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CHANCE CONSTRAINED MODEL OF WATER RESERVOIR: BOUNDS ON THE LONG-RUN DISTRIBUTION OF THE WATER STOCK

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ABSTRACT

In this model, treating water release as a deterministic decision variable facilitated the transformation of the chance constraints into deterministic form. This was done for a fairly generalized profit function and without assuming an a priori specific form for the decision rule. Moreover, an approximation for the long-run distribution of the stock of water in the reservoir was derived that provided reasonable bounds for the expected value of the distribution. Such an approximation facilitates the design of an insurance scheme that internalises the risk from the inflow's uncertainty. It also provides a rule of thumb against which a judgment as to whether too much or too little water is being stored.

CHANCE CONSTRAINED MODEL OF WATER RESERVOIR: BOUNDS ON THE LONG-RUN DISTRIBUTION OF THE WATER STOCK

The growth in population and rising level of industrialization in many arid and semi-arid parts of the world are increasing the demands for water. However, no corresponding change in the world supply of river water occurred. It has become a scarce resource, and active planning for water utilization is under way.

An important aspect of this planning is the distribution of the benefits of the rivers over time and among uses and users. Increasingly the construction of large reservoirs is becoming the vehicle to achieve and integrate these diverse objectives. Very few reservoirs are normally dedicated to achieve a single objective. Invariably, irrigation, power generation, flood control and recreation are among the objectives listed for any dam project. That does not mean there is no hierarchy imposed on these objectives by the planner. In fact, there may exist one or two prime objectives. The absence of explicit statements on this hierarchy has become a political expedient to appease the various groups affected by the construction of the dam. Model builders have reflected this hierarchy by directly including some variables in the objective function and others are formulated as constraints.

Some of these constraints are "soft," in the sense that they could be violated at a cost. This cost is dictated by the demand of the planner for these constraints to hold. The following analysis will focus on irrigation and power generation with soft constraints on the stock of water in the reservoir. These soft constraints reflect a trade-off between flood control and recreation purposes on the one hand and salinity control in the downstream on the other.

An often neglected aspect in the design of impounding reservoirs in arid and semi-arid regions where evaporation losses are significant is the trade-off between two opposing considerations:

1. There are benefits from assuring a more regular flow of water and hence a "better" distribution of the river benefit over time and among users and uses.
2. There are also costs imposed by the evaporation of the impounded water in the reservoir. These costs are significant. As Quirk and Burness point out [12] for a minor river such as the Colorado with an annual mean runoff of 13.5 million acre-feet per year, evaporation losses from existing reservoirs have already reached as high as 1.5 million acre-feet per year.

To produce an outflow pattern satisfying a given economic objective, the preceding trade-off is taken into consideration in ascertaining the relationship between the hydrology of a stream and the optimal decision rule. Moreover the long-run distribution of the water stock in the reservior will be derived. This distribution allows the selection of an insurance premium which takes the uncertainties of the water inflow into consideration.

Uncertainty will be revealed as the single most important factor affecting the optimal design and operation of a reservoir. Formally, this uncertainty may be reflected in the objective function, the constraints, or both. Consider the situation where the reservoir

manager is maximizing an n-period downstream profit function $\pi(y)$ of water releases $y = (y_1, y_2, \ldots, y_n)$. This maximization is subject to non-negativity and minimum pool level (R) constraints in every period i of the form:

$$r_i(x_{i-1} - y_i) + e_i \geq R \qquad (1)$$

where x_{i-1} is the stock of water at the start of period i,

y_i is the release at the start of period i (before e_i is observed),

e_i is the stochastic runoff in period i with known probability density function f_e, and

$1 - r_i$ is the evaporation loss in period i.

We can re-arrange (1) as follows:

$$r_i y_i \leq r_i x_{i-1} + e_i - R \qquad \forall\, i,\ i=1,\ldots,n$$

or in matrix form

$Ay \leq b$ where b_i is a function of the random variable e_i.

