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Supplementary Material

Dynamics of many-body localization in the presence of particle loss

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1. Benchmarking

To benchmark our algorithm and clarify the choice of our numerical parameters, we compare it to the exact time-evolution of Lindblad dynamics where available. The benchmarking is especially important in our case in order to certify that the results for dynamics including particle loss have converged.

The convergence of our results is controlled by two parameters, namely the bond dimension χ of the MPS matrices and the Trotter time step Δt for the time evolution. A larger bond dimension allows for a more exact representation of the density matrix in general, whereas the finiteness of the time step Δt causes errors of order $\mathcal{O}(\Delta t^3)$ in the time evolution separately. In this section we show that we can obtain convergence for the simulated times, in both χ and Δt, for the results reported in the main text.

We first consider the non-interacting case $U = 0$ shown in Fig. 1 for $N = 20$ sites and strong disorder ($h = 10J$). We track the imbalance $\mathcal{I}$ as a function of time for various bond-dimensions and for various Trotter step sizes used in the TEBD implementation. We find that for the non-interacting scenario, a large step size of $\Delta t = 0.1$ and a reasonable bond-dimension of $\chi = 100$ suffice for convergence of results to below a percent of the exact numerics. Choosing a large step size, $\Delta t = 0.1[1/J]$ for intermediate interactions allows us to reach longer simulation times with given (time-)resources.

The convergence of the TEBD data for the interacting cases with $U = 2$ and $U = 50$ are shown in Figs. 2 and 3. For the weak interactions $U = 2$ case, we observe good convergence of the results with step size $\Delta t = 0.1$ and $\chi = 100$, similar to the non-interacting case. For the results in the main text however, we have used the $\chi = 150$ results. Importantly, the TEBD results even for $\Delta t J = 0.1$ fluctuate symmetrically around the exact results, which is crucial for the fitting to a functional form. Due to the inclusion of loss (and possibly dephasing) we empirically find that bond dimensions can be kept relatively small when compared to closed system dynamics of pure states, most likely due to the reduced build-up of entanglement entropy.

In order for the Trotter decomposition to work, the step size should be at most

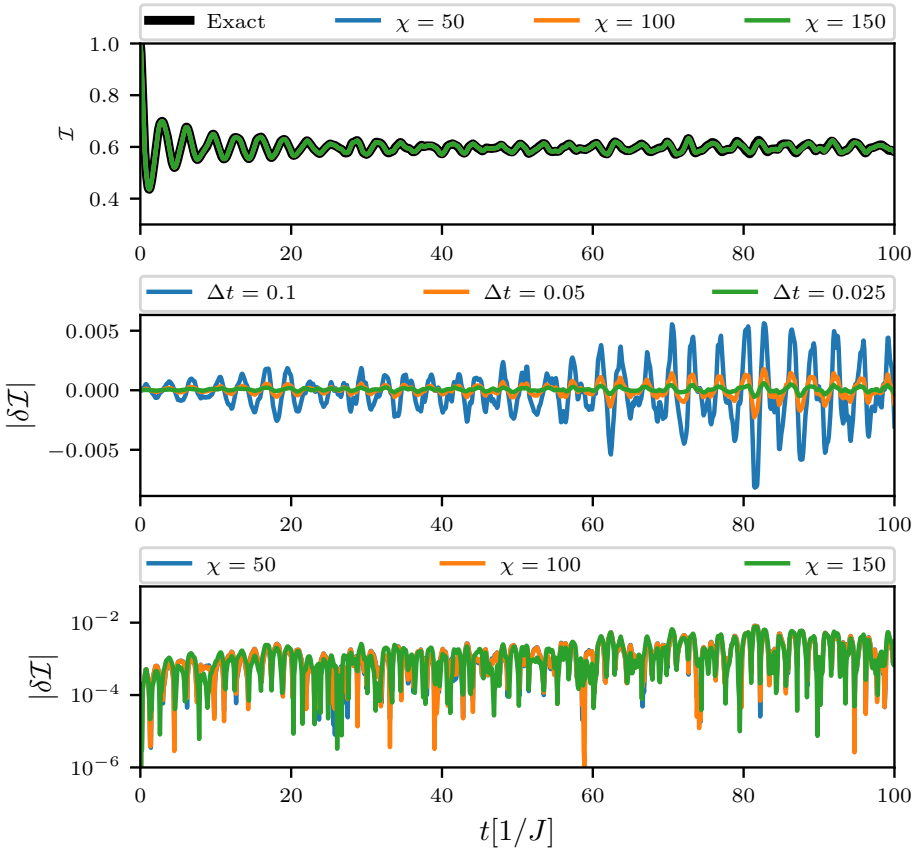

Figure 1. Time evolution of the imbalance for $U = 0$ and $N = 20$ comparing the exact result from the calculation of two-point functions [1] and TEBD. The top panel shows the imbalance $\mathcal{I}$ as a function of time, for various bond dimensions. The middle and lower panel show convergence in Δt (for fixed $\chi = 100$) and χ (for fixed $\Delta t = 0.1$), respectively. All of these results were obtained in the presence of particle loss ($\gamma_l = 0.02$, see main text).

