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THE EFFECT OF A RANDOM PLANNING HORIZON ON PRODUCTION AND INVESTMENT FOR PETROLEUM RESERVOIR -- A NOTE ON KULLER'S AND CUMMING'S MODEL

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SOCIAL SCIENCE WORKING PAPER 221

June 1978

ABSTRACT

There have been several formulations of models for crude oil production which tried to identify the elements of user cost and show their effect on production and investment decisions. In this chapter, previous results are extended by incorporating the uncertainty regarding the date of arrival of the backstop technology in the model. This uncertainty adds a new element to the user cost identified previously and is shown to affect the production and investment decisions.

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Introduction

Since the classical paper by Davidson [1], there have been many models which illustrate the role of user's costs in oil production. However, a paper by Kuller and Cumming offers the most comprehensive treatment of user costs by introducing the following assumptions:

1. Total recovery, as well as annual production rates from natural drive, depends not only on cumulative production, but also on the rate at which production has taken place.
2. The recoverable stock, as well as the production rate, depends on the time path of investment as well as on cumulative investment (i.e., the capital stock)

In their model, n firms are exploiting a given petroleum reservoir under centralized management which maximizes the expected profit function, Π, over a known planning horizon T, subject to constraints reflecting the above two assumptions and non-negativity. They identify four user cost elements: stock user costs, boundary user costs, user costs of capital consumption, and production user costs. Their policy prescriptions are simple: 1) produce at a rate which equates marginal net income to firm J and the user cost association with firm J's production, and 2) equate the marginal cost of investment (to firm J for capital-type k) with the marginal present value of the reservoir-wide benefits associated with such investment. The latter includes not only direct impacts on the marginal productivity of J's capital and J's future variable and boundary costs, but also external impacts on other firms' variable and boundary costs as well as on the recoverable stock [2].

This note will extend the results of Kuller and Cumming by introducing an additional source of randomness in the planning model, that which pertains to the planning period.

The Effect of Random Planning Horizon

One element in the decision matrix of the oil producer is uncertainty about the arrival date, T*, of the "backstop" technology that will replace hydrocarbon fuels as the principal source of energy. This uncertainty introduces another element into user cost and modifies the production decision of the producer. Assume that the central management of a field believes that T* is randomly distributed on the range $[0,\overline{T}]$. To facilitate comparison of these results with those obtained by Kuller and Cumming, assume further that their T corresponds to the expected value of T* in this framework.

Let K_t be the capital stock at period t; $R_t = (r_1, r_2, \ldots, r_t)$ is the history of production; $V_t = (v_1, v_2, \ldots, v_t)$ is the history of investment; C_t is the generalized cost function of period t. Then, let

$$C_t = C_t(R_t, V_t, K_t)$$

$$\frac{\partial C_t}{\partial r_\tau} \geq 0, \quad \frac{\partial C_t}{\partial v_t} \gtrless 0, \quad \underset{\tau \neq t}{\frac{\partial C_t}{\partial v_\tau}} \leq 0, \quad \frac{\partial C_t}{\partial K_t} \leq 0$$

r_{Jt} = the volume of petroleum extracted by firm J, J = 1, ..., n during period t

R_t = annual production rate by all firms during all periods, i.e., $R_t = (r_{11}, r_{21}, \ldots, r_{n1}, \ldots, r_{1n-1}, \ldots, r_{nt-1}, r_{1t}, \ldots, r_{nt})$

V_{Jkt} = gross investment by firm J in capital component k, k = 1, ..., q, during period t

V_t = gross investment for all capital components by all firms during the periods 1, ..., t

K_{Jkt} = firms J's stock of capital components k at the beginning of period t

$K_{Jt} = (K_{J1t}, \ldots, K_{Jqt})$

D_{Jkt} = net depreciation of firm J's stock of capital component k during period t

x = the recoverable stock

F_{Jt} = an upper (physical) bound on firm J's capacity to produce petroleum during period t

C_{Jt} = firm J's cost function during period t

β_t = a discount factor, $(1 + r)^{-t}$ where r is the appropriate discount rate

P_t = unit price of petroleum during period t

where[1]

$$\frac{\partial D_{Jkt}}{\partial v_{Jkt}} \le 0, \quad \frac{\partial D_{Jkt}}{\partial r_{J\tau}} \ge 0, \quad \frac{\partial D_{Jkt}}{\partial K_{Jkt}} \ge 0$$

$$\frac{\partial F_{Jt}}{\partial r_{i\tau}} \le 0, \quad \frac{\partial F_{Jt}}{\partial v_{i\tau}} \ge 0, \quad \frac{\partial F_{Jt}}{\partial k_{Jt}} \ge 0$$

$$\frac{\partial x}{\partial r_{i\tau}} \le 0, \quad \frac{\partial x}{\partial v_{i\tau}} \ge 0$$

$\tau = 1, \ldots, t$;

$i, J = 1, 2, \ldots, n$;

$k = 1, \ldots, q$

$1 \le t \le T$.

