Failure Localization in Power Systems via Tree Partitions

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ABSTRACT

Cascading failures in power systems propagate non-locally, making the control and mitigation of outages extremely hard. In this work, we use the emerging concept of the *tree partition* of transmission networks to provide an analytical characterization of line failure localizability in transmission systems. Our results rigorously establish the well perceived intuition in power community that failures cannot cross bridges, and reveal a finer-grained concept that encodes more precise information on failure propagations within tree-partition regions. Specifically, when a non-bridge line is tripped, the impact of this failure only propagates within well-defined components, which we refer to as *cells*, of the tree partition defined by the bridges. In contrast, when a bridge line is tripped, the impact of this failure propagates *globally* across the network, affecting the power flow on all remaining transmission lines. This characterization suggests that it is possible to improve the system robustness by *temporarily* switching off certain transmission lines, so as to create more, smaller components in the tree partition; thus spatially localizing line failures and making the grid less vulnerable to large-scale outages. We illustrate this approach using the IEEE 118-bus test system and demonstrate that switching off a negligible portion of transmission lines allows the impact of line failures to be significantly more localized without substantial changes in line congestion.

1. INTRODUCTION

Power system reliability is a crucial component in the development of sustainable modern power infrastructure. Recent blackouts, especially the 2003 and 2012 blackouts in Northwestern U.S. [1] and India [2], demonstrated the devastating economic impact a grid failure can cause. In even worse cases, where facilities like hospitals are involved, blackouts pose direct threat to people's health and lives.

Because of the intricate interactions among power system components, outages may cascade and propagate in a very complicated, non-local manner [3–5], exhibiting very different patterns for different networks [6]. Such complexity originates from the interplay between network topology and power flow physics, and is aggravated by possible hidden failures [7] and human errors [8]. This complexity is the key challenge for research into the modeling, control, and mitigation of cascading failures in power systems.

There are three traditional approaches for characterizing the behavior of cascades in the literature: (i) using simulation models [9] that rely on Monte-Carlo approaches to account for the steady state power flow redistribution on DC [5, 8, 10, 11] or AC [12, 13] models; (b) studying purely topological models that impose certain assumptions on the cascading dynamics (e.g., failures propagate to adjacent lines with high probability) and infer component failure propagation patterns from graph-theoretic properties [14–16]; (c) investigating simplified or statistical cascading failure dynamics [3, 17–19]. In each of these approaches, it is typically challenging to make general inferences across different scenarios due to the lack of structural understanding of power redistribution after line failures.

A new approach has emerged in recent years, which seeks to use spectral properties of the network graph in order to derive precise structural properties of the power system dynamics, e.g., [20–22]. The spectral view is powerful as it often reveals surprisingly simple characterizations of the complicated system behaviors. In the cascading failure context, a key result from this approach is about the *line outage distribution factor* [6, 23]. Specifically, it is shown in [21] that the line outage distribution factor is closely related to transmission graph spanning forests.

While this literature has yet to yield a precise characterization of cascades, it has suggested a new structural representation of the transmission graph called the *tree partition*, which is particularly promising. For example, [21] shows that line failures in a transmission system cannot propagate across different regions of the tree partition (for more background on the tree partition, see Section 2).

Contributions of this paper: *We prove that the tree partition proposed in [21] can be used to provide an analytical characterization of line failure localizability, under a DC power flow model, and we show how to use this characterization to mitigate failure cascades by temporarily switching off a small number of transmission lines.* Our results rigorously establish the well perceived intuition in power community that failures cannot cross bridges, and reveal a finer-grained concept that encodes more precise information on failure propagations within tree-partition regions. This work builds on the recent work focused on the line outage distribution factor, e.g., [6, 21, 24], and shows that the tree partition is a particularly useful representation of this factor, one that

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captures many aspects of how line failures can cascade.

Our formal characterization of localizability is given in Theorem 3. It shows that the impact of tripping a non-bridge line only propagates within well-defined components, which we refer to as cells, inside the tree partition regions. In constrast, the failure of bridge lines, in normal operating conditions, propagate globally across the network and impact the power flow on all transmission lines. In order to prove these results, we depend on properties of the tree partition proved in [21] as well as some novel properties derived in [25]. Further, we make use of the block decomposition of tree partition regions to completely eliminate the graph spanning forests among distinct cells, which in the spectral view means failure localization [21]. Lastly, we apply classical techniques from algebraic geometry to address potential pathological system specifications and establish our results.

