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CONSUMER MARKETS FOR WARRANTIES

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ABSTRACT

This paper considers consumer markets for warranties when consumers are imperfectly informed about both product and warranty prices and about which firms sell with warranties and which firms sell without warranties. We characterize necessary and sufficient conditions for existence of the various equilibrium configurations of price and warranty coverage that can arise in two paradigm cases; when all consumers prefer warranties and when none do. Our results suggest that firms will exploit imperfect information by charging noncompetitive prices as well as by offering less than ideal warranty coverage, and that the former practice may be more serious in many markets than the latter.

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Alan Schwartz* and Louis L. Wilde**

1. INTRODUCTION

A warranty is an insurance policy that sellers offer against product related harms. In the last two decades, this insurance increasingly has been made compulsory; sellers have been required to warrant concerning—i.e. to insure buyers against—various purchase risks. A related form of regulation requires sellers to give unusually clear explanations of contract terms relating to warranty coverage, in contrast to the explanations required of other contract terms. As examples of these rules: (a) Sellers cannot disclaim warranties or limit the remedies that would otherwise be available to buyers in the event warranties are breached if the product caused a buyer to incur personal injuries. Thus firms are required to insure buyers against such injuries; (b) Courts read promises to repair or replace defective parts of consumer durables as guarantees of perfect performance of the durables themselves. As one court recently explained:

> When the seller is given reasonable opportunity to correct the defect or defects, and the vehicle nevertheless fails to operate as should a new vehicle free of defects, the limited remedy [the promise to repair or replace parts] fails of its essential purpose. . . . The buyer may then invoke any of the remedies available under the Uniform Commercial Code, including the right to revoke acceptance of the goods. . . .[1]

In consequence of this rule, the buyer is allowed to recover back the price ("revoke acceptance") and to be compensated for any damages incurred, including the cost of renting another item when the product he bought was being serviced; (c) The Magnuson-Moss Warranty and Federal Trade Commission Improvement Act requires firms to "fully and conspicuously disclose in simple and readily understood language the terms and conditions" of consumer product warranties.[2] This statute also prevents sellers from disclaiming implied warranties if they have made written warranties respecting the products at issue.[3] The prohibition is meant to increase warranty coverage since implied warranties shift most purchase risks to sellers, and written warranties are commonly offered in connection with sales of durable goods; they usually obligate firms to repair or replace defective parts.

All of this regulation is often justified on the ground that the presence of "imperfect information" would otherwise enable firms to exploit consumers respecting warranty coverage, but the regulation actually preceded serious investigation of how imperfect information could affect warranty markets. Two distinct forms of imperfect information might disadvantage consumers respecting warranties. First, consumers may be unaware of true product failure probabilities because it is quite expensive to observe product attributes fully. If perceived failure probabilities differ from actual ones, consumers may demand more or less warranty coverage than they actually want. Second, consumers may be unaware of the full set of possible warranty prices and terms that the market could offer because they may perceive the costs of searching for desirable contracts to be high in relation to the gains. If consumers actually engage in little search, firms may have an incentive to offer less advantageous warranty coverage than better informed consumers would choose.

The regulation described above did not distinguish between these information concerns; instead, in the words of the Magnuson-Moss Act, it sought "to improve the adequacy of information available to consumers, prevent deception, and improve competition . . . ."[4] The few papers that constitute the relevant economics literature, in contrast, focus almost exclusively on the assumed inability of consumers to observe failure probabilities accurately (e.g. Corville and Hausman; Shapiro). These papers suppose that (i) this inability exists; (ii) search costs are zero or (iii) the seller is a monopolist, in which case search costs are irrelevant (see also Grossman). They then attempt to explain and predict warranty content. As we show below, omitting search costs from an analysis of warranties may yield seriously misleading conclusions. Hence, it is important to ask how warranty markets perform in the presence of costly search. This paper represents the beginning of an attempt to answer this question.

