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DEMAND UNCERTAINTY AND THE REGULATED FIRM*

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1. INTRODUCTION

In this paper, we investigate the impact of demand uncertainty on the choice of plant capacity by a regulated firm. Over the past few years, demand uncertainty has become a major element in the decision-making of utilities, and particularly in their decision-making with respect to capacity choices. In a recent study by SRI [1977], it was reported that to maximize expected consumers' surplus, more generating capacity was required for the electric utility industry when operating under demand uncertainty than under demand certainty.[1] This finding raises the question whether the structure of rate regulation of electric utilities provides the appropriate incentives for them to invest in more capacity under demand uncertainty then under certainty. The present paper addresses such questions.

The model of the regulated firm that is employed in this paper derives from the work of Joskow [1974] concerning the recent history of rate regulation in the electric utility industry. Briefly, Joskow reported that during the 1960s and the early 1970s, the impetus for rate reviews in the electric utility industry came mainly from the utilities and not from the public utility commissions (PUC's). Joskow's findings are corroborated by data for the period 1948–1978. A study of all rate cases involving electric utilities in the U.S. for that period shows that of a total of 363 cases, 350 of these were instances of utility-initiated rate reviews and only 13 were cases of PUC-initiated reviews.[2] From the point-of-view of the formal theory of the regulated firm, this strongly suggests that the Averch-Johnson model of a firm operating at or near an "allowed rate of return constraint" (with the allowed rate of return being in excess of the cost of capital to the firm) should be replaced by a model of price regulation in which attention is shifted to the "non-negativity of profits constraint", or, at least, this should be done with respect to the electric utility industry. With some elaborations, this is basically what we do in this paper, building on the approach adopted in a recent paper on a related topic.[3]

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[1] We should note that the SRI model differs from that of this paper. In particular, the SRI model permits brownouts and employs a fixed proportions technology.

[2] We are indebted to Donna Berry of Caltech for these data.
See the data in Table 1 of this paper.

[3] See Burness, Montgomery, and Quirk [1980].

2. A MODEL OF THE REGULATED FIRM

We posit a regulatory environment in which the PUC adopts an average cost pricing rule, under which the goal of regulation is to set prices for the regulated firm so that the firm earns just the market rate of return (the cost of capital) on its assets. We will ignore the difficulties that arise in attempting to determine a measure of the cost of capital, or in deciding on just which assets qualify for inclusion in the rate base of the firm and how they should be valued.

Let q denote quantity demanded for the regulated firm and let P denote the price per unit of output. K is the value of the capital employed by the firm (taken to be the same as the rate base of the firm), and L is a measure of the variable inputs employed by the firm. r is the rental per dollar of capital for the firm, and w is the cost per unit of the variable input. Profits denoted by π, are given by

(1) $$\pi = Pq - wL - rK.$$

Ideally, to achieve its objective, the PUC would choose P so that $\pi = 0$; that is, the PUC would act to set price equal to average cost. However, the PUC has neither the resources to engage in rate making on a continuous basis nor the incentive to do so. Following Joskow, we view the PUC as basically a passive agency, acting only when pressured to action. It acts to raise P when petitioned by a firm operating at a loss, and it acts to lower P when petitioned by consumer groups who can demonstrate that there are excessive profits on the part of the firm. Thus, regulatory review is triggered by either of two events:

(2) $$\pi < 0 \quad \text{or} \quad \pi > sK.$$

In this statement, s is some strictly positive rate of return sufficiently high so that the potential gains to consumer groups from rate review more than compensate for the organizational costs of putting together an effective intervention effort in the rate making process. s is *not* the "allowed rate of return" or the "fair rate of return" that is debated in regulatory hearings — the allowed or fair rate of return is r; the PUC engages in average cost pricing when it prices.[4]

Because the PUC uses average cost pricing whenever it sets prices at a formal regulatory hearing, there are strong incentives for regulated firms to avoid PUC instituted rate reviews called in response to protests by consumer groups. In fact, it makes sense for the regulated firm to institute review proceedings to reduce output prices when it appears that profits at the existing prices will exceed sK. By so doing, it avoids a confrontation with intervenors and has a better chance of preserving profits above the cost of capital. In fact, the data on rate hearings bear out this view of the regulatory process, as shown in Table 1.

[4] We will treat r as nonrandom, known, and fixed, to center attention on uncertainty in demand. To the extent that uncertainty about r enters into utility decision-making, the incentives for instituting rate reviews may be weakened.

TABLE 1
RATE CASES, U.S. ELECTRIC UTILITIES, 1948–1977

Period	No. of Rate Cases	Company Initiated			PUC Initiated No.
		No.	Rate Increases	Rate Decreases	
1948–1952	46	45	42	3	1
1953–1957	34	31	28	3	3
1958–1962	43	39	38	1	4
1963–1967	17	16	12	4	1
1968–1972	104	100	96	4	4
1973–1977	119	119	119	—	—
Totals	363	350	335	15	13

Source: These data were supplied by Donna Berry of Caltech, and are derived from *Public Utility Reports*.

Note that PUC initiated rate reviews were relatively rare, even during the 1948–1967 period when electric rates were falling for most of the country. In fact, Table 1 shows that company initiated reviews for rate decreases—while also rare—were more frequent than PUC initiated reviews. (All PUC initiated reviews were aimed at rate decreases). We should also point out that the steady fall in electric rates during the 1950s and 1960s —from an average rate of 2.77 cents per kwh in 1952 to a historic low of 2.10 cents per kwh in 1970— reflected in part "automatic" price decreases due to the declining block tariff structure of electricity prices together with the operation of fuel cost pass through rules. As Joskow has noted, consumer groups are no doubt more concerned with the average cost of electricity than with profits of utilities. In a more sophisticated version of our model, s would be taken to be a function of P to reflect this concern.

