

The Equivalence Principle at Work in Radiation from Unaccelerated Atoms and Mirrors

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Abstract

The equivalence principle is a perennial subject of controversy, especially in connection with radiation by a uniformly accelerated classical charge, or a freely falling charge observed by a supported detector. Recently, related issues have been raised in connection with the Unruh radiation associated with accelerated detectors (including two-level atoms and resonant cavities). A third type of system, very easy to analyze because of conformal invariance, is a two-dimensional scalar field interacting with perfectly reflecting boundaries (mirrors). After reviewing the issues for atoms and cavities, we investigate a stationary mirror from the point of view of an accelerated detector in “Rindler space”. In keeping with the conclusions of earlier authors about the electromagnetic problem, we find that a radiative effect is indeed observed; from an inertial point of view, the process arises from a collision of the negative vacuum energy of Rindler space with the mirror. There is a qualitative symmetry under interchange of accelerated and inertial subsystems (a vindication of the equivalence principle), but it hinges on the accelerated detector’s being initially in its own “Rindler vacuum”. This observation is consistent with the recent work on the Unruh problem.

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I. INTRODUCTION

Wolfgang Schleich is a master at combining quantum theory (particle or field) with relativity (special or general). It is a privilege to offer the present work in his honor.

In fact, the immediate impetus to this work came from a project in which Wolfgang is involved [1]. It concluded that atoms falling from outside through a cavity into a black hole emit acceleration radiation which to a distant observer looks much like Hawking black-hole radiation [2]. The derivation is a straightforward variant (following [3, 4]) of the Unruh–Wald [5, 6] analysis of the behavior of a uniformly accelerated detector. According to general relativity, however, the atom of [1] is in free fall (not accelerated), and the cavity is accelerated (supported against the gravitational field of the black hole). The fact of *relative* acceleration is critical to the result. A follow-up paper [7] perturbatively confirms this picture by showing in flat space-time that uniformly accelerated motion of a mirror can yield excitation of a two-level atom moving at constant velocity, with simultaneous emission of a real photon; in this calculation the photon mode considered is initially empty in the Rindler sense (i.e., with respect to field quantization based on the timelike Killing vector field of Lorentz boosts along the hyperbolic mirror trajectory).

The term “principle of equivalence” has been used to mean many different things. The version we have in mind here is that “uniform gravitational fields are equivalent to frames that accelerate uniformly relative to inertial [free-falling] frames” [8, p. 115]. Precisely what this means can be subtle. Physicists have argued for decades about whether and how the principle is respected in the electromagnetic radiation from a uniformly accelerated classical charge, or for a charge in free fall. (See, for instance, [9].) The issues raised there are certainly related to those involving Unruh detectors, but they are different in some important respects. Another type of model system, somewhat intermediate between charges and detectors, involves quantum field theories with reflecting boundaries that are allowed to accelerate [10]. For a massless scalar field in two-dimensional space-time, moving-mirror problems can be solved in closed form (nonperturbatively) and have cast great light on the more difficult problems of quantum field theory in external gravitational fields. Mirror models yield local expectation values of the stress-energy-momentum tensor, therefore providing more detailed information than calculations of transition amplitudes that require integration over entire worldlines. Additionally, lacking charge or internal structure, the moving mirrors provide a simple testing ground for general statements of equivalence.

In Sec. II we examine by gedankenexperiments the issues raised by absolute and relative acceleration of an atom and a resonant cavity, where the acceleration may or may not be gravitational. In Sec. IV we replace the atom by a mirror (in dimension $1 + 1$) and study the radiation produced, as manifested by the covariantly renormalized stress-energy-momentum tensor of the massless scalar field [11], which we review in Sec. III. The most unexpected conclusion — but one necessary to save the principle of equivalence — is that a stationary mirror radiates from the point of view of an accelerated observer, though only if that observer’s environment is initially some state similar to the “Rindler vacuum” [5, 12, 13]. (The mirror could equally well be moving at a constant velocity, but since we will be considering only one inertial mirror, we may work in its inertial frame, where it is *stationary*.) This effect of the mirror may be regarded as reflection of the negative energy density already present in the Rindler ground state. In Sec. V we observe that in essence the effect has already been calculated by Davies long ago [14], and then we draw some conclusions about the equivalence principle.

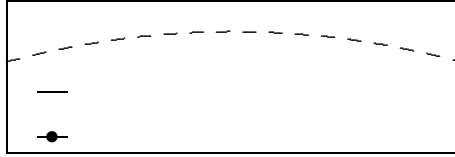


FIG. 1: An atom is in its ground state, with an unoccupied excited state above it. The fundamental mode of the cavity is drawn with a dashed curve to indicate that it is unoccupied.

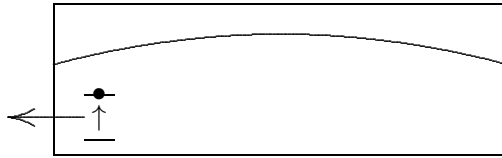


FIG. 2: An accelerated atom excites as an Unruh–DeWitt detector [5, 15], thereby emitting Unruh–Wald radiation.