We suppose a market for a product with an exogenous probability (π) of breaking and becoming useless; π is known to all. Firms can offer the good with a warranty or without one, but cannot do both. Consumers have preferences respecting warranty coverage, but do

not know, when they begin to search, which firms offer the good with a warran and which firms sell without one. Using these assumptions, we characterize necessary and sufficient conditions for existence of the various equilibria that could arise in two paradigm cases, when all consumers prefer warranties and when none do. In the former case, if enough consumers shop for warranty coverage, firms will offer warranties at competitive prices. Should fewer consumers than this shop, firms will provide warranties at supracompetitive prices if they have a comparative advantage (appropriately defined) at selling with warranties. When the comparative advantage runs the other way, firms will charge supracompetitive prices and deteriorate warranty coverage. Thi last outcome is less likely to occur if consumers strongly prefer warranties. When consumers do not prefer warranties at all--the second case--it turns out that warranties will never be offered; if enough consumers comparison shop, a competitive equilibrium occurs; otherwise, prices are too high. Also, these results apply to "quality" problems generally. A warranty is, in a formal sense, only a non-price product feature; hence, our conclusions can be considered to apply in any case in which "product quality" is a non-price feature over which consumers have homogeneous preferences. The implications of this interpretation are discussed in Part 4.

Part 2 of this paper briefly reviews the existing literature respecting warranties, focusing primarily on its utility for policy purposes. Part 3 then sets forth our model. Part 4 informally speculates about the consequences of relaxing some of the important assumptions on which the model rests, and briefly discusses the policy significance of our results. Respecting this, if it is supposed that regulation <u>on information grounds</u> is justified only when noncompetitive equilibria exist, then an important question is how competitive equilibria can be produced. Our model suggests that these equilibria are largely a function of the amount of comparison shopping in which consumers engage; hence, reducing the costs to consumers of comparing warranty coverage across firms seems wise. Also, the existence of imperfect information is commonly assumed to permit firms to exploit consumers by offering less preferred warranty coverage. Our results suggest that firms will also exploit consumers by charging excessive prices, and that pricing problems may be more serious in some markets than coverage problems. Some of the regulation described above seeks only to expand coverage and thus may be misconceived. Given the very preliminary nature of our analysis, these normative implications should be regarded more as interesting possibilities than as solid recommendations.

2. PRESENT WARRANTY THEORIES

Two theories constitute much of the literature respecting warranties. The first provides that a warranty signals the quality of the firm's product. (<u>e.g.</u>, Grossman; Spence). According to this theory, consumers cannot distinguish among competing products on the basis of quality, but believe that quality is positively correlated with warranty coverage. Also, the cost to firms of making warranties should vary inversely with product quality; the worse the product is,

the more expensive it will be to comply with warranties made respecting it. In consequence of these assumptions, warranty coverage should in fact correlate positively with product quality. Firms with good products will make extensive warranties that signal this fact, and firms with poor products will be unable to imitate these signals.

The signalling theory has three related difficulties. First, the theory presupposes a great deal of search, since firms have no incentive to send signals that will not be observed. Perhaps in consequence of this assumption, signalling papers commonly suppose search costs to be zero. We later suggest that the strength of this assumption may partly explain the second difficulty with the signalling theory, which is that it seems inconsistent with the data. For example, the theory predicts that firms with more durable products will make warranties that extend over longer time periods. Studies of reported legal cases and of actual warranties, in contrast, show that firms in given industries commonly make warranties effective for identical periods, and in all events for considerably less than the useful life of the product. (<u>e.g.</u>, Priest). Similarly, a positive correlation between warranty coverage and product reliability often seems difficult to detect. Thus, frequency of repair data such as that reported in Consumer Reports, sometimes shows wide variations among firms, but the products themselves trade under similar or identical warranties. Finally, many commercial buyers, particularly in industrial markets, seem able to distinguish among products on the basis of quality, yet warranties are as common in commercial markets as in consumer markets (Schwartz, 1977). Third, signalling explanations lend themselves uneasily to policy application. This is initially because signalling equilibria are notoriously unstable, so that it is difficult to derive from them criteria that would enable decisionmakers to evaluate real world markets. (e.g., Schwartz, 1981; Riley). Also, the welfare effects of signalling equilibria are very hard to evaluate. Such equilibria, when they exist, reflect only the sustained confirmation of a party's beliefs. Thus if consumers believe warranty coverage to correlate positively with product durability and if sellers with more durable products incur lower costs in making warranties than do sellers with less durable products, the former sellers have an incentive to make more extensive warranties. If they actually do so, a signalling equilibrium might arise in which warranty coverage varies directly with durability; in this event, the informational content that consumers attribute to the warranty signal is confirmed by the signals they see. This equilibrium would be efficient, however, only if the increased costs to firms of sending such warranty signals are less than the welfare gains to consumers of being able to distinguish more accurately among products on the basis of durability. This comparison is very difficult to make.

