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Models of the Firm and International Trade under Uncertainty

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One of the significant advances in economic theory has been the incorporation of uncertainty into the models used to investigate economic behavior. The explicit treatment of uncertainty has permitted economists to predict the behavior of economic agents operating in an uncertain environment and to explain, for example, the existence of insurance, stock markets, and forward exchange markets that have no necessary role in a deterministic world. One natural application of the economics of uncertainty has been to the study of international trade and exchange in which uncertainty regarding exchange rates and relative prices is a prominent feature of the environment of economic agents. The purpose of this paper is to frame the international trade results developed in the recent works of Wolfgang Mayer and Raveendra Batra in light of the current state of the theory of the firm under uncertainty. Before analyzing the effect of uncertainty on international trade, a perspective on the application of the economics of uncertainty to neoclassical theory will be presented with an emphasis on the theory of the firm.

One class of models into which uncertainty has been incorporated can be labeled as "entrepreneurial" models in which the firm is assumed to maximize the expected utility of profit.[1] The results from these models indicate, for example, that production decisions depend on the preferences and expectations of the entrepreneur and thus that the results of deterministic theory do not in general obtain. The question that remains, however, is whether these models are appropriate only for a firm owned by an entrepreneur or if they pertain to firms that are owned by more than one individual.

A second class of models assumes that markets exist such that contracts either can be traded contingent on each state of nature or such that there are as many securities as there are states.[2] The principal result of these "complete market" models is that consumers have sufficient opportunities to trade so that marginal rates of substitution are equated across states. In this case, the owners of the firm are unanimous in preferring that the firm maximize its market value using the established market prices to evaluate alternative production plans, and hence, the results of deterministic models obtain. When markets, however, are incomplete in the sense that there are more states than there are securities that can be traded, value maximization may not be in the best interests of the owners of a firm. Furthermore, the owners may not agree on an objective for a firm, since in an incomplete market consumers' marginal rates of substitution cannot be equated through their opportunities to trade.

A third class of models that deals with these issues was initiated by Peter Diamond's study of a model with an incomplete market structure in which the only opportunity consumers have to allocate risks is by trading the shares of firms in a securities market. Even though consumers are unable to equate their marginal rates of substitution in each state, they do equate their marginal rates of substitution between every pair of securities. Then, assume that the vectors (across states) of marginal returns from a production plan of a firm are contained in the subspace spanned by the return vectors of firms. Satisfaction of this spanning condition permits use of the marginal rates of substitution between securities to demonstrate that shareholders unanimously prefer that the firm maximize its market value. Applying this theory to a model

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[1]See Baron for an early survey of this class of model and Hayne Leland for a further contribution.

[2]See Kenneth Arrow and Gerard Debreu for the principal results for this class of model.

in which the firm is privately held but its securities are publicly traded, the usual results of deterministic theory are obtained with a securities market established certainty-equivalent used in place of the market prices that would be present in a complete market.

A complication in the use of these market models involves the separation of ownership from control. This separation raises the possibility that managers may believe that they are better informed than shareholders and may use their own judgements in evaluating alternative production plans. The incomplete market model to be considered here will be analyzed first under the assumption that the managers of a firm act in the best interests of the firm's shareholders, and then under the assumption that managers use their own or some chosen preferences and expectations in directing the firm.

Batra and Sandwip Das have considered an entrepreneurial model of a firm engaged in international trade, in which the firm maximizes the expected utility of profit in the presence of technological uncertainty. They conclude that risk aversion invalidates both the Heckscher-Ohlin and Rybczynski theorems. Mayer has analyzed an entrepreneurial model in which final commodity prices are uncertain. He demonstrated that with representative identical firms and free and costless entry, the Rybczynski and the Stolper-Samuelson theorems hold if the theorems are restated in terms of a "change in expected price, with higher central moments constant" (p. 797). He also concludes that the Factor-Price Equalization theorem obtains if the "utility functions and probability assessments of a given industry's firms are identical in both countries" (p. 804). These results are not contradictory in the context of entrepreneurial models, since Batra's model pertains to the "intermediate run," in Mayer's terminology, while Mayer considers an industry in long-run equilibrium.[3]