We use natural units, $c = 1 = \hbar$. For consistency with the original literature, such as [11], we use the metric signature in which the minus sign is associated with the spatial dimension.

II. FOUR EXPERIMENTS WITH AN ATOM AND A CAVITY

Consider a two-level atom in its ground state, inside a resonant cavity capable of acting as a photon detector (Fig. 1).

Experiment 1: Suppose that the atom is accelerated; is there a chance that the cavity mode will be excited?

Despite some remaining pockets of dissent, the answer is now generally agreed to be *yes*[6]: The atom may move to its excited state and emit a photon (Fig. 2).

Experiment 2: Now suppose that the atom is stationary and the cavity is accelerated; can the cavity be excited?

At this point a difference of opinion may emerge. One natural answer is, “Of course not. Nothing is happening to the atom.” (See Fig. 3(a).) In [1, 7] it is argued that the answer is *yes* (Fig. 3(b)). But for the moment let’s accept the negative answer and explore its consequences.

Let us now suppose that these accelerations are caused by a gravitational field (as opposed to, say, a rocket motor). In other words, the formerly accelerated bodies are now considered

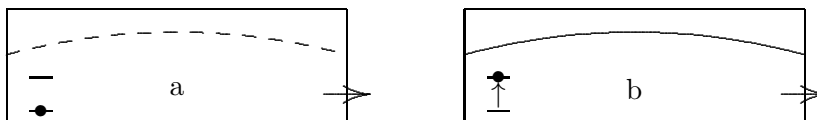


FIG. 3: Two opinions about thought experiment 2. (a) The atom is stationary, so it does not excite or radiate. (b) The atom is excited and radiates, much as in experiment 1.



FIG. 4: Two opinions about experiment 3 (among those holding the first opinion about experiment 2). (a) As far as atomic physics is concerned, it is the atom that accelerates. (b) According to general relativity, the atom is in an inertial frame and the cavity accelerates in the opposite direction.

in free fall, and the formerly stationary bodies are now *supported* by some force, to avoid falling.

Experiment 3: The atom is falling and the cavity is supported. Can the cavity be excited? Now the party of *no* in regard to experiment 2 may further divide into factions.

Answer 1: *Yes*. This is still experiment 1 (Fig. 4(a)). The radiation by the atom is a matter of electromagnetism and nonrelativistic quantum mechanics. The nature of the applied force doesn't affect that process.

Answer 2: *No*. This is still experiment 2 (Fig. 4(b)). By the equivalence principle, it is the atom that is at rest and the cavity that is accelerating.

Experiment 4: The atom is supported and the cavity is falling. What happens?

In this case the factions represented in Fig. 4 are likely to interchange their positions: In terms of atomic physics, the atom is at rest and should not radiate. In terms of relativity, the atom is accelerated and should radiate. The original party of *yes* in Fig. 3 will probably have no trouble continuing to answer *yes* in all cases. So there are three theories, offering *no-yes-no*, *no-no-yes*, and *yes-yes-yes* as predictions with respect to experiments 2, 3, 4.

The last two schools may have second thoughts when they consider that their position compels them to say that an atom at rest in a terrestrial laboratory has some probability of exciting and radiating. (In such remarks one is supposed to ignore the rotation of the earth. The issue concerns the radial force that keeps the atom from falling through the floor.) All this sounds hauntingly familiar to onlookers acquainted with the never-ending debates about radiation from a classical accelerating charge, in which case the final paradox is usually formulated, “Is it really true that a charge sitting on a table on the earth emits Larmor radiation?”

Sensing victory, the *no-yes-no* party now issues a rebuttal: “The rest of you are abusing the equivalence principle. Free fall in a gravitational field (metric $ds^2 = z^2 dt^2 - dz^2$, transverse dimensions ignored) is not really the same thing as acceleration in flat space-time (metric $ds^2 = dt^2 - dz^2$). Don't be misled by popularizations that say, ‘Special relativity shows that velocity is relative, and general relativity shows that acceleration is relative.’” The *no-no-yes* party agrees, claiming the equivalence only connects Experiments 1 and 4 and Experiments 2 and 3. Those critics are right, in that general relativity does not relativize acceleration in the sense that special relativity relativizes velocity. Experiments 1 and 2 are not exactly equivalent. More quantitatively, one should observe the behavior of worldlines under the *Rindler coordinate transformation* [16],

$$t = \rho \sinh \tau, \quad z = \rho \cosh \tau. \quad (1)$$

whose inverse is

$$\tau = \tanh^{-1} \left(\frac{t}{z} \right), \quad \rho = \sqrt{z^2 - t^2}. \quad (2)$$

The metric transforms as

$$ds^2 = dt^2 - dz^2 = \rho^2 d\tau^2 - d\rho^2. \quad (3)$$

The worldline $\rho = 1/a$ is the hyperbola $z^2 - t^2 = a^{-2}$, corresponding to *uniform acceleration* a . In contrast, the worldline $z = 1/a$ is the same as the curve

$$\rho = \frac{1}{a \cosh \tau}. \quad (4)$$

This formula describes how a *stationary* body is regarded by an accelerated observer.

