*Journal of Economic Literature*
*Vol. XXV (September 1987), pp. 1281–1306*

# General Equilibrium with Free Entry: A Synthetic Approach to the Theory of Perfect Competition*

*By* WILLIAM NOVSHEK
*Purdue University*
*and*
HUGO SONNENSCHEIN
*Princeton University*

*The essay is dedicated to the memory of Tjalling Koopmans (1910–1985). Our purpose has been to communicate to the nonspecialist an aspect of mathematical economics that has been developed since Koopman's masterful exposition of general equilibrium theory (1957). It is very difficult to do the job as well as Professor Koopmans, but he urged us all to try.*

## I. *Introduction*

THE TWO DISTINCT THEORIES of perfect competition, the Marshallian and the Arrow-Debreu-McKenzie (ADM), assume price-taking behavior as a fundamental. Filling blackboards with partial equilibrium diagrams, professors emphasize the Marshallian theory at the undergraduate level. As students progress, their teachers introduce them to the Arrow-Debreu-McKenzie theory, Gerard Debreu's *Theory of Value* (1959), the framework in which "highbrow theorists" explore the relation between perfect competition and economic efficiency. During the first half of this century, Marshallian analysis unquestionably dominated the theory of value. Although this probably remains true, a growing number of papers in the applied areas adopt a variant of the ADM framework. Only in this context can they discuss the importance of interactions among markets and the distribution of wealth.

The Marshallian and ADM theories have many striking and essential differences, for example:

1. The ADM theory specifies a fixed finite number of firms. The Marshallian theory postulates a pool of firms, any number of which may be active in the market.
2. The ADM theory postulates that the technology of each firm is convex, which rules out increasing returns to scale. The Marshallian the-

* We gratefully acknowledge the support of the National Science Foundation and our universities. We also thank Susan Elms for editorial assistance.

ory postulates that the average cost curve of each firm is U-shaped, based on the assumption of fixed costs of production or regions of increasing returns to scale.

3. The ADM theory assumes price-taking behavior regardless of the number of firms. The Marshallian theory assumes it only if the efficient scale is small relative to demand.
4. ADM theory, a general equilibrium theory, relates perfect competition and economic efficiency. Most of Marshallian analysis, partial equilibrium, ignores intermarket effects.
5. Finally, the ADM theory is a static one, to which the adjoinment of dynamics via a tâtonnement is not very satisfactory. The Marshallian analysis of equilibria is a dynamic one in the sense that entry and exit of firms cease in equilibria.

This essay explains a new theory of perfect competition, a synthesis of the ADM and Marshallian theories, and summarizes the recent work of many researchers (for example, Philippe Artzner, Carl Simon, and Hugo Sonnenschein 1986; Oliver Hart 1979; Andreu Mas-Colell 1974, 1983, 1986; William Novshek 1980; Novshek and Sonnenschein 1978, 1980, 1983, 1986a, 1986b; Kevin Roberts 1980; and Sonnenschein 1982). (See also the Symposium Issue, 1980 and the references therein.)

The new theory enables those economists who work in the world of U-shaped average cost curves and free entry to study intermarket effects and the decentralization and efficiency of perfect competition. We believe the traditional ADM theory dismays many Marshallians because it contains no role for marginal firms and insists on competitive behavior independent of the firm's ability to influence price. The synthetic theory allows both for marginal firms and for the firm to recognize its influence on price; it provides a precise general equilibrium framework for positive analysis and a framework in which to demonstrate the classical theorems of welfare economics. In contrast to both the ADM and Marshallian theories, we integrate into our analysis the leading classical explanation for price-taking behavior rather than assume it. We use the term *perfect competition* to describe a situation in which firms are arbitrarily small relative to their markets. Here firms perceive and take account of the price effect of their marketed quantities. As firms become small relative to the market, we observe in accordance with Cournot that their influence on price disappears and it is the limit of this that we call *perfect competition.* Because price-taking behavior is explained along the lines of the Cournot theory, what we offer might be better termed a *Cournot-Marshallian-ADM synthesis.*[1]

The Marshallian perspective enriches general equilibrium theory. A new condition effects a close correspondence between the general equilibrium model and standard intuition. Loosely speaking, it requires that prices provide the proper entry signals for firms and is a consequence of the dynamic aspect of Marshallian analysis. With each firm associated with the use of an unpriced and nondivisible resource, sometimes referred to as entrepreneurship, in equilibrium the returns to that factor must fall with entry and rise with exit. Such a condition might

[1] It is also important to acknowledge that small efficient scale and free entry, while sufficient to guarantee price-taking behavior in the limit, may not be necessary (see for example Michael Spence 1983). The extent to which free entry alone is sufficient (for the price-taking conclusion) is a much debated issue and is not discussed here. We believe that without the assumption of small efficient scale the case for price-taking behavior becomes less compelling.

seem axiomatic for partial equilibrium analysis where we cannot imagine its violation, but it is here that a general equilibrium perspective is important. In general equilibrium an increase in a commodity price, for example, a wage, has wealth effects via the changing value of the endowment that may increase the amount demanded of the commodity, for example, leisure, whereas in the typical partial equilibrium analysis of the firm such wealth effects are ignored. In general equilibrium the condition that prices provide the proper entry signals eliminates certain ADM equilibria. Hence, a combination of the Marshallian perspective and the general equilibrium perspective leads to better economics.

Before turning to the theory, a final word on the mathematical aspects of this paper. The papers on which we have based this essay are rather technical. Also, in its general form, the model we have in mind is more complicated than the model of equilibrium found in Debreu's classic *Theory of Value* (1959). In order to explain the results to the nonspecialist, we must forfeit either generality or precision. We will sacrifice generality and frame many of our arguments in a well-developed example. Although we loosely state our propositions, application to our example yields precision. Occasionally the included proofs hint at the arguments necessary for the general case. The interested reader can find proper statements and proofs in the references (in particular, Novshek and Sonnenschein 1986b).

## II. *The ADM and Marshallian Models*

To begin we present stylized versions of the ADM and Marshallian models in order to accentuate the relation between the ADM and Marshallian formulations. In particular, we will demonstrate that the ADM framework incorporates the standard Marshallian specification. Next, we will use these models to motivate our definition of partial equilibrium markets (and general equilibrium economies) in which firms are small. This notion lies at the heart of our definition of perfect competition: Perfectly competitive markets (perfectly competitive economies) are markets (economies) in which firms are arbitrarily small relative to the market (economy).

### A. *A Stylized Arrow-Debreu-McKenzie Model*

Our development follows Tjalling Koopmans' (1957) classic exposition of a Robinson Crusoe economy. There are two commodities, leisure and food. Robinson uses his leisure as an input to produce food according to constant returns to scale technology. Employing a convention of the Arrow-Debreu-McKenzie theory, we denote labor input as a negative quantity and food output as a positive one. Summarizing the technology with an appropriate choice of units, Figure 1 indicates that one unit of labor input yields one unit of food output. Robinson has 24 hours of leisure, his entire initial endowment of resources, to offer as labor input. Adding his leisure endowment to the technology set, we obtain the set $T$ of possible aggregate supplies. Each point in $T$, a bundle $(l, f)$, contains $l$ units of leisure and $f$ units of food and satisfies $f \leqq 24 - l$ (to be feasible, $l$ units of leisure implies no more than $24 - l$ units of labor, and food is produced 1:1 from labor).

Indifference curves, connecting equally preferred combinations of leisure and food, represent Robinson's preferences. In the economy of Figure 1, a unique "best attainable point" exists, which we have denoted by $x$.

A price system, a nonzero non-negative vector $(p_l, p_f)$ of commodity prices, defines the dollar value of each bundle

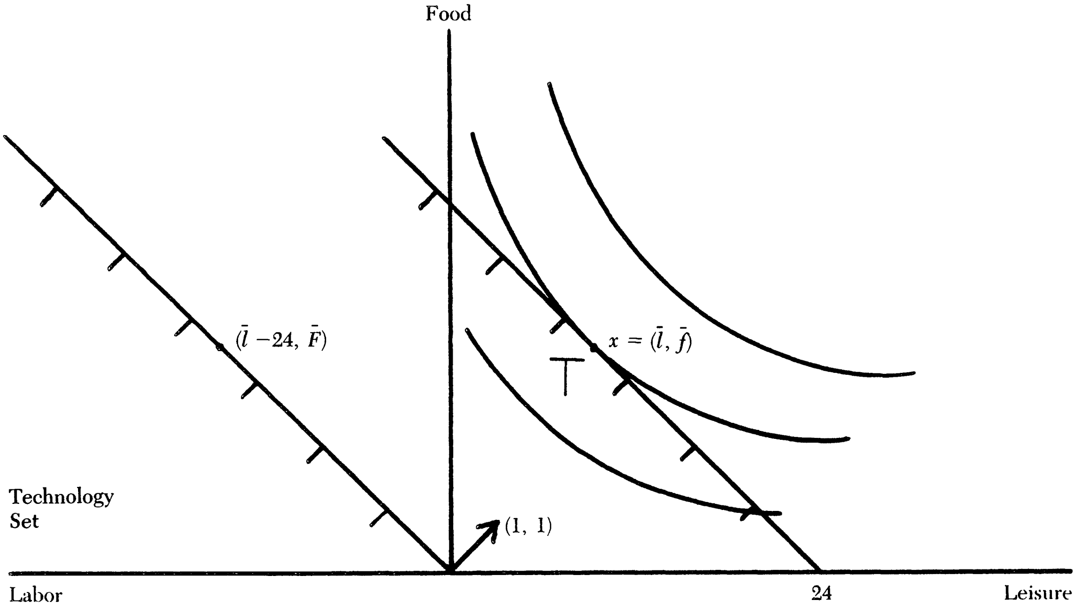


*Figure* 1

$(l, f)$ as $(p_l, p_f) \cdot (l, f) = p_l l + p_f f$. Given the price system (1, 1), the vector $x = (\bar{l}, \bar{f})$ maximizes the dollar value of supply at 24. Observe that $(\bar{l}, \bar{f})$ is not a unique maximizer of the dollar value of supply because each point of the northeast boundary of $T$ has the same value. If the value of the supply action is distributed to Robinson, then at prices (1, 1) he will be able to afford bundles with $l$ and $f$ non-negative that satisfy the budget inequality $p_l l + p_f f \leqq 24$. To maximize his utility Robinson demands $(\bar{l}, \bar{f})$, and so at prices (1, 1) the profit-maximizing supply and the utility-maximizing demand coincide. This is the unique equilibrium for the ADM model. Interpreting the technology set as that available to a competitive firm, and the economy as a private ownership one in which Robinson owns both the firm and the single scarce input labor, we can describe the equilibrium as such: At prices (1, 1) taking these prices as given the firm maximizes profit by purchasing $24 - \bar{l}$ units of labor and producing $24 - \bar{l}$ units of food. It pays $24 - \bar{l}$ for the labor and sells the food for $24 - \bar{l}$, which earns zero profit. From his ownership of the firm, Robinson receives no dividends. However, as a holder of labor resource he receives offers from the firm ($24 - \bar{l}$ units) and from himself viewed as a consumer ($\bar{l}$ units) for all 24 units and thus earns an income of 24. As a consumer, Robinson uses his 24 units of income to purchase $\bar{l}$ units of leisure and $\bar{f} = 24 - \bar{l}$ units of food. Both markets clear, and all accounts balance.