Briefly, our model of the regulatory process is one in which firms are the active participants and the PUC is passive. The firm petitions for a rate increase when profits are negative and receives an increase sufficient to achieve a zero profit level. The firm petitions for a rate decrease when profits exceed sK and receives a decrease sufficient to achieve the level sK, thus avoiding a PUC initiated review that would reduce profits to zero.

The model of this paper thus differs fundamentally from those employed in the traditional Averch-Johnson literature,[5] and from those appearing in the more recent literature on demand uncertainty.[6] We regard the regulated firm as subject to price regulation; the PUC (and not the firm) chooses the product price, in an environment in which both the non-negativity of profits constraint and a

[5] See for example, Averch and Johnson [1962], Bailey [1973], and Baumol and Klevorick [1970].

[6] See, for example, Pindyck [1981], Perrakis [Feb. 1976 and June 1976], Peles and Stein [1976], Rau [1979], Das [1980], and Bawa and Sibley [1980].

variant of the allowed rate of return constraint are relevant to the decision-making process of the utility. However, in recent years, it is certainly the non-negativity of profits, rather than the upper bound constraint, that has been binding. Moreover, our interpretation of the upper bound constraint is quite different from that of the traditional A-J literature.

To be explicit about both of these differences, consider an A-J approach applied to the case of a firm operating with *certainty* as to demand. The firm faces a given price set by the PUC, which is high enough to guarantee non-negative profits for the firm. Quantity is known once price is set. Then the regulated firm would choose its inputs, K and L, to maximize profit, subject to the upper bound on profits ($\pi \leq sK$) that the firm wants to satisfy in order to avoid triggering a rate review.

In the special case where the upper profit constraint is binding, then an *A-J* bias does occur.[7] But this is only a special case, since the price set by regulators may not generally allow the firm to earn profits great enough to reach the upper constraint. If, at the existing regulatory price, $\pi < sK$ for all choices of K and L, then the regulated firm will produce the required level of output in a cost minimizing fashion, i.e., it will operate on its efficient expansion path.

When we turn to the case of *uncertainty*, things are somewhat more complicated. Peles and Stein [1976] analyze an *A–J* type firm, showing that under additive uncertainty affecting demand the regulated firm chooses a larger stock of capital than the unregulated monopolist. But, under multiplicative uncertainty, the closer the allowed rate of return is to the actual cost of capital, the smaller the stock of capital. Perrakis [February, 1976] employs a similar model to show that under uncertainty the regulated firm will operate with excess capacity for some levels of demand, and that *ex ante* expected profits are less than the maximum allowed by regulators. In a later paper, Perrakis [June, 1976] extends his results to show that generally the *A-J* effect does not hold under uncertainty for arbitrary demand functions and probability distributions over disturbances, either in the case of risk neutral or risk averse firms. Pindyck [1980] presents a multi-period *A-J* model in which uncertainty again leads to ambiguity as to overcapitalization of costs.

The treatment to be presented here arrives at results similar to those summarized above; there is ambiguity about the relative capitalization levels under certainty and uncertainty and with respect to regulated and unregulated firms. But, the nature of the model leads to quite *different* explanations for the ambiguity.

[7] Formally, the firm operating under demand certainty will act to $\max_{(K,L)} \pi$, where $\pi = Pq - wL - rK$, subject to $\pi \leq sK$, $r < s < r_m$, $q = \phi(K, L)$ is a quasiconcave production function, and r_m is the unregulated monopoly rate of return. At an interior optimum, first order conditions are (a) $P\phi_K = -\lambda s/(1-\lambda) + r$ and (b) $P\phi_L = w$, where $0 \leq \lambda < 1$. If the upper constraint is binding, $\lambda > 0$, and hence, an $A-J$ bias exists since $\phi_K/\phi_L < r/w$. (Here it is ϕ that must be concave in L, rather than revenue as in the traditional $A-J$ literature.) If the regulator sets P so that constraint is not binding, $\lambda = 0$ implies that $\phi_K/\phi_L = r/w$, so that the firm is on the efficient expansion path.

In particular, if a regulated firm operating near the zero profit constraint faces an unexpectedly unfavorable demand or increase in costs, the firm can rely on the PUC to increase price to eliminate losses. Further, given a firm operating near the zero profit point, favorable demand or cost outcomes can lead to positive profits that, within limits, can be captured by the firm. Burness, Montgomery, and Quirk [1980] arrive at the conclusion that expected profit maximizing regulated firms tend to prefer risky (cost-plus) construction contracts for capital goods to safe (turnkey) contracts with the same expected cost.[8] These incentives are reversed when the firm operates near the allowed rate of return level, as is assumed in *A-J* models. The data in Table 1 clearly show that any model that concentrates on the upper bound profit constraint has serious flaws when used to explain the behavior of regulated electric utilities in the 1960's and 1970's.

The special features that characterize this model under demand uncertainty are taken up in the next section.

3. DEMAND UNCERTAINTY AND THE CHOICE OF CAPACITY

We next consider a decision-making environment for the firm in which demand is uncertain. The firm must choose its stock of capital K before demand is observed. It operates under a common carrier obligation so that output must be large enough to satisfy the quantity demanded. Thus, after demand is observed, the variable input L is chosen so as to satisfy the common carrier requirement. Let $\phi(K, L)$ denote the production function for the firm, where ϕ is strictly quasi-concave with positive marginal products and with third order differentiability.