The second warranty theory explains coverage by reference to the comparative advantages of firms and consumers in reducing the costs of or insuring against product defects. For example, firms will warrant against defects in refrigerator motors but will be reluctant to warrant against defects in refrigerator doors, or will warrant

against such defects for shorter time periods. This is because consumers have little expertise respecting the care of such motors and indeed seldom directly use them, while consumers can best influence the durability of doors. There plainly is a core of truth in this theory, and it explains some of what is observed. Commercial law, as an illustration, does not require firms to repair or replace defective parts; instead, it gives buyers an action for the damages that such defects could cause. Firms, however, frequently do make repair and replacement promises, apparently because it is more efficient for them rather than consumers to cure defects in new goods.

The comparative advantage theory, as it is usually set forth, also assumes search costs to be zero, and this creates two difficulties. First, suppose one specifies the respective comparative advantages of consumers and firms, predicts warranty content on the basis of these advantages, and then observes that actual coverage is too thin. The theory is not necessarily disconfirmed because, as we show below, when firms would do better by not making warranties than by making them when little search occurs, coverage may be deteriorated, even though firms have a comparative advantage over consumers at insuring against or preventing product defects. Second, suppose one observes the predicted coverage. The positive aspect of the theory then seems confirmed, but its normative implications remain uncertain. This is because if consumers engage in insufficient search, firms may be offering the "right" warranties but at supracompetitive prices. The comparative advantage theory thus should be evaluated in environments where information is costly to acquire. Section 3 sets out a model that begins this task.

3. A MODEL OF PRODUCT WARRANTIES UNDER IMPERFECT INFORMATION

Suppose that (i) large numbers of firms and consumers exist; (ii) a homogenous good is sold, with consumers buying one unit of this good (with or without a warranty) or none;[5] (iii) the good has a positive probability, π, of breaking and becoming totally useless after purchase; π is independent of the care with which the product is used, and known both to firms and consumers;[6] (iv) all firms produce this good with an identical technology, characterized by a fixed cost, F, a constant marginal cost, c, over some range [0,s], and an infinite marginal cost thereafter (s will thus be referred to as a "capacity constraint"); (v) firms choose a quantity to produce and a price to charge, and can offer the good with a warranty or without, but cannot do both; a warranty, in this model, consists of a promise to replace any defective product with a new one at no charge; (vi) offering the product with a warranty does not directly affect firms' marginal costs or the capacity constraint, but may require additional fixed costs, F', where $F' \geq 0$. These additional fixed costs may result from administrative or other expenses that a replacement program could cause; when $F' > 0$, we say that a warranty imposes "loading" costs on the firm.

A firm that does not offer warranties can produce and sell up to s units in any period, and faces a total cost curve given by $TC_N(x) = F + cx$ where $0 \leq x \leq s$. Average costs thus are

$AC_N(x) = c + (F/x)$ for $0 \leq x \leq s$; the "competitive price" for the good with no warranty is then defined by $p_N^* \equiv AC_N(s) = c + (F/s)$. A firm that offers warranties must plan for the replacement of defective goods when it decides how many units to sell (as opposed to how many units to produce). Since replacements also can be defective, the expected quantity that must be produced to "support" a sale of one unit is $1/(1 - \pi)$. Thus the total cost curve for a firm that sells with a warranty is $TC_W(x) = (F + F') + [cx/(1 - \pi)]$, where F' is the loading factor. This total cost curve is defined for $0 \leq x \leq s(1 - \pi) \equiv s_W$. Average costs thus are

$$\begin{aligned} AC_W(x) &= \{(F + F') + [cx/(1 - \pi)]\}/x \\ &= [c/(1 - \pi)] + [(F + F')/x] \end{aligned}$$

for $0 \leq x \leq s_W$, and the "competitive price" for the good with a warranty is defined by

$$\begin{aligned} p_W^* \equiv AC_W(s_W) &= [c/(1 - \pi)] + [(F + F')/s(1 - \pi)] \\ &= [1/(1 - \pi)][c + (F + F')/s] \end{aligned}$$

As this equation makes clear, the effective marginal cost for the good with a warranty is $c_W \equiv c/(1 - \pi)$.

