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SOME EXPERIMENTAL AMBIGUITIES WITH RESPECT TO THE CORE FOR MAJORITY RULE VOTING GAMES*

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In the context of spatial majority voting games, considerable experimental support exists for the core as a solution hypothesis when it exists (c.f. Berl, et al, 1976; Fiorina and Plott, 1978). Some recent experimentation, however, hints at possible problems in a finite alternative setting. Isaac and Plott (1978) report several such experiments in which subjects fail to adopt a core, although their experimental design uses a particular procedure of chairman control that might account for these results. Elsewhere (1979b) we report a series of vote trading experiments in which the core's success rate is less than fifty percent.

In this essay we present some additional experimental evidence to suggest that committee choice in simple majority rule games is not dictated solely by whether or not a Condorcet (core) point exists. We conclude that, in the experimental context of open and free discussion, the performance of the core is affected by the structure of alternative space, and also by the structure of the perceived dominance relation beneath the core in the social ordering.

Section 1 of this essay reviews the results of the vote

[^0]trading games and describes a series of experiments that appear|to resolve the question of why the core performs poorly there. Sedtion 2 presents an additional series of majority voting games with a relatively uncomplicated structure in which the core's success tate is only about 60 percent. Section 3 summarizes our alternative explanations of the core failures but concludes that an adequat the for incorporating such considerations is not presently available.

## 1. VOTE TRADING GAMES

In (1979b) we describe the experimental outcomes of sdvera vote trading games with and without cores. Briefly, in those games, subjects are given a list of "bills" (usually 5) and told that, bby majority rule, they must decide which bills to pass and which tc fail. If a bill is failed, they receive a payoff of zero from that bill while if a bill is passed their payoff is either positive dr negative, according to the value assigned to them for that bill The rules of the experiment allow the subjects to establish their own methods of disposing the bills. They can consider thd bil sequentially, or simultaneously as a package. Whenever a majority arrives at an agreement on some subset of the bills, they can enforce that agreement by signing an agreement card on those bills. They cannot, however reconsider decisions that have already been made.

Table 1 portrays a vote trading game with a core cortespo to passing only bills C and E, denoted FFPFP). Payoffs for a given individual, across bills, are additive so that, at the core, the
payoff vector ( $10,1,13,5,-8$ ) results.* This game is then modified in three ways. First, the payoffs of each player on bills $C$ and $E$ are multiplied by -1 so that the core is the more "obvious" outcome FFFFF. Second, the apparent vote trade between players 1 and 2 on bills A and B -- a trade that leads away from the core -- is eliminated by decreasing l's payoff from $B$ to -12. Third, various payoffs are adjusted to eliminate possible intransitive indifferences around the core. Table 2 sumarizes the results of these experiments.

Overall, we see that the core's success rate is only 45 percent and that the various modifications of the original game yield only modest effects. One possible explanation of the above results is that it is the separability of the alternative space in the vote trading experiment that leads to outcomes away from the core. Thus, in the experiment of Table 1 , the final alternative space consists of exactly $2^{5}=32$ possible outcomes, where each is a particular disposition of each of the five bills. Because of of the this set, subjects frequently attempt to disaggregate the decision by making agreements on only a subset of the bills. In the actual experiments, this may be done either for strategic reasons -- i.e. a player realizes he can do better by disaggregation --

[^1]
## TABLE 1

Payoffs for a Core Experiment (Core $=$ FFPFF)

| Bills |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Player | A | B | C | D | E |  |
| 1 | 10 | -2 | 5 | 4 | 5 |  |
| 2 | -2 | 10 | 5 | -5 | -4 |  |
| 3 | 4 | -8 | 5 | 3 | 8 |  |
| 4 | -8 | 4 | -3 | -5 | 8 |  |
| 5 | -5 | -5 | -4 | -10 | -4 |  |

Entries Denote Payoff if Bill is Passed

## TABLE 2

Success Rate of the Core*

|  | Original Game | Indifference <br> Modification | Vote Trade <br> Modification | All |
| :---: | :---: | :---: | :---: | :---: |
| Core $=$ FFFFF | $\frac{2}{7}$ | $\frac{5}{7}$ | $\frac{3}{7}$ | $\frac{10}{21}$ |
| Core $=$ FFPFP | $\frac{2}{7}$ | $\frac{4}{7}$ | $\frac{3}{7}$ | $\frac{9}{21}$ |
|  | $\frac{4}{14}$ | $\frac{9}{14}$ | $\frac{6}{14}$ | $\frac{19}{42}$ |