of order $1/E$ with E the larges energy scale in the closed system. While for $U = 2J$ all energy scales are of order one and thus $\Delta t \sim 1/10$ is sufficient, this is not the case anymore for $U = 50$. As expected, a relatively large step size of $\Delta t = 0.1$ causes the simulation to completely miss the dynamics. In Fig. 3 this is cleary visible, where in order to obtain convergence a stepsize of at least $\Delta t = 0.05$ is required. For the results reported in the main text we have used stepsize $\Delta t = 0.0025$.

1.1. Entanglement entropy

While we have seen in the previous section that for up to intermediate interactions a bond dimension $\chi \sim 100$ and a step size $\Delta t \sim 0.1[1/J]$ is sufficient for an accuracy of the imbalance of $< 1\%$, the operator space entanglement entropy suffers from a larger error, in particular for small coupling to the environment. For Figure 5 of the main manuscript, we have thus used a much smaller time step $\Delta t = 0.005[1/J]$, which again

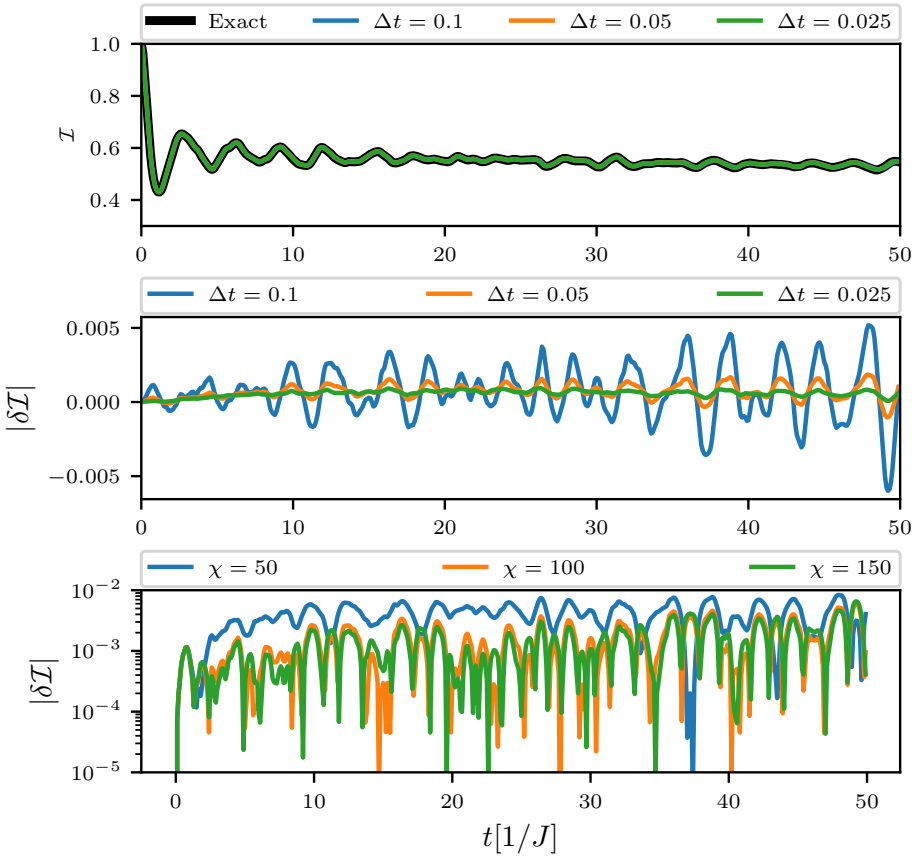

Figure 2. Time evolution of the imbalance for $U = 2$ and $N = 8$ comparing the exact result from the calculation of two-point functions and TEBD. The top panel shows the imbalance $\mathcal{I}$ for various Δt, whilst the middle and lower panel show convergence in Δt (for fixed $\chi = 150$) and χ (for fixed $\Delta t = 0.1$), respectively.

ensures convergence of the OSEE for the cases of finite coupling to a bath. There is, however, always a decrease of OSEE for a closed system simulated using MPO-TEBD, which is due to Landau-Zener-type 'errors' introduced by the simulation of a pure state as a density matrix. Figure 4 shows the OSEE for various Δt.

