S Supplementary Material

The supplementary material is organized as follows. Section S.1 shows that no index rule is optimal in the search problem considered in the main text. Section S.2 contains the proof of Propositions 4and Corollary 1, while Section S.3 contains the proof of Proposition 5. Section S.4 shows that Assumption 1 is without loss of generality. Section S.5 solves for the optimal policy when $N = 2$ and shows how Propositions 1-3 in the main text help simplify the taxonomy of the problem. Section T contains examples referenced in the main text, while Section U provides step-by-step derivations of expressions used in the analysis.

S.1 Indexability

I discuss formally why, unlike Weitzman's, the optimal policy in my model is not an index policy. To do so, I define the notion of an index, and an index rule. I then show that, under Assumption 1, no index rule is optimal even when $N = 1$. I finish the section with two remarks for the case of $N > 1$, which follow from the suboptimality of index rules. To keep the presentation simple, I assume that X_i, box i's set of possible prize realizations, is finite.

Formally, each box can be used to define a Markov decision process, with parameters as follows. Let $\delta \in [0, 1]$ denote the discount factor. The set of states is $S_i = \{\emptyset\} \cup X_i$, where $\{\emptyset\}$ represents that box i is uninspected, and x_i that prize $x_i \in X_i$ has been realized. The set of controls is $A_i = \{0, 1\}$, where $a_i = 0$ corresponds to taking box i without inspection. Transition probabilities are given by: $P(s_i = x_i|s_i = \emptyset, a_i = 1) = f_i(x_i)$, $P(s_i = x_i|s_i = \emptyset, a_i = 0) = 0$, $P(s_i = x_i|s_i = x'_i, a_i) = \mathbf{1}[x'_i = x_i]$. That is, if the agent inspects box i, it transitions to state x_i with probability $f_i(x_i)$; it does not transition when it is taken without inspection. Moreover, for all $x_i \in X_i$, state x_i is absorbing . Finally, payoffs are given by (i) $v(\emptyset, 0) = (1 - \delta)\mu_i$, (ii) $v(\emptyset, 1) = -k_i, v(x_i, 1) = (1 - \delta)x_i$, and (iii) $v(x_i, 0) = K$, for some $K < \min\{x_i : x_i \in X_i\}$. That is, (i) taking a box without inspection yields a payoff of μ_i, (ii) when the agent inspects the box, he pays its inspection cost, and when he returns to the box, he receives x_i, and (iii) when the agent inspects the box, he can't take it without inspection, so I assign a low payoff to $a_i = 0$ when the box is inspected. The agent maximizes his discounted expected sum of payoffs.

An index for box i is a function that depends on the state of box i; I denote it $\nu_i : S_i \mapsto \mathbb{R}$. An *index policy* for a set of boxes $\mathcal{N}$ is a policy that at each state chooses the box with the highest index.

In the environment under consideration, a slightly different definition of an index policy is needed. One needs to know both which box to choose next, and also whether to inspect it, or take it without inspection. Let $\nu_{i,a_i} : S_i \mapsto \mathbb{R}$ denote the index for box i for action a_i. An *index policy* chooses at each state the box with the highest $\max\{\nu_{\cdot,0}, \nu_{\cdot,1}\}$, and applies to it the action with the highest index.

Assume now that $N = 1$, and the box is uninspected. Let $\overline{z}$ denote the outside option. Suppose that $x_1^B < x_1^R$. If an index policy is optimal, then two things must be true. First, for $\overline{z} \leq x_1^B$, $\nu_{0,1}(\emptyset) \geq \nu_{1,1}(\emptyset)$ should hold, since box 1 should be taken without inspection. Second, for $\overline{z} \in (x_1^B, x_1^R)$, $\nu_{1,1}(\emptyset) \geq \nu_{0,1}(\emptyset)$, since box 1 should be inspected. Hence, it follows that $\nu_{0,1}(\emptyset) = \nu_{1,1}(\emptyset)$. Then, an index policy would imply that the agent is indifferent between inspecting box 1, and taking it without inspection, but this is not always the case. When the box is uninspected, what action is optimal depends on $\overline{z}$ (recall Proposition 0), but, by definition, the index cannot condition on this information.

Interestingly, when $x_1^R < x_1^B$, an index does exist, since for any $\overline{z}$, should the box be chosen, it can only be optimal to take it without inspection. To see this, define $\nu_{0,1} = \mu$, and $\nu_{1,1} < \mu$. Also, since $\overline{z}$ can be interpreted as a box with zero inspection cost, and probability 1 of yielding a prize of $\overline{z}$, one can define $\nu_{1,\overline{z}} = \nu_{0,\overline{z}} = \overline{z}$. In this case, the index policy is optimal. In fact, Glazebrook [1] shows that a sufficient condition for a stoppable superprocess[1] to be solvable by an index policy is that the optimal action with which to continue with box 1 does not depend on the value of $\overline{z}$, i.e., that $x_1^R < x_1^B$. However, if this holds for all boxes, the optimal policy is trivial: search finishes immediately, and the agent takes $\max\{\overline{z}, \max_{i\in\mathcal{N}} \mu_i\}$.

Remark S.1. When $N = 1$, the reservation and backup values, and the initial outside option, are enough to determine the optimal policy. However, the proof

[1] The Markov decision process defined above is a special case of a stoppable superprocess. Superprocesses are instances of restless bandits, which are shown to be PSPACE-hard in [2].

that no index rule is optimal when $N = 1$ suggests why the cutoffs are not enough to determine the optimal policy when $N > 1$.[2] The reason why more than the cutoff values matter to determine the optimal policy is that they don't necessarily determine the full "value" of a box. By the previous discussion, the value of a box depends on whether the box will be inspected, or taken without inspection. To see this, consider Problem 2 in Section 1. If only school A is available, it is optimal to accept school A without inspection. Now add school B, and note that it is worse than school A both to inspect *and* to take without inspection.[3] One would then expect that the optimal policy remains the same when adding school B. However, this is not the case, because what dominates taking school A without inspection is inspecting school B *and* then choosing, given x_B, whatever is best between inspecting or taking school A without inspection. Thus, the boxes' cutoffs alone are not enough to determine the optimal policy.

Remark S.2. A second difference between Weitzman's model and the one considered here is that, contrary to the stopping rule in Weitzman, stopping and taking a box without inspection is not a one-step look ahead rule. More precisely, in Weitzman's model stopping is optimal at decision node $(\mathcal{U}, z)$ if, and only if, for every $i \in \mathcal{U}$, it is optimal to stop at $(\{i\}, z)$. Clearly, if it is optimal to stop at $(\mathcal{U}, z)$, the agent should not find it optimal to inspect any box $i \in \mathcal{U}$, i.e., stopping being optimal at $(\{i\}, z)$ is a necessary condition for stopping to be optimal at $(\mathcal{U}, z)$. In Weitzman's model, it is also sufficient. However, in this search problem, it could be that for all $i \in \mathcal{U}$, stopping and taking a box without inspection is optimal at $(\{i\}, z)$, and yet this is not the optimal policy at $(\mathcal{U}, z)$. To see this, consider again Problem 2. Using equation (BV), it follows that $z = 0 < \min_{i\in\{A,B\}} x_i^B$. However, the optimal policy has the student visit school B first. This follows from the same observation as in Remark S.1: what dominates taking either school without inspection is the possibility of, after visiting school B, choosing optimally whether to use school A as an option to inspect, or to take without inspection.

[2]Section T.1 shows that two sets of boxes can share the same cutoffs, and yet have different optimal policies.

[3]Equations (RV)-(BV) can be used to show that $x_A^R > x_B^R > x_A^B > x_B^B$.

S.2 Proof of Proposition 4 and Corollary 1

Proposition 4. Fix a set $\mathcal{N} = \{1, ..., n\}$ of boxes. Assume that boxes can be labelled so that $[x_i^B, x_i^R]$ forms a monotone decreasing sequence in the set inclusion order. Then, for all $i, j \in \mathcal{N}$, such that $i < j$, $\Pi_{ij} \geq \max\{\Pi_{ji}, \mu_i\}$, and the optimal policy is an in Theorem 1.