(*Numerator is number of core outcomes, denominator is number of trials)
can be implemented by actually making the decision sequentially or it may occur through verbal commitments made and adhered to in|the negotiations with final decisions being' on all bills simultanepusly We know, of course, that such sequential vote trading can lead not only to noncore outcomes but to Pareto inferior outcomes (c.f. Riker and Brams, 1976). Our initial hypothesis, then, i:; that the deviations from the core reported in Table 2 can be attributed to this separability of the alternative space

To test this hypothesis we can construct a derivative "finite alternative" game. Specifically, with 5 bills, there|are 32 possible outcomes. Eliminating six outcomes that, a priori, are unlikely to be chosen and which occur infrequently in the bargationg, we can label the remaining 26 outcomes A through Z. Each letter, corresponds to a particular disposition of all five bills, while each player's preference across these alternatives is deduced from Table 1. The resulting preference configuration is given in Table (see Appendix A for the actual payoff schedules used and for the transformation relating the lettered alternatives of Table 3 to the bills of Table 1). To conduct the appropriate experiment usirg the preference orders in this table, we provide each player with these ordinal rankings and use procedures and instructions that require subjects use majority rule to choose one and only one alternative (letter). (A detailed description of the experimental design lof these "finite alternative" games can be found in McKelvey and Ordeshook [1979al.)

TABLE 3
Finite Alternative, Experiment F1

$$
\text { Core }=\mathrm{G}
$$

| Player 1 | Player 2 | Player 3 | Player 4 | Player 5 |
| :---: | :---: | :---: | :---: | :---: |
| X | Y | X | J | v |
| H | U | E | N | W,T |
| E | N | M | W | L, P |
| F, Q | L, S | Q | 2 | G |
| B | B | G | G | Y,J,I,O |
| I, 0 | J, R | F, $\mathrm{O}, \mathrm{H}$ | K, L, R | C |
| M | T | A | A | E,N |
| K, U | H,K | B, I | B, Y | A, D, U, K |
| R | I | D, R, W | M, O,V | B, M |
| G, P | G, Z | $\mathrm{N}, \mathrm{T}$ | T, E | S, Z, F, Q |
| D, A | D, v | P, K | S, H | R, X |
| N | E | Z, c | C, Q | H |
| s, 2 | F, P | U | U |  |
| T, W | M, W | J, v, s | D, P, X |  |
| C | c | Y | I |  |
| J, Y | 0, x | L | F |  |
| V | A |  |  |  |
| L | Q |  |  |  |

Preferences of Players (ranked from best to worst)

TABLE 4
Outcomes of Finite Alternative Experiment F1

| Outcome | Coalition |
| :--- | :--- |
| Core | 12345 |
| Core | 345 |
| Core | 345 |
| Core | 12345 |
| Core | 1345 |
| Core | 1345 |
| Core | 1345 |

We see from Table 4 now that, once forced to consider complete packages, the core prevails every time. It appears, then, that the results reported in Table 2 have a straightforward explanaton -- the complexity of the corresponding game and the myopic tendency to consider pairwise vote trading as against complete dispositions of all bills, leads away from the core. In the next section, however, we suggest that this explanation is incomplete.

## 2. SOME ADDITIONAL AMBIGUITIES

Note that in Table 3, the core (alternative G) stands at about the median of player 1 and $2^{\prime}$ s preference orders and distinctly above the median for players 3, 4, and 5. If subjects make interpersonal comparisons of utility based on the position of an alternative in the ordering, then the core appears to be a relatively "fair" outcome.

The experiments reported in this section were originally designed to attempt to verify, in a finite alternative setting, that the choice of the core point is independent of its "fairness" properties. This has already been verified in experiments where the alternative space has a spatial representation (c.f. Berl, et al [1976], and Fiorina and Plott [1978], where the core predicts well regardless of whether it is one player's ideal point or not). We initially expected the same results to hold here.

