and exploration of the role played by localized sources at finite distances from the test object.

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The objectives of this paper are to reconsider the problem of steady-state and transient coupling through a slab with transmitting and receiving antennas located at finite distances from the slab; to cast the problem in an elegant form; and to show that, at least in the case of a highly conductive slab, simple analytical solutions to the problem can be obtained. An important result of the paper is the determination of antenna positions and (in the transient regime) of incident waveforms that will yield a transmitted field practically the same as that produced by plane wave excitation.

Transmission through highly conductive slabs is certainly not a new problem. For plane wave steady-state excitation, transmission line techniques can easily be applied [4]. For pulsed plane wave excitations, the solution is also available [5]. The situation is much less satisfactory for the case we want to study. It is not the aim of this paper to provide a full bibliography on this subject (for a more complete bibliography see [6]). We note only that the first attempt to solve this problem was made in 1936 [7] by accommodating the classical results of Maxwell on eddy currents and thin shields to the case of two coaxial loops separated by a plane conducting sheet. Early studies on antenna coupling through plane shields were based on low-frequency [8], [9] or quasi-static [10] approximations, were mainly relative to loop excitation [8]-[10], and required numerical computation [8]-[12] of integral expressions for the transmitted field. Although the validity of the simple transmission line theory [4] for antennas at finite distances from the shield, or shields of finite extent, has been questioned [13], it appears that all expressions derived in the referenced literature resemble Schelkunoff's formulas [14].

Due to the symmetry of the problem it can easily be surmised that plane wave expansion techniques provide a powerful tool of analysis for an arbitrary type of excitation of an infinite slab. These techniques have been recently applied [14], [15] to the case of electric or magnetic dipole excitation in parallel (dipoles parallel to the slab) or coaxial (dipoles normal to the slab) configuration, by computing the transmitted field through the use of fast Fourier numerical programs. In this paper we shall use the same approach. However, we will show that, although the Fourier transformation of the fields is a logical intermediate step of the analysis, it is not needed in the final formulation of the solution. Indeed, the solution can be conveniently expressed in terms of a convolution integral, wherein the presence of the slab is described by an appropriate transfer function. Then, at least for antennas in coaxial configuration, the convolution integral can be analytically evaluated both in steady-state and transient regimes, and no numerical work is necessary. Inspection of the solution allows us to answer the original question about the influence of the finite antenna separation on measurements. After all the mathematical machinery has been worked out and simple, physically sound, understandable results are obtained, a discussion of the final results is presented in Section VI.

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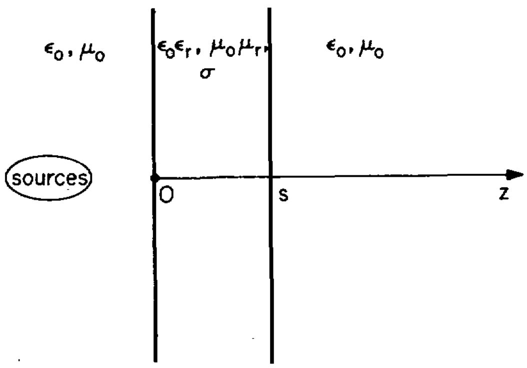

Fig. 1. Geometry of problem.

II. CIRCUIT-LIKE ANALYSIS OF ELECTROMAGNETIC TRANSMISSION THROUGH A SLAB

With reference to Fig. 1, let us consider an infinite slab of thickness s and characterized, in the frequency domain, by permittivity $\epsilon = \epsilon_0\epsilon_r$, permeability $\mu = \mu_0\mu_r$, and conductivity σ. We want to compute the field $\mathbf{E}^t$, $\mathbf{H}^t$ transmitted at $z > s$ along the z axis when the incident field $\mathbf{E}^i$, $\mathbf{H}^i$, i.e., the field produced by the sources when the slab is removed, is known at $z = 0$. For this purpose it is convenient to expand the incident field in a plane wave set, since the interaction of individual plane wave components with the slab can be conveniently taken into account.