Elhanan Helpman and Assaf Razin (1976a,b; 1978) have shown in the context of an incomplete market model that the opportunity to trade the shares of firms in a securities market is sufficient to yield all of the standard results of international trade. The model to be considered here will be analyzed both in the context of an incomplete securities market and of an extension of the entrepreneurial class of models. In the incomplete market model the firm will be assumed to act in the best interest of its shareholders, and shareholders will be shown to unanimously prefer that the firm acts to maximize its market value. Batra and Mayer assume, however, that the firm maximizes the expected utility of profit for some utility function and expectations, and find that in the intermediate run the behavior of the firm depends on the chosen utility function and expectations. The analogous assumption within the context of the model considered here is that because of the separation of ownership from control, the manager of the firm uses some chosen preferences and expectations to guide the firm. The second assumption under which the model will be analyzed is thus that the "manager cum entrepreneur" maximizes the expected utility of the total residual profits that accrue to shareholders after the purchase of treasury shares for an arbitrary strictly concave utility function and arbitrary expectations. The result in this case is that the firm will be operated in exactly the same manner as if it were operated directly in the best interests of shareholders. Furthermore, under either assumption the market equilibrium has the same properties as in a deterministic model, and all the standard theorems of international trade hold. Consequently, the reformulated entrepreneurial model leads to the same behavior as does the incomplete market model.

I. The Model

To examine the Batra and Mayer models, consider a two-sector model in which a firm in sector 1 is characterized by a technology utilizing factor inputs K_1 and L_1 of capital and labor, respectively, to produce an uncertain output X_1 of commodity 1 given by

$$X_1 = \alpha F_1(K_1, L_1)$$

where F_1 is a production function and α is a

[3]See Murray Kemp for concise statements of the theorems of international trade.

random variable, $\alpha \geq 0$. The output X_2 of commodity 2 produced by a firm in the second sector is deterministic and given by

$$X_2 = F_2(K_2, L_2)$$

Each sector is assumed to be composed of many firms, although only one firm in each sector will be designated and analyzed. The production functions F_1 and F_2 are assumed to be linear homogeneous and strictly concave so that

$$F_i(K_i, L_i) = L_i f_i(k_i), \qquad i = 1, 2$$

where k_i is the capital-labor ratio, $f_i' > 0$ and $f_i'' < 0$. The firms make their input decisions prior to observing α and are assumed to sell their output at the price determined in a perfectly competitive market. Letting the price of the output of the first sector expressed in terms of the second commodity be denoted by p,[4] the profit of a firm in sector 1 is

$$\pi_1 = p\alpha\, L_1 f_1(k_1) - wL_1 - rK_1$$

where w and r are the factor prices which are assumed to be determined in a competitive market.

Each firm's input decisions are made at the beginning of the period, α is then observed, and at the end of the period firms distribute their profits to their owners who then purchase commodities under certainty. Letting $\mathrm{U}^i(C_1^i, C_2^i)$ denote consumer i's ordinal utility function for consumption of the two commodities at the end of the period, his consumption problem is to choose (C_1^i, C_2^i) given his income $I^i(\alpha^0)$

$$\text{to } \underset{C_1^i,\, C_2^i}{\text{maximize}}\ \mathrm{U}^i(C_1^i, C_2^i)$$

$$\text{subject to } pC_1^i + C_2^i \leq I^i(\alpha^0)$$

where α^0 is a realization of α. The ordinary demand functions will be denoted by $C_1^i(I^i(\alpha^0), p)$ and $C_2^i(I^i(\alpha^0), p)$. Since at the end of the period when consumption decisions are made the supplies of the two commodities are fixed at X_1 and X_2, the equilibrium price $p(\alpha^0)$ is determined by the solution to the market-clearing conditions

$$\sum_i C_1^i(I^i(\alpha^0), p(\alpha^0)) = X_1$$

$$\sum_i C_2^i(I^i(\alpha^0), p(\alpha^0)) = X_2$$

Consequently, if the output of the first sector is uncertain, the relative price must be uncertain as indicated by Helpman and Razin (1976a). Batra assumed that the output price is constant when supply is uncertain but such an assumption is unwarranted. Furthermore, as will be demonstrated below, an uncertain price does not affect the standard results of international trade. Mayer considered an uncertain price assuming that it is determined "internationally."