Thus the problem becomes that of:

$$\text{Max } \pi(y) \qquad (2)$$

$$\text{Subject to } Ay \leq b = f(e) \qquad (3)$$

$$y \geq 0 \qquad (4)$$

where e, y, b: $n \times 1$ and A: $n \cdot n$.

There is a possibility that optimal decisions will lead to violation of the constraints because of very high or very low values of e. This is the basic problem posed by the nature of the random constraints.

At least three different types of characterizations are available in the optimization literature to cope with the random nature of the constraints. First, there is the penalty function approach [16] which introduces penalties for violating the random constraints. This is accomplished by adding the expected penalty costs to the objective function. For example, if there is a constant penalty cost $C_j > 0$ per unit violation of the jth constraint $a_j y \leq b_j$, and the violation of the constraint has a finite probability density function $\psi(z)$, then the total expected penalty cost is $CE[\psi(b - Ay)]$. The modified problem then becomes

$$\text{Max } \pi(y) - CE[\psi(b - Ay)], \text{ subject to } y \geq 0. \tag{5}$$

This method is actually related to two-stage programming under uncertainty [4].

Second, there is the truncated distribution approach which interprets the inequalities $a_i'y \leq b_i$ $(i = 1,2,\ldots,m)$ as a truncation of the probability distribution of b_i. For example, Sengupta [17] uses the χ^2 distribution for a truncated normal.

Thirdly, there is the chance constrained characterization [1], [2] which puts a reliability interpretation on the constraint, such as

$$\text{prob } (b_i \geq a_i'y_i) \geq \lambda_i, \quad 0 \leq \lambda_i \leq 1, \; i = 1,\ldots,m \tag{6}$$

by preassigning reliability (tolerance) measures λ_i up to which constraint violations are permitted. The λ_i can be varied parametrically to account for the different reliability levels. Alternatively, a reliability term can be added to the objective function and can be solved for an optimal set of λ_i's [14]. For example, the problem could be characterized as:

$$\text{Max } U(y,\lambda) = w_1\pi(y) + w_2 \sum_{i=1}^{m} \log \lambda_i \tag{7}$$

$$\text{Subject to } y \geq 0, \; 0 \leq \lambda_i \leq 1, \; 0 \leq w_j \leq 1 \tag{8}$$

$$\text{and} \quad 1 - F_i(a_i'y) \geq \lambda_i \quad \forall\, i, i=1,2,\ldots,m \tag{9}$$

where F_i is the cumulative distribution function of the random variable and w_j, $j = 1,2$ are weighting factors.

In the first version, where λ_i's are not derived optimally, the chance constraint is reduced to an equivalent deterministic constraint [2] by the use of the marginal distribution function of b_i: $\phi(b_i)$. The existence of a fractile $\bar{b}_i$ such that

$$P(b_i \geq a_i'y) \geq \lambda_i \iff \bar{b}_i(1 - \lambda_i) \geq a_i'y \tag{10}$$

makes this reduction possible. To facilitate this transformation in the reservoir models, the optimal decision rule is restricted to the class of linear functions [7, 8, 9]. Additionally, it is sometimes assumed that the random variable is distributed normally or truncated normal at zero [3, 17].

Linear Decision-Rule and Chance Constraint

Essentially the linear decision rule is a device to facilitate the transformation of chance constraints into equivalent deterministic forms while avoiding a difficult convolution problem [5]. To illustrate this, consider the situation where, at any period p the starting stock of water is x_{p-1}, and the inflow and discharge is e_p and y_p, respectively. Then the continuity equation, assuming no evaporation losses, is

$$x_p = x_{p-1} + e_p - y_p. \tag{11}$$

The deterministic equivalent for a chance constraint of the form

$P(x_p \leq x^u) \geq \alpha_1$, cannot be determined since the probability distribution of x_p is unknown even if the distribution of e_p is known. The linear decision rule, first used by Revelle et al. [13], defines x_p and y_p in terms of e_p by postulating that the optimal decision rule is of the form

$$y_p = x_{p-1} - a_p \quad \text{where } a_p \text{ is a decision variable.} \tag{12}$$

Since, from the continuity equation, $x_p = x_{p-1} + e_p - y_p$

then

$$x_p = e_p + a_p \tag{13}$$

and

$$y_p = e_{p-1} + a_{p-1} - a_p. \tag{14}$$

Since the distribution of e_p is known and a_p is a deterministic decision variable, (13) and (14) define the distribution of x_p and y_p. Hence, deterministic equivalents for the chance constraints:

$$P(x_p \leq x^u) \geq \alpha_1 \tag{15}$$

or

$$P(y_p \geq \bar{y}) \geq \alpha_2 \tag{16}$$

can be found.