2. Strong interactions / weak disorder

For the case of strong interactions, $U \gg t$, the dynamics starting from a perfect charge density wave can be frozen even in the case of weak disorder, i.e. when the system is not localized. This is particularly apparent in a system with odd number of sites when starting from the state $\psi_0 = |101\cdots101\rangle$ as is shown in the main text in the inset to figure 4. When considering a finite system with an even number of sites, the situation becomes, however, a little more intricate.

In this situation, it is instructive to start from a system without hopping and disorder. For $N = 6$, the lowest lying states $|101010\rangle$, $|101001\rangle$, $|100101\rangle$, and $|010101\rangle$

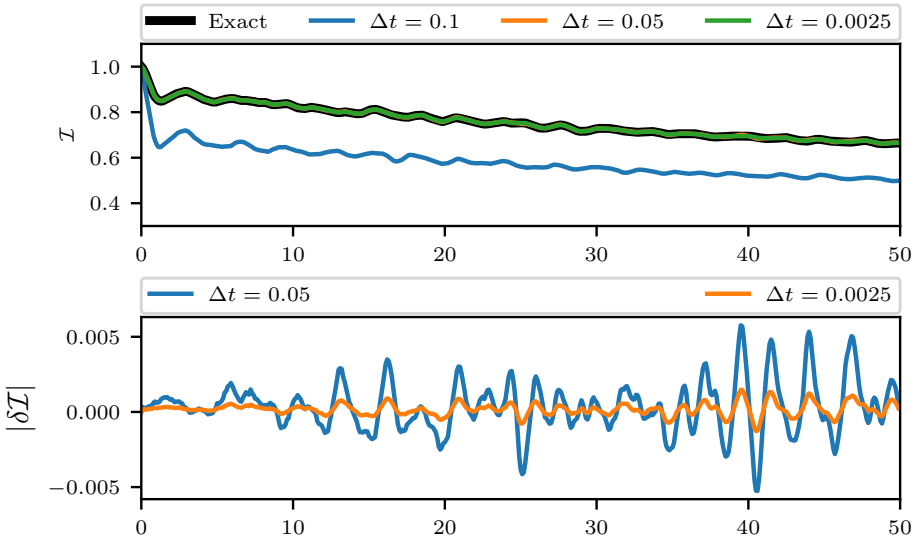

Figure 3. Time evolution of the imbalance for $U = 50$ and $N = 8$ comparing the exact result from the calculation of two-point functions and TEBD. The top panel shows the imbalance $\mathcal{I}$ for various Δt, and lower panel demonstrates that $\Delta t = 0.1$ (for fixed $\chi = 150$), as was used for the $U = 2$ results, is not enough to capture the correct dynamics.

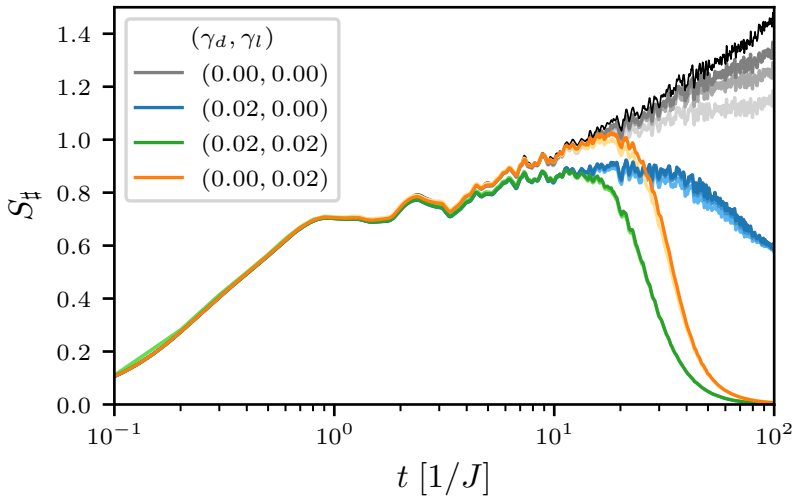

Figure 4. Operator space entanglement entropy for various bath couplings and $\Delta t = 0.1, 0.01, 0.005[1/J]$ (from light to dark). For the case of no coupling to the environment, also the (scaled) entanglement entropy of the pure state TEBD is shown for comparison. For $\Delta t \to 0$, the MPO-TEBD should yield this result.