Let x denote the values of a random variable that enters the demand function for output in a monotone increasing fashion, so that $q=q(x, P)$ where $\frac{\partial q}{\partial x}>0, \frac{\partial q}{\partial P}<0$. Let $f(x)$ be the pdf over x, while $F(x)$ is the cumulative distribution function.

Given P, then once x is observed, q is determined as well. Then the firm must satisfy the common carrier requirement (3).

$$\phi(K, L) = q. \tag{3}$$

Since L is chosen after q is observed, in such a way as to satisfy (3), it follows that

$$L = L(q, K), \tag{4}$$

where $\phi(K, L(q, K))=q$.

Given the price P, the choice of capital K, the level of demand q, and the choice of the variable input L, profits for the firm are given by

$$\pi(p, K, q) = Pq - wL(q, K) - rK, \tag{5}$$

[8] This bias still persists when there are costs associated with the regulatory process, so long as these transactions costs are not so large as to preclude instituting rate reviews in the face of unfavorable outcomes. See proposition 5 of Burness, Montgomery, and Quirk [1980].

where $q(x, P)=\phi(K, L(q, K))$.

We assume that regulatory price adjustment occurs instantaneously, i.e., we ignore regulatory lag. Let P_0 be the initial price facing the firm. The firm chooses K to maximize expected profits before the random variable x is observed. Once x is observed, then the price is adjusted instantaneously by the following rule:

$$(6) \qquad P = \begin{cases} P_0, & \text{if } 0 \leq \pi(P_0, K, q) \leq sK \quad \text{(no rate change)} \\ P_1, & \text{if } \pi(P_0, K, q) < 0 \quad \text{(firm seeks higher rate)} \\ P_2, & \text{if } \pi(P_0, K, q) > sK \quad \text{(firm seeks lower rate)}. \end{cases}$$

In (6), P_1 is defined by $\pi(P_1, K, q(x, P_1))=0$ and P_2 by $\pi(P_2, K, q(x, P_2))=sK$. We assume that the regulated price is always set on an inelastic region of demand so that it is always possible to increase profits for the firm by increasing its price.

Given K and P_0, define $q^i(K, P_0)$, $i=0, 1, 2, 3$, by

$$(7) \qquad \begin{aligned} \pi(P_0, K, q^0) &= \pi(P_0, K, q^3) = 0, \qquad q^3 > q^0 \\ \pi(P_0, K, q^1) &= \pi(P_0, K, q^2) = sK, \qquad q^2 > q^1. \end{aligned}$$

Further, let $x^i(P_0, K)$, $i=0, 1, 2, 3$, be those values of x such that $q^0=q(x^0, P_0)$, $q^1=q(x^1, P_0)$, $q^2=q(x^2, P_0)$, and $q^3=q(x^3, P_0)$. Figure 1 identifies these distinguished values of the qs, with π a maximum at $\tilde{q}$.

We note for future reference that, given P_0 with K,

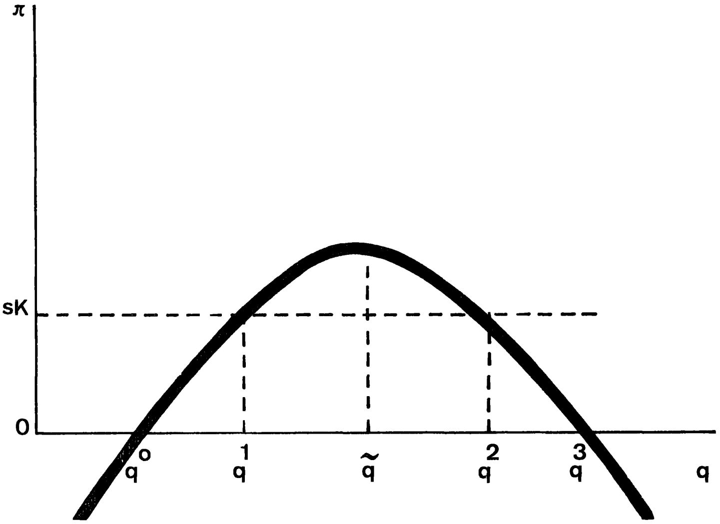

FIGURE 1

$$\frac{\partial \pi}{\partial q} = P - w \frac{\partial L}{\partial q}, \tag{8}$$

where from (3), we have $\partial L/\partial q = 1/\phi_L$, where $\phi_L = \partial \phi / \partial L$. Thus, $\partial \pi/\partial q > 0$ for $q < \tilde{q}$, $\partial \pi / \partial q < 0$ for $q > \tilde{q}$. Further,

$$\frac{\partial^2 \pi}{\partial q^2} = -w \frac{\partial^2 L}{\partial q^2} = w \frac{\phi_{LL}}{\phi_L^3} < 0, \tag{9}$$

assuming $\phi_{LL} = \partial^2 \phi / \partial L^2 < 0$, as we do.

Finally, we assume that the firm is an expected profit maximizer, taking into account in its choice of capital the regulatory price adjustments summarized in (6).