The interaction between markets above is explicit. For example, the wage determines the supply of labor and the demand for food through the relative price of leisure and food and the value of Robinson's initial endowment of labor. Developed with the classical theorems of welfare economics in mind, the ADM model of general equilibrium supplied precise conditions under which (*a*) equilibrium is efficient in the sense of Pareto

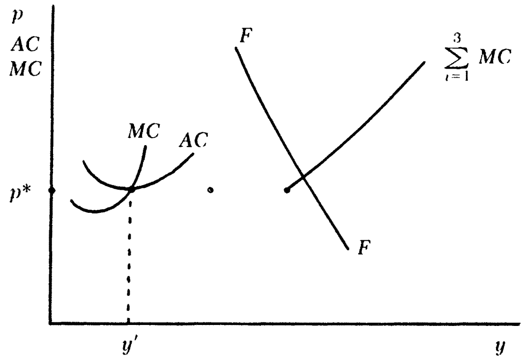


Figure 2.

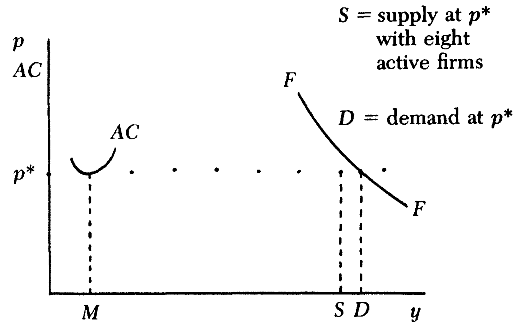


Figure 3a.

optimum, and (*b*) every Pareto optimum is an equilibrium after a suitable redistribution of ownership.

We emphasize that the ADM model does not consider the plausibility of price-taking behavior. The model offers descriptions of perfect competition for situations in which bilateral monopoly (one consumer and one producer) or single agent maximization (Robinson) should apply.

## B. *A Stylized Marshallian Model*

We begin with a familiar textbook figure (Figure 2) where we have labeled average cost, marginal cost, and demand $AC$, $MC$, and $FF$ respectively. All firms are identical and their number is fixed arbitrarily at 3. The aggregate supply is zero up to price $p^*(=$ minimum $AC)$; at $p^*$ supply is the indicated four point set (with gaps of length $y'$, the efficient scale, between the points), and above $p^*$ supply is the horizontal sum of the marginal cost curves.

If additional firms can obtain the technology represented by $AC$, then the aggregate supply shown is inconsistent with a situation in which profit-maximizing firms take prices as given. Note that at any price above $p^*$ any firm in the market may earn a positive profit, and so all firms should be active.[2] When each firm is maximizing profit ($p = MC$) and the number of firms is such that firms have little incentive to enter or exit, we say that the market is in equilibrium.

Marshall applies his model of perfect competition when efficient scale (minimum average cost output) is small relative to demand. Another strong justification for the price-taking assumption is the fact that the horizontal gaps in the supply function at price $p^*$ are small relative to demand. Here a particular number of active firms who maximize profit leave insufficient incentive for other firms to enter. The price in the market will exceed slightly $p^*$. Alternatively, we can consider an approximate equilibrium at price $p^*$, in which a finite number of firms maximize profit by offering the efficient scale output to the market, and demand very nearly matches supply. This is illustrated in Figure 3*a*.

In Figure 3*b* each firm achieves efficient scale at infinitesimal quantity $M$. We assume the existence of an unbounded mass of available firms. Let $D$ be demand at prices $p^*$. Each firm producing at efficient scale $M$ achieves an exact equilibrium at a mass $D/M$ of active firms. Clearly Figure 3*b* represents a natural limit of markets of the type consid-

[2] Clearly, with free entry, an exact price-taking equilibrium will exist only in the unlikely event that the value of demand at price $p^*$ is an integral multiple of the minimum average cost quantity.

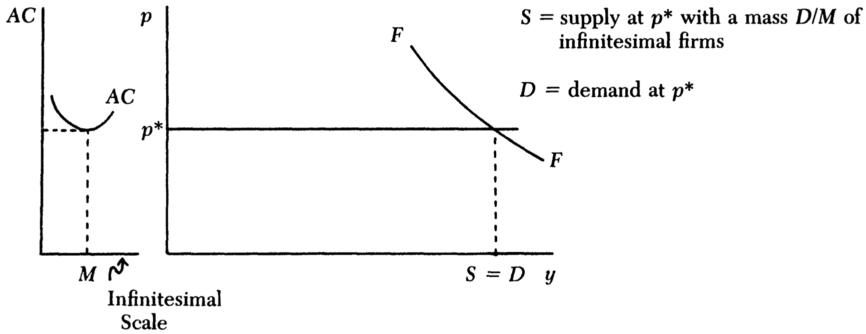


*Figure* 3b.

ered in Figure 3*a* as efficient scale becomes small relative to demand. In the equilibrium of Figure 3*b* a continuum of firms produce.

## C. *The Beginnings of the Synthesis*

Let us first place the Marshallian model in the framework of the ADM theory to demonstrate that the framework of the ADM theory captures the Marshallian ideas. The U-shaped average cost corresponds to the firm technology represented in the second quadrant of Figure 4. The efficient scale is $(-M, M)$, corresponding to the minimum average cost point in partial equilibrium. We obtain the set of feasible aggregate production possibilities as follows. We start with an unbounded number of potential identical firms (analogous to free entry), each with the given production technology. Suppose $n$ firms are active at each level of input. Select the allocation of inputs among firms that yields the maximum aggregate output. Next vary $n$ to obtain the maximum aggregate output given the level of input. Repeat this for each input level, to obtain the aggregate technology. Combine this with the initial endowment to construct $T$, the feasible set of aggregate supplies. Given the indifference curves depicted in the first quadrant of Figure 4, no price-taking equilibrium will obtain. Observe that the only price systems that lead to a positive output of food at a profit maximum take the form $(p, p)$. (Without loss of generality we will assume $p = 1$.) If the input price exceeds the output price, active firms operate at a loss; therefore all firms are inactive and production is (0, 0), i.e., no input and no output. Alternatively if the output price exceeds the input price, each firm would make a positive profit by using $M$ units of labor input to produce $M$ units of food output. Hence all firms would be active, which requires an unbounded amount of labor. Thus for any price-taking equilibrium with positive food production, input and output prices must be equal. At prices (1, 1) a consumer demands $z$, and each profit-maximizing firm supplies either (0, 0) or $(-M, M)$. Hence aggregate supply (including the endowment) must take on one of the values $[(24, 0), (24 - M, M), (24 - 2M, 2M) \ldots]$. In particular, the unique "best attainable point" $x$ is not an equilibrium of the system.

By limiting the number of firms to three (as in the Marshallian example), we can find a price system $(1, 1 + \epsilon)$ at which supply equals demand and excluded firms have little incentive to enter. Furthermore, the associated allocation approximates the "best point" $x$ as illustrated in Figure 5. Because the output price exceeds the input price, all three

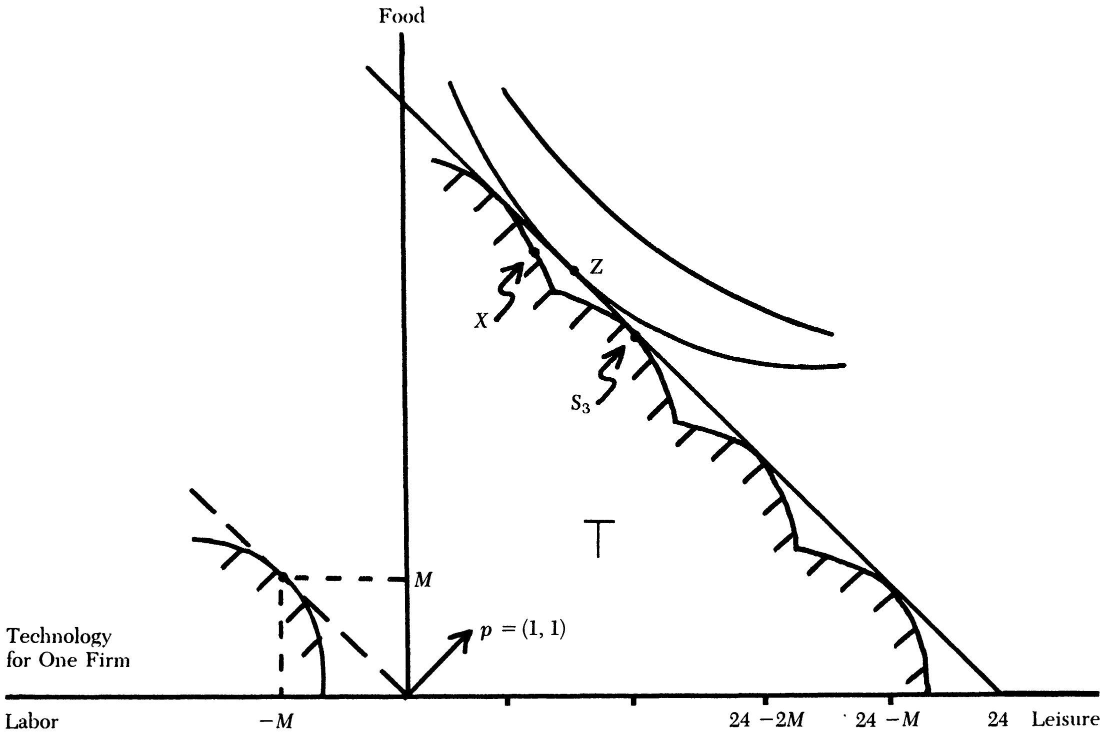


*Figure* 4

firms supply a positive amount. Point $z'$ in Figure 5 represents the price-taking aggregate production plan. His income from ownership of the labor resource, combined with his (positive) dividend from ownership of all three firms gives Robinson a budget line through $z'$, which is his optimal choice. So aggregate supply equals aggregate demand.

Alternatively, at prices (1, 1), we could consider an approximate price-taking equilibrium in which a finite number of firms produce positive quantities and maximize profit by offering the efficient scale output $(-M, M)$. In this situation demand nearly matches supply. Figure 4 essentially illustrates this where the gap between $s_3$ and $z$ is small. Once again demand almost equals the "best point" $x$.

In Figure 6 the efficient scale of each firm is an infinitesimal quantity and there is an unbounded mass of available firms. As in the Marshallian Figure 3*b*, an appropriate mass of active firms achieves an exact equilibrium at prices (1, 1) at which each produces at efficient scale. Figure 6 represents a natural limit of markets of the type considered in Figure 4 as the firms become small relative to the market. We point out the similarity with the Arrow-Debreu-McKenzie economy of Figure 1 where the equilibrium and the "best point" coincide.