Proof. Proposition 2 implies that, for $i < j$, $\max\{\Pi_{ij}, \Pi_{ji}\} \geq \mu_i$. It remains to show that $\Pi_{ij} \geq \Pi_{ji}$. Section U shows that:

$$\Pi_{ij} - \Pi_{ji} = \int_{x_j^R}^{+\infty} \int_{x_j^R}^{+\infty} (\min\{x_i^R, x_i, x_j\} - x_j^R) dF_j dF_i$$
$$+ \int_{-\infty}^{x_i^B} \int_{-\infty}^{x_i^B} (\max\{x_i, x_j, x_j^B\} - x_i^B) dF_j dF_i. \tag{S.1}$$

Hence, $[x_j^B, x_j^R] \subset [x_i^B, x_i^R]$ implies that $\Pi_{ij} \geq \Pi_{ji}$. □

Corollary 1. Assume $\{F_i\}_{i\in\mathcal{N}}$ is such that if $i < i'$, then F_i is a mean-preserving spread of $F_{i'}$. Moreover, assume $\forall i \in \mathcal{N} \quad k_i = k$. Then, $(\forall i, i' \in \mathcal{N})$, $i < i'$ implies that $[x_{i'}^B, x_{i'}^R] \subset [x_i^B, x_i^R]$.

Proof. It suffices to show that if $i < i'$, then $[x_{i'}^B, x_{i'}^R] \subseteq [x_i^B, x_i^R]$. To see this, rewrite equation (RV) for box i as:

$$k = \int_{x_i^R}^{+\infty} (x - x_i^R) dF_i(x) = \int_{-\infty}^{+\infty} \max\{x - x_i^R, 0\} dF_i(x),$$

and, note that, if F_i is a mean-preserving spread of $F_{i'}$, then:

$$k = \int_{-\infty}^{+\infty} \max\{x - x_i^R, 0\} dF_i(x) \geq \int_{-\infty}^{+\infty} \max\{x - x_i^R, 0\} dF_{i'}(x).$$

Since $\int_{x_i^R}^{+\infty} (x - x_i^R) dF(x)$ is decreasing in x_i^R, one concludes that $x_{i'}^R \leq x_i^R$. Likewise, rewrite equation (BV) as:

$$k = \int_{-\infty}^{x_i^B} (x_i^B - x) dF_i(x) = \int_{-\infty}^{+\infty} \max\{x_i^B - x, 0\} dF_i(x).$$

Using the mean-preserving spread assumption again, one obtains that $i < i'$ implies that:

$$k = \int_{-\infty}^{+\infty} \max\{x_i^B - x, 0\} dF_i(x) \geq \int_{-\infty}^{+\infty} \max\{x_i^B - x, 0\} dF_{i'}(x).$$

Since $\int_{-\infty}^{x_i^B} (x_i^B - x) dF(x)$ is increasing in x_i^B, one concludes that $x_i^B \leq x_{i'}^B$.

It follows that $[x_{i'}^B, x_{i'}^R] \subset [x_i^B, x_i^R]$. □

S.3 Proof of Proposition 5

I first establish a preliminary result on the cutoff values when the conditions in Proposition 5 hold:

Lemma S.1 (Cutoffs are linear in means)**.** Let x be a random variable such that $x \sim F(\cdot - \mu), E[x] = \mu$. Let k be the cost of inspecting the box with prizes distributed according to F. Then, $(\exists \underline{b}, \overline{b}) : x^B = \mu - \underline{b}, x^R = \mu + \overline{b}$.

Proof. I prove the statement for x^R, the other one follows immediately. Recall that:

$$k = \int_{x^R}^{+\infty} (x - x^R) dF(x - \mu).$$

I guess and verify that $x^R = \mu + \overline{b}$, for some $\overline{b} > 0$,

$$k = \int_{\mu + \overline{b}}^{+\infty} (x - \mu - \overline{b}) dF(x - \mu).$$

Let $u = x - \mu$ and perform a change of variables in the above expression:

$$k = \int_{\overline{b}}^{+\infty} (u - \overline{b}) dF(u). \tag{S.2}$$

It remains to show that equation (S.2) has a solution. Assumption 1 implies that if $\overline{b} = 0$, then $k < \int_0^{+\infty} u dF(u)$. On the other hand, as $\overline{b} \to \infty$, $\int_{\overline{b}}^{+\infty} (u - \overline{b}) dF(u) \to$

$0 < k$. Hence, since $g(b) = \int_b^{+\infty} (x - b) dF$ is continuous and decreasing in b, there exists $\overline{b} > 0$, such that the equality holds. This completes the proof. $\square$

Corollary S.1. Consider the same assumptions as before. If F is symmetric around 0 then $\overline{b} = \underline{b} = b > 0$

Proof. $b > 0$ follows from the condition that $x^B < \mu < x^R$. Now, recall the definition of x^B:

$$k = \int_{-\infty}^{x^B} (x^B - x) dF(x - \mu).$$

Replacing the assumptions made, one gets that the equation can be rewritten as:

$$k = \int_{-\infty}^{-\underline{b}} (-\underline{b} - u) dF(u),$$

where I changed variables by defining $u = x - \mu$. Also,

$$k = \int_{x^R}^{+\infty} (x - x^R) dF(x - \mu) = \int_{\overline{b}}^{+\infty} (u - \overline{b}) dF(u).$$

Now, symmetry of F implies that:

$$\int_{\overline{b}}^{+\infty} u dF(u) = -\int_{-\infty}^{-\overline{b}} u dF(u).$$

Hence, $(1 - F(\overline{b}))E[u|u \geq \overline{b}] = -F(-\overline{b})E[u|u \leq -\overline{b}]$ and $-(1 - F(\overline{b}))\overline{b} = -F(-\overline{b})\overline{b}$. Hence, $\overline{b} = \underline{b}$. $\square$

I am now ready to prove Proposition 5. It follows from equation (S.1) in Section

S.2 that:

$$\begin{aligned}
\Pi_{ij} - \Pi_{ji} &= \int_{-\infty}^{x_i^B}\int_{-\infty}^{x_i^B} (\max\{x_i, x_j, x_j^B\} - x_i^B) dF_i dF_j + \int_{x_j^R}^{+\infty}\int_{x_j^R}^{+\infty} (\min\{x_i, x_j, x_i^R\} - x_j^R) dF_i dF_j \\
&= (1 - F_i(x_i^R))(1 - F_j(x_i^R))(x_i^R - x_j^R) + \int_{x_j^R}^{x_i^R}\int_{x_i}^{+\infty} (x_i - x_j^R) dF_j dF_i \\
&+ \int_{x_j^R}^{x_i^R}\int_{x_j^R}^{x_i} (x_j - x_j^R) dF_j dF_i + (1 - F_i(x_i^R)) \int_{x_j^R}^{x_i^R} (x_j - x_j^R) dF_j \\
&+ F_i(x_j^B) F_j(x_j^B)(x_j^B - x_i^B) + F_i(x_j^B) \int_{x_j^B}^{x_i^B} (x_j - x_i^B) dF_j \\
&+ \int_{x_j^B}^{x_i^B}\int_{x_i}^{x_i^B} (x_j - x_i^B) dF_j dF_i + \int_{x_j^B}^{x_i^B}\int_{-\infty}^{x_i} (x_i - x_i^B) dF_j dF_i.
\end{aligned}$$

Perform the following change of variables. Let $u = x_i - \mu_i$, $\hat{u} = x_j - \mu_j$, and write $a = \mu_i - \mu_j \geq 0$. It follows that:

$$\begin{aligned}
G(a) &= \int_{b-a}^{b}\int_{u+a}^{+\infty} (u + a - b) dF(\hat{u}) dF(u) + \int_{b-a}^{b}\int_{b}^{u+a} (\hat{u} - b) dF(\hat{u}) dF(u) \\
&+ F(-b) \int_{b}^{b+a} (\hat{u} - b) dF(\hat{u}) + F(-b - a) \int_{-b}^{-b+a} (\hat{u} + b - a) dF(\hat{u}) \\
&+ \int_{-b-a}^{-b}\int_{u+a}^{-b+a} (\hat{u} + b - a) dF(\hat{u}) dF(u) + \int_{-b-a}^{-b}\int_{-\infty}^{u+a} (u + b) dF(\hat{u}) dF(u).
\end{aligned}$$

Note that $G(0) = 0$. I show that $(\forall a) G'(0) = 0, G''(a) = 0$. All of these together imply that $G(a) \equiv 0$.

$$\begin{aligned}
G'(a) = -[&\int_{b-a}^{b} F(-b - a) dF(u) + \int_{-b-a}^{-b} (F(-b + a) - F(u + a)) dF(u) \\
&- \int_{b-a}^{b} F(-u - a) dF(u)].
\end{aligned}$$

Note that $G'(0) = 0$. Moreover,

$$G''(a) = F(-b-a)f(b-a) - \int_{b-a}^{b} f(-b-a)dF(u) + (F(-b-a) - F(-b))f(-b-a)$$
$$+ \int_{-b-a}^{-b} (f(-b+a) - f(u+a))dF(u) - F(-b)f(b-a) + \int_{b-a}^{b} f(-u-a)dF(u) = 0,$$

where I used that $f(x) = f(-x), F(-x) = 1 - F(x)$ several times to cancel terms. This shows that $G(a) \equiv 0$.