Then the problem of the firm becomes:

$$\begin{aligned} \max_K W = & \int_{-\infty}^{x^0} \pi(P_1, q(x, P_1), K) f(x) dx \\ & + \int_{x^0}^{x^1} \pi(P_0, q(x, P_0), K) f(x) dx + \int_{x^1}^{x^2} \pi(P_2, q(x, P_2), K) f(x) dx \\ & + \int_{x^2}^{x^3} \pi(P_0, q(x, P_0), K) f(x) dx + \int_{x^3}^{\infty} \pi(P_1, q(x, P_1), K) f(x) dx. \end{aligned} \tag{10}$$

By the price adjustment rule (6), the first and last integrals are identically zero, while the third integral is simply $sK[F(x^2) - F(x^1)]$. Thus, the first order condition for maximizing W may be written as

$$\begin{aligned} \frac{\partial W}{\partial K} = & \int_{x^0}^{x^1} \pi_K(P_0, q(x, P_0), K) f(x) dx + s[F(x^2) - F(x^1)] \\ & + \int_{x^2}^{x^3} \pi_K(P_0, q(x, P_0), K) f(x) dx = 0 \end{aligned} \tag{11}$$

where $\pi_K = \partial \pi / \partial K$ and $\hat{K}$ is the optimal choice of K defined by a solution to (11).[9] In the work that follows, we will assume in addition that a regular max occurs, that is, $\pi_{KK} < 0$.

4. UNCERTAINTY AND THE *A-J* CAPITALIZATION BIAS

In contrast to the choice of capital for the regulated firm according to (11), we might consider the choice of K and L by an unregulated monopolist. Employing the same timing assumptions as above, the unregulated monopolist chooses K before x is known. After x is observed, the unregulated monopolist chooses P, which determines q, and then picks L to satisfy $\phi(K, L) = q$.

[9] In deriving (11), we use Leibnitz' rule, namely

$$\frac{d}{dy} \int_{\alpha(y)}^{\beta(y)} h(y, t) dt = \frac{d\beta}{dy} h(y, \beta) - \frac{d\alpha}{dy} h(y, \alpha) + \int_{\alpha(y)}^{\beta(y)} \frac{\partial h(y, t)}{dy} dt.$$

Because the underlying π function is continuous at the end points of the integrals, all terms of the form $(d\beta/dy)h(y, \beta)$ and $(d\alpha/dy)h(y, \alpha)$ cancel out in (11).

Hence, after x is observed, we have the problem

$$\max_P Pq(x, P) - wL(q, K) - rK$$

with first order condition

$$Pq_p + q - wL_q q_P = 0, \tag{12}$$

where $q_p = \partial q/\partial P$, $L_q = \partial L(q, K)/\partial q$. Let $P(K, x)$ denote the choice of P that satisfies (12), given K and x.

The *ex ante* problem is

$$\max_K \int_{-\infty}^{\infty} [P(K, x)q(x, P) - wL(q, K) - rK]f(x)dx$$

with the first order condition

$$\int_{-\infty}^{\infty} [qP_K + Pq_P P_K - wL_q q_P P_K - wL_K - r]f(x)dx = 0, \tag{13}$$

where $P_K = \partial P(K, x)/\partial K$, and $L_K = \partial L(q, K)/\partial K$.

Then (13) reduces to

$$- w\int_{-\infty}^{\infty} L_K f(x)dx = r. \tag{14}$$

We contrast (14) for the unregulated firm, with the first order condition for the regulated firm (15)

$$- w\{\int_{x^0}^{x^1} L_K f(x)dx + \int_{x^2}^{x^3} L_K f(x)dx\} + s[F(x^2) - F(x^1)] = r\{F(x^1) - F(x^0) + F(x^3) - F(x^2)\} \tag{15}$$

where x^0 satisfies $q(x^0, P_0) = q_0$, and similarly for x^1, x^2, x^3.

Clearly there is no unambiguous general result concerning an overcapitalization bias, independent of the functional forms of $f(x)$, ϕ, and $q(x, P)$, and of the value of P_0. In particular, as we have seen earlier, if P_0 is such that $\pi(P_0, x_1, K) = sK$ for some K with $f(x)$ a spike concentrating all probabilities at x_1, then an A-J bias arises (assuming ϕ is concave in L). And if $\max_K \pi(P_0, x, K) = \hat{\pi}$ satisfies $0 \leq \hat{\pi} < sK$ for some x then there is no A-J bias when all probabilities are concentrated at that value of x. Indeterminacy of bias given the certainty case implies indeterminancy under uncertainty.

5. UNCERTAINTY AND PLANT SIZE

Consider the profit function for the regulated firm, where we incorporate into the profit function the price adjustments that occur under the regulatory process (6). Let this function be denoted by $\pi^*(q, K)$. Then $\pi^*(q, K)$ is given by

$$(16) \qquad \pi^*(q, K) = \begin{cases} 0 \text{ if } q \leq q^0 \text{ or } q \geq q^3 \\ \pi(P_0, q, K) \text{ if } q^0 \leq q \leq q^1 \text{ or } q^2 \leq q \leq q^3 \\ sK \text{ if } q^1 \leq q \leq q^2. \end{cases}$$

Figure 2 displays the $\pi^*(q, K)$ function. The marginal profitability of capital can be written as