## D. *A Perfectly Competitive Economy*

We will apply the term *perfectly competitive economy* to a regime in which firms are arbitrarily small relative to their markets. We adopt the classical position that consumers have no market power. However, we could have provided a similar treatment in which we

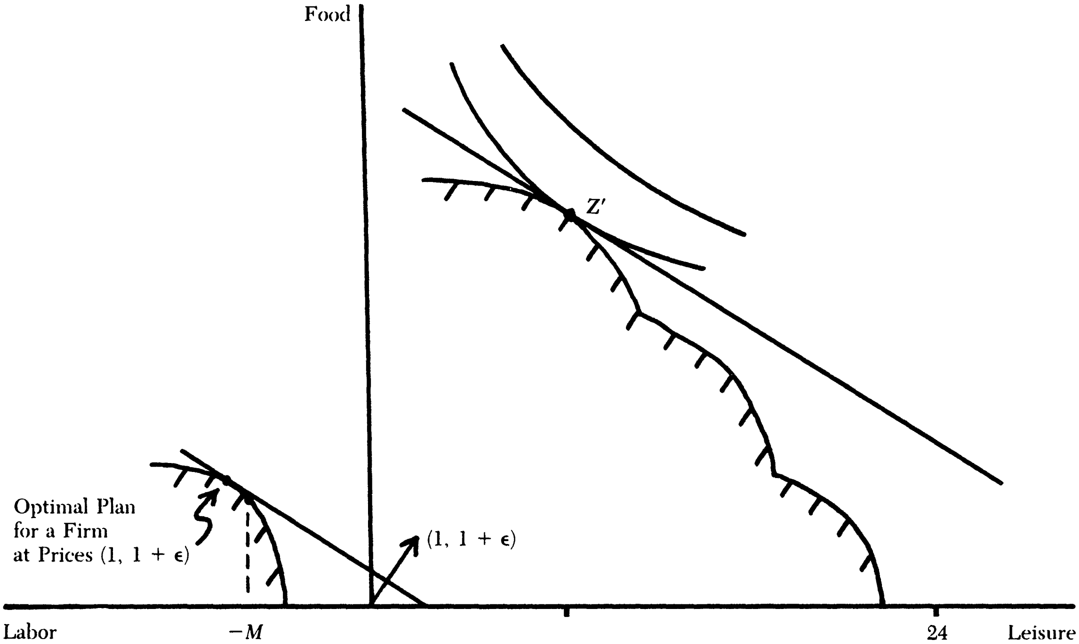


*Figure* 5.

view consumers and firms symmetrically. As in Marshallian theory, we assume that firms can freely enter and experience increasing returns to scale over some range of output to formalize the notion of the firm's being small relative to its market. In Figures 2 and 4 three firms, each producing at efficient scale, nearly satisfy the demand of consumers when price equals minimum average cost. By adding an identical twin for each original consumer to replicate demand, it would take six firms producing at efficient scale to satisfy demand approximately at the same price. Continuing in this manner each firm can be made arbitrarily small relative to its market. A perfectly competitive economy is a sequence of economies $[E(\alpha_k)]$ in which firms become arbitrarily small relative to their markets. In our formulation, $\alpha_k$ is a measure of firm size relative to the economy $E(\alpha_k)$, and we let the size of the firm in our economy diminish, $\alpha_k \rightarrow 0$. We denote the limit of the sequence by $E(0)$ in which firms are infinitesimal. With this interpretation the market in Figure 3*b* represents the limit of markets in Figure 3*a*. Similarly, the economy in Figure 6 is the limit of a sequence of economies in Figures 4 and 5.

In each economy of the sequence we will assume that firms correctly perceive the (typically non-negligible) effect on prices of their output, and maximize profit accordingly. Although nonconvexities in the firms' production sets lead to generic nonexistence of price-taking equilibrium (i.e., nonexistence for all but "knife-edge" cases), Cournot equilibrium with entry frequently exists for the Marshallian model if efficient scale is small. This suggests that the limit of Cournot equilibria of $[E(\alpha_k)]$ is a natural definition of an equilibrium for the perfectly competitive sequence $[E(\alpha_k)]$. In other words, we define perfectly competitive equilibrium as the limit of Cournot equi-

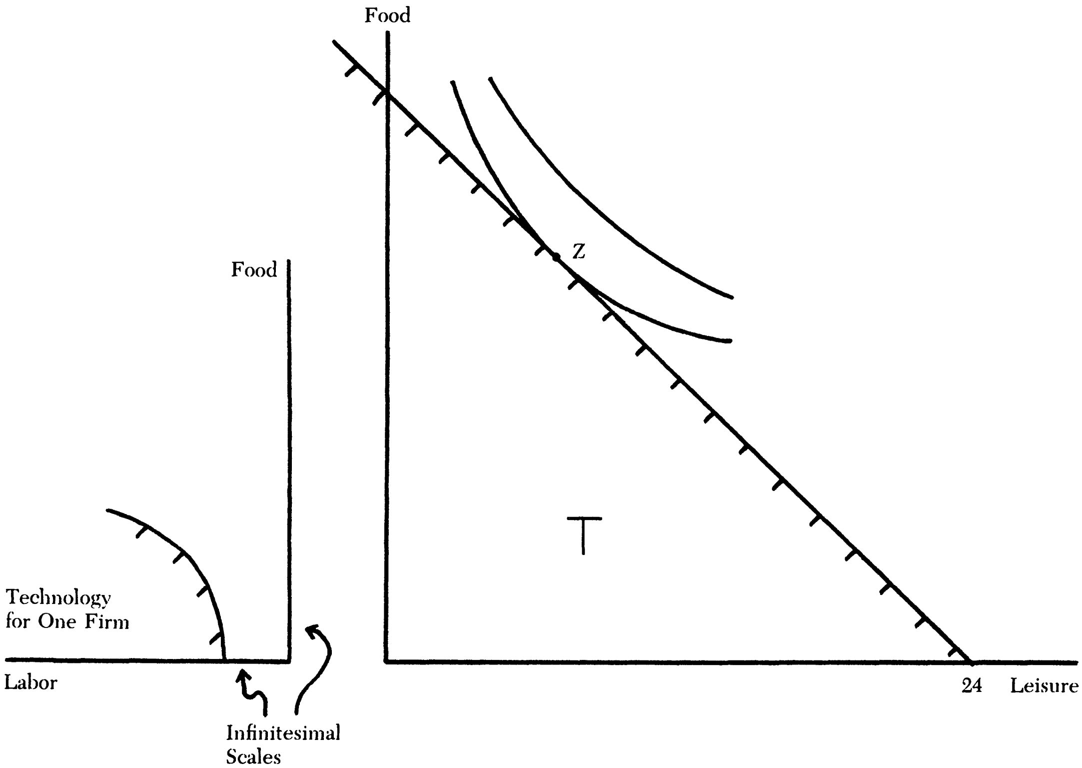


*Figure* 6.

libria with entry as firms become small relative to the market.

Not surprisingly we can characterize the perfectly competitive equilibria of the perfectly competitive sequence $[E(\alpha_k)]$ in terms of its limit economy $E(0)$ in which firms are infinitesimal. In light of this characterization we will show that firms in perfectly competitive equilibrium act "as if" they take prices as given in $E(0)$. On the other hand, we will show that other price-taking equilibria of $E(0)$ exist that are not the limit of Cournot equilibria with entry and thus not perfectly competitive equilibria of the sequence $[E(\alpha_k)]$.

Before continuing our exposition a historical note is in order. In their treatment of general equilibrium theory, Kenneth Arrow and Gerard Debreu (1954) concern themselves with the representation of the technology and behavior of the individual firm. Lionel McKenzie (1959), however, speaks of a competitive constant returns to scale aggregate technology. McKenzie assumes forthrightly that aggregate technology $T$ is a convex cone, which follows from the additivity and divisibility of basic production processes.[3] We say a production set has the additivity property if for any two possible production plans the joint production plan is also feasible. It has the divisibility property if any production plan can be proportionately scaled down. Additivity is regarded as axiomatic when *all* factors that affect production are listed so that there can be no underlying fixed factor.

[3] The assumption that $T$ is a convex cone with vertex at the origin means that for any two production plans $x$, $y$ in $T$, and for all non-negative numbers $a$, $b$, the production plan $ax + by$ is also in $T$.

Divisibility might be thought of as analogous to the assumption that commodities are infinitely divisible. McKenzie does not explicitly mention firms, but we might regard them as producing outputs from inputs according to one of an infinity of basic processes. When efficient scale is infinitesimal relative to the economy, these processes can be expanded or contracted continuously in the aggregate by varying the mass of firms using each process, and this corresponds to the situation in $E(0)$. Thus our foundations rest on the McKenzie interpretation of general equilibrium theory which we elaborate by explicitly modeling firms and by providing Cournot-like foundations for competitive behavior.

## III. *The Partial Equilibrium Synthesis*

An appropriate introduction to the notions of a perfectly competitive economy and perfectly competitive equilibrium is consideration of a partial equilibrium market. Besides being a natural stepping stone to our synthesis, it provides a rigorous foundation for the Marshallian theory. Explicitly, we establish the existence of equilibrium for the partial equilibrium market without either the need for introducing the approximate equilibrium notions described above or infinitesimal firms. Furthermore, firms correctly perceive their influence on price.

We first develop the notion of a *perfectly competitive sequence* of partial equilibrium markets. This is a sequence of Marshallian markets for a single homogeneous good where firms decrease in size. As suggested above, we define a *perfectly competitive equilibrium* as the limit of Cournot quantity setting equilibria-with-entry of the markets in the sequence.

Let us begin by returning to the Marshallian specification as in Figure 2. In order to simplify the analysis we assume some special structure for the cost function $C(y)$, namely that total costs are zero if production is zero while, if production is positive, costs consist of both strictly positive fixed costs $C_0$ and variable costs $v$. We assume variable costs increase with output at an increasing rate.

(*C*) $C(y) = 0$, if $y = 0$, and $C(y) = C_0 + v(y)$ if $y > 0$, where $C_0 > 0$, and for all $y \geqq 0$, $v' > 0$, $v'' \geqq 0$. We also assume average cost is minimized uniquely at $y = 1$.

An inverse demand function $F$ specifies demand by associating a price $[F(y)]$ with an amount $(y)$ placed on the market. We assume

(*F*) $F$ is twice continuously differentiable and $F(y) = C(1)$ implies $F'(y) \neq 0$.

These are regularity conditions that enable us to use the calculus in our analysis. $(C, F)$ specifies the basic Marshallian market. We assume there is no bound on the number of firms with access to a particular cost function. This captures the idea of free entry. Of course in equilibrium, demand will limit the number of firms using a technology.

A perfectly competitive sequence of markets $[M(\alpha_k)]$ is a sequence of markets in which firms become small relative to the market. Let $C$ be the cost function for a firm in market $M(1)$. The corresponding average cost is $AC_1(y) = C(y)/y$ for $y > 0$. We take a representative sequence of markets by rescaling the average cost functions: In market $\alpha$, the output $\alpha y$ has the same average cost as output $y$ in market 1. We can accomplish this by defining an $\alpha$-size firm corresponding to $C$ as a firm with cost function $C_\alpha(y) = \alpha C(y/\alpha)$. An $\alpha$-size firm has average cost $AC_\alpha(y) \equiv AC_1(y/\alpha)$, and attains minimum average cost uniquely at output $\alpha$. For each $\alpha > 0$, $C$, and $F$, we consider a market with a countable infinity of firms with technology $C_\alpha$ facing market inverse demand $F$. We denote this market by $M(\alpha)$. As $\alpha \rightarrow 0$, firms become small relative to the market, and the aggregate production possibilities converge to the constant returns to scale case diagrammed in Figures 3*b* and 6. Given the cost function $C$ and the inverse demand function $F$, we can define a perfectly competitive sequence of markets by any sequence of strictly positive real numbers less than or equal to one, where the sequence, representing firm size, shrinks to zero. We denote the limit market by $M(0)$. In particular, if the sequence of firm sizes is 1, ½, ⅓, . . . then the perfectly competitve sequence of markets may be thought of as resulting from repeated replication of demand followed by the representation of output in per-capita (actually per-replication) units.