Chance Constrained Formulation

The problem will be formulated as a chance constrained optimizing decision [4]. In particular, the constraint relating to the total recoverable stock becomes of the form

$$\text{Probability} \left\{ x(R_{T*}, V_{T*}) - \sum_{\tau=1}^{T} \sum_{J=1}^{n} r_{J\tau} \ge 0 \right\} = 1 .$$

[1]This "all or nothing" situation for the lifetime of the oil industry is unrealistic, since it is known that oil will command a positive price long after the emergence of the backstop technology.

And the problem is then:

$$\text{Max E} \left\{ \sum_{t=1}^{T*} \sum_{J=1}^{n} [P_t r_{Jt} - C_{Jt}(R_t, V_t, K_{Jt})]\beta_t \right\}$$

subject to

$$P \left\{ - \sum_{\tau=1}^{T*} \sum_{J=1}^{n} r_{J\tau} + x(R_{T*}, V_{T*}) \ge 0 \right\} = 1$$

$$K_{Jk,t+1} = K_{Jkt} - D_{Jkt}(r_{Jt}, v_{Jkt}, K_{Jkt})$$

$$r_{Jt} \le F_{Jt}(R_t, V_t, K_{Jt})$$

$$r_{Jt} \ge 0, \quad v_{Jkt} \ge 0 \qquad \forall \; J, k \text{ and } t, \; 0 < t < T^* .$$

Let T^* obey a probability mass function, γ_t, defined on $[0, \overline{T}]$ such that

$$\gamma_t > 0 \text{ for } 0 \le t \le \overline{T}, \quad \gamma_t = 0 \quad t \notin [0, \overline{T}]$$

and

$$\sum_{t=0}^{\overline{T}} \gamma_t = 1.$$

Define the probability that the "backstop" technology does not emerge in the period 0 to t by Φ_t, i.e., the probability that T is in the range t to $\overline{T}$ is

$$\Phi_t = \sum_{\tau=t}^{\overline{T}} \gamma_t .$$

Let $\overline{R}$ be the production plan for the entire period 0 to $\overline{T}$.

Thus,

$$E\{\pi(\bar{R})\} = \sum_{t=1}^{\bar{T}} \gamma_t \sum_{\tau=1}^{t} \sum_{J=1}^{n} [P_\tau r_{J\tau} - C_{J\tau}(R_\tau,V_\tau,K_{J\tau})]\ \beta_\tau$$

or, changing the order of summation:

$$E\{\pi(\bar{R})\} = \sum_{J=1}^{n} \sum_{t=1}^{\bar{T}} \Phi_t\ \beta_t[P_t r_{Jt} - C_{Jt}\ (R_t,V_t,K_{Jt})].$$

Let

$$S(T^*) = x(R_{T^*},V_{T^*}) - \sum_{J=1}^{n} \sum_{t=1}^{T^*} r_{Jt}\ .$$

Then the problem becomes:

$$\text{Max} \sum_{J=1}^{n} \sum_{t=1}^{\bar{T}} \Phi_t\ \beta_t[P_t r_{Jt} - C_{Jt}(R_t,V_t,K_{Jt})]$$

$$p(S(T^*) \geq 0) = 1$$

$$K_{Jk,t+1} = K_{Jkt} - D_{Jkt}\ (r_{Jt},v_{Jkt},K_{Jkt})$$

$$r_{Jt} \leq F_{Jt}\ (R_t,V_t,K_{Jt})$$

$$r_{Jt} \geq 0,\ v_{Jkt} \geq 0 \quad \forall J,k \text{ and } t\ .$$

But $p(S(T^*) \geq 0) = 1$, under the assumption that $\gamma_t > 0$ for $0 < t < \bar{T}$, is equivalent [3] (up to a set of γ_t-measure zero) to $S(t) \geq 0$ for all t. Thus, the Langrangian for the problem is:

$$L = \sum_{J=1}^{n} \sum_{t=1}^{\bar{T}} \Phi_t\beta_t[P_t r_{Jt} - c_{Jt}\ (R_t,V_t,K_{Jt})]$$