To address the issue of fairness, consider the experiment shown in Table 5, which has a nonempty core, alternative A. Note that A is "low" on some preference schedules (players 1 and 4), and is player $3^{\prime}$ 's ideal point. We report here two versions of this experiment. In the first, subjects are provided with complete ordinal information about everyones' preferences, while in the second, they are given no information on other subjects' preferences. Table 6 summarizes the results of thirty trials of this game, controlling for the experience of the subjects.

The results of these experiments provide something less than overwhelming support for the core. Overall, the core occurs in only 60 percent of the trials. The data also show two additional

TABLE 5
Finite Alternative Experiment F2 Core $=\mathrm{A}$

| Player 1 | Player 2 | Player 3 | Player 4 |
| :---: | :---: | :---: | :---: |
| J | K | A | L. |
| E | D | B | K |
| D | E | E | G |
| C | A | F | C |
| L | G | D | I |
| B | C | G | F |
| I | H | c | H |
| G | F | I | M |
| N | B | H | A |
| M | J | K | B |
| A | L | J | E |
| F | I | L | D |
| H | N | M | N |
| K | M | N | J |

Preferences of Players (ranked from best to worst)
patterns.
(1) Incomplete ordinal information about the preferences of other players leads to a higher success rate for the core.
(2) While the relationship is weak, and statistically insignificant, in the incomplete information games inexperienced subjects appear more likely to choose the core than experienced subjects.

Despite the fact that the core does not predict well in these experiments, deviations from the core do not seem to be explainable by considerations of fairness. Seven of the twelve failures correspond to the choice of alternative $E$, which benefits three subjects (subjects 1, 2, and 3) at the expense of the other two. Examination of the remaining failures shows that three failures at most (one $G$ and perhaps the two B's) can be classified as "fair". Thus, there must be some other explanation of the above results.

Before we attempt to interpret these results more fully, however, we can dismiss one simple explanation for the core's failures. Specifically, note that E is "good" for three adjacent players, 1, 2, and 3. We must consider the possibility, then, that E is chosen simply because its acceptability to a majority is more apparent on the payoff schedules. Note, in fact, that the frequency of $E$ declines appreciably when this visual clue is absent -- when players possess incomplete ordinal information about others' preferences. Table 7 reports a series of trials in which the preference orders of players 2 and 4 are switched. These results are somewhat equivocal. The relative frequency of $E$ decreases, but it still occurs three times out

TABLE 6
Results of Experiment F2


TABLE 7
Results of Experiment F2 with Preference Schedules Reversed

| Outcome |
| :--- |
|  |
| $\underline{A}$ |
| A |
| A |
| C |
| E |
| E |
| E |
| A |
| $(2,3,3,5)$ |
| $(2,3,4,5)$ |
| $(1,2,3,4)$ |
| $(1,3,4)$ |
| \% of Outcomes |
| in Core |

of the eight trials. The core succeeds 50 percent of the time versus 43 percent of the time in the previous experiments. None of these differences, however, are significant. Hence, we conclude that, even if there is a "visual effect," it is not the explanation for the failures of the core that we report here.

A more convincing explanation for these failures concerns the specific dominance structure of Experiment F2. Appendix B gives the dominance matrix for Experiment F2, which is illustrated in Figure 8. Note that $A$, the core point, is followed by a six element cycle set consisting of $\{E, B, C, G, D, F\}$, which in turn is followed by a five element cycle set consisting of $\{H, I, J, K, L\}$. These all beat N , which in turn beats the condorcet loser, M.

There are several things to note about the dominancel strudture of Figure 1. First, in the top cycle set below A, E is beaten only by B and, of course, A. Thus, E is stable against any other alternative except these two. Further, both $B$ and A beat $E$ by| only three votes, and require the support of exactly the same coalition -- namely $\{3,4,5\}$. Thus, not only is it hard to find an alterhative to beat $E$, but it is also difficult to find the right coalitioh to support a change. This is probably exacerbated by the fact thbt the coalition that must support a change to $A$ or $B--$ namely $\{\beta, 4,5\}$ is not the most obvious coalition to support these proposals. I The coalition includes the one player -- namely player 4 , for whom A or B are "low" on his ordering and are not very much better than 3 . A first glance at the payoff schedules of Table 5 suggests that a more natural supporting coalition for $A$ is $\{2,3,5\}$. In fact, when A actually occurs, it frequently is supported by player 2 (seel Table $6)$.