Cournot equilibrium requires that the quantity actions of firms maximize profit given the quantity actions of all other firms. In equilibrium no firm makes negative profit, because exit is possible and yields zero profit. Similarly, the assumption of an unbounded set of firms with access to the technology, only a finite number of which can be active (because of fixed costs), implies that in equilibrium no inactive firm can enter and make a positive profit. Stated precisely, a (pure strategy) Cournot equilibrium with entry for the market $M(\alpha)$ is an integer $n$ and a set of positive outputs $(y_1, y_2, \ldots, y_n)$ such that:

(*a*) $(y_1, y_2, \ldots, y_n)$ is an $n$-firm Cournot equilibrium (without entry); that is, for all $i = 1, 2, \ldots, n$, $F\left(\sum_{j\neq i} y_j + y_i\right)y_i - C_\alpha(y_i) \geqq F\left(\sum_{j\neq i} y_j + y\right)y - C_a(y)$ for all $y \geqq 0$. No active firm can choose another production plan $y$ and earn greater profit given the production plans of all other active firms.

(*b*) entry is not profitable; that is $F\left(\sum_{j=1}^{n} y_j + y\right)y - C_\alpha(y) \leqq 0$ for all $y \geqq 0$.

Cournot equilibrium with entry is an "exact" equilibrium of the model where firms do not take prices as given. Individually maximizing profit, firms (noncompetitively) supply exactly the quantity demanded by consumers at the Cournot equilibrium price $F(\Sigma\, y_j)$.

Finally, we define the equilibrium output of the perfectly competitive sequence $[M(\alpha_k)]$ as the limit of $\Sigma y_i(\alpha_k)$ where $[y_1(\alpha_k), \ldots, y_{n_k}(\alpha_k)]$ is a Cournot equilibrium with entry of the market $M(\alpha_k)$. These are called *perfectly competitive equilibria.*

Our definition of perfectly competitive equilibrium formalizes the idea that perfect competition represents a limiting case of regimes in which firms can influence price. Among economists interested in rigorous foundations our definition may not at first be acceptable. They object because some or all of the markets that form a perfectly competitive sequence may lack a Cournot equilibrium. Hence our definition is meaningless. If demand is downward sloping, however, then a Cournot equilibrium exists eventually in any sequence of markets $[M(\alpha_k)]$ forming a perfectly competitive sequence. By assuming that demand slopes downward our equilibrium concept will apply. Theorem 1 identifies the conditions "demand price equals minimum per unit cost" and "demand slopes downward" in the limit market $M(0)$ as the characteristics of perfectly competitive equilibria for a sequence $[M(\alpha_k)]$. Free entry and exit determine the mass of active firms endogenously. Average cost curves are U-shaped (and so firm technology is not convex). The theory does not assume price-taking behavior. Equilibrium is exact, production equals demand, and all firms maximize profit.

*Theorem 1.* (See Novshek 1980.) Given the cost function $C$ satisfying ($C$), the inverse demand function $F$ satisfying ($F$), and the perfectly competitive sequence $[M(\alpha_k)]$, the following conditions are equivalent:

(1a) $y^*$ is a perfectly competitive equilibrium for $[M(\alpha_k)]$, and

(1b) $F(y^*) = C(1)$ and $F'(y^*) < 0$.

For the case of partial equilibrium, the preceding result describes precisely our approach to perfect competition. Perfectly competitive equilibria are the limit points of Cournot equilibria of the Marshallian markets $[M(\alpha_k)]$.

This theorem establishes that these perfectly competitive equilibrium quantities in $M(0)$ equate the inverse demand price of consumers and minimum average cost and satisfy downward-sloping demand. For the case of globally downward sloping demand, the perfectly competitive equilibria of the sequence $[M(\alpha_k)]$ coincide with the unique Walrasian equilibrium in the derived constant returns to scale market $M(0)$. On the other hand, if inverse demand is not globally downward sloping,[4] then there may be Walrasian equilibria of the derived (constant returns to scale in the aggregate) market $M(0)$ that are not by our definition competitive equilibria. These equilibria fail because we require that entry and exit also be at rest.

Figure 7 illustrates this point. In Figure 7, the points $y^*$ and $y^{**}$ are equilibria of the perfectly competitive sequence $[M(\alpha_k)]$, but $\hat{y}$ is not, even though the demand price of consumers and minimum average cost coincide at $\hat{y}$. We ar-

[4] This becomes a more interesting possibility when demand is a function of both price and wealth (which in turn depends on price as in the derivation of the supply of labor).

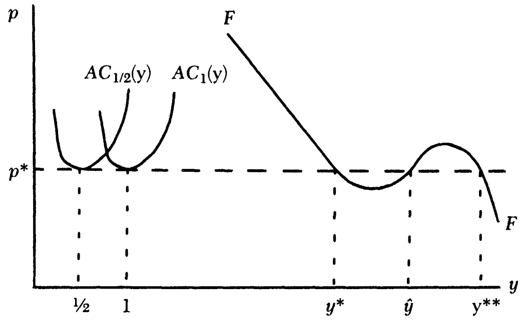


Figure 7.

gue that $\hat{y}$ does not constitute an equilibrium because an infinitesimal firm in $M(0)$ can enter and make a positive profit. First, observe that in any Cournot equilibrium for the market $M(\alpha)$ all firms make non-negative profit so aggregate output must lie in $[0, y^*]$ or $[\hat{y}, y^{**}]$. Second, inactive firms must not profit by producing $\alpha$, so aggregate Cournot equilibrium output plus $\alpha$ must lie in $[y^*, \hat{y}]$ or $[y^{**}, \infty)]$. This implies that aggregate output must lie in either the interval $[y^* - \alpha, y^*]$ or $[y^{**} - \alpha, y^{**}]$. For small $\alpha$ neither interval is near $\hat{y}$. In fact this is how one proves that $1a$ implies $1b$. The hard part of the theorem is to show that for $\alpha$ sufficiently small, if $F(y^*) = C(1)$ and $F'(y^*) < 0$, then $M(\alpha)$ has a Cournot equilibrium with entry with aggregate output in $[y^* - \alpha, y^*]$.

The above argument highlights the differences of our equilibrium concept and the Walrasian equilibrium in the limit market. In the Walrasian theory firms take prices as given, and in perfect competition we justify this as an approximation. The approximation applies so long as firms are small relative to their market and thereby have little influence in most any specification of strategic variables. Here the ability of each firm to affect price diminishes; however, in general each firm will have some influence that will affect the entry decision. If the entry of a firm will drive up price and make entry profitable, then inactive firms will take account of this effect and enter. In our model small firms correctly perceive their influence; therefore equilibrium requires that no firm can drive up prices by entering.

## IV. *The General Equilibrium Model, Preliminaries*

In this section we discuss the general equilibrium model on which we base our concept of perfectly competitive equilibrium. In the next section the model will be presented in the context of a simple example. This facilitates the exposition and should illustrate the close relationship between the partial equilibrium and general equilibrium models.

A perfectly competitive sequence of economies $[E(\alpha)]$ is analogous to a perfectly competitive sequence of markets. As $\alpha$ converges to 0 firms become arbitrarily small relative to the economy and in the limit economy, $E(0)$, are infinitesimal. Each economy $E(\alpha)$ has an unbounded set of potential firms. This provides our notion of free entry. In any equilibrium for $E(\alpha)$ only a finite number of firms can be active, so inactive firms always exist and can test the profitability of entry. No firms enter if they cannot gain positive profit by doing so.

In each economy $E(\alpha)$, we use Cournot-Nash equilibrium in quantities as our solution concept. As in Marshall, we treat consumers as a competitive, price-taking sector to focus on the role of firms and entry. In the Cournot tradition, quantity setting provides a tractable basis for our analysis and, as was the case in the partial equilibrium model, it avoids the obvious problem of nonexistence of equilibrium that arises when firms set prices in conditions of production under increasing returns. When firms consider a quantity action (a vector of input and output levels) they evaluate the corresponding profit using an "inverse demand" function $F$. In the next section we will carefully construct this function. The idea is this: To each vector of quantity actions of all firms, $y$, the "inverse demand" function associates a price vector, $p$, such that the competitive consumer sector's excess demand, given prices, $p$, and the income generated by the consumers' dividend payments (their fraction of the profit or loss for each firm), exactly matches the aggregate quantity action of the firms. Thus the payoff for each firm is a well-defined function of its own production plan and the production plans of other firms. Although some firms may be making losses, for any vector of production plans employed by the firms, prices adjust so that all markets clear. A Cournot equilibrium exists if (1) each firm takes a feasible action in its production set; and (2) each firm maximizes profits given $F$ and the actions of other firms. The assumption of nonconvex technologies (the general equilibrium analog of U-shaped average cost) implies that only a finite number of firms have nonzero actions in equilibrium. Note that inactive firms are available but cannot make positive profits by entry. Hence, the entry process is at rest in an equilibrium. Thus we have a description of Cournot equilibrium with free entry for each economy $E(\alpha)$.

For each economy $E(\alpha)$, consider the set of aggregate firm actions corresponding to Cournot equilibria of $E(\alpha)$ relative to "inverse demand" $F$. We define the perfectly competitive equilibria of the sequence of economies $[E(\alpha)]$ to be the limits of Cournot equilibria of the $E(\alpha)$ economies. Explicitly, a price vector $p^*$ and an aggregate production vector $y^*$ form a perfectly competitive equilibrium of the sequence $[E(\alpha)]$ provided that for an "inverse demand" selection $F$ satisfying certain conditions, $y^*$ is the limit of a sequence $[y(\alpha)]$ as $\alpha$ converges to zero, where $y(\alpha)$ is an aggregate production corresponding to a Cournot equilibrium of $E(\alpha)$ (relative to $F$). This coincides with the partial equilibrium model of Section III.

The partial equilibrium results in Section III depended on a condition of downward sloping demand. The results for general equilibrium will depend on an analogous condition, called DSD. Prices determined by the "inverse demand" selection $F$ must give proper entry signals. At a point satisfying the ADM equilibrium conditions, additional entry must lead to new prices at which the entrants make losses. Because input and output prices change, the DSD requirement is that the net effect of all the price changes leads to a loss for the entrant.

Walras had something similar to DSD in mind (Leon Walras [1874–77] 1954, p. 225):

> [U]nder free competition, if the selling price of a product exceeds the cost of the productive services for certain firms and a profit results, entrepreneurs will flow towards this branch of production or expand their output, so that the quantity of the product [on the market] will increase, its price will fall, and the difference between price and cost will be reduced; and, if [on the contrary], the cost of the productive services exceeds the selling price for certain firms, so that a loss results, entrepreneurs will leave this branch of production or curtail their output, so that the quantity of the product [on the market] will decrease, its price will rise and the difference between price and cost will again be reduced.

In the next section we will see how our framework allows us to analyze a logically precise general equilibrium model with nonconvex technologies where the number of active firms is determined endogenously and demand equals supply exactly. We do not assume price-taking behavior. Rather, when $\alpha$ converges to zero the ability of a firm to affect price becomes arbitrarily small. At any perfectly competitive equilibrium of the sequence, equilibrium production maximizes profit relative to the equilibrium prices. We will show that the equilibria of the perfectly competitive sequence satisfy all the conditions of an ADM equilibrium of the limit economy $E(0)$. Also, we will show that the classical welfare theorems still hold in our framework: every perfectly competitive equilibrium of the sequence is Pareto efficient and every Pareto efficient allocation for the sequence can be supported as a perfectly competitive equilibrium of the sequence.