$$- \sum_{t=1}^{\bar{T}} \sum_{J=1}^{n} [\sum_{k=1}^{q} \Delta_{Jk,t+1}\ \beta_{t+1}\ \{K_{Jk,t+1} - K_{Jkt} + D_{Jkt}\ (r_{Jt},v_{Jkt},K_{Jkt})\}$$

$$- \psi_{Jt}\beta_t\ \{r_{Jt} - F_{Jt}\ (R_t,V_t,K_{Jt})\}$$

$$-(\lambda\beta)_t\ \{\sum_{\tau=1}^{t} \sum_{J=1}^{n} r_{J\tau} - x(R_\tau,V_\tau)\}$$

$$+ \zeta_{Jt}\ \beta_t\ u_{Jt} + \sum_{J=1}^{n} \sum_{k=1}^{q} \sigma_{Jkt}\beta_t v_{Jkt}\ ]\ .$$

<u>Characteristics of Optimum Production Rates</u>

From the Langrangian expression:

$$\frac{\partial L}{\partial r_{Jt}} = (P_t\Phi_t\beta_t - \sum_{\tau=t}^{\bar{T}} \sum_{i=1}^{n} \frac{\partial c_{i\tau}}{\partial r_{Jt}}\ \beta_\tau\ \Phi_t)$$

$$- \sum_{k=1}^{q} \Delta_{Jk,t+1}\ \beta_{t+1}\ \frac{\partial D_{Jkt}}{\partial r_{Jt}} - \psi_{Jt}\beta_t + \sum_{\tau=1}^{\bar{T}} \sum_{i=1}^{n} \psi_{i\tau}\beta_\tau\ \frac{\partial F_{i\tau}}{\partial r_{Jt}}$$

$$- \sum_{t=1}^{\bar{T}} (\lambda\beta)_t\ (1 - \frac{\partial x}{\partial r_{Jt}}) + \zeta_{Jt}\beta_t = 0$$

or,

$$(P_t - \frac{\partial c_{Jt}}{\partial r_{Jt}})\ \Phi_t\beta_t = (\lambda\beta)_{\bar{T}}\ \ (1 - \frac{\partial x}{\partial r_{Jt}}) + \psi_{Jt}\beta_t$$

$$+ \sum_{\tau=t}^{\bar{T}} \sum_{i=1}^{n} \psi_{i\tau}\ \left|\frac{F_{i\tau}}{r_{Jt}}\right|\ \beta_\tau + \sum_{k=1}^{q} \Delta_{Jk,t+1}\beta_{t+1}\ \frac{\partial D_{Jkt}}{\partial r_{Jt}}$$

$$+ \sum_{\tau=t+1}^{\bar{T}} \frac{\partial c_{Jt}}{\partial r_{Jt}}\ \beta_\tau\ \Phi_i + \sum_{\tau=t}^{\bar{T}} \sum_{\substack{i=1 \\ i\neq J}}^{n} \frac{\partial c_{i\tau}}{\partial r_{Jt}}\ \beta_\tau\ \ \Phi_\tau$$

$$+ \sum_{\tau=1}^{\bar{T}-1} (\lambda\beta)_\tau\ (1 - \frac{\partial x}{\partial r_{i\tau}})\ \ \forall i,\ J = 1,\ \ldots,\ n;\quad 1 \leq t \leq \bar{T}\ .$$

Elements of User Costs

As in Kuller and Cumming, the following user cost components can be identified.

Stock User Costs for Firm J

λ measures the increase in net incomes from the reservoir associated with an incremental change in the endogenously determined stock; the stock user cost for firm J in period t is given by

$$\lambda\beta_{\overline{T}}\,(1 - \frac{\partial x}{\partial r_{Jt}})\,.$$

Boundary User Costs

Since $\Psi_{i\tau}$ measures the increase in net incomes which would result from an incremental relaxation of the restriction, the boundary user cost is given by

$$\Psi_{Jt}\beta_t - \sum_{\tau=t}^{\overline{T}} \sum_{i=1}^{n} \Psi_{i\tau} \left|\frac{\partial F_{ii}}{\partial r_{Jt}}\right| \beta_\tau\ .$$

User Costs of Capital Consumption

The multiplier $\Delta_{Jk,t+1}$ associated with the capital equation measures the marginal productivity of capital type k used by firm J in all future periods t+1, t+2, ..., T; the user costs of capital consumption is given by

$$\sum_{k=1}^{q} \Delta_{Jk,t+1}\beta_{t+1}\frac{\partial D_{Jkt}}{\partial r_{Jt}}\quad .$$