Accordingly, let $H_z^i(x, y, 0)$, $E_z^i(x, y, 0)$ be the z components of the field incident on the slab surface, with an assumed time dependence $\exp(j\omega t)$. The corresponding spectral components $h_z^i(u, v)$, $e_z^i(u, v)$ are given, at $z = 0$, by

$$h_z^i(u, v) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy\, H_z^i(x, y, 0) \cdot \exp(jux + jvy) \tag{2.1}$$

$$e_z^i(u, v) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy\, E_z^i(x, y, 0) \cdot \exp(jux + jvy). \tag{2.2}$$

At $z = s$, i.e., at the output of the slab, the spectral components $h_z^t(u, v)$, $e_z^t(u, v)$ will be linearly related to the incident components (2.1) and (2.2) in the case of a slab made of a linear material. Hence,

$$h_z^t(u, v) = t_H(u, v) h_z^i(u, v) \tag{2.3}$$

$$e_z^t(u, v) = t_E(u, v) e_z^i(u, v). \tag{2.4}$$

The transfer coefficients t_H, t_E can be easily computed for a homogeneous isotropic slab by noting that the transverse spectral components $\mathbf{h}_t(u, v)$, $\mathbf{e}_t(u; v)$ are related to the longitudinal ones $h_z(u, v)$, $e_z(u, v)$ via the following relations:

$$\mathbf{h}_t = \frac{\omega\epsilon e_z \kappa\times\hat{z} \mp w h_z \hat{z}\times(\kappa\times\hat{z})}{u^2 + v^2} \tag{2.5}$$

$$\mathbf{e}_t = \frac{-\omega\mu h_z \kappa\times\hat{z} \mp w e_z \hat{z}\times(\kappa\times\hat{z})}{u^2 + v^2} \tag{2.6}$$

wherein $\kappa = u\hat{x} + v\hat{y} + w\hat{z}$ and is the propagation vector referring to a Cartesian system of unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$, and the upper (lower) signs refer to waves propagating in the positive (negative) direction of the z axis. Equations (2.5) and (2.6) represent the total spectral field as a superposition of TE ($e_z = 0$) and TM ($h_z = 0$) parts. With the medium being identical at both sides of the slab, it is then evident that t_H coincides with the usual slab transmission coefficient for TE plane wave incidence and, similarly, t_E is the same as the slab transmission coefficient for TM plane wave incidence. Letting

$$w = \sqrt{\kappa^2 - (u^2 + v^2)},$$

$$w_s = \sqrt{\kappa^2\left(\epsilon_r + \frac{\sigma}{j\omega\epsilon_0}\right)\mu_r - (u^2 + v^2)}, \tag{2.7}$$

$$\gamma_H = \frac{w_s}{\mu_r w}, \qquad \gamma_E = \frac{j\omega\epsilon_0}{\sigma + j\omega\epsilon_0\epsilon_r}\,\frac{w_s}{w}, \tag{2.8}$$

we have

$$t(u, v) = \frac{4}{(1+\gamma)^2}\,\frac{\exp(-jw_s s)}{1 - \left(\dfrac{1-\gamma}{1+\gamma}\right)^2 \exp(-2jw_s s)} \tag{2.9}$$

wherein γ may be taken equal to γ_H or γ_E in order to obtain t_H or t_E, respectively, and $\kappa = \omega\sqrt{\epsilon_0\mu_0}$.

The spectral components h_z, e_z at any $z > s$ are equal to the corresponding values (2.3) and (2.4) at $z = s$ times the plane wave transfer function $\exp[-jw(z - s)]$. Accordingly, the z component $F_z^i(x, y, z)$ of the field transmitted at any arbitrary abscissa $z > s$ will be expressed in terms of the double Fourier integral

$$F_z^t(x, y, z) = \int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} dv\, f_z^i(u, v) t(u, v) \cdot \exp[-jw(z-s)] \cdot \exp(-jux - jvy) \tag{2.10}$$

wherein f_z^i may be taken equal to h_z^i or e_z^i and, correspondingly, the values of t_H or t_E should be used.