Nevertheless, we claim that the equivalence principle remains *qualitatively* valid in this situation of relative acceleration between an atom and a cavity, and that it dictates the *yes-yes-yes* conclusions. Transition-amplitude calculations [3, 4, 7, 17] back up these conclusions. Similar scenarios were discussed by Ginzburg and Frolov [18, Sec. 6], with similar conclusions; those authors emphasized the importance of being explicit about the choice of initial “vacuum” state in each scenario.

Remark: This is not the same as saying *yes-yes-yes* to the corresponding questions about classical charges. That situation turns out to be surprisingly muddled, even after the analogous quantum-field-theory questions have been settled. The classical charge and the Unruh atom differ in two major respects: The charge has no internal degrees of freedom, and the classical electromagnetic theory has no distinction between rival “vacuum” states like the one that forms the crux of the Unruh theory [5]. These differences turn out to make the classical-charge problem harder, not easier, to understand.

In the present paper we study primarily an accelerated perfect mirror, which has no internal degrees of freedom (by definition of “perfect”) but does have a variety of “vacuum” states in quantum field theory. We shall show that the answers to the analogous questions are *yes-yes-yes* when the initial states of the quantum field are appropriately chosen, thereby buttressing and generalizing the conclusions of [1, 7] with a “simplified” atom (the mirror).

Remark: In this section we have avoided the word “detector” as much as possible, because of its ambiguity. In the original analysis of experiment 2 by Unruh [5] the accelerated atom acts as a detector, responding to the quanta in the Rindler thermal bath that exists in the usual vacuum state of the quantum field (or in the ground state of the cavity, in our present scenario). In [1], on the other hand, the cavity is tuned to respond preferentially to a certain mode of the field, and hence it can detect quanta emitted by the atom. Both points of view are correct; they are complementary. In the cases of mirrors and charges, however, only the second interpretation makes sense.

III. THE AMAZING TRIVIALITY AND RICHNESS OF TWO-DIMENSIONAL MASSLESS QUANTUM FIELD THEORY

Massless (1+1)-dimensional models are the source of much of what we understand about quantum field theory in curved space or in the presence of boundaries or acceleration. This is so, even though massless (1+1)-dimensional free quantum field theory is very special, hence potentially misleading if one jumps to general conclusions too quickly. Its special

properties make it exactly solvable in most circumstances, and that, of course, is the source of its power.

In this paper we are primarily concerned with the effect of time-dependent boundaries introduced into flat (Minkowski) space-time. The pioneer paper on this topic is by Moore [10], who treated two mirror-like points bounding a finite spatial interval, thereby predicting the particle creation now often called “dynamical Casimir effect”. Independently, DeWitt [19] described the same effect for a single boundary. This theory was further developed and generalized by Fulling and Davies [11, 20] and applied to a model black hole by Davies, Fulling, and Unruh [21]. Davies [14, 22], in particular, recognized that the center of coordinates of a (3 + 1)-dimensional spherically symmetric star acts mathematically as a Dirichlet boundary that effectively accelerates away from outside observers if the star collapses to a black hole. Numerous papers since then have made additional contributions, of which [23] (which corrected an error in [22]) and [24–28] are especially significant; note also a long series of recent papers by Good et al., traceable from the most recent one [29].

The first special property of this theory is that *every two-dimensional manifold is locally conformally flat*: There are coordinates (nonunique) where the line element takes the simple forms

$$\begin{aligned} ds^2 &= C(dt^2 - dx^2) \\ &= C(u, v) du dv, \end{aligned} \tag{5}$$

for some function C on space-time, where the *null coordinates* are

$$u = t - x, \quad v = t + x. \tag{6}$$

In other words,

$$g_{uv} = \frac{1}{2}C, \quad g^{uv} = \frac{2}{C}, \tag{7}$$

and on-diagonal elements of the metric tensor are zero (thus raising or lowering indices converts u -components to v -components and vice versa). The lines of constant u or v are *light rays*. Any other such coordinate system must be just a relabeling of these rays: $u = f(u^*), v = g(v^*)$ leads to

$$\begin{aligned} ds^2 &= f'(u^*)g'(v^*)C(f(u^*), g(v^*)) du^* dv^* \\ &= C^*(u^*, v^*) du^* dv^* \end{aligned} \tag{8}$$

for some new conformal factor C^* .

The second special property is that *the massless wave equation is conformally invariant*: $0 = g^{\alpha\beta}\nabla_\alpha\nabla_\beta\phi = \frac{4}{C}\nabla_u\nabla_v\phi$, so $\nabla_u\nabla_v\phi = 0$ if and only if $\nabla_{u^*}\nabla_{v^*}\phi = 0$. (Note that a Klein–Gordon mass would ruin this property: $\nabla_{t^*}^2\phi - \nabla_{x^*}^2\phi + m^2\phi = 0$ becomes the nontrivial $\nabla_u\nabla_v\phi + C(u, v)m^2\phi = 0$.) Furthermore, the normal modes are elementary, simply built out of plane waves $e^{-ip_u u}$ and $e^{-ip_v v}$, or $e^{-i(\omega t - kx)}$ ($\omega = |k|$) and their negative-norm conjugates. The wave $e^{-ip_u u} = e^{-ip_u(t-x)}$ is right-moving, while $e^{-ip_v v} = e^{-ip_v(t+x)}$ is left-moving. *How* to build normal modes from these pieces depends on global geometry and boundary conditions. For a reflecting boundary at $x = 0$ one needs $\phi \propto e^{-i\omega t} \sin(kx)$, for instance.