Production User Costs

These user costs reflect the stock value of oil and gas to the

firm, contributing to output as natural forces of production, and are given by

$$\sum_{\tau=t+1}^{\overline{T}} \frac{\partial C_{ii}}{\partial r_{Jt}}\beta_\tau + \sum_{\tau=t}^{\overline{T}}\sum_{i=1}^{n}\frac{\partial C_{i\tau}}{\partial r_{Jt}}\beta_\tau$$

$$i,J=1,\ldots,n;\ 1 \leq t \leq \overline{T}\ .$$

However, a new user cost element is now introduced by the randomness of the planning horizon. This element will be termed "the boundary-time cost." It is equal to

$$\sum_{\tau=1}^{\overline{T}-1} (\lambda\beta_\tau)(1 - \frac{\partial x}{\partial r_{i\tau}})\ .$$

The Effect of the Optimal Production Rates

Comparing these first order conditions with those of Kuller and Cumming, the following can be noticed:

1) The net marginal benefit of producing one extra unit is decreased by a factor $\Phi_t(<1)$. This decrease causes the net marginal benefit curve to shift downward.

2) The effect of time-horizon uncertainty on marginal cost is indeterminate, and depends on the relative magnitudes of changes of opposite directions in the terms of the first order conditions equation. In comparison with the corresponding terms in Kuller and Cumming, the term

$$\sum_{\tau=1}^{\overline{T}}\sum_{i=1}^{n}\psi_{i\tau}\left|\frac{\partial F_{i\tau}}{\partial r_{Jt}}\right|\beta_t$$

is greater, because of the additional uncertainty.

The terms $(\lambda\beta)_{\overline{T}}\,(1 - \frac{\partial x}{\partial r_{Jt}})$ are smaller and the terms

$$\sum_{\tau=t+1}^{\overline{T}} \frac{\partial c_{i\tau}}{\partial r_{Jt}} R_\tau\, \Phi_\tau \quad \text{and} \quad \sum_{\tau=t}^{\overline{T}} \sum_{\substack{i=1 \\ i\neq J}}^{n} \frac{\partial c_{i\tau}}{\partial r_{J^t}} \beta_\tau\, \Phi_\tau$$

may increase or decrease depending on whether the extra terms in the summation which correspond to $\tau = T^*, T^* + 1, \ldots \overline{T}$ balance the reduction in each term of the summation caused by the weighting factor Φ_τ.

On the whole, if $\sum_{\tau=1}^{\overline{T}-1} (\lambda\beta)_\tau (1 - \frac{\partial x}{\partial r_i})$, the boundary time user cost, is sufficiently large, then the marginal user cost increases in comparison with that obtained from Kuller's and Cumming's formulation. This means, that a reduction in marginal benefit causes a reduction in production rate. In other cases, the effect on the production rate is ambiguous, since it depends on the shape and relative shifts in the marginal cost and marginal benefit curves.[2]

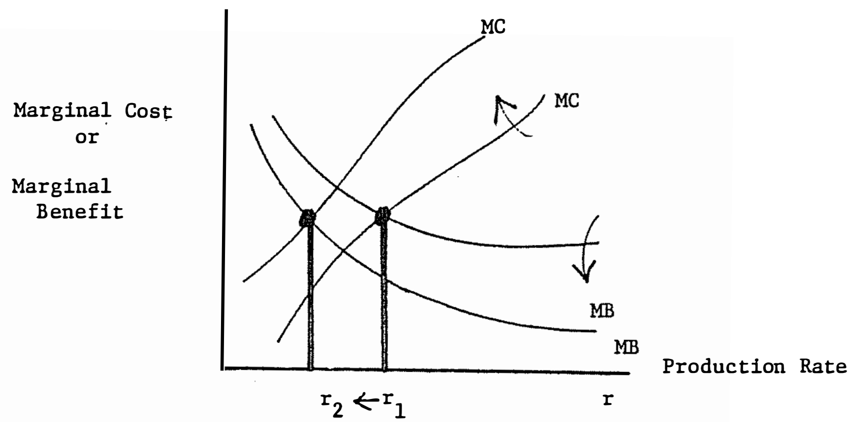

Figure 1

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[2] In comparing the effect of the introduction of the boundary time user cost on the production decision with that obtained from Kuller's and Cumming's formulation, it is here assumed that their T corresponds to the expected value of T* in this formulation. Thus, $\overline{T} > T$.