S.4 Boxes for which $x^R \leq x^B$ are never inspected in the optimal policy

This last subsection shows that, if there are boxes $i \in \mathcal{N}$ such that $x_i^R \leq x_i^B$, then, without loss of generality, box i is never inspected in the optimal policy. Therefore, for any such box $i \in \mathcal{N}$, it is either taken without inspection upon stopping search, or it is never used in the optimal policy. Moreover, note that only one such box can be taken without inspection conditional on stopping search. Then, by redefining x_0 to be whatever is best between the agent's initial outside option and the best of the boxes for which $x_i^R \leq x_i^B$, the analysis in the paper carries through by focusing on the boxes for which $x_i^B < x_i^R$.

Given a set of boxes $\mathcal{U}$, define:

$$\mathcal{U}^{B<R} = \{i \in \mathcal{U} : x_i^B < x_i^R\},$$
$$\mathcal{U}^{R\leq B} = \{i \in \mathcal{U} : x_i^R \leq x_i^B\}.$$

Given a decision node $(\mathcal{U}, z)$, I denote by $(\mathcal{U}', z')$,$\mathcal{U}' \subset \mathcal{U}, z' = z \circ z_{\mathcal{U}\backslash\mathcal{U}'}$ a generic decision node in which boxes in $\mathcal{U}\backslash\mathcal{U}'$ have been inspected, and prizes $z_{\mathcal{U}\backslash\mathcal{U}'}$ have been sampled.

Proposition S.1. *Let $\mathcal{U}$ be the set of boxes, and let z be a vector of realized prizes. Assume that $\mathcal{U}^{R\leq B} \neq \emptyset$. Then, there exists an optimal policy $\{\varphi^*, \sigma^*\}$ such that $(\forall(\mathcal{U}', z') : \mathcal{U}' \subseteq \mathcal{U} \wedge z' = z \circ \tilde{z}_{\mathcal{U}\backslash\mathcal{U}'})[\varphi^*(\mathcal{U}', z') = 1 \Rightarrow \sigma^*(\mathcal{U}', z') \notin \mathcal{U}'^{R\leq B}]$.*

Proof. The proof is by double induction in the cardinality of $\mathcal{U}$ and $\mathcal{U}^{R\leq B}$. Since $\mathcal{U}^{R\leq B} \subset \mathcal{U}$, then $|\mathcal{U}^{R\leq B}| \leq |\mathcal{U}|$. Induction is in $U = |\mathcal{U}|$, and n, where $|\mathcal{U}^{R\leq B}| =$

$\max\{U, n\}$. Let $P(U, n)$ denote the following predicate:

P(U,n): $(\forall z)(\forall \mathcal{U}) : |\mathcal{U}| = U, \mathcal{U}^{R\leq B} \neq \emptyset$, $|\mathcal{U}^{R\leq B}| = \max\{n, U\}$, the optimal policy satisfies the property in Proposition S.1.

I first show that $P(1, 1) = 1$, and then that if $P(U', n') = 1$ holds for $U' \leq U$, and $n' \leq n$, not both with equality, then $P(U, n) = 1$ holds.

P(1,1)=1:

Let $\mathcal{U} = \{i\}$ and let z denote the vector of already realized prizes. Since $U = n = 1$, then $\mathcal{U}^{R\leq B} = \{i\}$. I show that: $-k_i + \int \max\{x_i, \overline{z}\}dF_i \leq \max\{\mu_i, \overline{z}\}$. Suppose that $\overline{z} \geq \mu_i$. Then, since $i \in \mathcal{U}^{R\leq B}$, $x_i^R \leq \mu_i \leq \overline{z}$. Then,

$$-k_i + \int \max\{x_i, \overline{z}\}dF_i - \overline{z} = -k_i + \int_{\overline{z}} (x_i - \overline{z})dF_i(x_i) \leq 0,$$

since $\overline{z} \leq x_i^R$ (recall the derivation of equation (RV)), with equality only if $\overline{z} = x_i^R$. Now, suppose that $\mu_i > \overline{z}$. Then, $x_i^B \geq \mu_i > \overline{z}$, and it follows from (BV) that:

$$-k_i + \int \max\{x_i, \overline{z}\}dF_i - \mu_i = -k_i + \int_{-\infty}^{\overline{z}} (\overline{z} - x_i)dF_i(x_i) < 0.$$

P(U,n)=1:

Assume now that $(\forall U' \leq U)(\forall n' \leq n)$, not both with equality, $P(U', n') = 1$. I show that $P(U, n) = 1$. Let $\mathcal{U}$ be the set of boxes, $|\mathcal{U}| = U$, and let z denote the vector of already sampled prizes. Let $\mathcal{U}^{R\leq B} \subset \mathcal{U}$, $|\mathcal{U}^{R\leq B}| = \max\{U, n\}$. I use i to denote a box in $\mathcal{U}^{R\leq B}$, and j to denote a box in $\mathcal{U}\backslash\mathcal{U}^{R\leq B}$, whenever the latter is not empty.

I make two remarks. First, notice that if a box $j \in \mathcal{U}\backslash\mathcal{U}^{R\leq B}$ is inspected, then one moves to decision node $(\mathcal{U}', z \circ x_j)$, where $\mathcal{U}' = \mathcal{U}\backslash\{j\}, \mathcal{U}'^{R\leq B} = \mathcal{U}^{R\leq B}$, and $|\mathcal{U}'| = U - 1$, and $|\mathcal{U}'^{R\leq B}| = n$ (note that if there was $j \in \mathcal{U}\backslash\mathcal{U}^{R\leq B}$, then it can't be the case that $|\mathcal{U}^{R\leq B}| = U$). Since, by the inductive step, I know that $P(U - 1, n) = 1$, then there is an optimal policy in which boxes in $\mathcal{U}^{R\leq B}$ are not inspected in any continuation history. Second, if a box $i \in \mathcal{U}^{R\leq B}$ were to be inspected, then one moves to continuation history $(\mathcal{U}', z \circ x_i)$, where $\mathcal{U}' = \mathcal{U}\backslash\{i\}$, $\mathcal{U}'^{R\leq B} = \mathcal{U}^{R\leq B}\backslash\{i\}$, and $|\mathcal{U}'| = U - 1$, $|\mathcal{U}'^{R\leq B}| = \max\{U - 1, n - 1\}$. Since, by the inductive step, I know that $P(U-1, n-1) = 1$, then there is an optimal policy in which boxes in $\mathcal{U}'^{R\leq B}$ are not inspected in any continuation history. The first remark implies that to prove $P(U,n) = 1$ it remains to show that it is optimal not to inspect a box in $\mathcal{U}^{R\leq B}$ at decision node $(\mathcal{U}, z)$. The second remark will be used when computing the payoff of inspecting a box in $i \in \mathcal{U}^{R\leq B}$.

Given the above, I want to show that:

$$\max\left\{\overline{z}, \max_{i\in\mathcal{U}^{R\leq B}} \mu_i, \max_{j\in\mathcal{U}^{B<R}} \mu_j, \max_{j\in\mathcal{U}^{B<R}} \{-k_j + \int V^*(\mathcal{U}\backslash\{j\}, z\circ x_j)dF_j\}\right\}$$
$$\geq \max_{i\in\mathcal{U}^{R\leq B}} \{-k_i + \int V^*(\mathcal{U}\backslash\{i\}, z\circ x_i)dF_i\}, \tag{S.3}$$

where the LHS of the above expression denotes the payoff the agent can get by either stopping, and getting $\max\{\overline{z}, \max_{i\in\mathcal{U}^{R\leq B}} \mu_i, \max_{j\in\mathcal{U}^{B<R}} \mu_j\}$, or continuing search by inspecting a box in $\mathcal{U}^{B<R}$; the RHS denotes the payoff of inspecting a box in $\mathcal{U}^{R\leq B}$. The stars in V denote that the agent follows the optimal policy in the continuation histories, and the two remarks above apply, by the inductive step, to those histories. Note that I can write, for any box $i \in \mathcal{U}^{R\leq B}$:

$$\begin{aligned}
&-k_i + \int V^*(\mathcal{U}\backslash\{i\}, z\circ x_i)dF_i \\
&= -k_i + \int \max\left\{\begin{array}{c} x_i, \overline{z}, \max_{i'\in\mathcal{U}^{R\leq B}\backslash\{i\}} \mu_{i'}, \max_{j\in\mathcal{U}^{B<R}} \mu_j, \\ \max_{j\in\mathcal{U}^{B<R}}\{-k_j + \int V^*(\mathcal{U}\backslash\{i,j\}, z\circ x_i\circ x_j)dF_j\} \end{array}\right\} dF_i \\
&= -k_i + \int \max\left\{x_i, \max\left\{\begin{array}{c} \overline{z}, \max_{i'\in\mathcal{U}^{R\leq B}\backslash\{i\}} \mu_{i'}, \max_{j\in\mathcal{U}^{B<R}} \mu_j, \\ \max_{j\in\mathcal{U}^{B<R}}\{-k_j + \int V^*(\mathcal{U}\backslash\{i,j\}, z\circ x_i\circ x_j)dF_j\} \end{array}\right\}\right\} dF_i \\
&= \int_{x_i^R}^{+\infty} x_i^R + \max\left\{0, \max\left\{\begin{array}{c} \overline{z}, \max_{i'\in\mathcal{U}^{R\leq B}\backslash\{i\}} \mu_{i'}, \max_{j\in\mathcal{U}^{B<R}} \mu_j, \\ \max_{j\in\mathcal{U}^{B<R}}\{-k_j + \int V^*(\mathcal{U}\backslash\{i,j\}, z\circ x_i\circ x_j)dF_j\} \end{array}\right\} - x_i\right\} dF_i \\
&+ \int_{-\infty}^{x_i^R} \max\left\{x_i, \max\left\{\begin{array}{c} \overline{z}, \max_{i'\in\mathcal{U}^{R\leq B}\backslash\{i\}} \mu_{i'}, \max_{j\in\mathcal{U}^{B<R}} \mu_j, \\ \max_{j\in\mathcal{U}^{B<R}}\{-k_j + \int V^*(\mathcal{U}\backslash\{i,j\}, z\circ x_i\circ x_j)dF_j\} \end{array}\right\}\right\} dF_i,
\end{aligned}$$

where the first equality is by definition of the set of actions available to the agent, and I use the second remark above; the second equality is just a rearrangement of terms, and the third equality follows from using (RV) for box i.

Notice that the second term in the first integrand:

$$\max\left\{0, \max\left\{\begin{array}{c}\overline{z}, \max_{i'\in\mathcal{U}^{R\leq B}\backslash\{i\}}\mu_{i'}, \max_{j\in\mathcal{U}^{B<R}}\mu_j, \\ \max_{j\in\mathcal{U}^{B<R}}\{-k_j + \int V^*(\mathcal{U}\backslash\{i,j\}, z\circ x_i\circ x_j)dF_j\}\end{array}\right\} - x_i\right\},$$

is decreasing in x_i: the slope of $-x_i$ is -1, and the slope of the term in the $\max\{\cdot\}$ as a function of x_i is at most one (it would be 1 only if x_i is better than any of the terms in the $\max\{\cdot\}$ for all $x_i \in [x_i^R, +\infty]$). Thus, it follows that:

$$\begin{aligned}&\int_{x_i^R}^{+\infty} x_i^R + \max\left\{0, \max\left\{\begin{array}{c}\overline{z}, \max_{i'\in\mathcal{U}^{R\leq B}\backslash\{i\}}\mu_{i'}, \max_{j\in\mathcal{U}^{B<R}}\mu_j, \\ \max_{j\in\mathcal{U}^{B<R}}\{-k_j + \int V^*(\mathcal{U}\backslash\{i,j\}, z\circ x_i\circ x_j)dF_j\}\end{array}\right\} - x_i\right\} dF_i \\ &\leq \int_{x_i^R}^{+\infty} \max\left\{x_i^R, \max\left\{\begin{array}{c}\overline{z}, \max_{i'\in\mathcal{U}^{R\leq B}\backslash\{i\}}\mu_{i'}, \max_{j\in\mathcal{U}^{B<R}}\mu_j, \\ \max_{j\in\mathcal{U}^{B<R}}\{-k_j + \int V^*(\mathcal{U}\backslash\{i,j\}, z\circ x_i^R\circ x_j)dF_j\}\end{array}\right\}\right\} dF_i.\end{aligned}$$

Also,

$$\begin{aligned}&\int_{-\infty}^{x_i^R} \max\left\{x_i, \max\left\{\begin{array}{c}\overline{z}, \max_{i'\in\mathcal{U}^{R\leq B}\backslash\{i\}}\mu_{i'}, \max_{j\in\mathcal{U}^{B<R}}\mu_j, \\ \max_{j\in\mathcal{U}^{B<R}}\{-k_j + \int V^*(\mathcal{U}\backslash\{i,j\}, z\circ x_i\circ x_j)dF_j\}\end{array}\right\}\right\} dF_i \\ &\leq \int_{-\infty}^{x_i^R} \max\left\{x_i^R, \max\left\{\begin{array}{c}\overline{z}, \max_{i'\in\mathcal{U}^{R\leq B}\backslash\{i\}}\mu_{i'}, \max_{j\in\mathcal{U}^{B<R}}\mu_j, \\ \max_{j\in\mathcal{U}^{B<R}}\{-k_j + \int V^*(\mathcal{U}\backslash\{i,j\}, z\circ x_i^R\circ x_j)dF_j\}\end{array}\right\}\right\} dF_i,\end{aligned}$$

since the integrand is increasing in x_i. Putting all of this together, I conclude that for all $i \in \mathcal{U}^{R\leq B}$, the following holds:

$$\begin{aligned}&-k_i + \int V^*(\mathcal{U}\backslash\{i\}, z\circ x_i)dF_i \\ &= -k_i + \int \max\left\{\begin{array}{c}x_i, \overline{z}, \max_{i'\in\mathcal{U}^{R\leq B}\backslash\{i\}}\mu_{i'}, \max_{j\in\mathcal{U}^{B<R}}\mu_j, \\ \max_{j\in\mathcal{U}^{B<R}}\{-k_j + \int V^*(\mathcal{U}\backslash\{i,j\}, z\circ x_i\circ x_j)dF_j\}\end{array}\right\} dF_i \\ &\leq \max\left\{\begin{array}{c}x_i^R, \overline{z}, \max_{i'\in\mathcal{U}^{R\leq B}\backslash\{i\}}\mu_{i'}, \max_{j\in\mathcal{U}^{B<R}}\mu_j, \\ \max_{j\in\mathcal{U}^{B<R}}\{-k_j + \int V^*(\mathcal{U}\backslash\{i,j\}, z\circ x_i^R\circ x_j)dF_j\}\end{array}\right\}.\end{aligned}$$

But, then one concludes that, for all $i \in \mathcal{U}^{R\leq B}$:

$$\begin{aligned}
&\max\left\{\overline{z}, \max_{i\in\mathcal{U}^{R\leq B}} \mu_i, \max_{j\in\mathcal{U}^{B<R}} \mu_j, \max_{j\in\mathcal{U}^{B<R}} \{-k_j + \int V^*(\mathcal{U}\backslash\{j\}, z\circ x_j)dF_j\}\right\} \\
&\geq \max\left\{\begin{array}{c} x_i^R, \overline{z}, \max_{i'\in\mathcal{U}^{R\leq B}\backslash\{i\}} \mu_{i'}, \max_{j\in\mathcal{U}^{B<R}} \mu_j, \\ \max_{j\in\mathcal{U}^{B<R}}\{-k_j + \int V^*(\mathcal{U}\backslash\{i,j\}, z\circ x_i^R\circ x_j)dF_j\} \end{array}\right\} \\
&\geq -k_i + \int V^*(\mathcal{U}\backslash\{i\}, z\circ x_i)dF_i,
\end{aligned}$$

where the first inequality follows from $x_i^R < \mu_i$ for $i \in \mathcal{U}^{R\leq B}$, and the fact that taking box i without inspection and getting μ_i is always an option in the optimal policy in the first line, while not in the second. Moreover, note that for $i \in \mathcal{U}^{R<B}$, the first inequality is strict.

Since the above holds for each $i \in \mathcal{U}^{R\leq B}$, it follows that (S.3) holds, and, thus, $P(U, n) = 1$ □

S.5 Two boxes

To further the understanding of the difficulties involved when characterizing the optimal policy when the conditions of Section 4 do not hold, this section characterizes the optimal policy when there are two boxes. Hence, for the rest of the section, $\mathcal{N} = \{1, 2\}$, and the outside option is given by $\overline{z}$.