Consumers in this model are partitioned in two distinct ways. First, they pursue a fixed sample size shopping strategy, with Λ_1 consumers sampling one firm at random (from among all firms) before purchasing, and Λ_2 consumers sampling two firms at random (from among

<u>all</u> firms) before purchasing.[7] Define $A = A_1 + A_2$, $a_1 = A_1/A$ and $a_2 = A_2/A$. Consumers sample at random across firms because they do not know, when they begin to search, which firms sell with and which without warranties. Second, consumers are potentially differentiated according to their "taste" for warranties. A consumer's taste for a warranty can be a function of prices, income, attitude toward risk, perception of and ability to affect failure probabilities and so forth. Part 3A of this paper models the case when all consumers are sufficiently risk averse to prefer the product with a warranty, if they have the opportunity to purchase the product with or without a warranty, but always at the competitive price. This case is considered for two reasons: First, decisionmakers generally assume that consumers prefer warranty protection; it thus is useful to see how warranty markets work under this assumption. Second, an important reason why consumers might prefer not to buy warranties is that in some cases consumers have a comparative advantage at reducing failure probabilities; this reason is absent here, for we assume that consumers cannot influence the failure probability. Part 3B next considers the case when no consumer prefers a warranty. A warranty is in essence an insurance policy against product failure that the seller offers; in the situations that Part 3B explores, consumers prefer other goods to seller insurance. Because the model described here applies to any pair of heterogeneous goods, a more concrete way to interpret this second case follows from current law, which permits firms to describe extensive warranties as "full" but requires them to

describe less extensive warranties as "limited."[8] This second case thus can be conceived as modeling a market in which all consumers prefer limited warranties, although the technologu permits firms to offer full warranties. In both cases considered, each consumer is assumed to have a willingness to pay for the good with a warranty, h_W, and a different willingness to pay for the good without a warranty, h_N; the former willingness to pay is always higher than the latter.[9] In part 3A, where all consumers prefer warranties, we also require $h_W - p_W^* > h_N - p_N^*$; that is, the surplus generated by purchasing the good with a warranty at p_W^* exceeds the surplus obtained by purchasing the good without a warranty at p_N^*.

We let N_N be the number of firms that offer the good without a warranty and N_W be the number of firms that offer the good with a warranty; $N = N_N + N_W$, $n_N = N_N/N$ and $n_W = N_W/N$. Equilibrium in this model is then defined by a total consumer/firm ratio, $a = A/N$, a distribution of firms, (n_N, n_W), and two distributions of prices, one for the good with a warranty and one for the good without a warranty, such that (a) all consumers maximize their surplus given their shopping strategy, (b) given the distribution of firms and prices, all firms earn zero expected profits (entry is free), and (c) no firm can earn positive expected profits by changing its price or warranty coverage.

A. <u>All Consumers Prefer Warranties</u>

In this model, two markets must be considered, the market for the good with a warranty and the market for the good without one.

Each market can be nonexistent, competitive or noncompetitive. This yields nine possible outcomes. Ruling out the outcome when neither market exists, eight possibilities remain. We next present three theorems that summarize these possibilities in terms of whether the equilibrium is competitive or not, and whether imperfect information is exploited by noncompetitive pricing or poor warranty coverage (or both).

(a) All firms offer the product with a warranty at the competitive price; no firm offers the product without a warranty at any price.

When all consumers prefer warranties, the only possible competitive equilibrium occurs when the relevant product is sold with a warranty at the competitive price -- case 3A(a). This equilibrium will obtain only if the ratio of shoppers to total consumers in the market at issue is sufficiently high. Before presenting the theorem that proves this result, it will be helpful to define what we mean by the "comparative advantage" to firms of offering a product with or without a warranty. We define a comparative advantage by reference to the number of customers that a firm offering the product at the highest price consumers would be willing to pay would need to break even: If a firm, as a result of its cost structure and consumer preferences, would need fewer customers to break even offering the product at its highest price <u>with</u> a warranty than it would need offering the product without a warranty, we then say that in this particular market firms have a comparative advantage at selling with

warranties. Similarly, if a firm would need fewer customers at the break even point when offering the product at its highest price <u>without</u> a warranty, then we say that firms have a comparative advantage at selling without warranties. The relevant break-even demand for a firm of selling with warranties is:

$$\alpha_W = (F + F')/(h_W - c_W).$$

The relevant break-even demand for a firm of selling without warranties is:

$$\alpha_N = F/(h_N - c).$$

The relative sizes of α_W and α_N determine comparative advantage.