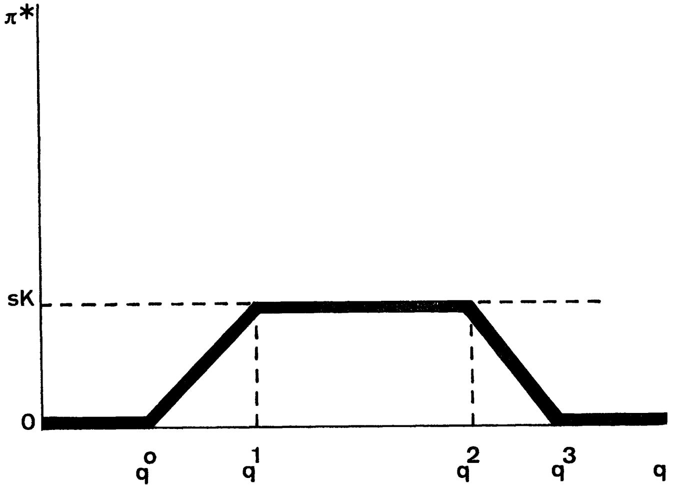

FIGURE 2

$$(17) \qquad \pi_K^* = \begin{cases} 0 \text{ if } q < q^0 \text{ or } q > q^3 \\ -wL_K - r \text{ if } q^0 < q < q^1 \text{ or } q^2 < q < q^3 \\ s \text{ if } q^1 < q < q^2 \end{cases}$$

where $L_K = -\phi_K/\phi_L$. Then it follows that

$$(18) \qquad \pi_{Kq}^* = \begin{cases} 0 \text{ if } q < q^0 \text{ or } q > q^3 \text{ or } q^1 < q < q^2 \\ -wL_{Kq} \text{ if } q^0 < q < q^1 \text{ or } q^2 < q < q^3 \end{cases}$$

where $L_{Kq} = -(\phi_L\phi_{KL} - \phi_K\phi_{LL})/\phi_L^3$.

In the general case, L_{Kq} is of indeterminate sign. However, if capital is a normal input, then $L_{Kq} < 0$ and the graph of π_K^* as a function of q appears as in Figure 3. Over the ranges (q^0, q^1) and (q^2, q^3), π_K^* is increasing. However, there is no presumption that the right and left-hand limits of π_K^* evaluated at any switching point (q^0, q^1, q^2, q^3) are equal. Thus, the jump discontinuities and general shape of the graph illustrate a "typical" situation so far as the π_K^* function is concerned. Finally, we note that the curvature of the π_K^* function over the intervals (q^0, q^1) and (q^2, q^3) depends on the sign of

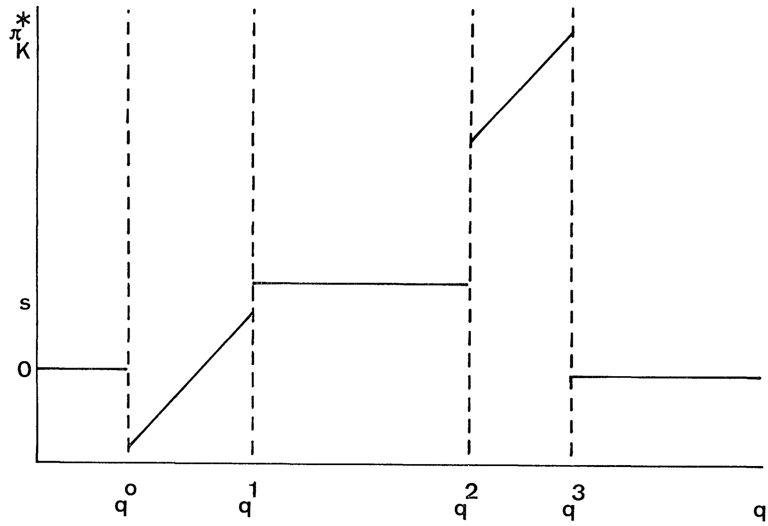

FIGURE 3

$$\pi_{Kqq} = w(\phi_L^2 \phi_{KLL} - \phi_K \phi_L \phi_{LLL} + 3\phi_K \phi_{LL}^2 - 3\phi_L \phi_{KL} \phi_{LL}) / \phi_L^5 \tag{19}$$

which is in general ambiguous.

We are now in a position to compare the choice of capital by a regulated firm operating under uncertainty with its choice under certainty. As we have seen, under uncertainty, the firm chooses $K = \hat{K}$ so that $W_k = 0$, that is,

$$E_f \pi_K^*(q, \hat{K}) = 0 \quad \text{or} \quad wE_f L_k = -r.$$

Under certainty, with demand given by $\bar{q} = E_f(q)$, the choice of the regulated firm is $K = \bar{K}$ such that $\pi_K^*(E_f(q; \bar{K}), \bar{K}) = 0$. By hypothesis, $W_{KK} < 0$ (there is a regular max) and further, $\pi_{KK} < 0$ so that $\pi_{KK}^* < 0$ where it is defined. Thus, we have an unambiguous ordering over the amounts of capital chosen iff

$$\begin{aligned} &0 = \pi_K^*(E_f(q; \bar{K}), \bar{K}) < E_f \pi_K^*(q, \bar{K}) \Longleftrightarrow \hat{K} > \bar{K} \\ \text{or,}\quad &0 = \pi_K^*(E_f(q; \bar{K}), \bar{K}) > E_f \pi_K^*(q, \bar{K}) \Longleftrightarrow \hat{K} < \bar{K}. \end{aligned} \tag{20}$$

Figure 4 illustrates the case where $\hat{K} > \bar{K}$.

By Jensen's Inequality, $\pi_K^*(E_f(q)) \geq E_f \pi_K^*(q)$ for all probability density functions f iff π_K^* is concave in q (and the inequality is reversed for all pdf's f iff π_K^* is convex in q). Since π_{Kqq}^* is generally ambiguous in sign over the intervals (q^0, q^1) and (q^2, q^3), and because of the other characteristics of π_K^* as shown in Figure 3, we have the following proposition.