## V. *The General Equilibrium Synthesis*

We adopted our general equilibrium model, with the exception of production sets (and our sequence of economies approach), from the standard Arrow-Debreu-McKenzie model. The economies $E(\alpha)$ are composed of consumers and firms. Each consumer receives an initial endowment of goods and has preferences over potential bundles of goods. For example, the first two diagrams of Figure 8 show representative indifference curves for two consumers, $A$ and $B$, in an economy with two commodities, leisure and food, each of which can be consumed only in non-negative amounts. Both consumers prefer to consume the two commodities in fixed proportions, person $A$ at 1:1, person $B$ at 1:2. A particular utility function that assigns utility equal to the minimum of $l$ and $f$ to a bundle with $l$ units of leisure and $f$ units of food represents person $A$'s preferences. Similarly, one that assigns utility equal to the minimum of $2l$ and $f$ to the bundle $(l, f)$ represents $B$'s. They receive identical endowments of (1, 0), containing one unit of leisure and no food.

To construct a general equilibrium analog of U-shaped average cost and free entry we differ from the standard assumptions on the producer sector in two important ways. Each firm's production

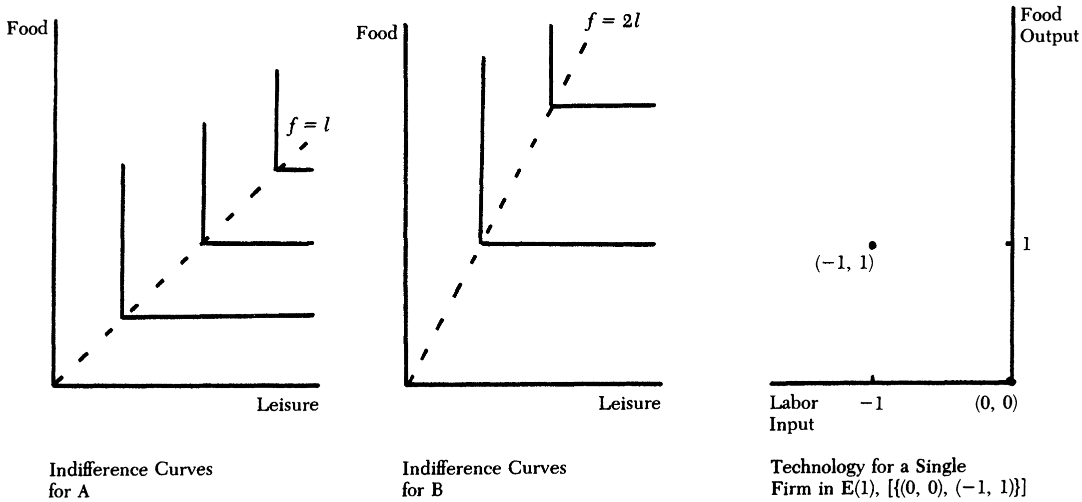


*Figure* 8.

set, the set of possible production plans for the firm,[5] has two components. The first is the origin (the production plan with no inputs and no outputs) and the second component is bounded away from the origin, compact and strictly convex. By *compact* we mean there is some number that is a bound for the magnitude of any input or output level in any feasible production plan (boundedness), and if a sequence of feasible plans has a limit, the limit is also feasible (closedness). By *strictly convex* we mean that for any two production plans in this component, any weighted average of the two plans is in the interior of this component. See Figure 9. This assumption provides a simple production set analog of U-shaped average cost in partial equilibrium. Inclusion of the origin in the production set guarantees free exit. For our example we will use the simplest version of this type of production, an on-off technology. The third diagram of Figure 8 shows the technology of a typical firm in $E(1)$. The firm has only two options. It can be inactive with production plan (0, 0), or it can use one unit of leisure as labor input (negative by convention) to produce one unit of food as output with production plan $(-1, 1)$. Observe that the production set $\{(0, 0), (-1, 1)\}$ contains all possible input-output vectors for the firm. The firm cannot scale the production level up or down to produce $(-2, 2)$ or $(-\frac{1}{2}, \frac{1}{2})$: there is an indivisibility in the production process at the firm level.

Our model differs from the standard ADM one in a second way. We assume that there is free entry with no bound on the number of possible firms. Each economy has an infinity of potential firms; a countable infinity of firms exist in each economy $E(\alpha)$, and a continuum of firms in the limit economy $E(0)$. In the Cournot equilibria of $E(\alpha)$ only a finite number of firms operate so that there will always be additional firms to check the profitability of entry.

By rescaling technologies in a manner analogous to the rescaling of cost in the partial equilibrium model of Section III, we generate a sequence of economies $[E(\alpha)]$ converging to a limit economy $E(0)$. For our example, each $E(\alpha)$ contains

[5] Each production plan is a vector with a complete specification of all inputs and outputs for that plan.

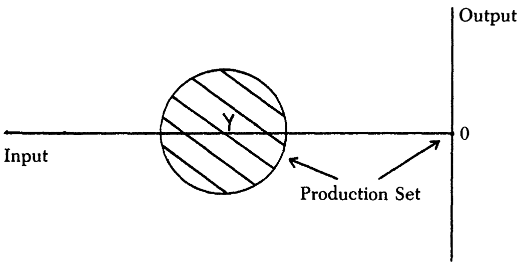


*Figure* 9.

a countable infinity of firms with production sets $\{(0, 0), (-\alpha, \alpha)\}$. Note that a firm's technology is small relative to the economy when $\alpha$ is small. The aggregate production set in $E(\alpha)$ is $\{(0, 0), (-\alpha, \alpha), (-2\alpha, 2\alpha), \ldots\}$: Aggregate production depends only on the number of active firms. The aggregate production set "converges" to the constant returns to scale technology that converts labor input into food output in the ratio 1:1. This constant returns to scale technology corresponds to the aggregate production set in the limit economy $E(0)$ with a continuum of infinitesimal firms. (As a closed, convex set this limit aggregate production set satisfies standard ADM assumptions for the production set of a single firm.)

For simplicity the consumer sectors of each $E(\alpha)$ and $E(0)$ coincide, and consumer $i$ owns a fraction of firm $t$ independent of the firm (i.e., if $\theta_{it}$ is the fraction of $t$ owned by $i$, then $\theta_{it} = \theta_i$ for all $t$). In our example consumer $A$ owns fraction $\theta_A$ of each firm while consumer $B$ owns $1 - \theta_A$ of each firm. Let $\theta_A = 3/4$. Thus $E(\alpha)$ "converges" to the limit economy $E(0)$ in terms of both consumer and producer sectors.

In order to define a Cournot equilibrium in $E(\alpha)$ we need a general equilibrium analog of a partial equilibrium inverse demand function so that firms can evaluate the profits corresponding to different actions. There is an important difference between partial and general equilibrium inverse demand functions. In partial equilibrium, dollar incomes remain constant while a single price varies. In contrast, the general equilibrium inverse demand function takes full account of the induced changes in income, through both changes in the value of endowments and in the received dividend payments.

The assumption of equality across firms of each consumer's ownership share implies that each consumer's wealth depends only on prices $p$ and aggregate production $y$, and not on the arrangement of production among firms. If consumer $i$ owns fraction $\theta_i$ of each firm and has endowment vector $w_i$, then at prices $p$ and the aggregate production $y$ the consumer receives dividend payment $\theta_i\, p \cdot y$ and has total income $p \cdot w_i + \theta_i\, p \cdot y$. (Because inputs are negative in $y$, the sum of individual prices times planned inputs or outputs is just the firm's profit at plan $y$ given prices $p$.) If his corresponding vector of demand is $D_i(p, p \cdot w_i + \theta_i\, p \cdot y)$ then the excess demand $D$ of the consumer sector (the sum of individual consumer's gross demands, minus the sum of resources owned as initial endowments by the consumer sector) is the function $D = \sum_i D_i(p, p \cdot w_i + \theta_i\, p \cdot y) - \sum_i w_i$. An "inverse demand" selection $F(y)$ is a function from aggregate production vectors $y$ to price vectors $F(y)$ that clear markets given the action of firms: $\sum_i D_i[F(y), F(y) \cdot w_i + \theta_i\, F(y) \cdot y] - \sum_i w_i = y$. This is the general equilibrium analog of partial equilibrium inverse demand. The corresponding partial equilibrium version as used in standard oligopoly theory would fix the income of consumer $i$ at $I_i$ rather than recognize that price changes affect income $[F(y) \cdot w_i + \theta_i\, F(y) \cdot y]$. Even after price normalization several price vectors may clear markets given $y$. We assume $F$ selects one of the price vectors.

We now illustrate an inverse demand function in terms of our example. Every feasible aggregate production plan assumes the form $(-t, t)$ where $t$ is nonnegative. Normalizing prices to sum to one (because only relative prices matter) the price vector is of the form $(1 - p, p)$ where $p$ lies between zero and one. With aggregate production $(-t, t)$ and price $(1 - p, p)$ consumer $A$ has income $(1 - p, p) \cdot (1, 0) + (3/4)(1 - p, p) \cdot (-t, t) = 1 - p + (3/4)t\ (2p - 1)$ and consumer $B$ has income $(1 - p, p) \cdot (1, 0) + (1/4)(1 - p, p) \cdot (-t, t) = 1 - p + (1/4)t\ (2p - 1)$. If $p$ is strictly between zero and one then consumer $A$ has demand vector $(I_A, I_A)$ and consumer $B$ has demand vector $[I_B/(1 + p), 2I_B/(1 + p)]$ where $I_i$ is the income of consumer $i$ (recall Figure 8).

The "inverse demand" $F(-t, t)$ for the example must yield prices that, together with the resulting incomes, generate aggregate excess demand $(-t, t)$ in the consumer sector, to match exactly the aggregate production plan. Solving for $F$ we find

$$F(-t, t) = \begin{cases} (0, 1) & 0 \leq t < 16/15 \\ [(15t - 16)/(6t - 4), \ (12 - 9t)/(6t - 4)] & 16/15 \leq t \leq 4/3 \\ (1, 0) & 4/3 < t \leq 2. \end{cases}$$

Observe that the inverse demand function depends on preferences and the initial distribution of wealth in the economy. In particular, when we consider a sequence of economies $[E(\alpha)]$, the inverse demand function is independent of $\alpha$.

Using "inverse demand" $F$, firms determine an equilibrium for $E(\alpha)$ just as in the partial equilibrium case. In general equilibrium the firms pick production plans to maximize profit, taking the production plans of all other firms as fixed. Free exit implies that in equilibrium no firm operates at a loss. Also, no inactive firm has an incentive to enter. Stated precisely, a Cournot equilibrium with entry for economy $E(\alpha)$ (relative to "inverse demand" $F$) is an integer $n$, the number of active firms, and a set of nonzero production plans $(y_1, y_2, \ldots, y_n)$ such that:

(*a*) $(y_1, y_2, \ldots, y_n)$ is an $n$-firm Cournot equilibrium (without entry); that is, for all $i = 1, 2, \ldots, n$, $y_i$ is a feasible production plan and $F\left(\sum_{j \neq i} y_j + y_i\right) \cdot y_i \geq F\left(\sum_{j \neq i} y_j + y\right) \cdot y$ for all feasible $y$, and

(*b*) entry is not profitable; that is $F\left(\sum_j y_j + y\right) \cdot y \leq 0$ for all feasible $y$.

Let $F$ be a (continuous) "inverse demand" function for the economy $E(1)$. We define a *perfectly competitive equilibrium* for the sequence of economies $[E(\alpha)]$ as a price vector $p^*$ and an aggregate production plan $y^*$ such that (1) $F(y^*) = p^*$ and (2) $y^*$ is the limit of aggregate production plans $y(\alpha)$ corresponding to Cournot equilibria with entry for economies $E(\alpha)$ (relative to $F$). Observe that every specification of perfectly competitive equilibrium includes an underlying inverse demand function.