Additionally, creation operators of normal modes in one frame in general correspond to superpositions of creation and annihilation operators in another; for instance $e^{ip_u u} \propto \sum_k [\alpha_{p_u k} e^{iku^*} + \beta_{p_u k} e^{-iku^*}]$ implies that the annihilation operator in the (u, v) -basis (b_{p_u}) can be written in terms of annihilation and creation operators in the (u^*, v^*) -basis (a_k and a_k^\dagger , respectively):

$$b_{p_u} = \sum_k [\alpha_{p_u k} a_k + \beta_{p_u k}^* a_k^\dagger]. \tag{9}$$

The vacuum annihilated by a_k (call it $|0_a\rangle$) does not have single excitations with respect to b_{p_u} (whose vacuum is $|0_b\rangle$): $\langle 0_b|b_{p_u}|0_a\rangle = 0$ while $\langle 0_a|b_{p_u}^\dagger b_{p_u}|0_a\rangle = \sum_k |\beta_{p_u k}|^2$. Therefore, we can already say that the radiation from a mirror is of a slightly different character from the atomic radiation in Refs. [1, 7].

Please note: The physics *does* depend on C . The physical world as a whole is not conformally invariant. Measuring instruments know about the metric tensor. If that were not so, all two-dimensional models would be just like static, flat space, and there would be no “effects” to be named after Unruh, Moore, and Hawking.

Example: To fit the Rindler coordinate system of Eqs. (1)–(3) into this framework, define ζ by $\rho = e^\zeta$. Then (3) takes (in 2-dimensional space-time) the manifestly conformally flat form

$$ds^2 = dt^2 - dz^2 = e^{2\zeta}(d\tau^2 - d\zeta^2), \quad (10)$$

and the stationary worldline (4) becomes

$$\zeta = -\ln(a \cosh \tau). \quad (11)$$

(Note that the coordinates (τ, ζ) — which range over all of \mathbb{R}^2 — cover only one quadrant, $z > |t|$, of the original space.) Identifying the x of the general formalism (6) with z , and the starred coordinates (8) there with barred ones here, one sees that

$$u = -e^{-\bar{u}}, \quad v = e^{\bar{v}}, \quad \text{where} \quad \bar{u} = \tau - \zeta, \quad \bar{v} = \tau + \zeta, \quad (12)$$

and the new conformal factor is

$$\bar{C}(\bar{u}, \bar{v}) = e^{\bar{v} - \bar{u}}. \quad (13)$$

What would be observable in our imaginary two-dimensional world? Presumably not particles, since following the textbook prescription for quantizing the field in the (τ, ζ) coordinates and in the (t, z) coordinates yield different results [5, 12]. The Minkowski vacuum is *not* the ground state of the Fock space built on Rindler modes, $\bar{\phi} \propto e^{i(-\omega\tau + k\zeta)}$. It is a mixed state of nonzero temperature proportional to the acceleration of a trajectory of fixed ζ . Unruh [5] observed that there is real physics, not just a mathematical artifact, in this nonstandard quantization: A detector at ζ observes a thermal bath at that temperature. But for a general coordinate system the quanta have no such clear physical interpretation, if they can be defined at all, and the real problem becomes not uniqueness of a natural particle interpretation, but existence. Therefore, the challenge to the subject in the 1970s was to define *field observables* that are independent of any particle concept and of any coordinate system (except, of course, for standard local transformations of tensor components).

Because the primary interest in field theory in curvilinear coordinates or curved spaces stems from gravitational physics, it was natural to concentrate on the *stress-energy-momentum tensor*, the source of the gravitational field, as the most important observable. In a standard Cartesian frame in two dimensions, this tensor has the structure

$$T_{\alpha\beta} = \begin{pmatrix} T_{tt} & T_{tx} \\ T_{xt} & T_{xx} \end{pmatrix},$$

where T_t^t has the interpretation of energy density, T_x^x that of pressure, and the off-diagonal components those of energy flux and momentum density. In generic index notation,

$$T_{\alpha\beta} = \partial_\alpha \phi \partial_\beta \phi - \frac{1}{2} g_{\alpha\beta} \partial_\lambda \phi \partial^\lambda \phi.$$

In null coordinates, we have

$$T_{uu} = \partial_u \phi \partial_u \phi, \quad T_{vv} = \partial_v \phi \partial_v \phi, \quad (14)$$

and, at least classically,

$$T_{uv} = \partial_u \phi \partial_v \phi - \partial_u \phi \partial_v \phi = 0.$$

T_{uu} has the interpretation of *rightward flux*, T_{vv} that of *leftward flux*. (Recall that $T^{uu} = \frac{4}{C^2} T_{vv}$, etc.)