<u>Theorem 1</u>: When all consumers prefer warranties, a necessary and sufficient condition for $n_N = 0$, $n_W = 1$, $\sigma = s_W$, and

$$G_W(p) = 0 \qquad \text{for} \qquad p < p_W^*$$
$$G_W(p) = 1 \qquad \text{for} \qquad p \geq p_W^*$$

to be an equilibrium, where $G_W(\cdot)$ is the distribution of prices in the market for the good with a warranty, is $a_1 s_W \leq \min(\alpha_W, \alpha_N)$.

<u>Proof of Theorem 1</u>: When the market for the product with a warranty is competitive and $n_N = 0$, expected demand at p_W^* must equal α_W. Hence firms enter until $\sigma \equiv \Lambda/N = s_W$. Consider whether a firm in the market for the product with a warranty would find it profitable to raise its price above p_W^*. Such a firm should charge h_W, the highest price a consumer would pay, since it could sell only to nonshoppers; any shopper who sees two prices, p_W^* and any price above this, would buy at

p_W^*. The potential deviant firm then would not raise its price if profits at h_W were nonpositive; that is, if $(A_1/N)(h_W - c_W) - (F + F') \leq 0$. Rearranging terms yields $(A_1/N) \leq (F + F')/(h_W - c_W) = \alpha_W$. Using $A/N = s_W$, this condition reduces to $a_1 s_W \leq \alpha_W$.

Suppose next that a firm wished to enter the market and offer the product without a warranty. Since all consumers prefer warranties when warranties are offered at their competitive price, p_W^*, this firm too would sell only to nonshoppers regardless of the price it charged. Hence, it should charge h_N, the highest price a consumer would pay for the product without a warranty. Once more, profits at this price would be nonpositive if $a_1 s_W \leq \alpha_N$. Q.E.D.

Theorem 1 is the more likely to be satisfied the larger is the ratio of shoppers to total consumers (a_2), the smaller is capacity (s_W), the larger are fixed costs and the smaller is the difference between consumers willingness to pay for the product (with or without a warranty) and the marginal cost of producing it. The latter two conditions refer to α_W and α_N, whose magnitudes increase with increases in fixed cost and decreases in the difference between willingness to pay and marginal cost.

(b) All firms offer the product with a warranty, but some or all firms charge noncompetitive prices; noncompetitive prices; no firm offers the product without a warranty.

Case 3A(b) occurs when too few shoppers exist to generate a competitive equilibrium, but firms have a comparative advantage at

selling with warranties ($\alpha_W < \alpha_N$). In this circumstance, market power arising from insufficient consumer shopping is exploited only through noncompetitive pricing, not by such pricing and by deteriorating warranty coverage; consumers will get the warranties they want, but will pay too much for them.

Theorem 2: When all consumers prefer warranties, a necessary and sufficient condition for $n_N = 0$, $n_W = 1$, $\sigma = \alpha_W/a_1$ and

$$
\begin{aligned}
G_W(p) &= 0 & &\text{for} \quad p < p_W^* \\
0 < G_W(p) &< 1 & &\text{for} \quad p_W^* \leq p < h_W \\
G_W(p) &= 1 & &\text{for} \quad h_W \leq p
\end{aligned}
$$

to be an equilibrium is $\alpha_W \leq \min\{a_1 s_W, \alpha_N\}$.

Proof of Theorem 2: When $n_N = 0$, the highest price in a noncompetitive equilibrium in the market for the good with a warranty must be h_W, since the firm charging the highest price gets only nonshoppers. Zero profits then implies $(A_1/N)(h_W - c_W) - (F + F') = 0$ or $\sigma = \alpha_W/a_1$. Consider $G_W(\cdot)$. Suppose it has a mass point at p_W^*, called G_W^*. Then expected demand at p_W^* equals A_1/N (the firm's share of nonshoppers) plus $(2/N)A_2[(1/2)G_W^* + (1 - G_W^*)]$ (the firm's share of shoppers). Zero profits at p_W^* implies

$$[(A_1/N) + (2/N)A_2[(1/2)G_W^* + (1 - G_W^*)]](p_W^* - c_W) - (F + F') = 0.$$

Solving for G_W^* and noting that $(F + F')/(p_W^* - c_W) = s_W$, yields

$$G_W^* = [(a_1 + 2a_2) - (s_W/\sigma)]/a_2.$$

Thus $G_W^* < 1$ requires

$$(a_1 + 2a_2) - (s_W/\sigma) < a_2,$$

which reduces to $\alpha_W < a_1 s_W$.