In our example, the use of the on-off technology greatly simplifies the analysis when looking for an equilibrium of the sequence: Each firm's profit depends only on whether it operates and the total number of active firms. In $E(\alpha)$, $N$ active firms result in each active firm receiving profit $\alpha$ if $N < 16/15\alpha$, $\alpha(14 - 12N\alpha)/(3N\alpha - 2)$ if $16/15\alpha \leq N \leq 4/3\alpha$, and $-\alpha$ if $4/3\alpha < N$. Inactive firms always have profit zero. To be a Cournot equilibrium each active firm must have non-negative profit, so in equilibrium $N \leq \frac{7}{6}\alpha$. On the other hand, in Cournot equilibrium inactive firms must not have an incentive to be active, so in equilibrium $N + 1 \geq \frac{7}{6}\alpha$. Each $E(\alpha)$ has a Cournot equilibrium with $N(\alpha)$ active firms where $N(\alpha)$ is the integer between $(\frac{7}{6}\alpha) - 1$ and $\frac{7}{6}\alpha$ (when $\frac{7}{6}\alpha$ is an integer there are two values of $N$ that work). The aggregate production plan in the Cournot equilibrium is $N(\alpha)(-\alpha, \alpha)$ which differs from $(-\frac{7}{6}, \frac{7}{6})$ by no more than $\alpha$ units of input and $\alpha$ units of output. Thus in our example, $p^* = (\frac{1}{2}, \frac{1}{2})$ and $y^* = (-\frac{7}{6}, \frac{7}{6})$ is a perfectly competitive equilibrium for the sequence of economies $[E(\alpha)]$. At these prices consumer $A$ chooses $(\frac{1}{2}, \frac{1}{2})$, consumer $B$ chooses $(\frac{1}{3}, \frac{2}{3})$, and supply equals demand. Note that this agrees with the ADM equilibrium of the limit economy $E(0)$.

Next we would like to prove the first classical theorem of welfare economics: Perfectly competitive equilibria for the sequence $[E(\alpha)]$ are efficient in the sense of Pareto. We begin by defining a Pareto efficient allocation for the perfectly competitive sequence $[E(\alpha)]$. First observe the way in which the sequence $[E(\alpha)]$ converges to $E(0)$: The demand sectors of each $E(\alpha)$ coincide with the demand sector in the limit economy $E(0)$, and the aggregate production sets in $E(\alpha)$ converge to the constant returns to scale aggregate production set in the limit economy $E(0)$. As a consequence we define efficiency for the perfectly competitive sequence $[E(\alpha)]$ partially in terms of the limit economy $E(0)$.

A vector listing the consumption of each consumer and an aggregate production plan is called an allocation; if it is feasible for $E(0)$ then we say it is feasible for the sequence $[E(\alpha)]$. Feasibility requires that aggregate consumption differ from aggregate production by exactly the aggregate endowment and also that aggregate production belong to the constant returns to scale aggregate production set of the limit economy $E(0)$. An allocation for the perfectly competitive sequence $[E(\alpha)]$ Pareto dominates an alternative allocation if it makes at least one consumer better off while leaving the remaining consumers at least as well off. A feasible allocation for the sequence $[E(\alpha)]$ is *Pareto efficient* if no feasible allocation Pareto dominates it.

*Proposition*. Equilibria of the perfectly competitive sequence $[E(\alpha)]$ are Pareto efficient.

The standard argument for this result, applied to our example, proceeds as follows. Recall that the perfectly competitive equilibrium for the sequence $[E(\alpha)]$ has corresponding allocation $(\frac{1}{2}, \frac{1}{2})$ for $A$, $(\frac{1}{3}, \frac{2}{3})$ for $B$, and aggregate production plan $(-\frac{7}{6}, \frac{7}{6})$ that yields zero profit at prices $(\frac{1}{2}, \frac{1}{2})$. This means that $A$ supplies one-half of a unit of leisure (as labor input to the production process) and consumes one-half of a unit of food, while $B$ supplies two-thirds of a unit of leisure and consumes two-thirds of a unit of food. In order to prove the proposition we must show that any allocation that Pareto dominates the equilibrium allocation is not feasible.

The set of bundles that $A$ considers at least as good as $(\frac{1}{2}, \frac{1}{2})$ is $X_A = \{l, f) | l \geq \frac{1}{2}, f \geq \frac{1}{2}\}$. Similarly, $X_B = \{l, f) | l \geq \frac{1}{3}, f \geq \frac{2}{3}\}$. To make one of $A$ and $B$ better off while keeping the other at least as well off would require in excess of $\frac{5}{6}$ units of leisure and in excess of $\frac{7}{6}$ units of food. But with only two units of leisure available in the economy, more than $\frac{7}{6}$ units of food must be produced from less than $\frac{7}{6}$ $(= 2 - \frac{5}{6})$ units of labor. One can see immediately that such a production plan is not feasible with the one unit of food for each unit of labor constant returns to scale aggregate production set of the limit economy $E(0)$.

In general, we must use a less direct argument to show that a Pareto-dominating allocation $a$ is not feasible. We combine the observation that, relative to the price system (½, ½), the production plan that matches aggregate net consumption in $a$ must yield a greater profit than does (−7/6, 7/6) with the fact that (−7/6, 7/6) is profit maximal in $E(0)$.

What are the properties of equilibria of the perfectly competitive sequence $[E(\alpha)]$? How are these equilibria related to the ADM (price-taking) equilibria of the limit economy $E(0)$? The answers agree with the partial equilibrium results of Theorem 1: The equilibria of the perfectly competitive sequence are those ADM equilibria of the limit economy that satisfy an additional condition, DSD. This condition is related to downward sloping demand in the partial equilibrium case.

Consider an equilibrium of the perfectly competitive sequence $(p^*, y^*)$ relative to the "inverse demand" function, $F$. In $E(\alpha)$, a firm has a production set comprised of two components, the no-production component and the production one and the "inverse demand" function is continuous in a neighborhood of $y^*$. In our example the latter component of the production set contained the single production plan, $(-\alpha, \alpha)$. Though we did not assume price-taking behavior, as $\alpha$ converges to zero a firm's ability to affect price becomes arbitrarily small. For $\alpha > 0$ the entry or exit of firms has an effect on price. In the limit this effect disappears and the aggregate production plan $y^*$ must maximize profit over the aggregate production set (a constant returns to scale cone) relative to prices $p^*$, and must satisfy $p^* \cdot y^* = 0$. (Because "inverse demand" is continuous near $y^*$, and the aggregate productions in the sequence of Cournot equilibria converge to $y^*$, if $p^* \cdot y^* > 0$ then, for small $\alpha$, some feasible action for a firm in $E(\alpha)$ would earn strictly positive profit at the Cournot equilibrium despite the price change due to entry by a single firm. Hence all firms should be active. Similarly, if $p^* \cdot y^* < 0$ then some active firms in the Cournot equilibria of $E(\alpha)$ make a loss, and should exit. If $p^* \cdot y^* = 0$ but $p^* \cdot y > 0$ for some $y$ in the aggregate production set then again some firms could not have been maximizing in $E(\alpha)$.) By an interchangeability lemma[6] (see Koopmans 1957, p. 13) adjusted for the fact that in the limit there is a continuum of firms, any decomposition of $y^*$ into feasible actions for the individual infinitesimal firms in $E(0)$ is such that *all* firms' actions must be profit maximizing over their production sets relative to prices $p^*$. Thus the actions of firms are *as if* they are price takers in equilibrium, and an equilibrium of the perfectly competitive sequence is an ADM equilibrium of the limit economy $E(0)$.

A slight modification of our example demonstrates the differences between the requirement that in the limit firms are unable to influence price, and the *assumption* of price-taking behavior. The equilibria of the perfectly competitive sequence must satisfy an additional condition, (weak) DSD. The DSD condition is necessary to ensure equilibrium in the entry decision. As $\alpha$ converges to zero, in the sequence of economies $[E(\alpha)]$ firms become arbitrarily small relative to the market and thus have arbitrarily small impact on prices. Therefore, any production plan $y$ in $\alpha Y$ with $p^* \cdot y < 0$ would not (for $\alpha$ sufficiently small) be able to change price enough to make a non-negative profit. Because max $p^* \cdot y = p^* \cdot$

[6] The interchangeability lemma of Koopmans states that if one maximizes separately a linear functional on two sets $A$ and $B$ then its maximum value on $A + B$ is simply the sum of the previous maximums. In less mathematical terms it states that centralized planning yields the same results as decentralized planning.

$y^* = 0$, no actions $\alpha y$ in $\alpha Y$ with $p^* \cdot y > 0$ are available. In our example, when an inactive firm becomes active it changes price by approximately $\alpha[\partial F/\partial y(y)]\ (-1, 1)'$ where $[\partial F/\partial y(y)]$ is the $2 \times 2$ matrix of partial derivatives. Because an active firm produces $\alpha(-1, 1)$, the entrant's profit minus the incumbent's profit before this additional entry is approximately $\alpha^2(-1, 1)\ [\partial F/\partial y(y)]\ (-1, 1)'$. For the case of $n$ commodities, the expression $\alpha^2 y'[\partial F/\partial y(y^*)]y$ is (approximately) the profit differential (above the profit made by an active firm before this additional entry) available to an inactive firm by switching to $\alpha y$ in $\alpha Y$ with $p^* \cdot y = 0$ in the $E(\alpha)$ equilibrium. Weak DSD requires that this profit differential be nonpositive. When weak DSD fails, inactive firms could not be profit maximizing for $\alpha$ sufficiently small. Thus firms would want to enter in $E(\alpha)$, and $y^*$ could not be a limit point of *any* sequence of equilibria of $E(\alpha)$ relative to $F$.

The DSD requirement, a static stability condition, is similar in spirit to Hicksian perfect stability (John Hicks 1939). Each considers changes in a single "market" at a time. Hicksian perfect stability requires that a fall in the price of a single commodity make demand exceed supply for that commodity (with or without other prices adjusting to clear other markets). The DSD condition applies to a change in the number of active firms of a particular type, with the corresponding change in aggregate production being a simultaneous change in several input and output commodities. Prices adjust to clear all markets given the new aggregate production. DSD requires that the profit for firms declines as more firms enter. Static equilibrium of the perfectly competitive sequence requires DSD.

We can demonstrate the consequences of failure of weak DSD in a slightly modified version of our example. Consider an example in which consumer $A$ owns one-fourth rather than three-fourths of each firm. For some aggregate production more than one price vector would clear the market, so the "inverse demand" $F$ must be a selection from these prices. In particular let us examine the ADM equilibrium $p^* = (1/2, 1/2)$, $y^* = (-7/6, 7/6)$ which is unchanged because there are zero profits in that equilibrium. Taking the "inverse demand" with $F[(-7/6, 7/6)] = (1/2, 1/2)$ and which is continuous near the ADM equilibrium $y^*$, we obtain $F[(-t, t)] = [(16 - 13t)/(4 - 2t), (11t - 12)/(4 - 2t)]$ for $12/11 \leq t \leq 16/13$. Notice that $p^* = (1/2, 1/2)$, $y^* = (-7/6, 7/6)$ cannot be an equilibrium for the perfectly competitive sequence $[E(\alpha)]$: No Cournot equilibrium for any small $\alpha$ has aggregate production corresponding to $t$ near $7/6$. Such a Cournot equilibrium would require the number of active firms, $N$, to be between $12/11\alpha$ and $16/13\alpha$. In that region each active firm earns $\alpha(12N\alpha - 14)/(2 - N\alpha)$, which is negative for $N < 7/6\alpha$. For $N \geq 7/6\alpha$, by engaging in production, an inactive firm changes from zero to strictly positive profit. Cournot equilibria cannot exist near $p^* = (1/2, 1/2)$, $y^* = (-7/6, 7/6)$ for any small $\alpha$ because weak DSD fails. The ADM equilibrium for $E(0)$ is not an equilibrium for the perfectly competitive sequence $[E(\alpha)]$.