PROPOSITION 1. *The regulated firm operating under demand uncertainty may choose a level of capital greater than, less than, or equal to the amount it would choose under certainty. The relative amounts of capital chosen under certainty*

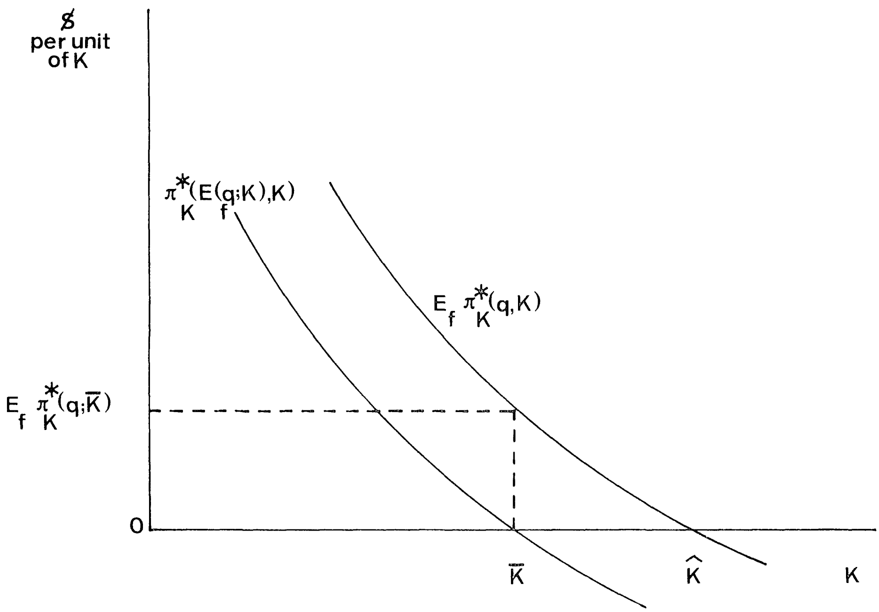

FIGURE 4

and uncertainty depend upon the specific properties of the production function and the probability density function over quantity demanded.

The interpretation of this proposition is best shown by reference to Fig. 3, which shows how the marginal profitability of capital (π_K^*) varies with quantity, which is uncertain. The regulatory system itself introduces discontinuities, so that π_K^* will generally be neither concave nor convex in quantity, which is why it is generally not possible to rank the choices of capital under certainty and uncertainty.

This raises the general question as to whether there are interesting conditions under which uncertainty has no effect on plant size. This occurs when given $\hat{K}$, the choice of capital under uncertainty, the probability density function is positive only over regions in which π_{Kqq} is zero. Formally, let $\hat{q}^0$, $\hat{q}^1$, $\hat{q}^2$, $\hat{q}^3$ satisfy condition (7) given $K=\hat{K}$. Let $q_L=\inf\{q|q=q(x, P_0) \text{ with } f(x)>0\}$ and q_u and sup $\{q|q=q(x, P_0) \text{ with } f(x)>0\}$.

If the firm's probability density function over demand is such that $q_u<\hat{q}^0$ or $q_L>\hat{q}^3$, then it is clear from inspection of Figure 2 and Figure 3 that profits are zero for every q such that $q=q(x, P_0)$ with $f(x)>0$ and π_{Kqq} is zero for every such q as well. In these cases the choice of capital under certainty is indeterminate, since profits are zero for all possible choices of capital. Hence, there is no definite ordering over capacity choice (certainty versus uncertainty) in these cases.

A more interesting possibility is that $\hat{K}$ will be selected by the firm such that $\hat{q}^1\leq q_L<q_u\leq\hat{q}^2$. In this case, the firm earns the maximum possible profits, $s\hat{K}$,

for every q such that $q=q(x, P_0)$ with $f(x)>0$; further, π^*_{Kqq} is zero for all such q as indicated by Figure 3. Then by Jensen's Inequality we would have $E_f(\pi^*_K(q), \hat{K})=\pi^*_K(E_f(q), \hat{K})$. Under these conditions, the firm chooses $\hat{K}(=\bar{K})$ as the largest value of K such that $\pi(q, K)=sK$ for all q satisfying $q=q(x, P_0)$ with $f(x)>0$. Thus, the choice of K would be the same under certainty as under uncertainty.

However, the argument fails because it can not be the case that $\hat{q}^1 \leq q_L < q_u \leq \hat{q}^2$. Proposition 2 shows why.

PROPOSITION 2. *The firm facing demand uncertainty never chooses a plant capacity $\hat{K}$ that guarantees the upper bound rate of return constraint ($\pi \leq s\hat{K}$) will be binding for all q such that $f(x)>0$. The profit maximizing plant size will always be larger than any plant size that guarantees that the constraint will be binding for all q such that $f>0$.*

PROOF OF PROPOSITION 2. Assume the contrary, i.e., at $\hat{K}$, $\hat{q}^1 \leq q_L < q_u \leq \hat{q}^2$. Then by the first order condition (11) when demand is uncertain (i.e., $x^2 > x^1$) we have $\frac{\partial W}{\partial K} = s(F(x^2)-F(x^1))>0$, and thus $\hat{K}$ cannot be optimal. Q.E.D.

On reflection, this result is quite consistent with intuition. For at least an infinitesimal increase of K beyond $\hat{K}$, the firm remains able to earn additional profits of $s(K-\hat{K})$, with only a very small probability that the firm will not be able to operate on the maximum profit constraint. Thus, $\hat{K}$ cannot be an optimum. Expected profits can be increased by increasing the plant size.

