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DAVID H. SHARP

WILLIAM G. WAGNER

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David H. Sharp* and William G. Wagner**

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* National Science Foundation Predoctoral Fellow, 1960-62.

**Hughes Doctoral Fellow, 1960-62.

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ABSTRACT

In this note we present an analysis of the leptonic decay modes of the K-meson in terms of a four-parameter representation of the two form factors $F_1(q^2)$ and $F_3(q^2)$ which describe the matrix element of the vector current of the weak interactions, $< \pi \, |V_\alpha| \, K >$. Such a representation, while general enough to take account of the violation of the $\Delta I = 1/2$ rule and the possible existence of two resonances in the K-π system, no longer permits unique predictions for the ratio of the electron to muon decay rate, or of the pion spectrum. We therefore suggest that experiments be carried out to determine the four unknown parameters, and theoretical attention be turned to relating these parameters to measurable quantities occurring in related processes.

Various other treatments of this problem which have appeared are obtained as special cases of the present treatment.

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According to the analysis of Bernstein, Fubini, Gell-Mann, and Thirring,[1] and of Kuang-Chao,[2] the Goldberger-Treiman relation[3] can be understood as a consequence of the following assumptions: (a) that the divergence of the axial vector current in β-decay is a highly non-singular operator, in the sense that its matrix elements obey unsubtracted dispersion relations, and (b) that these dispersion relations are dominated by the pion pole, at low momentum transfer.

The comparative success of this hypothesis invites its further application in the study of weak interactions. It would seem that the decay of a kaon into a pion plus leptons would provide a suitable process for this purpose. As has been pointed out by Bernstein and Weinberg[4] in the course of their discussion of a possible <u>scalar</u> resonance in the K-π system, the application of postulates of the type (a) and (b) above, lead in this case to clear-cut predictions of the branching ratio of muons to electrons and of the spectrum of the final state pion.

An $I = 1/2$ resonance in the K-π system has been discovered[5] at 880 MeV. Correlations in the production process led to the conclusion that it is vector. Assuming this, we call it the M-meson in conformity with Gell-Mann's terminology.[6] In view of the existence of this resonance, the idea of deriving a Goldberger-Treiman type formula for the decay $K \to \pi + \ell + \nu$ appeared sensible. Such a formula was obtained, assuming the dominance of the M-meson. However, no satisfactory agreement with experiment was obtained on the basis of this assumption. It led to the prediction that the form factor in the electron decay vanishes at the maximum pion energy; this behavior appears incompatible with the results of Brown et al.[7] in the case of K^{+}_{e3} decay, and of Luers et al.[8] for K^{o}_{2} decay. Moreover, it was found that the branching ratio of muons to electrons is ~ 3 : 5, in disagreement with the value 1.0 ± 0.2 found by Roe et al.[9] for K^{+} decays and the value 0.79 ± 0.19 found by Luers et al.[8] for K^{o}_{2} decays. Our theoretical results are in substantial agreement with those found by H. Chew.[10]

Aside from the shortcomings noted above, an analysis based on the dominance of the M-pole in both the K^{+} and K^{o}_{2} decays is vitiated if the $\Delta I = 1/2$ rule is not satisfied. That this is actually the case is indicated by the recent work of Ely et al.,[11] who report the violations of the $\Delta S = \Delta Q$ rule, and therefore also of the $\Delta I = 1/2$ rule, in the K_{e3} decay of neutral kaons.

Finally, very recent evidence[12] points to the existence of another resonance in the K-π system, and correlations in production at 880 Mev are disappearing as further statistics become available.

These circumstances force us to abandon the hope of using successfully a generalized Goldberger-Treiman relation in this problem. We present in this paper an analysis of these decays in terms of a more general formula involving four undetermined parameters. Such an analysis does not lead to definite predictions for experimental quantities, rather we suggest that experimental determinations of the parameters be carried out for the purpose of pinning down the structure of a successful theory of these decays.