The characterization we provide in Theorem 3 yields many interesting insights for the planning and management of power systems and, further, suggests a new approach for mitigating the impact of cascading failures. Specifically, our characterization highlights that switching off certain transmission lines *temporarily* in responds to the real-time injection profile can lead to more, smaller regions/cells, which localize line failures, thus making the grid less vulnerable against line outages. In Section 4, we illustrate this approach using the IEEE 118-bus test system. We demonstrate that switching off only a negligible portion of transmission lines can lead to significantly better control of cascading failures. Further, we highlight that this happens without significant increases in line congestion across the network.

2. PRELIMINARIES

We use the graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ to describe a power transmission network, where $\mathcal{N} = \{1, \ldots, n\}$ is the set of buses and $\mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ is the set of transmission lines. The terms bus/vertex and line/edge are used interchangeably. An edge in $\mathcal{E}$ between vertices i and j is denoted either as e or (i, j). We assume $\mathcal{G}$ is connected and simple, and assign an arbitrary orientation over $\mathcal{E}$ so that if $(i, j) \in \mathcal{E}$ then $(j, i) \notin \mathcal{E}$. The line susceptance of e is denoted as B_e and the branch flow on e is denoted as P_e. The susceptance matrix is defined to be the diagonal matrix $B = \text{diag}(B_e : e \in \mathcal{E})$.

We denote the power injection and phase angle at bus i as p_i and θ_i, and use n and m to denote the number of buses and transmission lines in $\mathcal{G}$. The vertex-edge incidence matrix of $\mathcal{G}$ is the $n \times m$ matrix C defined as

$$C_{ie} = \begin{cases} 1 & \text{if vertex } i \text{ is the source of } e \\ -1 & \text{if vertex } i \text{ is the target of } e \\ 0 & \text{otherwise.} \end{cases}$$

With the above notation, the DC power flow model can be written as

$$p = CP \tag{1a}$$

$$P = BC^T\theta, \tag{1b}$$

where (1a) is the flow conservation constraint and (1b) is Kirchhoff's and Ohm's Laws. The slack bus phase angle in θ is typically set to 0 as a reference to other buses. With this convention, the DC model (1) has a unique solution θ and P for each injection vector p such that $\sum_{j \in \mathcal{N}} p_j = 0$.

When a line e is tripped, the power flow redistributes according to the DC model (1) on the newly formed graph $\mathcal{G}' = (\mathcal{N}, \mathcal{E} \setminus \{e\})$. If $\mathcal{G}'$ is still connected, then the branch flow change on a line $\hat{e}$ is given as

$$\Delta P_{\hat{e}} = P_e \times K_{e\hat{e}},$$

where $K_{e\hat{e}}$ is the *line outage distribution factor* [23] from e to $\hat{e}$. It is known that this distribution factor is independent of the original power injection p and can be computed from the matrices B and C [23].

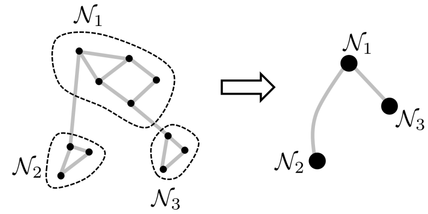


Figure 1: The construction of $\mathcal{G}_\mathcal{P}$ from $\mathcal{P}$.

If the new graph $\mathcal{G}'$ is disconnected, then it is possible that the original injection p is no longer balanced in the connected components of $\mathcal{G}'$. Thus, to compute the new power flow, a certain power balance rule $\mathcal{B}$ needs to be applied. Several such rules have been proposed and evaluated in literature based on load shedding or generator response [5,6,26–28]. In this work, we do not specialize to any such rule and instead opt to identify the key properties of these rules that allow our results to hold. With this more abstract approach, we can characterize the power flow redistribution under a class of power balance rules.