The solution to the consumer's end-of-period consumption problem determines his indirect utility function denoted by $U^i(I^i(\alpha), p(\alpha))$ which the consumer will use in making portfolio decisions. The uncertain price appears both as an argument of the utility function and as a determinant of income $I^i(\alpha)$, since the profit of firms depends on that price. The indirect utility function is assumed to be strictly concave in $I^i(\alpha)$, indicating risk aversion, and will be used to characterize the consumer's investment behavior.

A consumer i is assumed to be endowed with fixed quantities of labor L^i and capital K^i that can be sold to any firm at prices w and r, respectively. A consumer is also endowed with cash $\bar{y}^i$ and an ownership share in firm j denoted by $\bar{\gamma}_j^i$, where $\Sigma_i \bar{\gamma}_j^i = 1$, $j = 1, 2$. Consumers may sell their initial ownership shares or can purchase new shares γ_j^i in a securities market, which at the beginning of the period establishes the market values V_1 and V_2 of the two firms. The income $I^i(\alpha)$ available for consumption is obtained from savings y^i,[5] from the sale of labor and capital, and from share ownership which entitles the consumer to a share of profits or

$$I^i(\alpha) = \gamma_1^i \pi_1 + \gamma_2^i \pi_2 + y^i + wL^i + rK^i$$

[4]Batra uses p to denote the relative price of commodity 2 in terms of commodity 1. The alternative definition is used here because, as will be indicated, the price will be a random variable, and we wish to place all the uncertainty in the first sector.

[5]The interest rate is assumed to be zero. A positive interest rate will not alter the results.

Consumers are assumed to have expectations regarding $p\alpha$ that are expressed as a distribution function $G^i(p, \alpha)$, and expectations are assumed to be independent of the allocations made at the beginning of the period. That is, consumers and firms are small enough that their actions are not perceived to affect the relative commodity price.

At the beginning of the period the consumer has the portfolio problem

$$\text{to } \underset{\gamma_1^i, \gamma_2^i, y^i}{\text{maximize}} \; E^i U^i(I^i(\alpha), p(\alpha))$$

$$\text{subject to } y^i + \gamma_1^i V_1 + \gamma_2^i V_2 \leq \bar{y}^i + \bar{\gamma}_1^i V_1 + \bar{\gamma}_2^i V_2$$

where $\bar{\gamma}_1^i V_1 + \bar{\gamma}_2^i V_2$ is the value of the initial endowment of shares and $(\gamma_1^i V_1 + \gamma_2^i V_2)$ is the cost of purchasing the new portfolio. Solving the budget constraint for savings y^i and substituting into $I^i(\alpha)$, the portfolio optimality conditions are

$$E^i[U_1^i \cdot (\pi_1 - V_1)] = E^i[U_1^i \cdot (p\alpha L_1 f_1(k_1) - wL_1 - rL_1k_1 - V_1)] = 0$$

$$E^i[U_1^i \cdot (\pi_2 - V_2)] = E^i[U_1^i] \cdot (L_2 f_2(k_2) - wL_2 - rL_2k_2 - V_2) = 0$$

where U_1^i denotes $\partial U^i/\partial I^i(\alpha)$. Dividing the optimality conditions by $\int U_1^i \cdot dG^i(p\alpha)$ and defining the consumer's implicit price as

$$\rho^i(p, \alpha) = U_1^i \cdot g^i(p, \alpha)/(\int U_1^i \cdot dG^i(p, \alpha))$$

where $g^i(p, \alpha)$ is the density function corresponding to G^i, the optimality conditions can be rewritten as

$$(1) \quad (\int \rho^i(p, \alpha)(p\alpha)dpd\alpha)L_1 f_1(k_1) - wL_1 - rL_1k_1 = V_1$$

$$(2) \quad L_2 f_2(k_2) - wL_2 - rL_2k_2 = V_2$$

Since $\int \rho^i(p, \alpha)dpd\alpha = 1$, $\int V_1 \rho^i(p, \alpha)dpd\alpha = V_1$ and similarly for the other terms that do not depend on the realization of the random variables p and α. The implicit price $\rho^i(p^0, \alpha^0)$ is the amount consumer i would pay for a security that pays one dollar if and only if (p^0, α^0) obtains,[6] or equivalently, the marginal rate of substitution between a dollar if (p^0, α^0) obtains and a dollar obtained with certainty. A security such as cash that pays one dollar for any (p, α) outcome thus has a value of one dollar. A securities market equilibrium is assumed to exist.