<u>Characteristics of Optimum Investment Rates</u>

From the Langrangian expression:

$$\frac{\partial C_{Jt}}{\partial v_{Jkt}} \beta_t\, \Phi_t = -\,\Delta_{Jk,\,t+1}\, \beta_{t+1} \frac{\partial D_{Jkt}}{\partial v_{Jkt}} + \lambda\beta_{\overline{T}} \frac{\partial x}{\partial v_{Jkt}}$$

$$+ \sum_{\tau=t}^{\overline{T}} \sum_{i=1}^{n} \psi_{i\tau}\, \beta_\tau \frac{\partial F_{i\tau}}{\partial v_{Jkt}} - \sum_{\substack{i=1 \\ i\neq J}}^{n} \frac{\partial C_{it}}{\partial v_{Jkt}} \Phi_t \beta_t$$

$$- \sum_{\tau=t+1}^{\overline{T}} \sum_{i=1}^{n} \frac{\partial C_{i\tau}}{\partial v_{Jkt}} \Phi_\tau\, \beta_\tau + \sum_{\tau=1}^{\overline{T}-1} \lambda\beta_\tau \frac{\partial x}{\partial v_{Jkt}}$$

$$i, J = 1, \ldots n; \quad k = 1, \ldots q \quad 1 \leq t \leq \overline{T}.$$

These first order conditions state that the optimal level of firm J's investment in capital-type k during any t, $1 \leq t \leq \overline{T}$ is given by equating the present value of the marginal costs of such investment, adjust for the uncertainty of the planning horizon, to the aggregate benefits of the reservoir associated with such investment. The interpretation of the terms in the above expression follows closely that given by Kuller and Cummings's [2]. Comparing with their results note that the discounted marginal cost of the investment is reduced by a factor of $\Phi < 1$ and that the aggregate benefit to the reservoir as a whole has a new term as a result of the inclusion of uncertainty in the planning horizon. However, even if D, F, x and C are the same functions as those considered by Kuller and Cummings, the effect on the aggregate benefit of the reservoir is ambiguous. Only, if

$$\sum_{\tau=1}^{\bar{T}-1} \lambda\beta_\tau \frac{\partial x}{\partial v_{Jkt}}$$ is large enough to swamp all the changes in the

other terms on the right hand side of the first order conditions that the aggregate benefit increases at all levels of investments for all capital components. In this case the optimal investment level increases unambiguously.

See Figure 2.

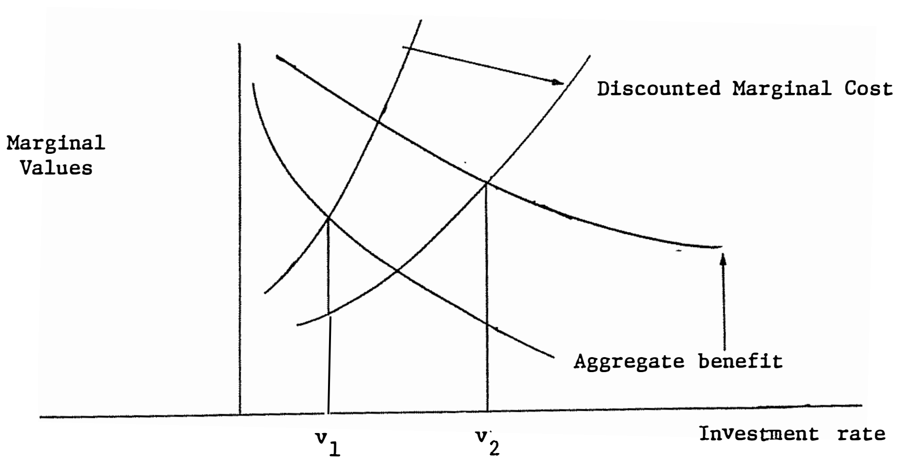

Figure 2

This note captures the effect of only one aspect of uncertainty, that which is related to the time of the emergence of the backstop technology. Other sources of uncertainty remain unexamined, such as uncertainty related to the price path and particularly the uncertainty regarding the prevailing price of the emerging alternatives. Moreover, a more realistic treatment should deal with the situation where:

a) the oil commands a positive price after the emergence of the backstop technology;

b) the strategic aspects provide the oil producers with a strategy of delaying the emergence of the alternative technologies.

The preceding analysis demonstrates that the theory of crude oil production is affected by incorporating the type of uncertainty considered in this note.

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