On the other hand, the spectral representation of the z components of the incident field (the slab is now removed) at any abscissa z is obviously the following:

$$F_z^i(x, y, z) = \int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} dv\, f_z^i(u, v) \exp(-jwz) \cdot \exp(-jux - jvy). \tag{2.11}$$

Comparison of (2.10) and (2.11) shows that the transmitted field can be computed as the double convolution of the incident field and the double Fourier transform of $t(u, v)$ $\exp(jws)$, hence

$$F_z^t(x, y, z) = \frac{1}{(2\pi)^2}\int_{-\infty}^{+\infty} dx' \int_{-\infty}^{+\infty} dy'\, F_z^i(x', y', z) \cdot T(x - x', y - y') \tag{2.12}$$

$$T(x, y) = \int_{-\infty}^{+\infty} du \int_{-\infty}^{+\infty} dv\, t(u, v) \exp(jws) \cdot \exp(-jux - jvy). \tag{2.13}$$

In the words of system theory F^t is identified with the output of a linear system described by the unit response function (2.13) and excited by the input F^i.

We further note that relations similar to (2.12) exist between the transmitted and incident transverse components of the field, as easily follows from (2.5) and (2.6). It is only necessary to decompose the incident field in its TE and TM parts and then to apply superposition.

III. THE AZIMUTHALLY SYMMETRIC CASE

A case of particular interest is obtained when the incident field does not depend on x and y separately but rather upon the transverse coordinate $\rho = \sqrt{x^2 + y^2}$. For instance, if the source is taken equal to an elementary electric or magnetic dipole parallel to the z axis at $P(0, 0, -d)$, then

$$F_z{}^i(x, y, z) = F_z{}^i(\rho, z) = -\frac{j\omega}{\kappa^2}[\kappa^2 \mathbf{A} + \nabla\nabla \cdot \mathbf{A}] \cdot \hat{z}$$

$$= -\frac{j\omega}{\kappa^2}\left[\kappa^2 A + \frac{\partial^2 A}{\partial z^2}\right] = \frac{j\omega}{\kappa^2}\frac{1}{\rho}\frac{\partial}{\partial \rho}\rho\frac{\partial A}{\partial \rho} \tag{3.1}$$

wherein

$$A(\rho, z) = C\frac{\exp(-j\kappa\sqrt{\rho^2 + (d+z)^2})}{\sqrt{\rho^2 + (d+z)^2}} \tag{3.2}$$

is an electric or magnetic vector potential, the source intensity being proportional to the constant C.

The integrals (2.12) and (2.13) can now be simplified by using the change of coordinates:

$$x = \rho\cos\phi, \quad y = \rho\sin\phi, \quad u = \xi\cos\psi, \quad v = \xi\sin\psi. \tag{3.3}$$

Accordingly,

$$T(x, y) = T(\rho) = \int_0^\infty \xi\, d\xi\, t(\xi) \exp(j\sqrt{\kappa^2 - \xi^2}s) \cdot \int_0^{2\pi} \exp[-j\rho\xi\cos(\psi - \phi)]\, d\psi$$

$$= 2\pi \int_0^\infty \xi\, d\xi\, J_0(\rho\xi) t(\xi) \exp(j\sqrt{\kappa^2 - \xi^2}s), \tag{3.4}$$

and the field transmitted on the axis is given by

$$F_z{}^t(0, 0, z) = \int_0^\infty \rho\, d\rho\, F_z{}^i(\rho, d+z) T(\rho)$$

$$= \int_0^\infty \xi\, d\xi \exp(j\sqrt{\kappa^2 - \xi^2}s) t(\xi) \cdot \int_0^\infty \rho\, d\rho\, F_z{}^i(\rho, d+z) J_0(\xi\rho) \tag{3.5}$$

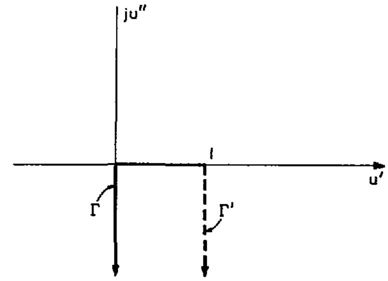

Fig. 2. Integration path in complex u plane.