The securities market ensures that the quantity $\int \rho^i(p, \alpha)p\alpha dpd\alpha$ will be the same for all consumers, since solving from (1) yields

$$(3) \quad \int \rho^i(p, \alpha)p\alpha dpd\alpha = (V_1 + wL_1 + rL_1k_1)/(L_1 f_1(k_1))$$

The right-hand side is independent of i, so $\int \rho^i(p, \alpha)p\alpha dpd\alpha$ is the same for all i. This quantity may be interpreted as the market price for a unit of "certain" production $L_1 f_1(k_1)$, since the numerator is the payment to factor inputs plus the rent V_1 to owners and the denominator is the certainty portion of output. Since the term in (3) is the value of a unit of certain output $L_1 f_1(k_1)$, it will be called the market certainty-equivalent price and will be denoted by p^*.

Suppose that there are two countries each having firms in both sectors. In the presence of an international securities market the consumer's portfolio problem remains unchanged except for the fact that the ownership shares which may be purchased may now be distinguished by country. A representative firm in each sector may still be designated and, from (1) and (2), it can be seen that the opportunity for consumers to trade in shares in both countries enables the market mechanism to operate so as to equate the marginal rates of substitution for securities among consumers in both countries, and thus the market certainty-equivalent p^* is the same for firms in these countries.[7]

II. Firm Input Decisions

In order to determine the optimal inputs, it is necessary to specify an appropriate criterion for the firm. Batra and Mayer assume

[6]The securities market is incomplete in this model, because there are more states than there are securities. Consequently, the implicit price $\rho^i(p, \alpha)$ for consumer i may differ from the implicit price $\rho^j(p, \alpha)$ for consumer j. If the markets were complete, the implicit prices for every consumer would be equal, since each would be able to make trades contingent on every state (p, α).

[7]As is customary in models such as these, it is assumed that there is no exchange rate uncertainty.

that the firm acts so as to maximize the expected utility of profit for some chosen utility function and expectations. Such an assumption is clearly warranted if the preferences are those of an entrepreneur who is the sole owner of the firm, but most firms are owned by shareholders and not by a single entrepreneur. Investor-owned firms are in principle to be operated in the best interests of their shareholders, but the separation of ownership from control suggests that the managers of firms may base their decision on their own preferences or on preferences that the manager believes are appropriate for his shareholders. Both the case in which the firm is operated in the interests of shareholders and the case in which the manager chooses some specific preferences to represent his or the shareholders' preferences will be considered. Given competitive behavior, the input decisions in both cases will be shown to be the same and to be functions only of prices observable in the markets. This is in contrast to the results of Batra and Mayer in which the optimal inputs depend on the preferences and expectations used.

A. *The Shareholder Interests Criterion*

With this criterion firms are assumed to act in the best interests of their shareholders. To determine a shareholder's preferences for the level of a firm's inputs, consider variations in consumer i's expected utility evaluated at a securities market equilibrium. Differentiating the expected utility evaluated at the optimal portfolio $\hat{\gamma}_1^i$, $\hat{\gamma}_2^i$, and $\hat{y}^i$ yields

$$(4)\quad \frac{\partial E^i\overline{U}^i}{\partial k_1} = \hat{\gamma}_1^i\,[(\textstyle\int \rho^i(p,\alpha)p\alpha dp d\alpha) L_1 f_1'(k_1) - rL_1] + (\overline{\gamma}_1^i - \hat{\gamma}_1^i)\frac{\partial V_1^i}{\partial k_1}$$

$$(5)\quad \frac{\partial E^i\overline{U}^i}{\partial L_1} = \hat{\gamma}_1^i\,[(\textstyle\int \rho^i(p,\alpha)p\alpha dp d\alpha) f_1(k_1) - rk_1 - w] + (\overline{\gamma}_1^i - \hat{\gamma}_1^i)\frac{\partial V_1^i}{\partial L_1}$$

where $\partial V_1^i/\partial L_1$ and $\partial V_1^i/\partial k_1$ are i's forecast of the change in the market value of the firm and $\partial E^i\overline{U}^i/\partial k_1 \equiv (\partial E^i U^i/\partial k_1)/(EU_1^i)$ and similarly for L_1. Since the consumer knows that a securities market equilibrium will be established for any level of inputs contemplated, the forecasts may be determined by differentiating (1) to obtain