The field equation implies the *conservation law* $\partial_\alpha T^{\alpha\beta} = 0$. Written out in null coordinates, this is

$$\partial_u T_{vv} = -\partial_v T_{uv} - C^{-1} \partial_v C T_{uv} \quad (15)$$

and the analogous equation for $\partial_v T_{uu}$. Classically, and in quantum field theory in flat space-time, we have $T_{uv} = 0$ and hence T_{vv} independent of u and vice versa. In quantum field theory with spatial curvature, however, T_{uv} will turn out to be nonzero.

To provide a partial explanation of these claims, we step back a bit to the generic conformally flat metric,

$$ds^2 = C^*(u^*, v^*) du^* dv^* \quad (16)$$

(cf. (5) and (8)). In what follows we shall assume that either (u^*, v^*) range over all of \mathbb{R}^2 , or that $u^* = t^* - x^*$, $v^* = t^* + x^*$, $-\infty < t^* < \infty$, and $0 < x^* < \infty$ with the boundary condition $\phi(t^*, 0) = 0$ imposed on the quantum field. In those two situations the textbook quantization can be carried out, using spatial eigenfunctions e^{ikx^*} or $\sin(kx^*)$, respectively; for details see [11, 20]. One has then a Fock vacuum relative to the starred coordinate system, and one can calculate expectation values in it, such as $\langle T_{vv} \rangle = \langle (\partial_v \phi)^2 \rangle$. As always, divergences must be removed. The prescription (inherited from the 1970s) is that this must be done *in a covariant manner* that reduces to the known right answer in flat, or initially flat, space-time and also satisfies the conservation law, $\nabla_\alpha T^{\alpha\beta} = 0$. The covariant prescription uses the metric structure of flat space, since it requires expanding the two-point function $\langle \phi(t, x) \phi(t', x') \rangle$ in terms of the geodesic separation between the two points; hence it is *not* conformally invariant. The conservation requirement resolves some ambiguities in the renormalization prescription, and it forces the famous *trace anomaly*:

$$T_\alpha^\alpha = \frac{4}{C} T_{uv} = -\frac{R}{24\pi}, \quad (17)$$

where R is the *Ricci curvature scalar*,

$$R = \frac{4}{C^3} (C \partial_u \partial_v C - \partial_u C \partial_v C) = -\frac{4}{C} \partial_u \partial_v (\ln C) = -\square_g (\ln C). \quad (18)$$

In view of (15), (17) says that R acts as a *source* in first-order differential equations satisfied by T_{uu} and T_{vv} (an insight due to Unruh [21]). R itself is not dynamical (for a given geometry), but it influences how $T_{\alpha\beta}$ propagates.

The upshot of these calculations [11, 20, 21] is

$$\begin{aligned} \langle T_{\alpha\beta} \rangle &= \theta_{\alpha\beta} - \frac{1}{48\pi} R g_{\alpha\beta}, \\ \theta_{uu} &= \frac{1}{24\pi C^2} [C \partial_u^2 C - \frac{3}{2} (\partial_u C)^2], \\ \theta_{vv} &= \frac{1}{24\pi C^2} [C \partial_v^2 C - \frac{3}{2} (\partial_v C)^2], \\ \theta_{uv} &= 0. \end{aligned} \quad (19)$$

(When two mirrors are present, the spectrum of normal modes becomes discrete, but the only effect of that is to add to (19) a term representing Casimir stress-energy. We expect to treat that situation in future work [30].)

In the Rindler example, with conformal factor (13), one gets from (19) that

$$\langle T_{\bar{v}\bar{v}} \rangle = \theta_{\bar{v}\bar{v}} = -\frac{1}{48\pi} = \langle T_{\bar{u}\bar{u}} \rangle. \quad (20)$$

In a local orthonormal frame aligned with the curvilinear coordinates, this is

$$\langle T_{\tau}^{\tau} \rangle = -\frac{1}{24\pi} e^{-2\zeta} = -\frac{1}{24\pi\rho^2} = -\langle T_{\zeta}^{\zeta} \rangle. \quad (21)$$

(Recall that ρ is the distance from the horizon at $x = |t|$, or $\zeta = -\infty$.) The stress is traceless and the energy is negative. This Rindler-space vacuum energy is singular at the horizon. In the true Minkowski vacuum state, it is cancelled by the positive energy of the Unruh thermal bath.

In summary, for any conformal coordinate system (in a two-dimensional space-time) that has the right global properties to permit the standard Fock-space construction to be performed, there results a “vacuum” state peculiar to that coordinate chart, along with creation operators to generate all the other states of definite particle number. There is a conserved renormalized energy-momentum tensor operator, independent of which conformal coordinate system is chosen, whose expectation value in the Fock vacuum of any particular such construction is given by (19). If the space-time is flat ($R = 0$), the off-diagonal component in the null frame, T_{uv} , is zero; more generally, that component is proportional to the trace of the tensor, which is independent of the state (a fixed multiple of R).

The role of coordinate systems in this discussion may appear suspicious: does it not fly in the face of the modern understanding of general relativity? No. Coordinates are just a tool to make calculations feasible. The real point is *which initial state* of the field is assumed in a calculation. It is a special property of the two-dimensional massless field that any conformal coordinate system defines a state (in the globally hyperbolic space-time region covered by those coordinates) that has many of the formal properties of the usual vacuum state of a free field. Barceló et al. [27] have shown how the construction can be reexpressed in manifestly covariant terms, replacing the distinguished coordinate system by a distinguished timelike vector field.