In this problem only the vector current of the weak interactions enters, because the K is treated as pseudoscalar. We shall write for the

T-matrix,

$$T = (G/\sqrt{2}) < \pi\, |V_\alpha|\, K > \bar{u}_\ell\, \gamma_\alpha\, (1 + \gamma_5)\, u_\nu \tag{1}$$

with the current expressed in terms of form factors as:

$$(G/\sqrt{2}) < \pi\, |V_\alpha|\, K > \; = (-i/\sqrt{2}) \left[F_1(q^2)\, (P^\pi + P^K)\alpha + F_3(q^2)\, (P^\pi - P^K)\alpha\right] \tag{2}$$

In Eq. (2), P^K and P^π are the four-momenta of the K and π, and $q = P^\pi - P^K$. In the rest frame of the K, the decay rate is

$$\Gamma(K \to \pi + \ell + \nu) = (16\pi^3\, m_K^2)^{-1} \int_{-(m_K - m_\pi)^2}^{-m_\ell^2} p_\pi(q^2)\, J(q^2)\, dq^2 \tag{3}$$

where

$$2m_K\, p_\pi(q^2) = \left\{(m_K^2 - m_\pi^2 - q^2)^2 + 4m_K^2\, q^2\right\}^{1/2} \quad , \tag{4}$$

and

$$24(1 + m_\ell^2/q^2)^{-2}\, J(t) =$$

$$F_1^2 \left\{2\left[(m_K^2 - m_\pi^2)^2 + q^2\,(2m_K^2 + 2m_\pi^2 + q^2)\right] - m_\ell^2 \left[(2m_K^2 + 2m_\pi^2 + q^2) + 4\,(m_K^2 - m_\pi^2)^2/q^2\right]\right\} - 6\, F_1\, F_3\, m_\ell^2\, (m_K^2 - m_\pi^2) - 3m_\ell^2\, q^2\, F_3^2 \tag{5}$$

Note that for electron decays, only the form factor F_1^2 is likely to enter, because m_ℓ^2 is such a small factor.

Since $-6.6\, m_\pi^2 < q^2 < 0$, while the threshold for the K-π system is $q^2 = -20.7$, we propose that the experimental data on the spectrum be analyzed on the basis of the following linearized expressions for the form factors,[13)]

$$F_1(q^2) = A + C(q^2/m_\pi^{\ 2}) \tag{6}$$

$$F_3(q^2) = B + D(q^2/m_\pi^{\ 2}) \tag{7}$$

In terms of these expressions we find the following formulae for the decay rates (given in units of the mass of the charged pion):

$$\Gamma(K \to \pi + e + \nu) = (48\pi^3\ m_K)^{-1}\,(35.9\ A^2 + 125.4\ AC + 173.4\ C^2)\ , \tag{8}$$

$$\Gamma(K \to \pi + \mu + \nu) = (48\pi^3\ m_K)^{-1}\,(23.4\ A^2 + 128.6\ AC - 215.1\ C^2 - 64.9\ CD + 15.8(AD + BC) - 4.51\ AB + 0.676\ B^2 - 5.54\ BD + 12.6\ D^2)\ . \tag{9}$$

There are no convincing theoretical arguments for the elimination of any of the parameters in the form factors. We propose that experimental data be analyzed so as to determine the constants A, B, C, and D, and that future theoretical studies be aimed at relating these parameters to measurable quantities entering into other strong and weak interaction processes.

In conclusion, we wish to obtain some information about A, B, C, and D by analyzing existing data.

For this purpose, we suppose that the constant terms dominate the form factors. We may then express the ratio of rates as

$$\Gamma(K \to \pi + \mu + \nu/\Gamma(K \to \pi + e + \nu) = 0.652 + 0.126\xi + 0.0189\xi^2 \tag{12}$$

in terms of a parameter

$$\xi = -F_3/F_1 = -B/A \tag{13}$$

In the case of K^+ decay, this ratio 1.0 ± 0.2 gives either -8.7 ∓ 1.0 or $+2.1 \pm 1.0$ for ξ. Recent experiments by J. M. Dobbs et al.[14] indicate that $\xi = -8.7 \mp 1.0$, while those of J. L. Brown et al.[15] support the opposite conclusion. One convenient measure of the ratio $-C/A$ follows from a determination of the pion spectrum in K_{e3} decay. Early measurements[7] give $-0.05 < -C/A < 0.25$. For the neutral kaon, one finds for ξ either -7.5 ∓ 1.4 or 1.0 ± 1.4, while the data of Luers et al.[8] indicate that[16] $0 < -C/A < 0.20$. Finally, we note that when the ratios are accurately fixed, the magnitude of the parameters A, B, C, and D may be found from the decay rates.

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16. In connection with this result, it is of interest to note that the effect of an intermediate boson in the theory of weak interactions is indistinguishable from the effect of the form factors.