Previous models which used the linear decision rule within the framework of chance constraints formulation have two major shortcomings. First, the formulation of chance constraints implies that the continuity equation applies only probabilistically since there is positive probability that the constraints may be violated but their models do not specify what happens when the constraints are violated. Secondly, there exists no guarantee that the linear decision rule is actually optimal among all possible classes of bounded functions.

The model in this paper is a chance constraint formulation

with the assumption of a linear decision rule dropped. The optimal policies and the long-run distribution of the reservoir content will be investigated using the Chebychef inequality to bound the probability of a general distribution of the inflow. This general distribution is assumed, however, to have a known mean and variance.

A Chance Constrained Model: Deterministic Equivalent Approach

Consider a reservoir of infinite size, the problem is to maximize over a T period planning horizon a net discounted benefit function subject to chance constraints. Formally:

$$\max_{\substack{0 \leq y_p \leq y_{max} \\ p=1,\ldots,T}} \sum_{p=1}^{T} \beta^{p-1} \pi(y_p) \tag{1}$$

$$\text{Subject to } P(x_p \leq x^u) \geq \alpha_1 \quad , \forall\ p = 1,2,\ldots,T \tag{2}$$

$$P(x_p \geq x^m) \geq \alpha_2 \quad , \forall\ p = 1,2,\ldots,T \tag{3}$$

$$x_p = rx_{p-1} \pm e_p - ry_p \geq 0 \quad \forall\ p = 1,2,\ldots,T \tag{4}$$

where x^u is the usable capacity, fixed by law to provide for flood control or some other considerations. x^m is the minimum head required for power generation. Alternatively, x^m can be determined by environmental considerations such as wildlife preservation or, perhaps more importantly, salinity control downstream. α_i is the maximum tolerance level associated with the ith constraint and x_p is the storage level at the end of period p (measured from the start of the planning period). y_p and e_p are the release and inflow in period p, respectively. β is an appropriate discount rate. Finally,

$$r = 1 - k, \quad 0 < r < 1 \tag{5}$$

where k is the percentage evaporation from the reservoir. For simplicity, the salvage value function at the end of the horizon is assumed to be zero. $\pi(y_p)$ is a strictly concave profit function such that

$$\pi(y_p) = 0 \iff y_p = 0 \text{ or } y_p = y_{max}.$$

It is assumed that $\frac{\partial\pi}{\partial y_p}$ at 0 and y_{max} are finite, and that there exists y_0, $0 < y_0 < y_{max}$ such that $y_p < y_0 \implies \frac{\partial\pi}{\partial y_p} > 0$, $y_p > y_0 \implies \frac{\partial\pi}{\partial y_p} < 0$, $y_p = y_0 \implies \frac{\partial\pi}{\partial y_p} = 0$, as shown in Figure 1. e_p is assumed independent and identically distributed with mean μ and variance σ.