wherein the order of integration has been reversed. Upon substitution of (3.1) in (3.5) the inner integral can be evaluated by repeated integration by parts as follows:

$$\int_0^\infty \rho\, d\rho\, F^i(\rho, d+z) J_0(\rho\xi)$$

$$= -\frac{j\omega C}{\kappa^2}\xi^2 \int_0^\infty \frac{\rho\exp(-j\kappa\sqrt{\rho^2 + (d+z)^2})}{\sqrt{\rho^2 + (d+z)^2}} J_0(\rho\xi)\, d\rho$$

$$= -\frac{\omega C}{\kappa^2}\xi^2 \frac{\exp[-j\sqrt{\kappa^2 - \xi^2}(d+z)]}{\sqrt{\kappa^2 - \xi^2}}, \tag{3.6}$$

the last expression stemming from a known Fourier–Bessel transform [16]. Note that $\sqrt{\kappa^2 - \xi^2} = -j\sqrt{\xi^2 - \kappa^2}$ for $\xi^2 > \kappa^2$ and that we have implicitly assumed $k \neq 0$ in this section.

The formal expression for the z component of the field transmitted through the slab is now the following:

$$F_z{}^t(0, 0, z) = \omega C\kappa \int_\Gamma (1 - u^2) t(u) \exp(-j\kappa l u)\, du \tag{3.7}$$

wherein $l = d - s + z$, the integration path Γ is depicted in Fig. 2, and the substitution $\kappa^2 - \xi^2 = \kappa^2 u^2$ has been used.

IV. THE CASE OF AN ELECTRIC PLANE SHIELD—STEADY-STATE EXCITATION

A case particularly interesting for applications is obtained when $\mu_r = 1$, $\sigma \gg \omega\epsilon_0\epsilon_r$, i.e., when a highly conducting nonmagnetic slab is used as a shielding screen. As already noted in Section I, this is an important configuration in shielding theory and practice. The solution to this problem is available in numerical form [14], [15] for prescribed sinusoidal time variation and arbitrary spatial dependence for the fields, and in analytical form [5] for prescribed plane wave excitation and arbitrary time variation.

The case of a magnetic dipole excitation is considered first. The expression for $t(u)$ pertinent to this case is the following:

$$t_H(u) = \frac{4u\sqrt{\alpha^2 + u^2}}{(u + \sqrt{\alpha^2 + u^2})^2} \cdot \frac{\exp(-j\sqrt{\alpha^2 + u^2}\kappa s)}{1 - \left[\frac{u - \sqrt{\alpha^2 + u^2}}{u + \sqrt{\alpha^2 + u^2}}\right]^2 \exp(-2j\sqrt{\alpha^2 + u^2}\kappa s)} \tag{4.1}$$

$$\alpha^2 = \frac{\sigma + j\omega\epsilon_0(\epsilon_r - 1)}{j\omega\epsilon_0} \cong \frac{\sigma}{j\omega\epsilon_0}. \tag{4.2}$$

It is noted that $t_H(u)$ exhibits no singularity in the lower right quadrant of the complex u plane, so that the integration path Γ can be freely deformed therein, e.g., in the new path Γ' (see Fig. 2). When (4.1) is substituted in (3.7) it is noted that we can neglect u^2 with respect to α^2 provided the integrand is negligible when $u > |\alpha|$. Accordingly, when $\kappa l|\alpha| \gg 1$ the integral (3.7), specified to the case at hand, becomes

$$H_z{}^t(0, 0, l) = -j\omega C\kappa \frac{4}{\alpha} \frac{\exp(-j\alpha\kappa s)}{1 - \exp(-2j\alpha\kappa s)} \exp(-j\kappa l) \cdot \int_0^\infty v(-jv^2 + 3v + 2j) \exp(-\kappa l v)\, dv, \tag{4.3}$$

and the origin of coordinates is now in correspondence to the source. The integral is now straightforward to evaluate and can be conveniently normalized to the value of the incident field $H_z{}^i(0, 0, l)$. We have

$$\frac{H_z{}^t(0,0,l)}{H_z{}^i(0,0,l)} = \left[\frac{4}{\alpha}\frac{\exp(-j\alpha\kappa s)}{1-\exp(-2j\alpha\kappa s)}\right]\left[\frac{1+\dfrac{3}{j\kappa l}+\dfrac{3}{(j\kappa l)^2}}{1+\dfrac{1}{j\kappa l}}\right] = t_0(\alpha, \kappa s)\Omega_H(\kappa l). \tag{4.4}$$