$$(6)\quad (\textstyle\int \frac{\partial \rho^i(p,\alpha)}{\partial k_1} p\alpha dp d\alpha) L_1 f_1(k_1) + (\textstyle\int \rho^i(\mathrm{p},\alpha)p\alpha dp d\alpha) L_1 f_1'(k_1) - rL_1 = \frac{\partial V_1^i}{\partial k_1}$$

$$(7)\quad (\textstyle\int \frac{\partial \rho^i(p,\alpha)}{\partial L_1} p\alpha dp d\alpha) L_1 f_1(k_1) + (\textstyle\int \rho^i(p,\alpha)p\alpha dp d\alpha) f_1(k_1) - rk_1 - w = \frac{\partial V_1^i}{\partial L_1}$$

As is usual in models in which there is an incomplete set of markets for risk sharing, each consumer will in general perceive a change in his implicit prices as given in the first term of (6) and (7). If however, changes in the inputs of one firm have a negligible effect on the availability of inputs of other firms and all consumers perceive that the profit and market value of a firm is independent of the decisions of any other firm, the change in the certainty-equivalent price can be shown to be zero. Since there are many firms in each industry, consumer i's forecast of the change in the market value of another firm in the first industry will be zero, which implies that

$$\int \frac{\partial \rho^i(p,\alpha)}{\partial k_1} p\alpha dp d\alpha = \frac{\partial p^*}{\partial k_1} = 0$$

$$\int \frac{\partial \rho^i(p,\alpha)}{\partial L_1} p\alpha dp d\alpha = \frac{\partial p^*}{\partial L_1} = 0$$

Thus each consumer may perceive a change in his implicit prices but this variation is constrained, since he acts as a price taker with respect to the market certainty-equivalent price p^*. This is analogous to the usual pure competition assumption.

With this result (6) and (7) may be substituted into (4) and (5) to obtain

$$(8)\quad \frac{\partial E^i U^i}{\partial k_1} = \overline{\gamma}_1^i \frac{\partial V_1^i}{\partial k_1}$$
$$= \overline{\gamma}_1^i((\textstyle\int \rho^i(p,\alpha)p\alpha dp d\alpha) L_1 f_1'(k_1) - rL_1)$$
$$= \overline{\gamma}_1^i(p^* L_1 f_1'(k_1) - rL_1)$$

$$(9)\quad \frac{\partial E^i U^i}{\partial L_1} = \bar{\gamma}_1^i \frac{\partial V_1^i}{\partial L_1}$$
$$= \bar{\gamma}_1^i ((\int \rho^i(p,\alpha) p\alpha dp d\alpha) f_1(k_1) - w - rk_1)$$
$$= \bar{\gamma}_1^i (p^* f_1(k_1) - w - rk_1)$$

All consumers who are initial shareholders ($\bar{\gamma}_1^i > 0$) will thus be unanimous with respect to a change in k_1 and L_1, since they each use the market certainty equivalent p^* given in (3) and thus the right sides of (8) and (9) depend only on shareholder characteristics through their ownership share.[8]

The factor input levels unanimously preferred by all initial shareholders may be determined from (8) and (9) and satisfy

$$(10)\quad p^* f_1'(k_1) - r = 0$$

$$(11)\quad p^* f_1(k_1) - rk_1 - w = 0$$

For the firm in sector 2 the analogous conditions are

$$(12)\quad f_2'(k_2) - r = 0$$

$$(13)\quad f_2(k_2) - rk_2 - w = 0$$

An equilibrium in the factor markets requires that the factor rewards be the same for both firms,

$$(14)\quad p^* f_1'(k_1) = f_2'(k_2)$$

$$(15)\quad p^* f_1(k_1) - k_1 f_1'(k_1)) = f_2(k_2) - k_2 f_2'(k_2)$$

and that resources are fully employed:

$$(16)\quad K = \sum_i K^i = L_1 k_1 + L_2 k_2$$

$$(17)\quad L = \sum_i L^i = L_1 + L_2$$

An equilibrium is assumed to exist such that positive amounts of both commodities are produced.