For purposes of comparison with Proposition 2, it is of interest to characterize somewhat more finely the choice of capacity by the regulated firm operating under certainty and subject to upper and lower constraints on profits. Proposition 3 identifies certain basic properties of the choice of capacity.

PROPOSITION 3. *Given a regulated firm operating under demand certainty and with $\pi(P_0, \mathrm{Eq}(P_0), K)>0$ for some K, then* (i) *the firm never chooses $K=\bar{K}$ such that P_0 is changed;* (ii) *the firm selects $\bar{K}$ as the largest K such that $\pi(P_0, \mathrm{Ep}(P_0), K)=sK$, if such a choice is feasible, and* (iii) *if no K satisfies $\pi(P_0, \mathrm{Eq}(P_0), K)=sK$, then the firm acts to minimize cost in satisfying the common carrier requirement at P_0.* (*For proof see the Appendix.*)

This proposition is again quite in line with intuition. In a world of certainty, and with an inelastic demand, the firm would not want a lower price since that would reduce revenues, and more importantly, profits. A higher price is not desirable since the firm could only receive such a price if it would earn no supernormal profits at the higher price. Thus, the firm will operate to maximize profits given P_0, and hence given revenues as well. Parts (ii) and (iii) of the proposition follow directly. The firm will want to select capital that permits it to operate on the maximum profit constraint if that is possible. If such a choice of capital is not possible, then profit maximization is equivalent to cost minimization since revenues are fixed.

The comparison of capacity choices by the regulated firm between the certainty and uncertainty cases is now complete. The firm facing demand uncertainty will not necessarily choose a larger plant capacity than it would in the uncertainty case. Under uncertainty, the firm does not choose a plant size that makes the upper constraint binding for all q that may be observed; under demand certainty, the firm will choose a capacity that makes the upper constraint binding where this is feasible, but it will not choose a plant size such that a change from P_0 occurs, so long as $\pi(P_0, \mathrm{Eq}(P_0), K)>0$ for some K. If $\pi(P_0, \mathrm{Eq}(P_0), K)\leq 0$ for every K, then the choice of plant size is indeterminate, with every such choice leading to zero profits for the firm.

It is clear from all that has been said so far that only under very special assumptions can anything definite be said about the amount of capacity that will be chosen under uncertainty relative to that chosen under certainty. A natural question would be whether there exists a set of sufficient conditions under which it is possible to order the levels of capacity chosen under certainty and uncertainty. Our examination of this question leads us to Proposition 4.

PROPOSITION 4. *Given any interval* (q_L, q_u), *necessary and sufficient conditions that the choice of capital* $\hat{K}$ *under uncertainty be less* (*greater*) *than the choice of capital* $\bar{K}$ *under certainty for all probability density functions* $f(\cdot)$ *over the interval are*

(i) $\hat{q}^0\leq q_L<q_u\leq\hat{q}^1$ *or* $\hat{q}^2\leq q_L<q_u\leq\hat{q}^3$, *and*
(ii) π_K^* *be concave* (*convex*) *in* q *over the interval.*

Proposition 4 thus has the virtue of providing both necessary and sufficient conditions under which $\hat{K}$ and $\bar{K}$ can be ordered. It also indicates that only in the case when the regulated firm acts as an unregulated firm (i.e., no price adjustments occur as the firm hits a regulatory floor or perceives pressure from the possibility of earning a return so high that a rate review would be triggered by interest groups) that we can say anything definite about the relative amounts of capital that would be installed under certainty and uncertainty.

6. PREFERENCES BETWEEN CERTAINTY AND UNCERTAINTY

One might ask whether a firm subject to the non-negativity of profits constraint and the upper bound constraint on profits prefers certainty to uncertainty. We compare expected profits under demand uncertainty with profits under certainty given that demand is the expected value of demand under uncertainty. As above, let q_L and q_u be the inf and sup of the values of q. Let $\hat{K}$ denote the level of capital that maximizes expected profits under uncertainty, and let $\bar{K}$ denote the level of capital that maximizes profits under certainty. Let $\hat{q}^0$, $\hat{q}^3$ denote the values of q^0, q^3, given P_0 and $\hat{K}$.

Then, from Jensen's Inequality and Figure 2, we have:

PROPOSITION 5. *Preferences of an expected profit maximizing firm between demand certainty and uncertainty depend on the subjective probability beliefs of the firm with respect to demand. In particular,*

(i) *if* $\hat{q}^0 \leq q_L$ *and* $q_u \leq \hat{q}^3$, *then the regulated firm prefers certainty to uncertainty*;

(ii) *if* $q_L < \hat{q}^0$ *or* $\hat{q}^3 < q_u$, *then there exist probability density functions such that uncertainty is preferred to certainty, and there exist probability density functions such that this preference is reversed.* (For proof, see the Appendix).

A graphical explanation of this proposition can be provided with the aid of Figure 2. When condition (i) holds, the profit function π^* is concave in q, which is uncertain. Hence, the firm prefers certainty to uncertainty. However, when (ii) holds, then π^* is neither concave nor convex, and thus no ranking is possible.

7. CONCLUSIONS

Our analysis of the impact of demand uncertainty on the choice of plant capacity by a regulated firm does not provide a definitive answer to the question: "Will the regulated firm choose more capacity when faced with demand uncertainty than it does when demand is certain?" Instead, generally the ranking of capacity choices depends upon the subjective probability distribution over demand and on the form of the demand and production functions. The same indeterminacy surfaces with respect to preferences of the regulated firm between demand certainty and uncertainty. However, certain characteristics of capacity choices under demand certainty and uncertainty can be identified; those are summarized in Propositions 2 and 3 above.