It is noted that the first bracketed term $t_0(\alpha, \kappa s)$ is just the plane wave transmission coefficient under normal incidence and appropriate to a highly conducting screen. The second term $\Omega(\kappa l)$ depends on the mutual distance l between transmitting and receiving points and approaches 1 when $\kappa l \gg 1$. Accordingly, it follows that a simple plane wave transmission coefficient can be used for evaluating shielding effectiveness provided that transmitting and receiving antennas are a few wavelengths apart.

On the other hand, when κl is small, $\Omega_H(\kappa l) \cong 3/j\kappa l$, and

$$\frac{H_z{}^t(0,0,l)}{H_z{}^i(0,0,l)} \cong \frac{3}{j\kappa l} t_0(\alpha, \kappa s) = \frac{\exp(-j\alpha\kappa s - j\pi/4)3\delta}{[1-\exp(-2j\alpha\kappa s)]\sqrt{2}\, l}, \tag{4.5}$$

wherein δ is the skin depth of the screen. Note that (4.5) is valid provided that $\delta/l \ll 1$; otherwise the assumption $\kappa|\alpha|l \gg 1$ is no longer met.

The case of an electric dipole excitation can be treated similarly. We have

$$t_E(u) = \frac{4\alpha^2 u\sqrt{\alpha^2+u^2}}{(\alpha^2 u + u + \sqrt{\alpha^2+u^2})^2} \cdot \frac{\exp(-j\sqrt{\alpha^2+u^2}\,\kappa s)}{1-\left(\dfrac{\alpha^2 u + u - \sqrt{\alpha^2+u^2}}{\alpha^2 u + u + \sqrt{\alpha^2+u^2}}\right)^2 \exp(-2j\sqrt{\alpha^2+u^2}\,\kappa s)}. \tag{4.6}$$

We can now neglect u^2 with respect to α^2 without serious limitation in the validity of the results. The integral corresponding to (4.3) is the following:

$$E_z{}^t(0,0,l) = \omega C\kappa t_0(\alpha, \kappa s) \cdot \left\{\int_1^\infty \frac{\exp(-j\kappa l u)}{u}\, du + j\int_0^\infty (1-ju)\exp(-\kappa l v)\, dv\right\} \tag{4.7}$$

which can be easily evaluated to yield

$$\frac{E_z{}^t(0,0,l)}{E_z{}^i(0,0,l)} = t_0(\alpha, \kappa s)\Omega_E(\kappa l)$$

$$\Omega_E(\kappa l) = \tfrac{1}{2}\frac{1 + j\kappa l + (j\kappa l)^2 \exp(j\kappa l)[\mathrm{Ci}(\kappa l) - j\,\mathrm{Si}(\kappa l)]}{1+\dfrac{1}{j\kappa l}} \tag{4.8}$$

wherein the cosinus integral $\mathrm{Ci}(x)$ and sinus integral $\mathrm{Si}(x)$ functions [17] do appear.

It is again noted that $\Omega_E(\kappa l) \to 1$ when $\kappa l \gg 1$, as easily follows upon use of the asymtotic series expansions [17] of the functions $\mathrm{Ci}(x)$ and $\mathrm{Si}(x)$, so that (4.8) reduces again to the plane wave transmission coefficient $t_0(\alpha, \kappa s)$ provided that transmitting and receiving antennas are a few wavelengths apart. On the contrary, when κl is small, a proper series expansion [17] shows that $\Omega_E(\kappa l) \cong j\kappa l/2$ and

$$\frac{E_z{}^t(0,0,l)}{E_z{}^i(0,0,l)} \cong \frac{j\kappa l}{2} t_0(\alpha, ks) = j\frac{\exp(-j\alpha\kappa s + j\pi/4)}{1-\exp(-2j\alpha\kappa s)}(\kappa l)^2 \frac{\delta}{\sqrt{2}\, l}. \tag{4.9}$$