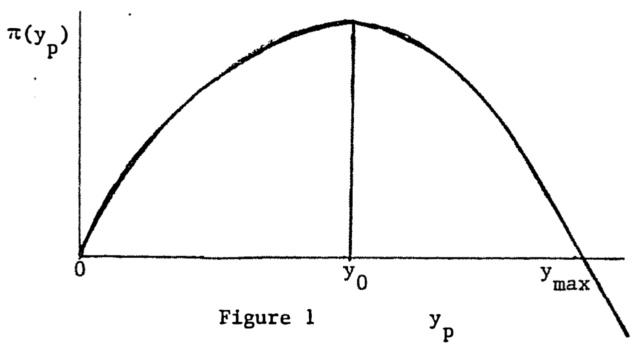

Figure 1

In the following, the deterministic equivalent of the stochastic problem is found, using the method developed by Charnes and Cooper [5]. The deterministic problem is then solved for the optimal policy $(y_1^*, y_2^*, \ldots, y_T^*)$ over the planning horizon. Next the implication of this policy is examined within the original random context of the problem. In particular, the effect of this deterministic policy on the distribution of the stock of water is investigated when the planning horizon is extended indefinitely and the random setting of the problem is restored. This method has some problems which will be mentioned later. Finally, the distribution of the water stock, developed here, is only an approximation, as will be explained in detail below.

A Proposition

There exists a unique optimal solution $y_1^*, y_2^*, \ldots, y_T^*$ to the reduced equivalent deterministic planning problem of the original chance constraint of equations (1) - (4) if $(x^u - x^m) \geq \left(\frac{1}{\sqrt{\beta_1}} - \frac{1}{\sqrt{\alpha_2}}\right)\frac{\sigma}{\sqrt{(2-k)}}$.

The implementation of this policy yields a family of approximate long term distributions for the water stock in the reservoir given by $\psi^j(\mu_{xp}^j, \sigma_{xp}^2)$ where

$$x^m - \frac{\sigma}{\sqrt{k\alpha_2(2-k)}} \leq \mu_{xp}^j \leq x^u - \frac{\sigma}{\sqrt{k\beta_1(2-k)}}, \ \forall\ j.$$

Proof

The deterministic equivalents for the chance constraints will be developed first. Consider (2): $P(x_p \leq x^u) \geq \alpha_1$ or equivalently

$$P(x_p \geq x^u) \leq \beta_1, \quad \text{where } \beta_1 = 1 - \alpha_1. \tag{6}$$

But from the continuity equation, we have $x_p = rx_{p-1} + e_p - ry_p^*$ where y_p^* is the optimal release in period p. Hence,

$$x_p = r^p x_0 - \sum_{i=1}^{p} r^{p-i+1} y_i^* + \sum_{i=1}^{p} r^{p-i} e_i. \tag{7}$$

Or,

$$x_p = r^p x_0 - y^*(p) + E_p \tag{8}$$

where

$$y^*(p) = \sum_{i=1}^{p} r^{p-i+1} y_i^*, \tag{9}$$

$$E_p = \sum_{i=1}^{p} r^{p-i} e_i. \tag{10}$$

Then

$$E_p \sim g(\mu_p, \sigma_p) \tag{11}$$

where

$$\mu_p = \frac{\mu(1 - r^p)}{1-r} \tag{12}$$

$$\sigma_p^2 = \frac{\sigma^2(1 - r^{2p})}{1 - r^2}. \tag{13}$$

Thus from (6) we have

$$P(r^p x_0 - y^*(p) + E_p \geq x^u) \leq \beta_1$$

or, equivalently

$$P\left(\frac{x^u - r^p x_0 + y^*(p) - \mu_p}{\sigma_p} \leq \frac{E_p - \mu_p}{\sigma_p}\right) \leq \beta_1. \tag{14}$$

Define K_{β_1} by

$$P\left(K_{\beta_1} \leq \frac{E_p - \mu_p}{\sigma_p}\right) = \beta_1. \tag{15}$$

Then (14) implies

$$K_{\beta_1} \leq \frac{x^u - r^p x_0 + y^*(p) - \mu_p}{\sigma_p} \tag{16}$$

However, by Chebychef's inequality,

$$P\left(K_{\beta_1} \leq \frac{E_p - \mu_p}{\sigma_p}\right) \leq \frac{1}{K^2_{\beta_1}}.$$

Therefore, (15) $\Longrightarrow$ $\beta_1 \leq \frac{1}{K^2_{\beta_1}} \Longrightarrow K_{\beta_1} \leq \frac{1}{\sqrt{\beta_1}}$ (17)

Substitution in (16) for $K_{\beta_1} = \frac{1}{\sqrt{\beta_1}}$ we have

$$x^u - r^p x_0 + \sum_{i=1}^{p} r^{p-i+1} \bar{y}_i^* - \mu_p - \frac{\sigma_p}{\sqrt{\beta_1}} \geq 0. \tag{18}$$

This is a more stringent constraint than the original deterministic equivalent constraint which would have resulted from using the actual distribution of e_p rather than the Chebychef bound. Alternatively, sharper bounds such as Markov, or special case bounds [4] could be used to develop deterministic equivalents for the chance constraints in this problem.