The conclusions we have derived rest upon a model of the regulated firm that is based on Joskow's observations concerning the regulatory process, and in particular, on a model which recognizes the relevancy of non-negativity of profits as a constraint on the price-setting process. While the analysis assumes expected profit maximizing firms, the major conclusions concerning indeterminacy of preferences between uncertainty and certainty, and concerning the ranking of capacity choices, clearly extend to the case of risk averse firms as well.

APPENDIX

PROOF OF PROPOSITION 3. To establish (i), note that if $\pi(P_0, \text{Eq}(P_0), K) > 0$ for some K, then P is never increased, since P is increased only when the PUC acts, and when the PUC acts, profits are set equal to zero. In effect, the firm chooses K and $\tilde{P}$ to maximize profits, where $\tilde{P} = P_0 - P_2$, $\tilde{P} \geq 0$. Formally, the firm solves the problem

$$\max_{(K,\tilde{P})} \pi(P_0 - \tilde{P}, \text{Eq}(P_0 - \tilde{P}), K) = R(P_0 - \tilde{P}) - wL(\text{Eq}(P_0 - \tilde{P}), K) - rK$$

subject to $wL(\mathrm{Eq}(P_0-\tilde{P}),\ K)+(s+r)K-R(P_0-\tilde{P})\geq 0$, where $R(P_0-\tilde{P})$ is revenue, that is, $R(P_0-\tilde{P})=(P_0-\tilde{P})\mathrm{Eq}(P_0-\tilde{P})$. The constraint states that profits cannot exceed sK. We form the Lagrangean H, with the multiplier λ corresponding to the constraint. For simplicity, the arguments of R and L are suppressed. Thus,

$$H = (R-wL-rK) + \lambda(wL+(s+r)K-R).$$

The first order conditions include the following at an interior optimum $(K>0)$:

$$H_K = -wL_K - r + \lambda(wL_K+(s+r)) = 0, \quad \text{and}$$

$$H_{\tilde{P}} = R_{\tilde{P}} - wL_{\tilde{P}} + \lambda(wL_{\tilde{P}} - R_{\tilde{P}}) \leq 0,\ \tilde{P}H_{\tilde{P}} = 0,\ \tilde{P}\geq 0.$$

We wish to show that $\tilde{P}=0$. Assume the contrary so that $\tilde{P}>0$. Then $H_{\tilde{P}}=0$ implies that $(1-\lambda)(R_{\tilde{P}}-wL_{\tilde{P}})=0$. $L_{\tilde{P}}>0$ and $R_{\tilde{P}}<0$ since demand is inelastic. Thus, $1-\lambda=0$. But from $H_K=0$, this implies $s=0$. This contradiction establishes (i).

It remains to establish (ii) and (iii). With $\tilde{P}=0$, revenue is fixed at $R(P_0)$ with output being $\mathrm{Eq}(P_0)$. Suppose that the firm can minimize the cost of producing $\mathrm{Eq}(P_0)$ without violating the upper bound constraint. If so, $\lambda=0$ and the firm acts as an unconstrained profit maximizer given P_0.

On the other hand, if minimizing cost violates the upper bound constraint, the firm increases K to meet the constraint rather than change P_0. Thus, (ii) and (iii) hold. Q.E.D.

Proof of Proposition 4. Because of possible jump discontinuities, it is not possible to rank $\hat{K}$ and $\bar{K}$ for pdf's extending over intervals containing switching points. Further, over the intervals $q<\hat{q}^0$, $q>\hat{q}^3$, profits are zero for all choices to K, hence no ranking is possible. By Proposition 2, the choice of $\hat{K}$ such that $\hat{q}^1\leq q\leq \hat{q}^2$ is inconsistent with expected profit maximization. Hence, a necessary condition for ranking $\hat{K}$ and $\bar{K}$ is condition (i), while condition (ii) is necessary by Jensen's Inequality. When condition (i) holds, then the first order condition determining $\hat{K}$ is simply

$$\int_{x^0}^{x^1} \pi_K(P_0,\ q(x,\ P_0),\ \hat{K})f(x)dx = 0, \quad \text{or}$$

$$\int_{x^2}^{x^3} \pi_K(P_0,\ q(x,\ P_0),\ \hat{K})f(x)dx = 0,$$

which uniquely determines $\hat{K}$, given $\pi_{KK}<0$ over the interval as has been assumed earlier. By Jensen's Inequality, the ranking between $\hat{K}$ and $\bar{K}$ is determined according to the concavity or convexity or π_K in q.

Proof of Proposition 5. To prove (i), we use Jensen's Inequality and the fact that π^* is concave in q on the interval $(q_L,\ q_u)$ given $\hat{K}$ (see Figure 2). Thus,

$$E_f(\pi^*(q,\ \hat{K})) < \pi^*(E_f(q),\ \hat{K}) < \pi^*(E_f(q),\ \bar{K})$$

for all pdf's f. The first inequality is strict, since Proposition 2 shows that $q^1 \leq q_L < q_u \leq q^2$ is not possible at $\hat{K}$. The second inequality simply expresses the fact that $\bar{K}$ is the maximizer of profits under certainty.

For (ii), we note that with $q_L < \hat{q}^0$ or $\hat{q}^3 < q_u$, the function π^* is neither concave nor convex in q on (q_L, q_u), hence, no definite preference can be stated without knowledge of the specific properties of the production function, the pdf over demand, and the way in which demand responds to price.

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