V. THE CASE OF AN ELECTRIC PLANE SHIELD—TRANSIENT EXCITATION.

We have shown in Section IV that the steady-state z components of the field transmitted through a highly conductive plane shield are given by

$$F_z{}^t(0,0,l) = F_z{}^i(0,0,l)t_0(\alpha, \kappa s)\Omega(\kappa l). \tag{5.1}$$

It is then evident that the z component of the transient transmitted field can be obtained by time-convolving the transient z components of the incident field with the inverse Fourier transforms of $t_0(\omega)$ and $\Omega(\omega)$, let us say $T_0(t)$ and $\Omega(t)$. Use of Laplace inversion tables [18] shows that

$$T_0(t) = 2\sqrt{\frac{\tau}{\pi}} \sum_{1}^{\infty}{}_n \frac{\exp(-n^2\eta/t)}{t^{5/2}}(2n^2\eta - t) = 4\sqrt{\frac{\tau}{\pi}} \sum_{1}^{\infty}{}_n \frac{d}{dt}\frac{\exp(-n^2\eta/t)}{t^{1/2}} = 4\sqrt{\frac{\tau}{\pi}} \sum_{1}^{\infty}{}_n \frac{d}{dt}S_0(t) \tag{5.2}$$

where $\tau = \epsilon_0/\sigma$ and is the relaxation time of the material of the shield, and $\eta = s^2/(c^2\tau)$ and is the diffusion time through the shield thickness.

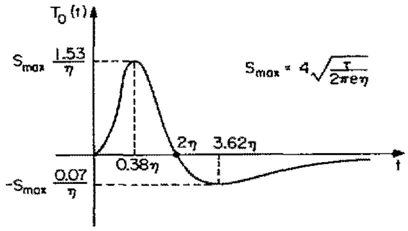

Fig. 3. Qualitative behavior of first series term of function $T_0(t)$.

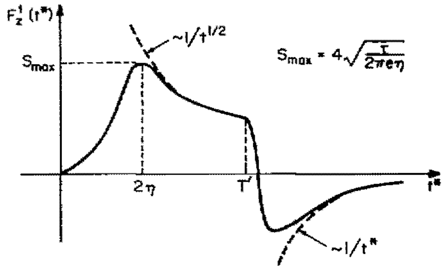

Fig. 4. Qualitative behavior of pulsed field of time duration T' after transmission through a highly conductive slab.

A qualitative behavior of the first term $n = 1$ of $T_0(t)$ is given in Fig. 3, wherein $S_{\max} = 4\sqrt{\tau/2\pi e\eta} = S_0(2\eta)$ and is the maximum value of the function $S_0(t)$ (see also Fig. 4), where in $S_{\max}$ "e" is the Neper's constant.

The behavior of successive terms of the series (5.2) is similar to that depicted in Fig. 3. The maxima occur at later times and their absolute values are smaller by the factor $\exp[-2.6(n^2 - 1)]/n^3$. Accordingly, they can be safely neglected, and we can take only the first term of the series (5.2).

After some algebra Laplace inversion [19] of the two functions $\Omega(\omega)$ leads to

$$\Omega_H(t) = \delta(t) + \frac{3 - \exp(-t/T)}{T} U(t) \tag{5.3}$$

$$\Omega_E(t) = \delta(t) - \frac{2}{T} \exp(-t/T) U(t) + \frac{3}{T} \int_0^{t/T} \frac{\exp(-u)}{\left(1 + \frac{t}{T} - u\right)^4} du \tag{5.4}$$

where $\delta(t)$ and $U(t)$ are the Dirac and the unit step function, respectively, $T = l/c$ and is the free-space transit time from the transmitting to the receiving antenna.