Similarly, the equivalent deterministic form for (3) is found to be

$$x^m - r^p x_0 + \sum_{i=1}^{p} r^{p-i+1} y_i^* - \mu_p - \frac{\sigma_p}{\sqrt{\alpha_2}} \leq 0 \tag{19}$$

Thus the problem is transformed into

$$\max_{\substack{0 \le y_p \le y_{max} \\ p=1,2,\ldots,T}} \sum_{p=1}^{T} \beta^{p-1} \pi(y_p) \tag{20}$$

subject to (18) and (19).

Note that (18) and (19) can be rewritten as

$$y_p \ge \frac{1}{r} [-x^u + r^p x_0 - \sum_{i=1}^{p-1} r^{p-i+1} y_i^* + \mu_p + \frac{\sigma_p}{\sqrt{\beta_1}}] \tag{21}$$

$$y_p \le \frac{1}{r} [-x^m + r^p x_0 - \sum_{i=1}^{p-1} r^{p-i+1} y_i^* + \mu_p + \frac{\sigma_p}{\sqrt{\alpha_2}}] . \tag{22}$$

The solution will be determined next. The Langrangian for the problem in (20-21) is given by:

$$L = \sum_{p=1}^{T} \beta^{p-1} \{\pi(y_p) - C(\bar{x})\} + \sum_{p=1}^{T} \lambda_{1p} [-x^u + r^p x_0 - \sum_{i=1}^{p} y_i^* r^{p-i+1} + \mu_p + \frac{\sigma_p}{\sqrt{\beta_1}}] + \sum_{p=1}^{T} \lambda_{2p} [x^m - r^p x_0 + \sum_{i=1}^{p} y_i^* r^{p-i+1} - \mu_p - \frac{\sigma_p}{\sqrt{\alpha_2}}]. \tag{23}$$

Ignoring the nonnegativity constraints on the y's, the first order conditions are given by

$$\beta^{p-1} \frac{\partial \pi}{\partial y_p} - \sum_{i=p}^{T} (\lambda_{2i} - \lambda_{1i}) \sigma^{i-p} = 0 \tag{24}$$

$\forall p$, p = 1, 2, . . ., T.

This is the usual marginality condition; the discounted marginal benefit from a particular choice of water release y* must be equal to the total discounted marginal cost which results from that choice. The other first order conditions are:

$$-x^u + r^p x_0 - \sum_{i=1}^{p} y_i^* r^{p-i+1} + \mu_p + \frac{\sigma_p}{\sqrt{\beta_1}} \le 0 \tag{25}$$

(strict inequality implies $\lambda_{1p}^* = 0$);

$$-x^m - r^p x_0 + \sum_{i=1}^{p} y_i^* r^{p-i+1} - \mu_p - \frac{\sigma_p}{\sqrt{\alpha_2}} \le 0 \tag{26}$$

(strict inequality implies $\lambda_{2p}^* = 0$);

$$\text{and } y_p^*, \lambda_{1p}, \lambda_{2p}^* \ge 0. \tag{27}$$

Differentiating the first order condition (24) with respect to y_p:

$$\frac{d^2L}{dy_p^2} = \beta^{p-1} \frac{d^2\pi}{dy_p^2} \tag{28}$$

But $\frac{d^2\pi}{dy_p^2} < 0$ by strict concavity of π,

$$\text{therefore } \frac{d^2L}{dy_p^2} < 0 . \tag{29}$$

Thus the solution to (24), y_p^*, is unique.