Given that Proposition 0 characterizes the optimal continuation when there is one box left for inspection, I only need to determine which of the following three alternatives yields the highest payoff to characterize the optimal policy for two-boxes: (i) stop, taking $\max\{\overline{z}, \mu_1, \mu_2\}$, (ii) inspect box 1 first, and apply the optimal policy in Proposition 0 to box 2, and (iii) inspect box 2 first, and apply the optimal policy in Proposition 0 to box 1. Let Π_1 denote the payoff of (ii), and Π_2 denote the payoff of (iii).[4]

Proposition S.2 below describes the optimal policy when $\mathcal{N} = \{1, 2\}$:

Proposition S.2. *Fix a set of boxes $\mathcal{N} = \{1, 2\}$, and let $\overline{z}$ denote the outside*

[4] Π_i is the payoff from inspecting box i first, and: (i) if $\max\{x_i, \overline{z}\} > x_j^R$ stop, and take $\max\{x_i, \overline{z}\}$, (ii) if $\max\{x_i, \overline{z}\} \in [x_j^B, x_j^R]$ inspect box j, and take $\max\{x_i, x_j, \overline{z}\}$, (iii) if $\max\{x_i, \overline{z}\} < x_j^B$ stop, and take μ_j

option. Assume without loss of generality that $x_2^R < x_1^R$. The following is the optimal policy:

1. *If $\overline{z} > x_1^B$ and $\overline{z} > x_2^B$, then the optimal policy is given by Weitzman's rule.*
2. *If $x_1^B < x_2^B$, then it is optimal to inspect box 1 first. The optimal continuation policy is given by Proposition 0.*
3. *If $x_2^B < x_1^B$, $\overline{z} < x_1^B$, and $\mu_1 \leq x_2^R$, it is optimal to inspect at least one box. If $\Pi_1 > \Pi_2$, box 1 is inspected first; otherwise, box 2 is inspected first. In both cases, the optimal continuation is as in Proposition 0.*
4. *Otherwise, if $x_2^B < x_1^B$, $\overline{z} < x_1^B$, and $x_2^R < \mu_1$, it is optimal to inspect box 1 first if $\Pi_1 > \max\{\Pi_2, \mu_1\}$, to inspect box 2 first if $\Pi_2 > \max\{\Pi_1, \mu_1\}$; otherwise, box 1 is taken without inspection. If search does not stop, the optimal continuation policy is as in Proposition 0.*

Item 1 follows from Proposition 1, and item 2 follows from Proposition 4 in the main text. When $x_2^R < x_1^R$ and $x_2^B < x_1^B$, Proposition 2 allows us to simplify the taxonomy by considering two cases: $\mu_1 \leq x_2^R$ and $x_2^R < \mu_1$. In the first case (item 3), the agent only has to decide which box to inspect next, i.e. the optimal policy is determined by $\max\{\Pi_1, \Pi_2\}$. In the second case (item 4), the agent has to choose either to stop, taking box 1 without inspection, or which box to inspect next.

To determine the optimal policy in item 3 and 4 above, I now analyze the differences $\Pi_1 - \Pi_2$, $\Pi_2 - \mu_1$, and $\Pi_1 - \mu_1$. The first determines the optimal policy in item 3, and all three determine the optimal policy in item 4.

Consider first $\Pi_1 - \Pi_2$. It is immediate, if somewhat tedious, to show that it is given by:[5]

$$\begin{aligned}\Pi_1 - \Pi_2 &= \int_{x_2^R}^{+\infty}\int_{x_2^R}^{+\infty} (\min\{x_1^R, x_2, x_1\} - x_2^R)dF_2dF_1 \\ &+ \int_{-\infty}^{x_1^B}\int_{-\infty}^{x_1^B} (\max\{x_1, x_2, \max\{x_2^B, \overline{z}\}\} - x_1^B)dF_2dF_1\end{aligned} \tag{S.4}$$

[5] Equations (S.4)-(S.6) are derived in Appendix U for completeness.

Recall I am assuming that $x_1^R > x_2^R$, and $x_1^B > x_2^B$, so that the first term in (S.4) is positive, and the second is negative. Equation (S.4) shows that inspecting first box 1 has a benefit, which is given by the possibility of obtaining higher prizes, net of inspection costs, and a cost, which is given by the possibility of obtaining really low prizes, in which case keeping box 1 to take without inspection would act as a buffer. A somewhat loose intuition is that the higher the backup value of box 1, or the higher the reservation value of box 2, the higher the cost of inspecting box 1 first, and hence the optimal policy would start with box 2.[6]

Proposition S.3 below characterizes when $\Pi_1 \leq (\geq)\Pi_2$. In what follows, denote by $\overline{\mathbb{R}}$ the extended real line. In what follows, I prove the following result:

Proposition S.3. *Assume $\mathcal{N} = \{1, 2\}$. Under the assumptions of item 3 in Proposition S.2, there exists $x_O \in \overline{\mathbb{R}}$ such that if $x_1^B \leq x_O$, then box 1 is inspected first; if $x_1^B > x_O$ box 2 is inspected first.*

Proof. To show the first part, use equation (S.4) to define the function $f_O : [x_2^B, +\infty) \mapsto \mathbb{R}$ as:

$$
\begin{aligned}
f_O(y) &= \int_{x_2^R}^{+\infty}\int_{x_2^R}^{+\infty} (\min\{x_1^R, x_2, x_1\} - x_2^R) dF_2 dF_1 \\
&+ \int_{-\infty}^{y}\int_{-\infty}^{y} (\max\{x_1, x_2, \max\{x_2^B, \overline{z}\}\} - y) dF_2 dF_1
\end{aligned}
$$

Note that $f_O(x_1^B) = \Pi_1 - \Pi_2$, and $f_O(x_2^B) > 0$ since $x_2^R < x_1^R$. Now define:

$$x_O = \inf\left\{y \in [x_2^B, +\infty) : f_O(y) \leq 0\right\}$$

I now check that f_O is decreasing in y. Then, using the convention that $\inf \emptyset = +\infty$, I show that $x_O \in \overline{\mathbb{R}}$ is well-defined. To show that f_O is decreasing, consider

[6] The intuition is loose because some changes in x_1^B (x_2^R) may change also x_1^R (x_2^B).

$y' > y \geq x_2^B$:

$$f_O(y) - f_O(y') = \int_{-\infty}^{y} \int_{-\infty}^{y} (y' - y) dF_2 dF_1 + \int_{-\infty}^{y} \int_{y}^{y'} (y' - \max\{x_1, x_2, \max\{x_2^B, \overline{z}\}\}) dF_2 dF_1$$
$$+ \int_{y}^{y'} \int_{-\infty}^{y'} (y' - \max\{x_1, x_2, \max\{x_2^B, \overline{z}\}\}) dF_2 dF_1 \geq 0$$

Hence, x_O is well-defined. Hence, if $x_1^B \leq x_O$, it follows that $\Pi_1 \geq \Pi_2$. $\square$

Equation (S.4) alone determines the optimal policy when $x_2^R < x_1^R, x_2^B < x_1^B, \mu_1 \leq x_2^R$. When $\mu_1 > x_2^R$, by Proposition 2, the agent may find it optimal to stop, and take box 1 without inspection. Hence, I also need to compare Π_1 to μ_1, and Π_2 to μ_1.

Consider first the choice of whether to inspect box 2 first, or take box 1 without inspection. It is immediate that if $x_2^R > \mu_1 (> x_1^B > \overline{z})$, then stopping cannot be optimal: inspecting box 2 and then taking box 1 without inspection whenever $x_2 < \mu_1$ certainly dominates stopping and taking box 1 without inspection. It is also immediate that if $x_2^R < x_1^B$, then stopping dominates inspecting box 2 first: x_2^R is the maximum prize the agent expects to get from box 2 after inspection, while x_1^B is the lowest prize the agent expects to get from box 1 when taking it without inspection. To sharpen this intuition, note that the difference $\Pi_2 - \mu_1$ is given by:

$$\Pi_2 - \mu_1 = -k_2 + \int_{x_1^B}^{+\infty} \int_{-\infty}^{+\infty} \begin{pmatrix} \max\{x_2, \min\{x_1^R, \max\{x_1, x_2\}\}\} \\ - \min\{x_1^R, \max\{x_1, x_1^B\}\} \end{pmatrix} dF_1 dF_2 \tag{S.5}$$

When $x_2 < x_1^B$, box 1 is taken without inspection, after inspecting box 2, and this determines the integration limits in the outer integral in (S.5). Recall from equation (2) that when taking box 1 without inspection, the agent expects to gain no more than x_1^R, and no less than x_1^B, and this determines the second term in the integrand. The first term is the gain from inspecting box 2 first, followed by inspecting box 1: by not taking box 1 without inspection, the agent gets the possibility of getting the prize inside box 2, though this comes at the cost of paying k_2.