Convolution of (5.2) with the $\delta(t)$ terms of (5.3) and (5.4) just reproduce the function $T_0(t)$. Convolution with the other terms may become significant only after a time of order T. Accordingly, if the incident field has a time duration small compared with T, i.e., its spatial length is small compared with the in-between antennas distance l, then the time dependence of the transmitted field is simply given by the time convolution of the incident signal and the function $T_0(t)$. This transmitted field is the same as would be obtained for the case of plane wave excitation. Accordingly, the result is obtained that the finite distance between antennas plays no significant role if the incident waveform is sufficiently short in time. For instance, if the incident signal is a pulse of unit amplitude and time duration T', then

$$F_z{}^t(0, 0, l, t^*) = 4\sqrt{\frac{\tau}{\pi}}\ \frac{\exp\left(-\frac{\eta}{t^*}\right)}{\sqrt{t^*}}, \quad t^* \leqslant T' \tag{5.5}$$

$$F_z{}^t(0, 0, l, t^*) = 4\sqrt{\frac{\tau}{\pi}} \left\{ \frac{\exp\left(-\frac{\eta}{t^*}\right)}{\sqrt{t^*}} - \frac{\exp\left(-\frac{\eta}{t^* - T'}\right)}{\sqrt{t^* - T'}} \right\}, \quad t^* > T' \tag{5.6}$$

where $t^* = t - (l/c)$ and is the retarded time. A qualitative sketch of (5.5) and (5.6) is given in Fig. 4 for $T' > 2\eta$. When $T' \leqslant 2\eta$, then the transmitted field is just given by (5.2) times T'.

VI. CONCLUSIONS AND PRACTICAL CONSIDERATIONS

We have considered the problem of the transmission of steady-state and transient electromagnetic waves through a slab. An analytical solution has been obtained for the case of a linear homogeneous isotropic highly conducting infinite slab excited by collinear electric or magnetic dipoles. The transmitted z components of the field are expressed as the product (steady-state case) or the convolution (transient case) of the corresponding incident field components and a two-term factor. In the frequency domain the first term of this factor (see (5.1)) is exactly the transmission coefficient of a plane wave normally incident on the slab. The second term takes into account the finite distance between the transmitting and receiving antennas and becomes significant only when this distance is of the order of, or smaller than, the free-space wavelength (steady-state case) or the spatial length of the incident pulse (transient case). It is therefore possible to obtain plane wave excitation results even when sources (and receivers) are located at finite distances. For this all that is needed is the proper choice of distance between antennas.

It is certainly true that these results have been obtained under the conditions that the transmitting antenna is a dipole oriented normal to the slab, that the transmitted field is computed along the axial direction of the dipole, and that only the z components of the field are used in the comparison. However, we believe that our analysis has a more general validity. For instance, results of the collinear configuration can easily be extended to transmitted field points off the axis. We should only substitute

$$J_0(\xi\sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos\phi'}) \tag{6.1}$$

for $J_0(\xi\rho)$ in (2.12). Then expansion [20] of the Bessel function (6.1) and integation in ϕ' gives

$$F_z{}^t(\rho, l) = \omega C\kappa \int_\Gamma (1 - u^2) t(u) J_0(\kappa\rho\sqrt{1 - u^2}) \cdot \exp(-j\kappa l u)\, du \tag{6.2}$$

which is the generalization of (3.7) to the case $\rho \neq 0$. Then $\partial F_z{}^t(\rho, l)/\partial\rho = 0$ for $\rho = 0$, which implies that the results of our analysis are certainly also valid in the neighborhood of the axis. Furthermore, use of Maxwell's equations, with (6.2) as the longitudinal field, shows that the same is true for transverse fields. In this extension, however, the simplifying assumptions used in the body of this paper should be checked again. Should further study show that the above considerations can be extended to more complicated geometries, all simulation studies for shielding purposes might be worth reconsidering.