The conditions in (10) and (11) are identical to those for a firm in a deterministic world and do not depend on the characteristics of any consumer. This results because the securities market allows all consumers to make trades until the marginal return p^* for a dollar of investment per unit of certain output is the same. Furthermore, the equilibrium market value of the firm is zero, since multiplying (11) by L_1 yields

$$(18)\quad 0 = p^* f_1(k_1) - w - k_1 r = p^* L_1 f_1(k_1) - wL_1 - rL_1 k_1 = V_1$$

This is the same result as in the deterministic case for a competitive firm with a linear homogeneous production function.

The above demonstration indicates that Batra's intermediate-run case and Mayer's long-run equilibrium are identical for investor-owned firms in the sense that a competitive firm with a linear homogeneous production function has a zero market value. The opportunity for consumers to allocate their risks in a security market provides firms with the needed information p^* to plan their inputs efficiently, and constant returns to scale ensure that no firm may earn an excess return and thus that the market value is zero.

B. *A Managerial Model*

Separation of ownership from control of a firm suggests that managers may not wish to or may not be able to determine how the firm can be operated in the best interests of its shareholders. The manager then may choose some representative utility function and expectations to use in decision making. Since one alternative open to any firm is to purchase its own shares and hold them as treasury shares, the manager will be assumed in this "manager cum entrepreneur" model to maximize the expected utility of the net (after payment for treasury shares) cash flow per share that accrues to the outstanding shares. Viewing the payment for treasury shares as a cost, this is equivalent to maximizing the expected utility of earnings per share. Letting γ_1^* denote the percentage of the shares purchased by the firm and held in its treasury, the cash flow per share is $\pi_1^* = (\pi_1 - \gamma_1^* V_1)/(1 - \gamma_1^*)$. If the manager employs a concave utility function $U^*(\pi_1^*)$ and expecta-

[8]The same result obtains if each firm faces a "firm-specific" technological risk, since the vector of returns for a production plan of a firm is a multiplicative factor of the return vector for any other production plan. When the profit of a firm is not linear in the random variables, a market certainty equivalent cannot be determined as in (3).

tions denoted by the distribution function $G^*(p, \alpha)$, the first-order conditions for γ_1^*, L_1, and k_1 are

$$(19)\ E^*[U^{*\prime} \cdot (-V_1(1-\gamma_1^*) + \pi_1 - \gamma_1^* V_1)] = E^*[U^{*\prime} \cdot (\pi_1 - V_1)] = 0$$

$$(20)\ E^*[U^{*\prime} \cdot (p\alpha f_1(k_1) - w - rk_1)] = 0$$

$$(21)\ E^*[U^{*\prime} \cdot (p\alpha L_1 f_1'(k_1) - rL_1)] = 0$$

where the market value of the firm is assumed to be unaffected by the firm's share purchases.[9] The manager's (or firm's) implicit prices $\rho^*(p, \alpha)$ may be defined as

$$\rho^*(p, \alpha) = U^{*\prime} \cdot g^*(p, \alpha)/(\textstyle\int U^{*\prime} \cdot dG^*(p, \alpha))$$

and the manager's certainty equivalent is seen to be the same as that given in (3). Dividing (19), (20), and (21) by $E^*U^{*\prime}$ and substituting from (19) gives the conditions in (10) and (11), so the managerial model yields the same results as the model in the previous section.

The difference between the results of this model and that of Batra is that here the firm may purchase treasury shares, and this requires the manager to utilize the same market certainty-equivalent used by all shareholders. Consequently, the manager chooses the same levels of inputs as those that result from acting directly in the best interests of shareholders. Trading in the securities market allows the manager to align his marginal rate of substitution with that of all consumers, and hence, the optimal inputs depend only on market observable prices. Batra does not permit the manager to trade in a securities market, and thus, the optimal levels of k_1 and L_1 depend on the manager's preferences and expectations.

III. The Theorems of International Trade

Batra and Das conclude that when firms maximize their expected utility of profit the Heckscher-Ohlin and the Rybczynski theorems do not obtain. Mayer finds that these theorems hold when he assumes that entry is free and that all firms are identical, but he concludes that the Factor-Price Equalization theorem does not hold because "factor returns are crucially dependent on the utility functions and probability assessments of firms" (p. 803). When the shares of firms may be traded in a securities market, the factor returns depend only on the market observable certainty-equivalent as indicated in (10)–(13). The purpose of this section is to briefly indicate that the standard theorems of international trade obtain in this case.