Equation (S.5) resembles the equation that determines the reservation value for box 2, but where now the outside option is μ_1. As the previous intuition suggests, as long as it is worth inspecting box 2 (i.e., x_2^R is high compared to x_1^B), the above expression should favor inspecting at least one box.

Finally, it remains to compare Π_1 and μ_1. The difference $\Pi_1 - \mu_1$ can be written as:

$$\Pi_1 - \mu_1 = \int_{-\infty}^{x_2^R} \int_{-\infty}^{+\infty} \min\{x_2^R, \max\{x_1, x_2, \max\{x_2^B, \overline{z}\}\}\} - \max\{x_1, x_1^B\} dF_2 dF_1 \tag{S.6}$$

The difference between Π_1 and μ_1 is that by inspecting box 1 first, the agent retains the option of inspecting box 2 (the first term in the integrand), while he loses the option to take box 1 without inspection (the second term in the integrand). The equation resembles the computation of the backup value of box 1, but with an inspection cost of 0. When the agent inspects box 1 first, he gives up the backup value of box 1; hence, if box 2 is sufficiently good for search, the possibility of searching with box 2 may compensate for this. This, in turn, favors inspecting at least one box over stopping, and taking box 1 without inspection.

Proposition S.4 below characterizes formally the optimal policy in item 4:

Proposition S.4. *Under the assumptions of item 4 in Proposition S.2, there exist $x_O, x_1^S, x_2^S \in \overline{\mathbb{R}}$ such that the following is the optimal policy:*

1. *If $x_1^B \leq \min\{x_2^S, x_O\}$, then box 1 is inspected first.*
2. *If $x_O < x_1^B \leq x_2^S$, box 2 is inspected first.*
3. *If $x_2^S < x_1^B \leq x_O$, inspect box 1 if $x_2^R \geq x_1^S$, take box 1 without inspection, otherwise.*
4. *If $x_1^B > \max\{x_2^S, x_O\}$, take box 1 without inspection.*

In case search does not stop, the continuation policy is as in Proposition 0.

Proof. To prove Proposition S.4, I need to consider $\Pi_2 - \mu_1$, and $\Pi_1 - \mu_1$. In order to determine the sign of $\Pi_2 - \mu_1$, use (S.5) to define the function $f_{2S}(y)$:

$$f_{2S}(y) = -k_2 + \int_{y}^{+\infty}\int_{-\infty}^{+\infty} \begin{pmatrix} \max\{x_2, \min\{x_1^R, \max\{x_1, x_2\}\}\} \\ -\min\{x_1^R, \max\{x_1, y\}\} \end{pmatrix} dF_1 dF_2$$

Note that $f_{2S}(x_1^B) = \Pi_2 - \mu_1$. Define x_2^S as follows:

$$x_2^S = \inf\{y \in (-\infty, x_2^R] : f_{2S}(y) \leq 0\} \tag{S.7}$$

I show that: (i) $f_{2S}(y)$ is decreasing in y, and (ii) $f_{2S}(x_2^R) < 0$. Then, one can conclude that x_2^S is well-defined. To show (i), consider $y' > y$:

$$f_{2S}(y) - f_{2S}(y') = \int_{y'}^{+\infty}\int_{-\infty}^{+\infty} \min\{x_1^R, \max\{x, y'\}\} - \min\{x_1^R, \max\{x_1, y\}\} dF_1 dF_2$$
$$+ \int_{y}^{y'}\int_{\infty}^{+\infty} \max\{x_2, \min\{x_1^R, \max\{x_1, x_2\}\} - \min\{x_1^R, \max\{x_1, y\}\} dF_1 dF_2 \geq 0$$

where the inequality follows from: $\max\{x_2, \min\{x_1^R, \max\{x_1, x_2\}\} \geq \max\{y, \min\{x_1^R, \max\{x_1, y\}\}\} \geq \min\{x_1^R, \max\{x_1, y\}\}$ when $x_2 \geq y$. In order to show (ii), evaluate f_{2S} at $y = x_2^R$ and use equation (RV) to write:

$$\begin{aligned} f_{2S}(x_2^R) &= -k_2 + \int_{x_2^R}^{+\infty}\int_{-\infty}^{+\infty} \begin{pmatrix} \max\{x_2, \min\{x_1^R, \max\{x_1, x_2\}\}\} \\ -\min\{x_1^R, \max\{x_1, x_2^R\}\} \end{pmatrix} dF_1 dF_2 \\ &= -\int_{x_2^R}^{+\infty} (x_2 - x_2^R) dF_2 + \int_{x_2^R}^{+\infty}\int_{-\infty}^{+\infty} \begin{pmatrix} \max\{x_2, \min\{x_1^R, \max\{x_1, x_2\}\}\} \\ -\min\{x_1^R, \max\{x_1, x_2^R\}\} \end{pmatrix} dF_1 dF_2 \\ &= \int_{x_1^R}^{+\infty}\int_{x_2^R}^{+\infty} (x_2^R - \min\{x_1^R, x_1\}) dF_1 dF_2 + \int_{x_2^R}^{x_1^R}\int_{x_2^R}^{+\infty} (x_2^R - \min\{x_2, x_1\}) dF_1 dF_2 < 0 \end{aligned}$$

The proofs of (i) and (ii) show that x_2^S is well-defined. Hence, as long as $x_1^B \leq x_2^S$, it follows that $\Pi_2 - \mu_1 \geq 0$.

Use (S.6) to define the function $f_{1S}(y)$, given by:

$$f_{1S}(y) = \int_{-\infty}^{y}\int_{-\infty}^{+\infty} \min\{y, \max\{x_1, x_2, \max\{\overline{z}, x_2^B\}\}\} - \max\{x_1, x_1^B\} dF_2 dF_1.$$

Define x_1^S to be:

$$x_1^S = \sup\{y \in [x_1^B, +\infty) : f_1(y) \leq 0\} \tag{S.8}$$

I now show that f_{1S} is increasing in y, for $y \geq x_1^B$. Then, following the convention that $\sup \emptyset = -\infty$, I obtain that x_1^S is well-defined. To show f_{1S} is increasing whenever $y \geq x_1^B$, consider $y' > y \geq x_1^B$:

$$f_{1S}(y') - f_{1S}(y) = \int_y^{y'} \int_{-\infty}^{+\infty} \min\{y', \max\{x_1, x_2, \max\{\overline{z}, x_2^B\}\}\} - \max\{x_1, x_1^B\} dF_2 dF_1$$
$$+ \int_{-\infty}^{y} \int_{-\infty}^{+\infty} \min\{y', \max\{x_1, x_2, \max\{x_2^B, \overline{z}\}\}\} - \min\{y, \max\{x_1, x_2, \max\{x_2^B, \overline{z}\}\}\} dF_2 dF_1$$

and note the above difference is non-negative. Note that whenever $x_2^R \geq x_1^S$, we obtain that $\Pi_1 - \mu_1 \geq 0$.

The result in Proposition S.4 follows from the above observations. □

Equations (S.4)-(S.6) and the discussion above show that, even in the case $N = 2$, it is not always simple to determine the optimal policy by just looking at the boxes' cutoff values. This, in turn, highlights the value of the conditions in Section 4, which allow us to characterize the optimal policy by only looking at these cutoffs, and thus retain tractability which is useful for applications.

T Examples

T.1 *Cutoffs don't determine the optimal policy if $N \geq 2$*

Examples 1 and 2 demonstrate the claim made in Section S.1:

Example 1. Suppose $\mathcal{N} = \{1, 2\}$, and $X_1 = X_2 = \{0, 2, 10\}$. Assume first that $P(X_1 = 2) = P(X_2 = 2) = 0.2$, and $P(X_1 = 10) = 0.7, P(X_2 = 10) = 0.5$, so that $F_1 >_{FOSD} F_2$. Assume that $k_1 = k_2 = 1$. It can be shown that $x_1^B = \frac{14}{3} > x_2^B = 2.8$, and $x_1^R = \frac{60}{7} > x_2^R = 8$. Note that after inspecting box i, search always stops: the agent takes the inspected box when $x_i = 10$, and takes box j without inspection whenever $x_i \leq 2$. Since $\mu_1 < x_2^R$, inspecting box 2 first dominates taking box 1 without inspection; moreover, inspecting box 2 first dominates inspecting box 1 first since: $8.62 = 0.7 \times 10 + 0.3 \times \mu_2 < 0.5 \times 10 + 0.5 \times \mu_1 = 8.7$.