Prices give the wrong entry signals in this example because of the general equilibrium income effects in $E(\alpha)$. The entry of an additional firm will result in a lower price for labor relative to food yielding profit for the entering firm. The necessary additional labor is obtained by lowering the wage. Because $A$ owns only a small fraction of each firm, the reduction in the value of his leisure endowment (as labor) dominates his extra dividend (all active firms have strictly greater profit after the entry of an additional firm) and his income falls. Thus he reduces his demand for both leisure and food equally. On the other hand $B$ owns a large fraction of each firm, and his larger dividend dominates the reduction in value of his leisure endowment. Consumer $B$ receives a larger income and demands both more leisure and more food, but in the ratio 1:2. The extra labor needed for production by the entrant and to compensate for reduced labor by $B$ comes from $A$. $B$ consumes the extra food produced and the extra food available because of the reduced demand by $A$. Though each consumer and firm is well behaved in a partial equilibrium sense, the general equilibrium income effects lead to an analog of upward sloping demand providing wrong entry signals.

The second welfare theorem is a more difficult theorem in our context. According to the ADM second welfare theorem, under suitable assumptions, every Pareto efficient allocation can be supported as an ADM equilibrium by redistributing wealth; however, not every ADM equilibrium of the limit economy $E(0)$ is an equilibrium of the perfectly competitive sequence $[E(\alpha)]$. In our context we must redistribute initial endowments and ownership shares so that prices give correct entry signals (DSD holds). Otherwise, entry and exit might not cease near the Pareto efficient allocation, and therefore that allocation would not be a limit of Cournot equilibria of $E(\alpha)$, that is, an equilibrium of the perfectly competitive sequence $[E(\alpha)]$.

Despite the extra difficulties imposed by the requirement that prices give correct entry signals, it remains true that under rather general conditions (such as no externalities) a Pareto efficient allocation can be supported as an equilibrium of a perfectly competitive sequence. To do so, the redistribution of endowments and ownership shares must not only generate the required wealth level for each consumer but also ensure that prices give correct entry signals.

*Proposition.* For every Pareto efficient allocation of the perfectly competitive sequence $[E(\alpha)]$, there exists a redistribution of ownership and initial endowments so that this allocation is a perfectly competitive equilibrium for the sequence $[E'(\alpha)]$ obtained from $[E(\alpha)]$ by the redistribution.

We can demonstrate this result in terms of our example. The first step is to find all Pareto efficient allocations. Set a utility level $\bar{u}$ for consumer $A$. Maximize the utility of $B$ subject to the constraints that the leisure-food allocation to $A$ yields utility level at least $\bar{u}$, and the allocation to $A$ and $B$ is feasible given their aggregate initial endowment and the constant returns to scale technology of the aggregate production set of $E(0)$. Vary the utility level assigned to consumer $A$ to trace out all Pareto efficient allocations.

By the nature of consumer $A$'s preferences (recall Figure 8), his utility depends on the minimum of the amounts of leisure and food he obtains. If his allocation has different amounts of leisure and food, then the excess amount of one above the other is wasted. Because that excess added to $B$'s bundle might increase the utility of consumer $B$, it should be clear that for this special example we can fix consumer $A$'s bundle, with equal amounts of leisure and food, rather than fixing his utility level. If $A$ receives bundle $(1 - d, 1 - d)$, then a total of $2 - 2d$ units of leisure are used (including the $1 - d$ units to produce the food consumed by $A$) so $2d$ units of leisure remain. The problem is to maximize $B$'s utility given those $2d$ units of leisure and the technology that turns labor into food in a 1:1 ratio. Any bundle $(2d - f, f)$ with $f$ between zero and $2d$ can be assigned to $B$, and the problem resembles a standard partial equilibrium consumer choice problem (see Figure 10) with solution $(l, f) = (2d/3, 4d/3)$. As $d$ varies be-

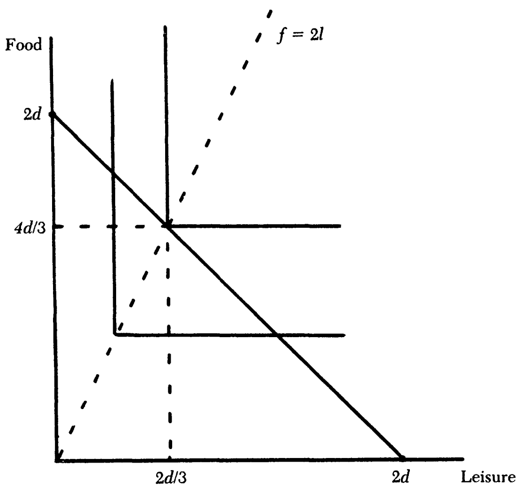


*Figure* 10.

tween zero and one we trace out all Pareto efficient allocations, with $d = 0$ corresponding to $B$'s receiving nothing, and $d = 1$ corresponding to $A$'s receiving nothing. Observe that as $d$ varies, the aggregate production $(-1 - d/3, 1 + d/3)$ also varies.

With constant returns to scale technology, every ADM equilibrium will have zero profit and prices $p^* = (\frac{1}{2}, \frac{1}{2})$. Thus the Pareto efficient allocations can be supported as ADM equilibrium of $E(0)$ only by assigning ownership of $2 - 2d$ units of the aggregate leisure endowment to $A$ and the remaining $2d$ units to $B$. This generates the required wealth levels so that each consumer can purchase the appropriate bundle with prices $(\frac{1}{2}, \frac{1}{2})$. The ownership share of firms is irrelevant for the ADM equilibria.

To support these Pareto efficient allocations as equilibria of a perfectly competitive sequence $[E'(\alpha)]$ we must also assign ownership of $2 - 2d$ units of the aggregate leisure endowment to $A$ and $2d$ units to $B$ in order to generate appropriate wealth levels. However, we must also assign the ownership share of the firms so that prices give the correct entry signals in $E(\alpha)$. The equilibrium corresponding to $d$ must have aggregate production $(-1 - d/3, 1 + d/3)$ and prices $p^* = (\frac{1}{2}, \frac{1}{2})$. Solving for the "inverse demand" selection $F$ which is continuous near $(-1 - d/3, 1 + d/3)$ and satisfies $F[(-1 - d/3, 1 + d/3)] = (\frac{1}{2}, \frac{1}{2})$ we obtain

$$F[(-t,t)] = [(3t+\theta t-4)/(2d+2\theta t-2), \quad (2d + \theta t - 3t + 2)/(2d + 2\theta t - 2)]$$

for $t$ near $1 + d/3$ where $\theta$ is the fraction of each firm owned by consumer $A$. In $E(\alpha)$, when aggregate output is near $(-1 - d/3, 1 + d/3)$ the entry of an additional firm changes the profit of each active firm by approximately $(-\alpha, \alpha) \cdot \partial F/\partial t\, [(-1 - d/3, 1 + d/3)]\, \alpha = -9\alpha^2/(\theta d + 3\theta + 3d - 3)$ ($t$ changes by $\alpha$ when a firm enters) which must be negative for DSD to hold. To give correct entry signals, we must redistribute ownership shares so that consumer $A$ owns fraction $\theta > 3(1 - d)/(d + 3)$ of each firm. This is possible for each Pareto efficient allocation corresponding to a $d$ value greater than zero. That this can be done in general depends on a revealed preference argument.[7]

It is important to observe that our version of the second welfare theorem requires more flexibility of transfers than the corresponding ADM result (Debreu 1959, p. 95). In that theory all of the required redistribution can be achieved by means of a single commodity, which may be thought of as "government wealth transfer." Here more than a single commodity may need to be redistributed because the allocation of all of the endowments is important, not just the purchasing power. Leonid Hurwicz (1959) refers to the property that every

[7] At $d = 0$ we hit the constraint $\theta \leq 1$. With $d = 0$ and $\theta = 1$ we have essentially a one-person economy; consumer $A$ owns all the aggregate endowment and all of the firms. Because of the kink in the indifference curves in our special example, $F$ cannot be made continuous near $(-1, 1)$ for this case. Thus in this case there is a Pareto efficient allocation that cannot be achieved as a perfectly competitive equilibrium of a sequence $[E'(\alpha)]$; however, this is an artifact of the fact that preferences are not differentiable.

optimum is an equilibrium (after redistribution) as "unbiasedness." Our analysis suggests a bias of the competitive mechanism beyond that which government transfers can correct. This bias can be corrected, but the correction in general requires more than the redistribution of a single commodity. From the viewpoint presented here, the competitive mechanism has the bias that it can seek out only those optima that, with the means of transfer at hand, give the necessary entry signals.

## VI. *Dynamics*

Marshall and Walras examined a long-run equilibrium in which all factors vary freely and "flow toward that branch of production" where they can realize profits. In the short run the returns to fixed factors do not necessarily equalize. At each moment, prices clear markets, but these prices reflect only the relative scarcity of the variable factors. Walras described the process from which these prices derive by a tâtonnement.

In each $E(\alpha)$ firms correctly perceive the prices, $F(y)$, that will prevail given any aggregate production $y$. Thus our analysis presupposes an anticipated adjustment of prices to clear markets, a tâtonnement for the exchange economy generated by a fixed production vector $y$. In each Cournot equilibrium firms anticipate that the price vector $F(y)$ will result from production $y$. At the equilibrium the prevailing price is the right price; that is, it equates supply and demand. Thus, no firm actually changes output in an equilibrium. We could have assumed that firms formed a subjective $F(y)$, but we required that $F(y)$ actually clear markets. For our model of dynamics we assume that the adjustment of prices to clear markets is instantaneous relative to the speed at which firms are able to change production levels.

Following the standard Marshallian framework, we next consider the quantity adjustments of a fixed number of active firms to achieve a short-run equilibrium. In $E(\alpha)$ we assume a continuous Cournot dynamics. Assuming all other firms' outputs are fixed, each active firm changes its output continuously. In the limit economy $E(0)$, each firm is infinitesimal and this dynamics agrees with that generated by firms viewing price as fixed. In both $E(0)$ and $E(\alpha)$ the firms do not account for the adjustments of other firms or the changing of price over time. We prohibit exit in the short run, so that an active firm cannot produce zero. It must produce from the nonzero component of its production set (e.g., some inputs may be fixed in the short run so that even if all outputs are zero some inputs are used). If $F$ is continuously differentiable and $\alpha$ is small, then the firm has little effect on price. Thus the incentives and behavior of the firm in the short-run dynamic for $E(\alpha)$ "converge to" that in the short-run dynamic for $E(0)$.

We build a bridge between the short run and the long run in the following manner. For concreteness, let us refer to the fixed factor as entrepreneurship. After each short-run adjustment we assume that entrepreneurship flows toward higher profits; that is, the number of active firms in each industry changes because of their profitability. Following such a period of factor movement, a new short-run equilibrium arises with associated prices, which lead to new incentives for factors to move, and so on. From this standpoint the tâtonnement occurs quickly relative to the short-run quantity adjustment of firms, which in turn takes place faster than the entry and exit adjustment of the entrepreneurial factor. As in the short-run adjustment, in $E(\alpha)$, recognizing only their own effect on price, firms enter or leave, so again, in $E(0)$ this entry dynamic agrees with that generated by viewing price as fixed. In both the short and long run, firms behave myopically. The entry-exit decision is a choice of production set $\alpha Y$ or $\{0\}$ (recall Figure 9). If $F$ is continuously differentiable and $\alpha$ is small, then the firm's decision has little effect on price. Thus the incentives and behavior of the firm in the long-run dynamic for $E(\alpha)$ "converge to" that in the long-run dynamic for $E(0)$.