IV. RADIATION FROM A STATIONARY MIRROR

With the tools defined in Sec. III, we can address the problem of a stationary mirror “from the point of view of an accelerated observer.” By this we do not mean just that the observer uses the Rindler hyperbolic coordinates [16]; of course, in principle a calculation can be done in any coordinate system, with equivalent results. Rather, we mean that the initial state of the quantum field is a vacuum associated with the timelike Killing vector field of the Rindler system. Ideally this environment would be induced by doing experiments inside a cavity with perfectly reflecting walls, which has been uniformly accelerated for all time, and has been refrigerated to remove all excitations of the scalar field. A cavity of length L has a discrete set of modes with eigenfrequencies $\omega_n \propto n/L$ as measured with the Rindler timelike Killing vector ∂_{τ} ; all these modes are initially unoccupied. We are interested in

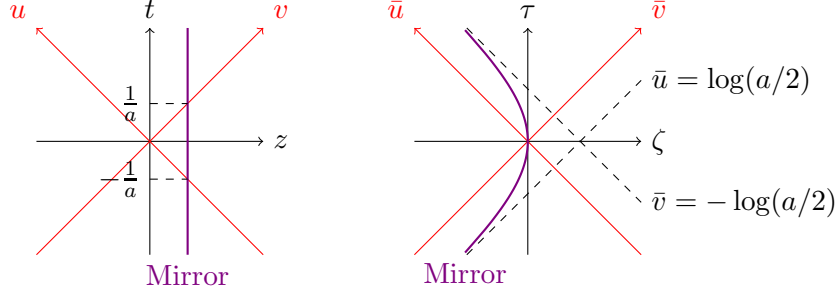


FIG. 5: A stationary mirror at $z = 1/a$ in Minkowski coordinates (left) enters the right Rindler wedge for $-1/a < t < 1/a$. In this interval, the trajectory of the mirror in Rindler coordinates is pictured (right).

what occurs when the mirror passes through the cavity, not the effects of the finite size, so we take $L \gg 1/a$, leaving us with the entire Rindler wedge at zeroth order. Therefore, we will focus on how the stationary mirror excites the Rindler vacuum in the part of the wedge to the right of the mirror.

Within the Rindler wedge, we assume that the cavity is initially in the Rindler vacuum defined by the normal modes $e^{-i\omega\bar{u}}$ and $e^{-i\omega\bar{v}}$ with (\bar{u}, \bar{v}) defined in (12).

As for the mirror's trajectory, it enters and leaves the Rindler wedge as seen in Fig. 5. In flat space, the trajectory is characterized by its constant spatial coordinate $z = 1/a$, or by its path in (Minkowski) null coordinates $[U(t), V(t)] = (t - 1/a, t + 1/a)$. Transforming to Rindler coordinates using (12), we can find the path in Rindler null coordinates:

$$\begin{aligned}\bar{U}(t) &= \log a - \log(1 - at), \\ \bar{V}(t) &= \log(1 + at) - \log a.\end{aligned}\tag{22}$$

(We use capital letters for the functions that define a particular trajectory, and the corresponding lower-case letters for the associated coordinates in a chart. Formulas (22) are equivalent to (4) or (11).)

At $t = -1/a$, the mirror enters the Rindler wedge. At this point, notice that $\bar{U}[(-1/a)^+] = \log(a/2)$ while $\bar{V}[(-1/a)^+] \rightarrow -\infty$. Similarly, when the mirror exits the Rindler wedge at $t = 1/a$, $\bar{V}[(1/a)^-] = -\log(a/2)$ and $\bar{U}[(1/a)^-] = \infty$. These indicate past and future null asymptotes as pictured in Fig. 5.

Now, we can use conformal transformations of the form mentioned in (8) along with the remarkable property that the massless wave equation is conformally invariant to find a new set of coordinates (\hat{u}, \hat{v}) with the following property: the mirror is at the *constant* coordinate position $\hat{x} = 0$ defining time and space coordinates (\hat{t}, \hat{x}) by $\hat{u} = \hat{t} - \hat{x}$ and $\hat{v} = \hat{t} + \hat{x}$. Once this coordinate transformation is found, the normal modes are simply $\phi \propto e^{-i\omega\hat{t}} \sin(k\hat{x})$, and the analysis leading to (19) applies.

There is in fact a continuum of such coordinate transformations $(\bar{u}, \bar{v}) \mapsto (\hat{u}, \hat{v}) = (p(\bar{u}), q(\bar{v}))$ (one of which returns us to Minkowski coordinates), and they all must satisfy

$$p[\bar{U}(t)] - q[\bar{V}(t)] = 0,\tag{23}$$

which is just a restatement of $\hat{x} = 0$. For the accelerated cavity defined with the right-moving normal modes $e^{-i\omega\bar{u}}$ and left-moving normal modes $e^{-i\omega\bar{v}}$, the left-movers should

remain equally spaced and unaffected while the right-movers will be distorted by the mirror for $u > \log(a/2)$ (they have been reflected off of the mirror). Physically then, we expect $q(\bar{v}) = \bar{v}$ and (23) can subsequently be solved such that formally

$$p = \bar{V} \circ \bar{U}^{-1}, \quad (24)$$

and for our particular case,

$$p(\bar{u}) = \log(2 - ae^{-\bar{u}}) - \log(a), \quad \bar{u} > \log(a/2) \quad (25)$$