For a power network $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, a collection

$$\mathcal{P} = \{\mathcal{N}_1, \mathcal{N}_2, \cdots, \mathcal{N}_k\}$$

of subsets of $\mathcal{N}$ is said to form a *partition* of $\mathcal{G}$ if $\mathcal{N}_i \cap \mathcal{N}_j = \emptyset$ for $i \neq j$ and $\cup_{i=1}^k \mathcal{N}_i = \mathcal{N}$. For any partition, we can define a reduced multi-graph $\mathcal{G}_\mathcal{P}$ from $\mathcal{G}$ as follows. First, we reduce each subset $\mathcal{N}_i$ to a super node (see Figure 1). The collection of all super nodes forms the node set for $\mathcal{G}_\mathcal{P}$. Second, we add an undirected edge connecting the super nodes $\mathcal{N}_i$ and $\mathcal{N}_j$ for each pair of $n_i, n_j \in \mathcal{N}$ with the property that $n_i \in \mathcal{N}_i$, $n_j \in \mathcal{N}_j$ and n_i and n_j are connected in $\mathcal{G}$. Note that multiple ledges are added when multiple pairs of such n_i, n_j exist. Unlike the graph $\mathcal{G}$ to which we assign an arbitrary orientation (and thus is a directed graph), the reduced multi-graph $\mathcal{G}_\mathcal{P}$ is undirected.

Definition 1. *A partition $\mathcal{P} = \{\mathcal{N}_1, \mathcal{N}_2, \cdots, \mathcal{N}_k\}$ of $\mathcal{G}$ is said to be a **tree partition** if the reduced graph $\mathcal{G}_\mathcal{P}$ forms a tree.*

Definition 2. *Given a tree partition $\mathcal{P} = \{\mathcal{N}_1, \mathcal{N}_2, \cdots, \mathcal{N}_k\}$, the sets $\mathcal{N}_i$ are called the **regions** of $\mathcal{P}$. An edge $e = (w, z)$ with both endpoints inside $\mathcal{N}_i$ is said to be **within** $\mathcal{N}_i$. If e is not within $\mathcal{N}_i$ for any i, then we say e forms a **bridge**.*

In [25], we derived a set of results regarding the reducibility, uniqueness, and computational complexity of tree partitions. In particular, it is shown that each graph $\mathcal{G}$ has a unique irreducible tree partition and this particular partition can be computed in linear time. Thus, to simplify the terminology, whenever we say the tree partition of $\mathcal{G}$ in the sequel, we always refer to its irreducible partition.

3. SUMMARY OF RESULTS

Our main result applies in contexts where the system is operating under *normal* conditions, i.e., when the following

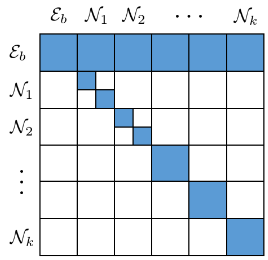


Figure 2: Non-zero entries of the $K_{e\hat{e}}$ matrix (as represented by the dark blocks) for a graph with tree partition $\{\mathcal{N}_1, \mathcal{N}_2, \cdots, \mathcal{N}_k\}$ and bridge set $\mathcal{E}_b$. The small blocks represent cells inside the regions.

two assumptions are satisfied: (a) the injection is *island-free*; and (b) the grid is *participating* with respect to its power balance rule. Moreover, to address certain pathological cases, we add a perturbation drawn from certain probability measure μ to the line susceptances and assume μ is absolutely continuous with respect to the Lebesgue measure $\mathcal{L}_m$ on $\mathbb{R}^m$. We redirect the readers to our online report [25] for formal definitions of these terminologies and proof of our result.

Theorem 3. *For a power network operating under normal conditions, $K_{e\hat{e}} \neq 0$ almost surely in μ if and only if:*

1. *$e, \hat{e}$ are within a common tree partition region and $e, \hat{e}$ belong to the same cell; or*
2. *e is a bridge.*

This result highlights that, for a practical system, the tree partition encodes rich information on how the failure of a line propagates through the network. We emphasize that: (i) the condition that μ is absolutely continuous with respect to $\mathcal{L}_m$ is satisfied by almost all practical probability models for such perturbations; and (ii) the conditions that the injection is island-free and the grid is participating are satisfied in typical operating scenarios. Therefore, the conditions posed in Theorem 3 are satisfied in practical settings.

Figure 2 shows how the tree partition is linked to the sparsity of the $K_{e\hat{e}}$ matrix through Theorem 3. It suggests that, compared to a full mesh transmission network consisting of single region/cell, it can be beneficial to *temporarily* switch off certain lines so that more regions/cells are created and the impact of a line failure is localized within the cell in which the failure occurs. We study this network planning and design opportunity in Section 4.