The first step in the development is to show that there is a one-to-one relationship between the labor-capital factor-price ratio $\omega = w/r$ and the commodity-price ratio. From (10), (11), (12), and (13) the factor-price ratio is

$$(22)\quad \omega = f_i(k_i)/f_i'(k_i) - k_i, \qquad i = 1, 2$$

Differentiation yields

$$(23)\quad \frac{d\omega}{dk_i} = \frac{-f_i''(k_i) f_i(k_i)}{(f_i'(k_i))^2} > 0$$

which is the desired result. The factor rewards condition in (14) can be differentiated to obtain

$$(24)\quad \frac{dp^*}{d\omega} f_1'(k_1) + p^* f_1''(k_1) \frac{dk_1}{d\omega} = f_2''(k_2) \frac{dk_2}{d\omega}$$

so

$$(25)\quad \frac{dp^*}{d\omega} / p^* = \frac{f_2''}{f_2'}\frac{dk_2}{d\omega} - \frac{f_1''}{f_1'}\frac{dk_1}{d\omega}$$

(substituting from (14))

$$= -\frac{f_2'}{f_2} + \frac{f_1'}{f_1}$$

(substituting from (23))

$$= -\frac{1}{\omega + k_2} + \frac{1}{\omega + k_1}$$

(substituting from (22))

With the usual factor-intensity assumption it is evident that an increase in the factor-price ratio results in an increase in the market certainty-equivalent price p^* if the second good is more capital intensive than the

[9]The initial shareholders are indifferent to the purchase of the treasury shares, since the firm pays the market price. In this formulation the treasury shares are not utilized except to alter the cash flow per share to the remaining shareholders.

first. In contrast to Batra's result this holds for any strictly concave utility functions for consumers or for the managers in an entrepreneurial model and not just for the class of utility functions exhibiting decreasing absolute risk aversion as he finds. In contrast to Mayer's analysis the market certainty-equivalent is not stated in terms of a "change in expected price, with higher central moments constant" but instead is based on values that are readily observable in the securities market.

In a nonstochastic model, the Rybczynski theorem states that with a constant relative commodity price and inelastic factor supplies an increase in the supply of a factor increases the output of the commodity that uses that factor more intensively and reduces the output of the other commodity. Under uncertainty this theorem must be examined with a constant relative certainty-equivalent price, since as shown earlier, the presence of an international stock market guarantees that p^* will be the same in all countries. To analyze the effect of a change in the supply of K, it is first necessary to consider how the uncertain output of the first sector is to be treated. The output X_1 is given by $X_1 = \alpha L_1 f_1(k_1)$, so that an increase in $(L_1 f_1(k_1))$ due to an increase in K will result in an increase or decrease in actual output for any realization of α. The analysis will thus be made for the certainty portion $X_1^* = L_1 f_1(k_1)$ of output (and for $X_2 = L_2 f_2(k_2)$).[10] Differentiation yields

$$\frac{\partial X_1^*}{\partial K} = L_1 f_1'(k_1)\frac{\partial k_1}{\partial K} + f_1 \frac{\partial L_1}{\partial K} \tag{26}$$

$$\frac{\partial X_2}{\partial K} = L_2 f_2'(k_2)\frac{\partial k_2}{\partial K} + f_2 \frac{\partial L_2}{\partial K} \tag{27}$$

To determine the derivatives on the right sides of (26) and (27), totally differentiate (16), (17), (14), and (15), and set $dL = 0$ $(= dL_1 + dL_2)$ to obtain the following system of equations

$$\begin{pmatrix} k_1 - k_2 & L_1 & L_2 \\ 0 & p^* f_1'' & -f_2'' \\ 0 & -p^* k_1 f_1'' & k_2 f_2'' \end{pmatrix} \cdot \begin{pmatrix} dL_1 \\ dk_1 \\ dk_2 \end{pmatrix} = \begin{pmatrix} dK \\ 0 \\ 0 \end{pmatrix}$$

[10]Batra analyzes a change in the *ex post* output.