Example 2. Modify the above example as follows. Box 1 is the same as before. Instead, box 2 is such that $X_2 = \{0, 9\}$, $P(X_2 = 9) = \frac{921}{1250}$, and $k_2 = \frac{14}{9}$. It is immediate to show that cutoffs are exactly the same as the ones above. However, the optimal policy now inspects box 1 first; search stops if $X_1 = 10$, and the agent gets $X_1 = 10$, while box 2 is taken without inspection when $X_1 \leq 2$.

T.2 Example footnote 3 in Section 1

Below, I present an example where, unlike Problem 2 in Section 1, the worst prize in both boxes is the same, and where, like Problem 2, the agent inspects first the box with the lowest reservation value.

Example 3. Assume the agent has an outside option $z = 0$. Table 1 describes the prize distribution, and inspection costs of boxes A and B:

A	Prize	0	1	5	Inspection cost
	Probability	0.10	0.80	0.10	0.10
B	Prize	0	0.5	4.3	Inspection cost
	Probability	0.2	0.55	0.25	0.10

Table 1: Prize distribution for each box

It can be verified that $x_A^R = 4 > x_B^R = 3.9$, $x_A^B = 1 > x_B^B = \frac{1}{2}$, and $\mu_2 = 1.35 > \mu_1 = 1.3$. Thus, in Weitzman's model, the agent inspects box A first; if $x_A = 5$, search stops, and, if $x_A < 5$, he inspects box B, and takes $\max\{x_A, x_B\}$.

In the model considered here, by Proposition 0, after inspecting box A, the agent inspects box B only when $x_A = 1$; if $x_A = 5$, search stops and the agent takes x_A, and when $x_A = 0$ he takes box B without inspection. If, instead, he starts with box B, box A is never inspected: if $x_B = 4.3$, search stops, and he takes x_B, while if $x_B \in \{0, 0.5\}$, he takes box A without inspection. That is, he takes box A without inspection when $x_B \leq \frac{1}{2}$ even if box A may contain a prize worse than $\frac{1}{2}$. This is because the agent assigns a high probability to $x_A = 1$; this is reflected in box A's backup value. The combined effect of saving on inspection costs when box B has a low enough prize and the "certainty" of a not so low prize from box A imply inspecting box B first is optimal.

T.3 Assumption 1: example.

I use the example in Problem 1 in Section 1 to illustrate Assumption 1 in the main text. It is worth noting that the analysis after the statement of Proposition 0 in Section 2.2 provides an alternative way of deriving Assumption 1, and it makes explicit that the value of the information for the agent is maximal at $\overline{z} = \mu$.

Example 4. Consider again Problem 1. School A has prizes $X_A = \{1, 2, 5\}$, where $P(X_A = 1) = P(X_A = 5) = \frac{1}{4}$. Contrary to Section 1, I consider an arbitrary cost k of visiting school A.

Consider first calculating the reservation value. If $x_A^R \geq 2$, then it solves:

$$k = \frac{1}{4}(5 - x_A^R) \Leftrightarrow x_A^R = 5 - 4k.$$

Note that it has to be that $k \leq \frac{3}{4}$ for $x_A^R \geq 2$. Otherwise, x_A^R solves:

$$k = \frac{1}{4}(5 - x_A^R) + \frac{1}{2}(2 - x_A^R) \Leftrightarrow x_A^R = \frac{9 - 4k}{3}.$$

Since x_A^R must satisfy that $x_A^R \geq 1$, then $k \leq \frac{3}{2}$.

Similarly, one can calculate school A's backup value. Analogous steps to the above yield:

$$x_A^B = \begin{cases} 4k + 1 & \text{if } k \leq \frac{1}{4} \\ \frac{5+4k}{3} & \text{if } k \in (\frac{1}{4}, \frac{5}{2}] \end{cases}.$$

Figure 1 below plot the backup and reservation values of school A as a function of k. It is worth noting that three properties of the figure are true beyond the specifics of the example: (i) x^B is increasing in k, (ii) x^R is decreasing in k, and (iii) when they coincide, they do so at μ. That (i) holds follows from noting that, the more expensive the information from a box is, the more incentives the agent has to leave it to take without inspection. Similarly, this implies that (ii) holds.

Property (iii) follows from the observation made in Section **??**.

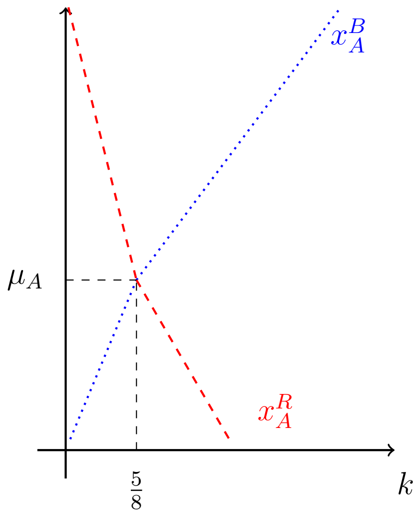

Figure 1: Reservation (red, dashed) and backup (blue, dotted) values as a function of k.

Note that for $k = \frac{5}{8}$, $x_A^R = x_A^B = \mu_A = \frac{5}{2}$. It is easy to calculate that $5/8 = \frac{1}{2}(\mu_A - 2) + \frac{1}{4}(\mu_A - 1)$, where the latter is the upper bound for k in Assumption 1. For $k < \frac{5}{8}$, it follows that $x_A^B < \mu_A < x_A^R$, and for $k > \frac{5}{8}$, $x_A^R < \mu_A < x_A^B$.

When $k > \frac{5}{8}$, and $x_A^R < x_A^B$, regardless of the value of $\overline{z}$, school A is never visited. To see this, note that when $\overline{z} > \mu$, it also holds that $\overline{z} > x_A^R$, and hence it is optimal to stop and take $\overline{z}$; likewise, when $\overline{z} \leq \mu$, it also holds that $\overline{z} < x_A^B$, and hence it is optimal to stop and accept school A without first visiting it.

U Equations S.4-S.6

I derive equation (S.4) for the case $\overline{z} < x_2^B$ and $\overline{z} < x_1^B$; the case $x_2^B < \overline{z} < x_1^B$ is analogous. Given the assumptions, it follows that:

$$\Pi_i = -k_i + \int_{-\infty}^{x_j^B} \mu_j dF_i(x_i) + \int_{x_j^B}^{x_j^R} \left(-k_j + \int \max\{x_i, x_j\} dF_j \right) dF_i(x_i) + \int_{x_j^R}^{+\infty} x_i dF_i(x_i)$$

When $x_1 \in [x_2^B, x_2^R]$, and $x_2 \in [x_1^B, x_1^R]$ both policies give payoff $\max\{x_1, x_2\} - k_1 - k_2$. Hence, this part cancels when taking the difference:

$$\begin{aligned}\Pi_1 - \Pi_2 = &\int_{x_1^R}^{+\infty}\left(-k_1 + \int_{x_2^R}^{+\infty}(x_1 - x_2 + k_2)dF_1 + \int_{-\infty}^{x_2^B}(\mu_2 - x_2 + k_2)dF_1\right)dF_2 \\ &+ \int_{x_1^B}^{x_1^R}\left(\int_{x_1^R}^{+\infty} k_2 dF_1 + \int_{x_2^R}^{x_1^R}(x_1 - \max\{x_1, x_2\} + k_2)dF_1 + \int_{-\infty}^{x_2^B}(\mu_2 - x_2 + k_2)dF_1\right)dF_2 \\ &+ \int_{-\infty}^{x_1^B}\left(\begin{array}{c} -k_1 + \int_{x_2^R}^{+\infty}(x_1 - \mu_1 + k_2)dF_1 + \int_{x_2^B}^{x_2^R}\max\{x_1, x_2\} - \mu_1 dF_1 \\ + \int_{-\infty}^{x_2^B}(\mu_2 - \mu_1 + k_2)dF_1 \end{array}\right)dF_2\end{aligned}$$