4. LOCALIZING CASCADING FAILURES

Our findings highlight a new approach for improving the robustness of the network. More specifically, Theorem 3 suggest that it is possible to localize failure propagation by *temporarily* switching off certain transmission lines. This creates more, smaller areas where failure cascades can be contained. We remark that the lines that are switched off are still part of the system. In cases where the newly created bridges are tripped, some of these lines should be switched on so the system are still connected. The examples presented below are preliminary and we are still investigating how to optimally tradeoff the increased robustness from localized failures and the yet also increased vulnerability from having more bridges.

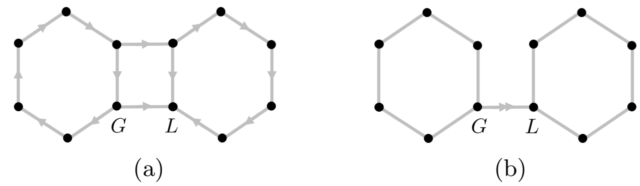


Figure 3: (a) A double-ring network. G is the generator bus and L is the load bus. Arrows represent the original power flow. (b) The new network after removing an edge. Arrows represent the new power flow.

It is reasonable to expect that such an action may increase the stress on the remaining lines and, in this way, worsen the network congestion. In fact, one may expect that improved system robustness obtained by switching off lines *always* comes at the price of increased congestion levels. In this section, we argue that this is not necessarily the case, and show that if the lines to switch off are selected properly, it is possible to *improve the system robustness and reduce the congestion simultaneously*. We corroborate this claim by considering first a small stylized example and then an IEEE test system.

4.1 Double-Ring Network

Consider the double-ring network in Figure 3(a), which contains exactly one generator and one load bus. The original power flow on this network is also shown in Figure 3(a). Suppose we switch off the upper tie-line. The new network and the redistributed power flow are shown in Figure 3(b). In this example, by switching off one transmission line, the circulating flows inside the hexgons are removed and the overall network congestion is decreased. In fact, it is easy to show that the topology in Figure 3(b) minimizes the sum of (absolute) branch flows over all possible topologies.

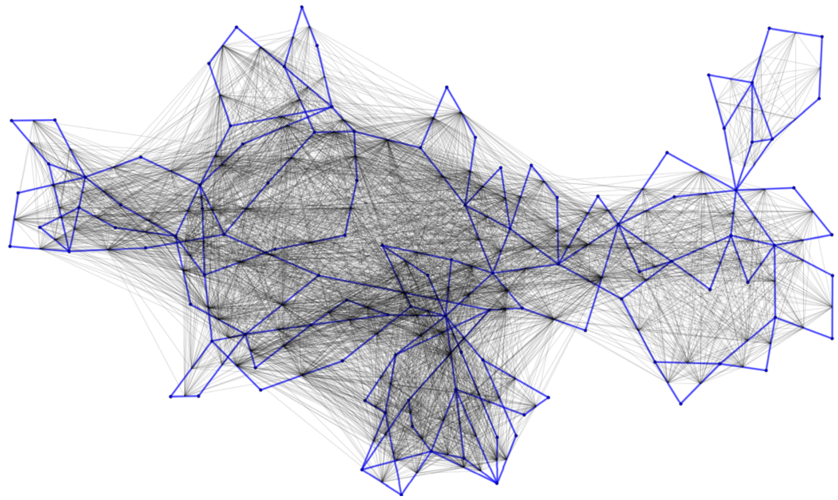
4.2 IEEE test system

In the simple example above, removing a line provides improvements in both robustness and congestion. Now, we move to the case of a more realistic network, the IEEE 118-bus test system. In this case, we also see that line removals can improve robustness without more than minor increase in congestion.