Some few practical notes are now in order. Reference is made to a copper slab ($\sigma = 5.8 \times 10^7$ S/m) of thickness $s = 1$ mm, so that $\tau = 1.52 \times 10^{-19}$ s and $\eta = 70$ μs. Only the plane wave transmission coefficient will be considered. For incident pulses of unit amplitude and time duration $T' \ll \eta$, the peak of the transmitted field is equal to $2.7 \times 10^{-7}\ T'/\eta$, therefore linearly decreasing with the bandwidth ($\sim 1/T'$) of the signal. In the sinusoidal excitation case the attenuation due to the mismatch $4|\alpha|$ equals that due to the damping inside the slab material $\exp(-|\alpha|\kappa s/\sqrt{2})$ at the frequency $f = 0.72$ MHz. At this frequency the transmitted field is equal to 11×10^{-12} times the incident one. At higher frequencies the signal is decreasing exponentially with the square root of the frequency.

For moderate antenna spacings it is noted that the transmitted field can be computed using the plane wave transmission coefficient only when the attenuation is very high. However, this may not be the case if even small apertures exist in the screen. Accordingly, we believe it is worthwhile to extend the analysis presented in this paper to other canonical problems, which are amenable to the same analytical approach. Among those, we list the problem of an infinite conductive screen with a single hole and that of a conductive screen with a regular lattice of equal small apertures. The former problem can take advantage of the solution of a plane wave diffraction by apertures in conducting screens [21]–[23] and, eventually, of symmetrization procedures [24]. The latter could make use of the artifical dielectric theory [25] properly accommodated to this single sheet problem.

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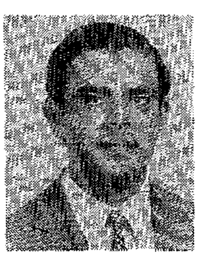

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Absorption of Energy from a Large Amplitude Electromagnetic Pulse by a Collisionless Plasma

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Abstract—A series of experiments in which an electromagnetic pulse (EMP) is propagated through a nitrogen plasma are discussed. The pulse has the general characteristics of an EMP. The pulse is observed as it emerges from the plasma as a function of the plasma parameters. As the electron number density increases, it is found that energy is increasingly absorbed from the pulse, a process due to joule heating. In addition, at higher number densities, ringing of the pulse occurs. The nitrogen pressure in these experiments is sufficiently low so that collisions play only a minor role. Also developed is a theoretical model based on the fluid transport equations. This theory predicts that the electrons of the plasma are attaining a temperature of about 15 eV in that part of the system where the dc magnetic field is about 80 G. More importantly, it is able to predict the output pulse quite well under the conditions that the ambient nitrogen pressure and the electron number density are low. The theory appears to fail as these parameters are increased.

I. INTRODUCTION

THE ELECTROMAGNETIC pulse (EMP) generated by a nuclear detonation has been the subject of extensive study. Computer codes have been developed which attempt to model the generation and propagation of the EMP due to both a high-altitude detonation [1]-[7] and a low-altitude detonation [8]-[11]. In some of these codes, the current density term which appears in Maxwell's equations due to conduction currents of secondary electrons is modeled as $\sigma \bar{E}$, where σ is a conductivity that is spatially and time dependent. This model is satisfactory as long as the thermalization time, i.e., the time for the average electron velocity to reach steady state after the application of a step function electric field [12], is short compared to the time that the EMP electric field changes appreciably. It is generally believed that collision frequencies of electrons in air below 40 km are great enough so that this condition is satisfied.

A more complete description of conduction currents of secondary electrons is given by the familiar fluid transport equations for mass, momentum, and energy [13], [5]. We shall hereafter refer to these equations collectively as the swarm model [5].

Knight [6] has made the point that the validity of these various models needs to be checked by a method that does not contain their inherent limitations. He suggests a Monte Carlo computer model. However, an even more satisfactory method would be to check these models experimentally. It is the purpose of this paper to present some preliminary experimental results which are compared to predictions of the electron swarm model.

A theoretical treatment of the propagation of an EMP-type electromagnetic pulse through a plasma has been developed by one of the authors using the electron swarm model [14] and is summarized in Section II. Subsequently, at the Plasma Laboratory at the University of Arizona, a machine has been built in which an EMP-type electromagnetic pulse is allowed to propagate through a nitrogen plasma [15]-[18]. In this machine one

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