To determine the radiation leaving the mirror to the right ($z > 1/a$), we calculate the expectation value of the energy momentum tensor $\langle T_{\mu\nu} \rangle$ with (19). As previously discussed, this requires the conformal factor in the hatted coordinates,

$$ds^2 = \hat{C}(\hat{u}, \hat{v}) d\hat{u} d\hat{v} = f'(\hat{u}) \bar{C}(f(\hat{u}), \hat{v}) d\hat{u} d\hat{v}, \quad (26)$$

where $f = p^{-1}$. If we define the functional

$$F_x(f) = \frac{\partial_x^2 f(x)}{f(x)} - \frac{3}{2} \left(\frac{\partial_x f(x)}{f(x)} \right)^2, \quad (27)$$

then we have

$$\langle T_{\hat{u}\hat{u}} \rangle = \frac{1}{24\pi} F_{\hat{u}}[\hat{C}], \quad \langle T_{\hat{v}\hat{v}} \rangle = \frac{1}{24\pi} F_{\hat{v}}[\hat{C}], \quad \langle T_{\hat{u}\hat{v}} \rangle = \langle T_{\hat{v}\hat{u}} \rangle = 0. \quad (28)$$

It can then be checked that

$$F_{\hat{u}}(\hat{C}) = f'(\hat{u})^2 F_{f(\hat{u})}(\bar{C}) + F_{\hat{u}}(f'). \quad (29)$$

Additionally, if p is the inverse of f , then $F_{\hat{u}}(f') = -f'(\hat{u})^2 F_{f(\hat{u})}[p']$. Written in terms of the \bar{u} , this implies

$$F_{\hat{u}}(\hat{C}) = \frac{1}{p'(\bar{u})^2} [F_{\bar{u}}(\bar{C}) - F_{\bar{u}}(p')]. \quad (30)$$

The stress-energy tensor in the original Rindler coordinates can be found by using the coordinate transformation $\langle T_{\bar{u}\bar{u}} \rangle = p'(\bar{u})^2 \langle T_{\hat{u}\hat{u}} \rangle$. Therefore,

$$\langle T_{\bar{u}\bar{u}} \rangle = -\frac{1}{48\pi} \frac{a^2 e^{-2\bar{u}}}{(2 - ae^{-\bar{u}})^2}, \quad \langle T_{\bar{v}\bar{v}} \rangle = -\frac{1}{48\pi}, \quad \bar{u} > \log(a/2). \quad (31)$$

A further simple transformation takes us back to (null) Minkowski coordinates, where we find

$$\begin{aligned} \langle T_{uu} \rangle &= -\frac{1}{48\pi} \frac{a^2}{(2 + au)^2} \quad \text{for } u > -\frac{2}{a}, \\ \langle T_{vv} \rangle &= -\frac{1}{48\pi} \frac{1}{v^2} \quad \text{for } v > 0. \end{aligned} \quad (32)$$

In these formulas, notice first the (negative) flux moving to the left, $\langle T_{\bar{v}\bar{v}} \rangle$ or $\langle T_{vv} \rangle$; it is well known for the Rindler vacuum in the absence of a mirror. In that case there would be a similar flux to the right, $\langle T_{uu} \rangle = -\frac{1}{48\pi} \frac{1}{a^2}$, which is divergent at $u = 0$. This term has

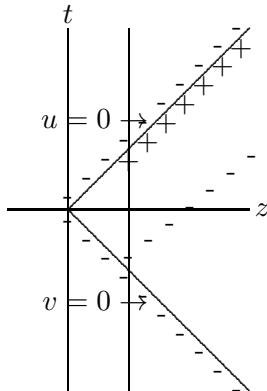


FIG. 6: In the absence of the mirror, the Rindler vacuum possesses a negative vacuum stress (20), (21) concentrated near the horizon (here indicated outside the horizon for visual clarity, but actually inside). When the mirror is introduced at $z = 1/a$ and the region to its right is initially in a Rindler vacuum, an outgoing burst of positive stress cancels the Rindler stress along the future horizon, while the incoming stress along the past horizon is reflected into the region from the point where the mirror enters the Rindler wedge.

been removed by the presence of the mirror, but it also seems to have been reproduced at an earlier retarded time, $u = -2/a$. A physical explanation of this early negative-energy burst is that the divergent incoming flux $\langle T_{vv} \rangle$ has reflected off the mirror at $z = 1/a$. On the other hand, the cancellation of the divergence at $u = 0$, present in the normal Rindler vacuum, might have been predicted because of energy conservation (for a mirror stationary in Minkowski space) and the absence of any incoming flux at $v = 2/a$ to be reflected. See Fig. 6.

Any realistic experiment is limited not only in space (by cavity walls) but also in time. The Rindler horizons and the accompanying singularities, consequently, are not actually inside the experimental region.