The above considerations argue that E is, in some sepse, stable, and it is difficult to move from $E$ to the core, A. Moreover further inspection of the dominance structure for this game rejeals that, with the exception of alternative $N$, A beats each altern ative only three votes, and player 3 is always one of the three who nust support a change to $A$. Thus, to get to A from any other point in space (except $N$ ), player 3 must support the move. While it might that it is always in player $3^{\prime}$ 's interest to support such a move, this is true only if player 3 is aware that this proposal is stable. If player 3 suspects that $A$ is not stable, then by proposing almove
to $A$, he runs the potential risk of sacrificing whatever he may be making from the prevailing proposal. If, for example, the prevailing proposal is E, the gain can be fairly small (see Appendir A).

The preceeding arguments suggest that the dominance structure in Experiment F2 account for the core failures. They create a dominance pattern that makes E "stable", and that make it necessary to have player $3^{\prime \prime}$ s support to obtain the core outcome. Given the fact that there are cycles below the core, a weak or timid player 3 may assume that A is also unstable, and hence be unwilling to push for it. The result is that the outcomes will tend to be distributed in the top cycle below the core, rather than at the core.

If we consider the core failures, we note that they are, in fact, all in the top cycle below the core, distributed, roughly in accordance with the vulnerability of these proposals (see Table 8).

The preceeding explanation also appears to account for the differences we observe due to information and experience. The effect of incomplete information seems to be that subjects are then forced to internalize the relevant preferences of other players and, in doing so, learn better the dominance relations in the game. They are also forced to consider all alternatives in the process of collecting information, and do not have the visual signal of alternative $E$ being "high" on the list for a majority.

The second pattern may be attributed to the fact that experienced players expect cycles and instability -- based on having played unstable games earlier -- and, as a result, assume that every alternative can be beaten by another. What we frequently

Figure 1:
Dominance Structure of Experiment F2


## TABLE 8

Distribution of Core Failures over Top Cycle Below A

|  | Outcome |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | E | B | C | G | D | F |
| Frequency |  |  |  |  |  |  |
| Vulnerability* | 7 | 2 | 0 | 1 | 0 | 2 |
|  | 2 | 3 | 4 | 4 | 4 | 4 |
|  |  |  |  |  |  |  |

*Vulnerability $=$ \# of proposals that are majority preferred to given proposal
observe, in fact, is subjects becoming "trapped" in a cycle fust beneath the core and, rather than attempting to break the cy申le, they attempt to negotiate the most advantageous outcome in it:

## 3. CONCLUSIONS

This essay reports on thirty seven experimental tribls several finite alternative games with a core, in addition to kevie 42 vote trading core games. In the vote trading games, the cpre prevails only 45 percent of the time. In the "finite alternative" the core prevails a modest 60 percent of the time. This sumary however, disguises much interesting variation. First, two distinct finite alternative games are considered: In one, namely the game is equivalent to the vote trading game, the core prevails 100 perce of the time while in the other, the core prevails only 57 percent the time. Second, in the latter game, the core's success rate is 52 percent with experienced subjects and 64 percent with inexperiep players. Finally, with complete ordinal preference information, th core is chosen 43 percent of the time in the second game, whereas with in complete information this rate increases to 75 percent

We conclude that there are at least two reasons for the departures from the core: Complexity of the alternative space, and the structure of the dominance relation. Unfortunately, above explanations seem to account for the deviations observed here these are ad hoc explanations generated from the particular
problems seen in these two games. Since there is no theory that incorporates such considerations, there is no way of ascertaining a priori, whethere these factors are relevant in a given experiment, and if so, exactly how they will affect the outcome. Further, we do not know if there might not be other reasons for departures from the core that have not been observed in the experiments run to date. In short, we must conclude that the extent to which the core is a reasonable prediction about choice depends on a great many things that we have not yet begun to understand or appreciate fully.