The determinant D of the coefficient matrix is $D = -p^* f_1'' f_2''(k_2 - k_1)^2 < 0$, and the solutions to the equations are

$$\frac{dL_1}{dK} = -\frac{dL_2}{dK} = \frac{1}{D} p^* f_1'' f_2''(k_2 - k_1) = \frac{1}{k_2 - k_1}$$

$$\frac{dk_1}{dK} = \frac{dk_2}{dK} = 0$$

Evaluating the conditions in (26) and (27) yields

$$\frac{\partial X_1}{\partial K} = -\frac{f_1}{k_2 - k_1} \tag{28}$$

$$\frac{\partial X_2}{\partial K} = \frac{f_2}{k_2 - k_1} \tag{29}$$

If the first sector is more capital intensive than the second $(k_1 > k_2)$, then $\partial X_1/\partial K > 0$ and $\partial X_2/\partial K < 0$ which establishes the Rybczynski theorem. Technological and price uncertainty do not affect this basic result because the opportunity to trade in a securities market is sufficient to yield an observable market certainty-equivalent that may be used to plan input levels.

Batra and Das find, in contrast, that the Rybczynski theorem does not obtain because the relationship between the factor-price ratio and the commodity-price ratio depends in their model on the endowments in the economy. Mayer reaches a similar conclusion in his analysis of the intermediate run. In the model considered here the relationships given in (28) and (29) are independent of endowments, and, consequently, firms are able to plan their inputs using only information available in the securities and factor markets.

The Heckscher-Ohlin theorem follows directly from the Rybczynski theorem when the consumption patterns are identical in two countries, so a country exports the commodity that uses the more abundant factor more intensively. Also, the proof of the Stolper-Samuelson theorem is directly analogous to the proof in the certainty case. Differentiating (18) and the analogous condition for sector 2 yields

$$0 = f_1 - \frac{\partial w}{\partial p^*} - k_1 \frac{\partial r}{\partial p^*}$$

$$0 = - \frac{\partial w}{\partial p^*} - k_2 \frac{\partial r}{\partial p^*}$$

Solving gives

$$\frac{\partial r}{\partial p^*} = \frac{-f_1}{k_2 - k_1} \tag{30}$$

and

$$\frac{\partial w}{\partial p^*} = \frac{k_2 f_1}{k_2 - k_1}$$

Comparing (30) to (28) establishes the reciprocity theorem that

$$\frac{\partial X_1}{\partial K} = \frac{\partial r}{\partial p^*}$$

Similarly,

$$\frac{\partial X_1}{\partial L} = \frac{\partial w}{\partial p^*}$$

Converting (30) to elasticities and using (11) yields

$$\frac{dlogr}{dlogp^*} = - \frac{p^*}{r} \left(\frac{f_1}{k_2 - k_1}\right)$$

$$= - \frac{w + rk_1}{r(k_2 - k_1)} = - \frac{\omega + k_1}{k_2 - k_1}$$

$$\frac{dlogw}{dlogp^*} = \frac{p^* k_2}{w} \left(\frac{f_1}{k_2 - k_1}\right)$$

$$= \frac{k_2(w + rk_1)}{w(k_2 - k_1)} = \frac{k_2(\omega + k_1)}{\omega(k_2 - k_1)}$$

Consequently, if capital is used more intensively in sector 1 ($k_1 > k_2$), the real reward to capital increases with an increase in the relative certainty-equivalent price of the first commodity while the real reward to labor decreases. This establishes the Stopler-Samuelson theorem.

With irreversible factor intensities the factor-price equalization theorem will also hold as long as consumers in both countries can trade in the same securities and commodity markets. This is in contrast to Mayer's conclusion that the factor-price equalization theorem does not hold unless all "utility functions and probability assessments of a given industry's firms are identical in both countries" (p. 804).

IV. Conclusions

Although technological uncertainty will result in price uncertainty, the existence of an international securities market is sufficient to yield the standard theorems of international trade, since the opportunity to allocate risks in a securities market permits consumers to equate their marginal rates of substitution between shares and savings so as to establish a market certainty-equivalent that the firm may use to plan its inputs. By using the information implicit in stock market data in making their decisions, the firm will operate in the stockholders' interest, even when ownership is divorced from management. The key to this analysis is the existence of the market certainty-equivalent for the uncertain factors in the model. Given this and competitive behavior, the theorems of international trade result as in a deterministic model. When a market does not exist in which the certainty-equivalent can be established, the standard results will not in general obtain.

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