Denoting the right-hand side of (21) and (22) by $\underline{y}_p$ and $\bar{\bar{y}}_p$ respectively, it follows that

$$y_p \geq \underline{y}_p = \frac{1}{r}[-x^u + r^p x_0 - \sum_{i=1}^{p-1} y_i^* r^{p-i+1} + \mu_p + \frac{\sigma_p}{\sqrt{\beta_1}}] \qquad (30)$$

$$y_p \leq \bar{\bar{y}}_p = \frac{1}{r}[-x^\mu + r^p x_0 - \sum_{i=1}^{p} y_i^* r^{p-i+1} + \mu_p + \frac{\sigma_p}{\sqrt{\alpha_2}}] \qquad (31)$$

and

$$y_{max} \geq \bar{\bar{y}}_p \geq y_p^* \geq \underline{y}_p \geq 0 \iff (x^u - x^m) \geq \sigma_p\left(\frac{1}{\sqrt{\beta_1}} - \frac{1}{\sqrt{\alpha_2}}\right). \qquad (32)$$

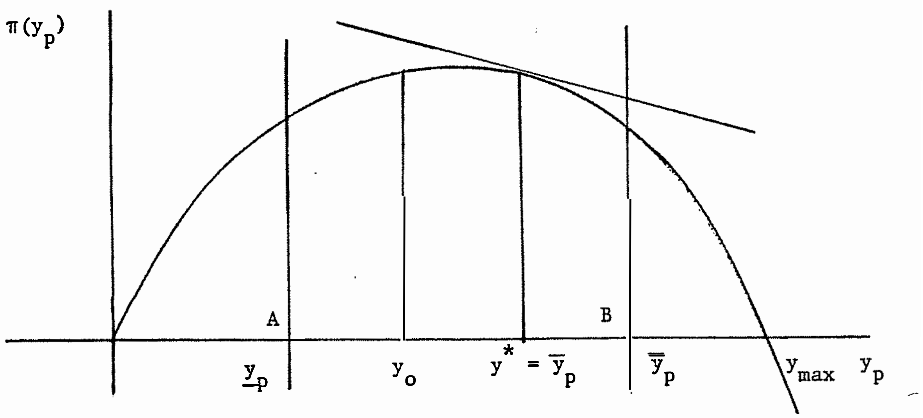

Figure 2

In this case, y_p^* lies in the closed convex interval {AB} in Figure 2. On the other hand, if the choice of α_1 and β_1 is such that

$$x^u - x^m < \sigma_p\left(\frac{1}{\sqrt{\beta_1}} - \frac{1}{\sqrt{\alpha_1}}\right). \qquad (33)$$

Then (21) and (22) cannot hold simultaneously.

Let $\bar{y}_p$ denote the solution to (24). Thus,

$$y_p^* = \begin{cases} \bar{\bar{y}}_p & \text{if } \lambda_{2p}^* > 0 \\ \bar{y}_p & \text{if neither } \lambda_{1p}^*, \lambda_{2p}^* > |0| \\ \underline{y}_p & \text{if } \lambda_{1p}^* > 0 \end{cases} \qquad (34)$$

Figure 3 illustrates the nature of the solution of (24).

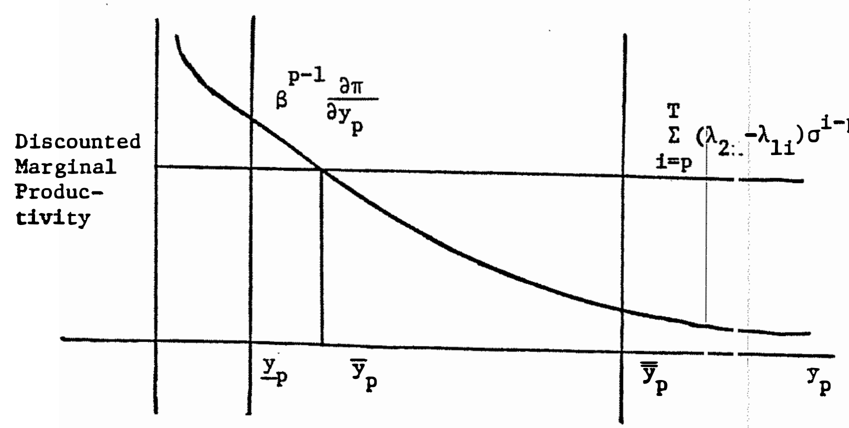

Figure 3

The Long-Run Distribution of x_p

For an infinite size reservoir, the probability of a spillover is zero. Moreover, if μ is large and we start with $x_0 = x^u$, the probability of empty reservoir is, also, very small.