APPENDIX A

This appendix contains payoff schedules for experiments and F2 as well as the transformation relating F1 to the vote tradin experiment. The following list gives that transformation. In this list, the first entry in each row represents the alternative in F1, the second and third entries represent the corresponding alternativ in the two versions of the vote trading experiments. The first vote trading experiment is that in Table 1 , with Core $=$ FFPFP. The second is the vote trading experiment described in the text with Core $=$ FFFFF.

| A - DE | - CD | N - bCE | B |
| :---: | :---: | :---: | :---: |
| B - ABCE | - AB | $0-\mathrm{AE}$ | - AC |
| C - D | - CDE | P - A | ACE |
| D - CD | - DE | Q - ADE | - ACD |
| E - ACE | - A | R - bCDE | BD |
| F - ACD | - ADE | S - BCD | - bDE |
| G - CE |  | T - C |  |
| H - abcie | - ABD | U - ABC | - ABE |
| I - AC | - AE | v- $\phi$ | CE |
| J - BE | - BC | W - E | - C |
| K - AbE | - ABC | X - ACDE | - AD |
| L - B | - bCE | $\mathrm{Y}-\mathrm{BC}$ | - BE |
| M - CDE | D | Z - BDE | - BCD |

Table Al gives the payoff schedules for experiment F1. In each of these experiments, the actual schedules used for each subject were drawn randomly from three possible payoff schedules: the master schedule, which is the schedule in column 1 for each player a schedule in which all payoffs were doubled, and a schedule in which all payoffs are halved (except for player 4, whose third schedule is the master schedule multiplied by a factor of form). This is the same as the procedures for generating the payoff schedules in the vote trading experiments. The initial endowment (whose magnitude was unknown to the subject until termination of the experiment) for each version of the payoff schedule is given in the column $\alpha$ at the bottom of the table.

The payoff schedules for Experiment F2 were generated using exactly the same procedures as those in McKelvey and Ordeshook [1979a]. They are given in Table F2. Again, in each experiment for each subject, one payoff schedule was drawn randomly from the three schedules listed.

TABLE Al
PAYOFF SCHEDULES FOR EXPERIMENT F1

table a2
PAYOFF SCHEDULES FOR EXPERIMENT F2


APPENDIX B
Dominance Matrix for
Experiment F2

|  | A | B | C | D | E | F | G | H | I | J | K | L | M | N | $\mathrm{n}(\mathrm{x})$ | 1(x) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A |  | 2 | 2. | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 1 | 2 | 0 |
| B | 3 |  | 3 | 2 | 2 | 3 | 2 | 2 | 1 | 2 | 2 | 2 | 1 | 0 | 3 | 3 |
| C | 3 | 2 |  | 3 | 3 | 2 | 4 | 1 | 0 | 2 | 2 | 2 | 0 | 1 | 4 | 4 |
| D | 3 | 3 | 2 |  | 3 | 3 | 2 | 2 | 1 | 2 | 2 | 2 | 1 | 1 | 3 | 4 |
| E | 3 | 3 | 2 | 2 |  | 2 | 2 | 2 | 1 | 2 | 2 | 2 | 1 | 1 | 3 | 2 |
| F | 3 | 2 | 3 | 2 | 3 |  | 3 | 1 | 2 | 1 | 2 | 2 | 1 | 1 | 3 | 4 |
| G | 3 | 3 | 1 | 3 | 3 | 2 |  | 1 | 1 | 2 | 2 | 2 | 0 | 1 | 3 | 4 |
| H | 3 | 3 | 4 | 3 | 3 | 4 | 4 |  | 3 | 2 | 2 | 2 | 1 | 1 | 4 | 8 |
| I | 3 | 4 | 5 | 4 | 4 | 3 | 4 | 2 |  | 3 | 2 | 4 | 0 | 1 | 5 | 9 |
| J | 3 | 3 | 3 | 3 | 3 | 4 | 3 | 3 | 2 |  | 3 | 1 | 1 | 1 | 4 | 9 |
| K | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 |  | 3 | 1 | 2 | 3 | 10 |
| L | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 1 | 4 | 2 |  | 0 | 1 | 4 | 9 |
| M | 3 | 4 | 5 | 4 | 4 | 4 | 5 | 4 | 5 | 4 | 4 | 5 |  | 3 | 5 | 13 |
| N | 4 | 5 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 3 | 4 | 2 |  | 5 | 12 |

Matrix of $n(x, y)$ for
Experiment 2

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[^0]:    *Prepared for delivery at the Public Choice Convention, March 17-19, 1979 Charleston, South Carolina.

[^1]:    * Subjects are given an endowment to cover possible losses. Subjects do not know the magnitude of their endowment during the negotiations, but only know it is a predetermined fixed amount that will be revealed to them at the termination of the experiment. Further details on the exact experimental design can be found in McKelvey and Ordeshook [1979b].