are degenerate. Introducing hopping, we can map the subspace spanned by these states to a non-interacting one-dimensional hopping model. The system will thus explore all the states available in this subspace leading to a decaying imbalance in the long-time limit. Again introducing disorder, the resulting model is a one-dimensional system

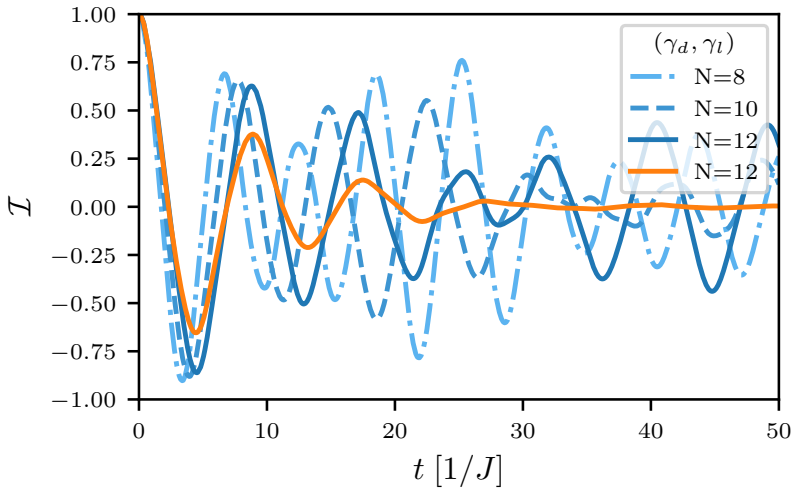

Figure 5. Decay of an initial imbalance in a system with even number of sites with strong interaction ($U = 50J$) but weak disorder ($h = 0.1J$). The imbalance oscillates around a small value with periodicity increasing with increasing system size. The inclusion of loss (orange line) leads to a slightly faster decay at short times, but mainly results in a suppression of the oscillation amplitude.

with random on-site disorder, hence localized. As this localization is rather weak for small (real-space) on-site potentials, the imbalance might still fluctuate around a very small value. Additional particle loss then reduces this value to zero and suppresses fluctuations, see figure 5. Note that the periodicity of the oscillation is not given by the hopping amplitude t of the original model anymore, but grows with system size (figure 5). For an infinite system, this periodicity becomes infinitely long. Thus, the system will effectively be frozen even for even system size and loss will lead to a reduction of the imbalance as in the inset in figure 4 of the main text.

3. Bosonic versus Fermionic

Figure 6 shows a comparison of the imbalance decay for Fermions (as considered in our work) and Bosons. For only dephasing, Fermions and Bosons are formally equivalent, and thus there is no difference between the two cases. For the cases of loss, however, the imbalance in the case of Bosons decays slightly faster. Note that the difference is rather small, as we start from a state at almost infinite temperature.

References

[1] Mark H Fischer, Mykola Maksymenko, and Ehud Altman. Dynamics of a many-body-localized system coupled to a bath. *Phys. Rev. Lett.*, 116:160401, Apr 2016.

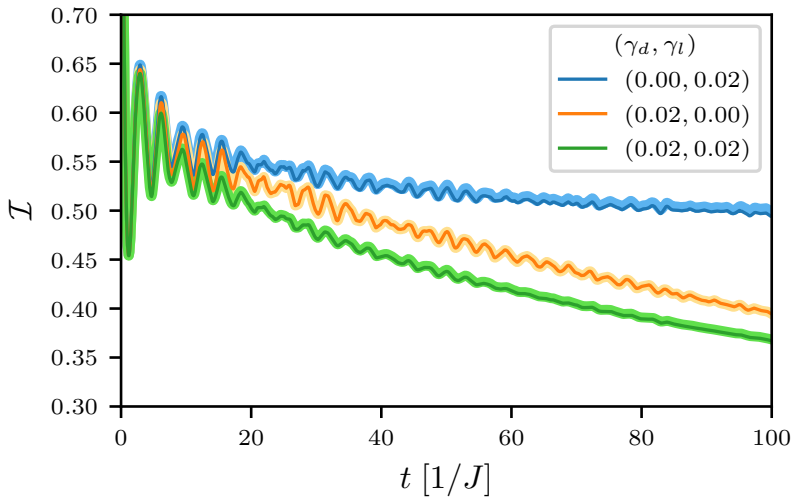

Figure 6. Decay of an initial imbalance comparing Fermions (thicker, lighter color) and Bosons (dark). While the case of pure dephasing is formally equivalent for Fermions and Bosons, the imbalance in the case of Bosons decays slightly faster, when losses are present.