From (12) and (13) as p is increased, $r^p x_0 \to 0$, $\mu_p \to \mu/k$ and

$$\sigma_p^2 \to \frac{\sigma^2}{k(2-k)} \tag{35}$$

Hence from (7)

$$x_p \to \psi(\mu/k - \sum_{i=1}^{p} r^{p-i+1} y_i^*, \frac{\sigma^2}{k(2-k)}). \tag{36}$$

However, if (32) holds $(x^u - x^m) \geq \sigma_p(\frac{1}{\sqrt{\beta_1}} - \frac{1}{\sqrt{\alpha_2}})$. That is, when the "adjusted" variability of the stream flow is small in comparison with the usable capacity,

$$\underline{y}_i \leq y_i^* \leq \overline{\overline{y}}_i, \qquad \forall i = 1, 2, \ldots, p. \tag{37}$$

Hence,

$$\sum_{i=1}^{p} r^{p-i+1} \underline{y}_i \leq \sum_{i=1}^{p} r^{p-i+1} y_i^* \leq \sum_{i=1}^{p} r^{p-i+1} \overline{\overline{y}}_i \quad . \tag{38}$$

From (25) and (26) we have

$$\sum_{i=1}^{p} r^{p-i+1} \underline{y}_i = -x^u + r^p x_0 + \mu_p + \frac{\sigma_p}{\sqrt{\beta_1}} \tag{39}$$

and

$$\sum_{i=1}^{p} r^{p-i+1} \overline{\overline{y}}_i = -x^m + r^p x_0 + \mu_p + \frac{\sigma_p}{\sqrt{\alpha_2}} \quad . \tag{40}$$

When $p \to \infty$ then $\mu_p \to \frac{\mu}{k}$, $\sigma_p \to \frac{\sigma}{\sqrt{k(2-k)}}$ and $r^p x_0 \to 0$.

Therefore,

$$\sum_{i=1}^{p} r^{p-i+1} \underline{y}_i \to -x^u + \frac{\mu}{k} + \frac{\sigma}{\sqrt{k\beta_2(2-k)}} \tag{41}$$

and

$$\sum_{i=1}^{p} r^{p-i+1} \overline{\overline{y}}_i \to -x^m + \frac{\mu}{k} + \frac{\sigma}{\sqrt{k\alpha_2(2-k)}} \quad . \tag{42}$$

Hence,

$$-x^u + \frac{\mu}{k} + \frac{\sigma}{\sqrt{k\beta_1(2-k)}} \leq \sum_{i=1}^{p} r^{p-i+1} y_i^* \leq -x^m + \frac{\mu}{k} + \frac{\sigma}{\sqrt{k\alpha_2(2-k)}} . \tag{43}$$

Thus, the long term distribution of x_p belongs to a class of distribution functions $\psi_j(\mu_{xp}^j, \sigma_{xp}^2)$ where

$$\sigma_{xp}^2 = \frac{\sigma^2}{k(2-k)} \tag{44}$$

and μ^j_{xp} is bounded as follows

$$x^m - \frac{\sigma}{\sqrt{k\alpha_2(2-k)}} \leq \mu^j_{xp} \leq x^u - \frac{\sigma}{\sqrt{k\beta_1(2-k)}}. \qquad (45)$$

Notice that there exist β_1 small enough so that

$$x^m - \frac{\sigma}{\sqrt{k\alpha_2(2-k)}} = x^u - \frac{\sigma}{\sqrt{k\beta_1(2-k)}} . \qquad (46)$$

In this case,

$$\mu_{xp} \to x^u - \frac{\sigma}{\sqrt{k\beta_1(2-k)}}. \qquad (47)$$

In general, however, (47) holds if: 1) the value of r is large enough, and 2) the nature of the solutions y^*_i, which is bounded above, makes the sequence $s_p = \sum_{i=1}^{p} r^{p-i+1} y^*_i$ a nondecreasing sequence. In this case, $s_p \to \bar{s}$ [30] and

$$x_p \to \psi(\mu_{xp}, \sigma^2_{xp}) , \qquad (48)$$

where σ^2_{xp} is given by (44).