V. DISCUSSION

A. The work of Davies

The 1975 paper of Davies [14] has usually been regarded as a precursor of the 1976 paper of Unruh [5], in which the Minkowski-to-Rindler Bogolubov transformation was derived and was somehow attributed to the presence of a stationary reflecting boundary. Close examination, however, shows that [14] is actually doing something quite different from [5] (or [12], usually cited in the same context) and closer to the present paper.

Davies started from the observation (developed further in [22]) that the origin of a spherical coordinate system in dimension $(3 + 1)$ acts after separation of variables as a mirror boundary in dimension $(1 + 1)$, and that in a collapsing black hole that boundary “accelerates” in Schwarzschild coordinates. He then followed the black-hole calculation of Hawking [2] step by step to show that a stationary mirror in Minkowski space radiates as seen by a Rindler observer. This calculation is based on a Bogolubov transformation. In Rindler coordinates the trajectory of the mirror is, of course, the one described by (4), (11), or (22).

In [22] the problem was revisited on the basis of the intervening development [11, 20, 21] of a physical interpretation of the theory in terms of expectation values of the renormalized stress tensor. As reviewed in Sec. III, this approach usually avoids explicit Bogolubov transformations. The paper [22] contains the statement, “[I]t is now clear that the static mirror in Rindler space [in [14]] does not actually produce energy,” and for that reason attention was shifted to the same trajectory (11) but regarded as embedded in the Minkowski background metric ($\eta \rightarrow z$ and $\tau \rightarrow t$); in that case it was shown that the trajectory did produce radiation.

In view of the present paper, however, it is now clear that the retreat from [14] was unwarranted and was based on a failure to distinguish between Minkowski and Rindler vacuums as initial state. Both Minkowski and Rindler space are flat, so the conservation law (15) applies with vanishing right-hand side (because of (17)). Therefore, any outgoing radiation at future null infinity can be propagated back to the mirror (unlike in the case of the model black hole in [21]); the only difference between the emissions of mirror (11) in Minkowski space and in Rindler space is geometrical factors related to the introduction of local orthonormal frames, and it is not possible for the “radiation” to vanish in one problem and not the other. The resolution of this apparent paradox is that the Rindler-space calculation (whether by the method of [14] or that of [22] and our Sec. IV) tacitly starts from *the Rindler vacuum as initial state*. Thus Davies’s Hawking-style calculation revealed real radiation from the stationary mirror, embodied in Bogolubov transformation coefficients that turned out to be identical with those of [5, 12]. What he could not have known at the time is that the Rindler vacuum contains a negative stress that (near $u = 0$) precisely cancels the stress tensor of his radiation. (In both [14] and [22] attention was focused on the far future, where the relevance to the black-hole situation is greatest. Therefore, the emission at $u \approx -2/a$ in (32) was not noted.)

B. The status of the equivalence principle

The mirror scenario in Sec. IV is clearly analogous to “experiment 2” in Sec. II. The role of the atom is played by the stationary mirror that enters and exits the Rindler wedge. An accelerating optical cavity within that wedge carries with it its own vacuum, and our calculations indicate that in that case, excitations as measured by the stress-energy tensor are created; cavity modes are excited! From the perspective of the cavity, the mirror enters violently, modifying $\langle T_{\mu\nu} \rangle$. On the other hand, a stationary observer will see from the outset that the cavity begins with some $\langle T_{\mu\nu} \rangle$ that is reflected off the mirror. These conclusions are analogous to the *yes-yes-yes* conclusions for the atomic gedankenexperiments.

One already knows that an accelerating mirror radiates into a stationary vacuum [11, 22]. Our converse result thus shows that this remains true for an accelerated (Rindler) vacuum and stationary mirror. Insofar as this principle extends to *all* physical phenomena, it resolves the conundrum of which bodies to call “accelerated” in gravitational problems: those which accelerate relative to the relevant vacuum. Its primary practical implication is that a uniformly accelerated frame can legitimately be treated as a “static” frame set in a gravitational field; it is this fact that is properly called the *qualitative equivalence principle*. This conclusion is of great importance because near a black hole (or in any nontrivial static gravitational field) the accelerated frame is the physically simpler one, as compared to a frame in free fall (which does not experience static conditions).

Our simple, nonperturbative result verifies that phenomena analogous to the radiation by

unaccelerated atoms deduced in [1] and [7] can indeed happen. If one assumes the qualitative equivalence principle in generality, such atomic radiation is unsurprising, indeed necessary for consistency (“*yes-yes-yes*”). On the other hand, if one already accepts the logic of [1] and [7], then the analogous study in the present paper adds to the evidence that the principle holds in generality.

Two limitations on this picture are important to note. First, the situations are not *quantitatively* symmetric. For example, a stationary mirror in an accelerated cavity is not mathematically the same thing as an accelerated mirror in a stationary cavity, as has been understood for a long time [11, 22]. Second, both mirror and atom [1, 7] studies, and the general detector analyses of [18], show that the stationary object does not radiate (in the precise way described by the equivalence principle) to the accelerated observer or detector unless the latter is initially in its Rindler vacuum, or as close to a Rindler vacuum as the modified conditions (e.g., cavity walls) permit. How such a state could be realized in practice is a difficult separate question.

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