The DSD condition suggests a dynamic theory of convergence to the long-run equilibrium, in which the realignment of the entrepreneurial factor plays a significant role. Because DSD is a necessary condition for equilibrium of the perfectly competitive sequence $[E(\alpha)]$, no infinitesimal firm in $E(0)$ can enter with positive profit at an equilibrium. However, we can conceive of a model where out of a long-run equilibrium the entry and exit of firms in a sector is proportional to the returns to the entrepreneurial factor available there. This leads us to consider the stability of the equilibrium introduced in the previous sections. It relates to the questions of whether returns to the homogeneous entrepreneurial factors tend to equalize and whether myopic profit-seeking behavior moves an economy toward a Pareto optimum. In addition, it is relevant to the viability of a planning procedure in which central planners increase production in the most profitable sectors.

Our previous argument that when $\alpha$ is small, both the short- and long-run dynamics in $E(\alpha)$ will be similar to the dynamics in $E(0)$ assures us that we lose no generality by examining $E(0)$ alone. Hence, for simplicity we will discuss the dynamics in terms of the limit economy $E(0)$. Our three stages of dynamics are as follows:

1. instantaneous adjustment of prices to clear markets given any aggregate production $y$;
2. output adjustment by a fixed number (mass) of firms, each viewing price as fixed at each instant, to reach a short-run equilibrium; and
3. entry and exit at a rate proportional to the (firm) profit levels in each industry to reach a long-run equilibrium.

The partial equilibrium market $M(0)$ used in Section III adequately illustrates the dynamics that we have in mind. We add to the hypotheses from that section the condition that $F$ is nonincreasing and there exists a unique $y^*$ such that $F(y^*) = C(1)$. In this case there is a unique equilibrium.

Introduction of the dynamics is as follows. For each aggregate output $y$, $F(y)$ gives market-clearing prices, the adjustment process of which is stable because $F$ is nonincreasing. In the short run an active firm must produce a positive quantity whether or not the positive profit-maximizing action yields positive profit. Each active firm increases output whenever the current price exceeds the marginal cost at the current output level and decreases output when current marginal cost exceeds price. This adjustment is also stable because $F$ is nonincreasing and all firms have nondecreasing marginal cost. For each mass of active firms $\mu$, the short-run equilibrium price associated with $\mu$, $p(\mu)$, is determined as follows:

1. Supply, $S[p(\mu)]$, is the integral of the profit-maximizing actions of the $\mu$ active firms at price $p(\mu)$, and
2. $F[S(p(\mu)] = p(\mu)$; that is, the inverse demand of supply at $p(\mu)$ is $p(\mu)$, or short-run supply equals demand at $p(\mu)$.

Let $\pi(\mu)$ denote the profit of each active firm when there are $\mu$ active firms in short-run equilibrium. If DSD holds, then a mass of active firms $\mu$ less than (greater than) the mass of active firms in the equilibrium of the perfectly competitive sequence receives positive (negative) profit. Thus in this case the long-run equilibrium is globally stable under the long-run adjustment process: $d\mu/dt = \pi(\mu)$. Let us pause to interpret this. Suppose that we start with the initial mass of active firms $\mu(0)$. The prices, $p[\mu(0)]$, provide signals for entry. Over time the mass of active firms changes, inducing a continuum of short-run equilibria. The mass of active firms adjusts toward the final equilibrium at which firms earn zero profits and produce at efficient scale.

To examine these ideas in more detail we return to our original general equilibrium example. Recall consumer $A$'s ownership share of firms is $\theta$. For $\theta > 3/7$ there is a unique equilibrium for $[E(\alpha)]$, the unique ADM equilibrium of $E(0)$, and DSD is satisfied. For any initial mass of active firms that leaves consumers in their consumption sets when each active firm produces at the efficient point $(-1, 1)$, the dynamics defined by $d\mu/dt = \pi(\mu)$ converge to the unique equilibrium $\mu^* = 7/6$. On the other hand, if $\theta < 3/7$, then the unique ADM equilibrium remains $\mu^* = 7/6$, but this allocation is not even locally stable. It is not locally stable for either the first-stage tâtonnement or for the third-stage entry dynamic using the "inverse demand" given in Section V. See Lars Svensson (1984).

The example at hand lends itself to a planning interpretation. Suppose that the central authority provides licenses for opening or closing facilities, but aside from this allows prices to be flexible. In the example we start with a certain mass of licensed facilities. If $\theta > 3/7$, then the procedure that has facilities open when profits are positive and closed when there are losses converges to the unique Pareto optimal allocation.

Let us summarize the dynamics. Prior to this section we were concerned with static analysis. We treated time in the spirt of Debreu's *Theory of Value*, by dating all commodities and opening all markets only once for the purpose of deciding on "who was to deliver what to whom and on what date." In this section the analysis is dynamic and temporary equilibrium in spirit. Markets open for today's exchange. The prices determined in those markets determine profits, which induce factor movements and a different distribution of firms tomorrow. The new distribution of immoble factors makes tomorrow's prices different from today's and so profits differ, etc. In our temporary equilibrium analysis we assume that firms are myopic; they do not consider how prices will change over time. Even with such nearsightedness, for our example, when prices provide the correct entry signals and so equilibrium exists, equilibrium is globally stable. This of course provides a strong link between the existence and stability theorem, and we consider this to be very much in the spirit of both Walrasian and Marshallian analysis.

## VII. *Conclusion*

As we have shown, with the right model it is not so difficult to travel between the world of the general equilibrium theorist and the Marshallian, partial equilibrium analyst. Furthermore, there are substantial gains from trade! A rigorous general equilibrium framework incorporates the Marshallian model, with marginal firms and U-shaped average costs. Perfect competition pertains to markets where firms are relatively small. In this case their ability to influence price disappears and price-taking behavior results from our theory. This is because in perfectly competitive environments the production sector mimics the behavior of a manager who is unable to influence price. Notice the enhancement of the classical welfare theorems: Competition leads to the number of active firms consistent with Pareto efficiency; it does not require the assumption of convex technology for the second welfare theorem (this is in part because Cournot equilibrium does not require convexity); it highlights the effects of the distribution of wealth on the entry signals conveyed by prices and so on. Finally, the assumption that profits induce entry (and losses induce exit) plays a major role in the analysis and suggests a classical but much neglected dynamics that fits well with the Marshallian vision.

Our analysis suggests that we need not be content with a general equilibrium theory that excludes marginal firms and free entry and hypothesizes price-taking behavior for firms without regard to their strategic opportunities. At the same time we need not settle for a theory of value that does not account for intermarket effects and relies on indirect measures, such as consumer and producer surplus, for its welfare economics. We should be all the more suspicious of a theory that loosely specifies behavior and equilibrium. The theory exposited here does have its special features, but it is precise, and it allows us to encompass both the Marshallian framework and the classical welfare theorems. With this theory less compromise between descriptive relevance and mathematical precision is required, and as a result there should be better communication between the "highbrow" and the "bread and butter" theorists.

## References


ARROW, KENNETH AND DEBREU, GERARD. "Existence of an Equilibrium for a Competitive Economy," *Econometrica*, July 1954, *22*, pp. 265–90.

ARTZNER, PHILIPPE; SIMON, CARL P. AND SONNENSCHEIN, HUGO. "Convergence of Myopic Firms to Long-Run Equilibrium via the Method of Characteristics," in *Models of economic dynamics*. Ed.: HUGO SONNENSCHEIN. Berlin: Springer-Verlag, 1986, pp. 157–83.

DEBREU, GERARD. *Theory of value*. NY: Wiley, 1959.

HART, OLIVER. "Monopolistic Competition in a Large Economy with Differentiated Commodities," *Rev. Econ. Stud.*, Jan. 1979, *46*(1), pp. 1–30.

HICKS, JOHN R. *Value and capital*. Oxford: Clarendon, 1939.

HURWICZ, LEONID. "Optimality and Informational Efficiency in Resource Allocation Processes," in *Mathematical methods in the social sciences*. Eds.: KENNETH ARROW, SAMUEL KARLIN, AND PATRICK SUPPES. Stanford: Stanford U. Press, 1959, pp. 27–46.

KOOPMANS, TJALLING. *Three essays on the state of economic science*. NY: McGraw-Hill, 1957.

MAS-COLELL, ANDREU. "Algunas Observaciones sobre la Teoria del Tâtonnement de Walrasen Economias Productivas," *Anal. Econ.*, Enero-Junio 1974, *21–22*, pp. 191–224.

———. "Walrasian Equilibria as Limits of Noncooperative Equilibria. Part I: Mixed Strategies," *J. Econ. Theory*, June 1983, *30*(1), pp. 153–70.

———. "Notes on Price and Quantity Tatonnement Dynamics," in *Models of economic dynamics*. Ed.: HUGO SONNENSCHEIN. Berlin: Springer-Verlag, 1986, pp. 49–68.

MCKENZIE, LIONEL. "On the Existence of General Equilibrium for a Competitive Market," *Econometrica*, Jan. 1959, *27*(1), pp. 54–71.

NOVSHEK, WILLIAM. "Cournot Equilibrium with Free Entry," *Rev. Econ. Stud.*, Apr. 1980, *47*(3), pp. 473–86.

NOVSHEK, WILLIAM AND SONNENSCHEIN, HUGO. "Cournot and Walras Equilibrium," *J. Econ. Theory*, Dec. 1978, *19*(2), pp. 223–66.

———. "Small Efficient Scale as a Foundation for Walrasian Equilibrium," *J. Econ. Theory*, Apr. 1980, *22*(2), pp. 243–55.

———. "Walrasian Equilibria as Limits of Noncooperative Equilibria. Part II: Pure Strategies," *J. Econ. Theory*, June 1983, *30*(1), pp. 171–87.

———. "Quantity Adjustment in an Arrow-Debreu-McKenzie Type Model," in *Models of economic dynamics*. Ed.: HUGO SONNENSCHEIN. Berlin: Springer-Verlag, 1986a, pp. 148–56.

———. "Non-cooperative Marshallian-like Foundations for General Equilibrium Theory," in *Contributions to mathematical economics in honor of Gerard Debreu*. Eds.: WERNER HILDENBRAND AND ANDREU MAS-COLELL. Amsterdam: North-Holland, 1986b.

ROBERTS, KEVIN. "The Limit Points of Monopolistic Competition," *J. Econ. Theory*, Apr. 1980, *22*(2), pp. 256–78.

SONNENSCHEIN, HUGO. "Price Dynamics Based on the Adjustment of Firms," *Amer. Econ. Rev.*, Dec. 1982, *72*(5), pp. 1088–96.

SPENCE, MICHAEL. "*Contestable Markets and the Theory of Industry Structure:* A Review Article," *J. Econ. Lit.*, Sept. 1983, *21*, pp. 981–90.

SVENSSON, LARS. "Walrasian and Marshallian Stability," *J. Econ. Theory*, Dec. 1984, *34*(2), pp. 371–79.

SYMPOSIUM ISSUE. "Noncooperative Approaches to the Theory of Perfect Competition," *J. Econ. Theory*, Apr. 1980, *22*.

WALRAS, LEON. *Elements of pure economics*. Trans.: WILLIAM JAFFE. London: Allen & Unwin, [1874–77] 1954.