(This ends the proof of the proposition).

In this model, treating water release as a deterministic decision variable facilitated the transformation of the chance constraints into deterministic form. This was done for a more generalized profit function and without assuming an a priori specific form for the decision rule. Moreover, an approximation for the long-run distribution of the stock of water in the reservoir was derived that provided reasonable bounds for the expected value of the distribution.

Such an approximation facilitates the design of an insurance scheme that internalises the risk from the inflow's uncertainty It also provides a rule of thumb against which a judgment as to whether too much or too little water is being stored.

REFERENCES

1. Charnes, A. and Cooper, M. J. "Chance-Constrained Programming," Management Science, Vol. 6, No. 1 (October 1959):73-80.

2. _______. "Deterministic Equivalents for Optimizing and Satisfying Under Chance Constraints," Operation Research, Vol. 11, No. 1 (January 1963):18-39.

3. _______. "Chance Constraints and Normal Deviates," Journal of American Statistical Association, Vol. 57, No. 297 (March 1962):134-140.

4. Dantzig, G. B. "Linear Programming Under Uncertainty," Management Science, Vol. 1 (1955):197-206.

5. Eisel, L. M. "Chance Constrained Reservoir Model," Water Resources Research (April 1972):339-347.

6. _____. "Comments on the Linear Decision Rule in Reservoir Management and Design," Water Resources Research, Vol. 6, No. 4 (1970):1239-1241.

7. Louks, D. and Dorfman, P. "An Evaluation of Some Linear Decision Rules in Chance-Constrained Models for Reservoir Planning and Operation," Water Resources Research, Vol. 11, No. 6, (December 1975):777-782.

8. _____. "Some Comments on Linear Decision Rules and Chance Constraints," Water Resources Research, Vol. 6, No. 2 (April 1970).

9. Nayak, S. C. and Arora, S. R. "Linear Decision Rule: A Note on Control Volume Being Constant," Water Resources Research, Vol. 10, No. 4 (August 1974):637.

10. . "Optimal Capacities for a Multi-reservior System Using The Linear Decision Rule," Water Resources Research, Vol. 7, No. 3 (June 1971):485.

11. Prekopa, A. "On the Probability Distribution of the Optimum of a Random Linear Program," Society for Industrial and Applied Mathematics, Journal on Control and Optimization, Vol. 4, No. 1 (1966):211-222.

12. Quirk, J., and Burness, H. S. "The Colorado River, Water Rights and Allocation," Environmental Quality Laboratory, California Institute of Technology, December 1976.

13. Revelle, C., Joeres, E., and Kirby, W. "The Linear Decision Rule in Reservoir Management and Design: Development of the Stochastic Model," Water Resources Research, Vol. 5, No. 4, (1969):767-777.

14. _______. "Linear Decision Rule in Reservoir Management and Design: Performance Optimization," Water Resources Research, Vol. 6, No. 4 (August 1970):1033-1044.

15. Rudin, W. Principles of Mathematical Analysis. McGraw Hill, 1953.

16. Segupta, J. K. "Safety First Rules Under Chance-Constrained Linear Programming," Operation Research, Vol. 17, No. 1 (January 1969):112-132.

17. _______. "A Generalization of Some Distribution Aspects of Chance-Constrained Linear Programming," Economic Models, Estimation and Risk Programming, (New York: Springer-Verlag 1962), 15.

18. Sobel, M. "Reservoir Management Models," Water Resources Research, Vol. 11, No. 6 (December 1975):767-776.

19. Young, G. "Finding Reservoir Operating Rules," Proceding of the American Society of Civil Engineers, Journal of Hydrolics Division, No. 6 (November 1967):297-321.