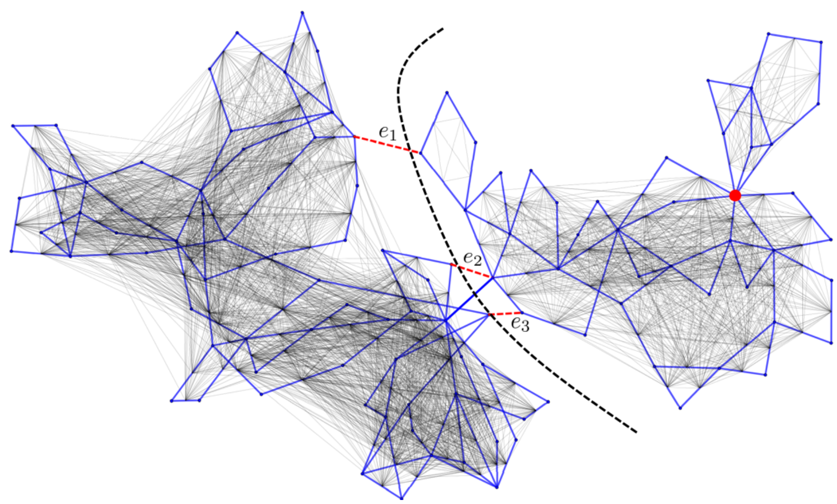
In our experiments, the system parameters are taken from the Matpower Simulation Package [29] and we plot the influence graphs among the transmission lines to demonstrate how a line failure propagates in this network[1]. More specifically, in the influence graph we plot, two edges e and $\hat{e}$ are connected if the impact of tripping e on $\hat{e}$ is not negligible (we use $|K_{e\hat{e}}| \geq 0.005$ as a threshold). In Figure 4(a), we plot the influence graph of the original network. It can be seen that this influence graph is very dense and connects many edges that are topologically far away, showing the non-local propagation of line failures within this network.

Next, we switch off three edges (indicated as e_1, e_2 and e_3 in Figure 4(b)) to obtain a new topology that has a bridge and whose tree partition now consists of two regions of comparable size. The new influence graph is shown in

[1]The original IEEE 118-bus network has some trivial "dangling" bridges that we remove (collapsing their injections to the nearest bus) to obtain a more transparent influence graph.



(a) Original influence graph.

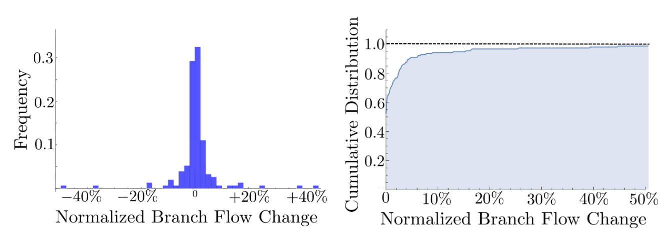


(b) The influence graph after switching off e_1, e_2 and e_3. The black dashed line indicates the failure propagation boundary defined by the tree partition.

Figure 4: Influence graphs on the IEEE 118-bus network before and after switching off lines e_1, e_2 and e_3. Blue edges represent physical transmission lines and grey edges represent connections in the influence graph.

Figure 4(b). One can see that, compared to the original influence graph in Figure 4(a), the new influence graph is much less dense and, in particular, there are no edges connecting transmission lines that belong to different tree partition regions.

It is also of interest to see how the network congestion is impacted by switching off these lines. To do so, we collect statistics on the difference between the branch flows in Figure 4(b) and those in 4(a). In Figure 5(a), we plot the histogram of such branch flow differences normalized by the original branch flow in Figure 4(a). It shows that roughly half (the exact percentage is 47.41%) of the transmission lines have higher congestion yet the majority of these branch flow increases are negligible. To more clearly see how much the congestion becomes worse on these lines, we plot the cumulative distribution function of the normalized positive branch flow changes, which is shown in Figure 5(b). One can see from the figure that 90% of the the branch flows increase by no more than 10%.



(a) (b)

Figure 5: (a) Histogram of the normalized branch flow changes. (b) Cumulative distribution function of the positive normalized branch flow changes. Note that the curve intercepts the y-axis since 52.59% of the branch flows decrease.

5. CONCLUSION

This work can be extended in several directions. First, we provide an analytical characterization of power flow redistribution when a line fails, and our results are generalizable to bus failures. It is of interest to understand how these two types of failures interact. Second, we demonstrate in our case studies that by switching off certain transmission lines, grid robustness can potentially be improved. It would be useful if the selection of such lines can be optimized for a certain objective function, such as the sparsity of the influence graph or the total load loss when some critical lines are tripped. Third, to fully capture the cascading failure dynamics, both the power flow redistribution and the line capacities are relevant. It is important to investigate how line capacities can be incorporated to our framework.

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