Replace $k_1 = \int_{x_1^R}^{+\infty}(x_1 - x_1^R)dF_1$ to obtain:

$$\begin{aligned}\Pi_1 - \Pi_2 = &\int_{x_1^R}^{+\infty}\left(\int_{x_1^R}^{+\infty}(x_1^R - x_2 + k_2)dF_1 + \int_{x_2^R}^{x_1^R}(x_1 - x_2 + k_2)dF_1 + \int_{-\infty}^{x_2^B}(\mu_2 - x_2 + k_2)dF_1\right)dF_2 \\ + &\int_{x_1^B}^{x_1^R}\left(\int_{x_1^R}^{+\infty} k_2 dF_1 + \int_{x_2^R}^{x_1^R}(x_1 - \max\{x_1, x_2\} + k_2)dF_1 + \int_{-\infty}^{x_2^B}(\mu_2 - x_2 + k_2)dF_1\right)dF_2 \\ + &\int_{-\infty}^{x_1^B}\left(\begin{array}{c} \int_{x_1^R}^{+\infty}(x_1^R - \mu_1 + k_2)dF_1 + \int_{x_2^R}^{x_1^R}(x_1 - \mu_1 + k_2)dF_1 \\ + \int_{x_2^B}^{x_1^B}\max\{x_1, x_2\} - \mu_1 dF_1 + \int_{-\infty}^{x_2^B}(\mu_2 - \mu_1 + k_2)dF_1 \end{array}\right)dF_2\end{aligned}$$

Replace $\mu_1 = \int_{x_1^R}^{+\infty} x_1^R dF_1 + \int_{x_1^B}^{x_1^R} x_1 dF_1 + \int_{-\infty}^{x_1^B} x_1^B dF_1$ to obtain:

$$\begin{aligned}\Pi_1 - \Pi_2 = &\int_{x_1^R}^{+\infty}\left(\begin{array}{c} \int_{x_1^R}^{+\infty}(x_1^R - x_2 + k_2)dF_1 + \int_{x_2^R}^{x_1^R}(x_1 - x_2 + k_2)dF_1 \\ + \int_{-\infty}^{x_2^B}(\mu_2 - x_2 + k_2)dF_1 \end{array}\right)dF_2 \\ + &\int_{x_1^B}^{x_1^R}\left(\int_{x_1^R}^{+\infty} k_2 dF_1 + \int_{x_2^R}^{x_1^R}(x_1 - \max\{x_1, x_2\} + k_2)dF_1 + \int_{-\infty}^{x_2^B}(\mu_2 - x_2 + k_2)dF_1\right)dF_2 \\ + &\int_{-\infty}^{x_1^B}\left(\int_{x_2^R}^{+\infty} k_2 dF_1 + \int_{x_2^B}^{x_1^B}(\max\{x_1, x_2\} - x_1)dF_1 + \int_{-\infty}^{x_2^B}(\mu_2 - x_1^B + k_2)dF_1\right)dF_2\end{aligned}$$

The above can be written as:

$$\Pi_1 - \Pi_2 = \int_{x_1^R}^{+\infty} (1 - F_1(x_2^R))k_2 + \left(\begin{array}{c} \int_{x_1^R}^{+\infty} (x_1^R - x_2) dF_1 + \int_{x_2^R}^{x_1^R} (x_1 - x_2) dF_1 \\ + \int_{-\infty}^{x_2^B} (\mu_2 - x_2 + k_2) dF_1 \end{array} \right) dF_2$$
$$+ \int_{x_1^B}^{x_1^R} \left((1 - F_1(x_2^R))k_2 + \int_{x_2^R}^{x_1^R} (x_1 - \max\{x_1, x_2\}) dF_1 + \int_{-\infty}^{x_2^B} (\mu_2 - x_2 + k_2) dF_1 \right) dF_2$$
$$+ \int_{-\infty}^{x_1^B} \left((1 - F_1(x_2^R))k_2 + \int_{x_2^B}^{x_1^B} (\max\{x_1, x_2\} - x_1) dF_1 + \int_{-\infty}^{x_2^B} (\mu_2 - x_1^B + k_2) dF_1 \right) dF_2,$$

and replacing $k_2 = \int_{x_2^R}^{+\infty} (x_2 - x_2^R) dF_2$, it follows that:

$$\Pi_1 - \Pi_2 = \int_{x_1^R}^{+\infty} \left(\int_{x_1^R}^{+\infty} (x_1^R - x_2^R) dF_1 + \int_{x_2^R}^{x_1^R} (x_1 - x_2^R) dF_1 + \int_{-\infty}^{x_2^B} (\mu_2 - x_2 + k_2) dF_1 \right) dF_2$$
$$+ \int_{x_2^R}^{x_1^R} \left(-(x_2 - x_2^R) + \int_{x_2^R}^{x_1^R} (x_1 - \max\{x_1, x_2\}) dF_1 + \int_{-\infty}^{x_2^B} (\mu_2 - x_2 + k_2) dF_1 \right) dF_2$$
$$+ \int_{x_1^B}^{x_2^R} \left(\int_{-\infty}^{x_2^B} (\mu_2 - x_2 + k_2) dF_1 \right) dF_2$$
$$+ \int_{-\infty}^{x_1^B} \left(\int_{x_2^B}^{x_1^B} (\max\{x_1, x_2\} - x_1) dF_1 + \int_{-\infty}^{x_2^B} (\mu_2 - x_1^B + k_2) dF_1 \right) dF_2.$$

Finally, replace $\mu_2 = -k_2 + \int \max\{x_2, x_2^B\} dF_2$ to obtain:

$$\Pi_1 - \Pi_2 = \int_{x_1^R}^{+\infty} \left(\int_{x_1^R}^{+\infty} (x_1^R - x_2^R) dF_1 + \int_{x_2^R}^{x_1^R} (x_1 - x_2^R) dF_1 \right) dF_2$$
$$+ \int_{x_2^R}^{x_1^R} \left(-(x_2 - x_2^R) + \int_{x_2^R}^{x_1^R} (x_1 - \max\{x_1, x_2\}) dF_1 \right) dF_2$$
$$+ \int_{-\infty}^{x_1^B} \left(\int_{x_2^B}^{x_1^B} (\max\{x_1, x_2\} - x_1) dF_1 + \int_{-\infty}^{x_2^B} (\max\{x_2, x_2^B\} - x_1^B) dF_1 \right) dF_2.$$

Rearranging terms one obtains equation (S.4).

To obtain equation (S.5), use the expression for Π_2 to obtain:

$$
\begin{aligned}
\Pi_2 - \mu_1 &= -k_2 + \int_{x_1^R}^{+\infty} x_2 dF_2 + \int_{x_1^B}^{x_1^R} (-k_1 + \int \max\{x_1, x_2\} dF_1) dF_2 + \int_{-\infty}^{x_1^B} \mu_1 dF_1 - \mu_1 \\
&= -k_2 + \int_{x_1^R}^{+\infty} (x_2 - \mu_1) dF_2 + \int_{x_1^B}^{x_1^R} \left(\begin{array}{c} \int_{x_1^R}^{+\infty} x_1^R dF_1 + \int_{-\infty}^{x_1^R} \max\{x_1, x_2\} dF_1 \\ - \int \min\{x_1^R, \max\{x_1, x_1^B\}\} dF_1 \end{array} \right) dF_2 \\
&= -k_2 + \int_{x_1^R}^{+\infty} (x_2 - \mu_1) dF_2 + \int_{x_1^B}^{x_1^R} \left(\int \min\{x_1^R, \max\{x_1, x_2\}\} - \min\{x_1^R, \max\{x_1, x_1^B\}\} dF_1 \right) dF_2 \\
&= -k_2 + \int \left(\int \max\{x_2, \min\{x_1^R, \max\{x_1, x_2\}\}\} - \min\{x_1^R, \max\{x_1, x_1^B\}\} dF_1 \right) dF_2,
\end{aligned}
$$

where the second equality comes from canceling μ_1 when $x_2 < x_1^B$, replacing $k_1 = \int_{x_1^R}^{+\infty} (x_1 - x_1^R) dF_1$ in the second term, and replacing $\mu_1 = \int \min\{x_1^R, \max\{x_1, x_1^B\}\} dF_1$, and the rest follows from rearranging terms.

Equation (S.6) follows similar steps as above, but I replace $\mu_1 = -k_1 + \int_{x_1^R}^{+\infty} x_1 dF_1 + \int_{-\infty}^{x_1^R} \max\{x_1, x_1^B\} dF_1$ in the first step to cancel the term $-k_1 + \int_{x_2^R}^{+\infty} x_1 dF_1$ in Π_1.

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