We must also consider whether a firm would enter the market for the good without a warranty. To calculate profits in that market, it is necessary to know the explicit form of the price distribution, $G_W(\cdot)$, on $(p_W^*, h_W]$. Expected profits at price p are

$$\pi_W(p) = [(A_1/N) + (2/N)A_2[1 - G_W(p)]](p - c_W) - (F + F').$$

Zero profits then gives

$$G_W(p) = 1 - \{[(F + F')/\sigma(p - c_W)] - a_1\}/2a_2.$$

Suppose next that a firm enters the market for the good without a warranty at a price q. To calculate the expected demand for the firm at this price, we first note that the expected demand from nonshoppers is A_1/N. Respecting shoppers, consider a price q' for the product with a warranty, such that $h_N - q = h_W - q'$; that is, the surplus to a consumer of buying with a warranty at q' is equal to the surplus obtained by buying without a warranty at q. A shopper then will purchase from the firm selling without a warranty at p = q only if his or her other observation is at $p > q'$ (recall that all other firms sell with warranties). The probability that the consumer's other observation is $p > q'$ is $1 - G_W(q') = 1-(h_W - h_N + q)$. A firm that enters without a warranty at price q thus has an expected demand from

shoppers of $(2/N)A_2[1 - G_W(h_W - h_N + q]$. Hence

$$\pi_N(q) = [(A_1/N) + (2/N)A_2[1 - G_W(h_W - h_N + q)]](q - c) - F$$
$$= [(F + F')(q - c)/(h_W - h_N + q - c)] - F,$$

whence

$$\pi_N'(q) = (F + F')(h_W - h_N)/(h_W - h_N + q - c)^2 > 0.$$

Because profits for a firm entering without a warranty are increasing in q, entry would not occur if $\pi_N(h_N) \leq 0$. But

$$\pi_N(h_N) = [(F + F')(h_N - c)/(h_W - c_W)] - F,$$

so that $\pi_N(h_N) \leq 0$ if and only if $\alpha_W \leq \alpha_N$. Q.E.D.

Theorem 2 states that when consumers prefer warranties imperfect information will be exploited by noncompetitive pricing in the market for the product with a warranty (i.e., there is no coverage problem) if and only if the proportion of nonshoppers is relatively high ($\alpha_W \leq a_1 s_W$) and firms have a comparative advantage at offering the product with a warranty ($\alpha_W < \alpha_N$).

(c) Some (possibly all) firms offer the product without a warranty and at noncompetitive prices.

This case occurs when too few consumers shop to generate a competitive equilibrium and firms have a comparative advantage at selling without warranties. If few enough consumers shop, all firms offer the product without a warranty and at noncompetitive prices.

Case 3A(c) is the least desirable normatively, for consumers who prefer warranties often fail to get them and in addition pay too much for the product. This case is less likely to arise if a fair number of consumers shop; should enough shopping occur, then even though firms have a comparative advantage at selling without warranties, at least some firms will be induced to offer the product with a warranty, although not at the competitive price.

Theorem 3: When all consumers prefer warranties, a necessary and sufficient condition for $0 < n_N < 1$, $0 \leq n_W < 1$, $\sigma = \alpha_N/a_1$,

$$\begin{aligned} G_N(p) &= 0 && \text{for} && p < p_N^* \\ 0 \leq G_N(p) &< 1 && \text{for} && p_N^* \leq p < h_N \\ G_N(p) &= 1 && \text{for} && h_N \leq p \end{aligned}$$

and

$$\begin{aligned} G_W(p) &= 0 && \text{for} && p < p_W^* \\ 0 \leq G_W(p) &\leq 0 && \text{for} && p_W^* \leq p < h_W \\ G_W(p) &= 1 && \text{for} && h_W \leq p \end{aligned}$$

to be an equilibrium, where $G_W(\cdot)[G_N(\cdot)]$ is the distribution of prices in the market for the good with [without] a warranty, is $\alpha_N \leq \min\{\alpha_W, a_1 s_W\}$. Furthermore, $n_W = 0$ if and only if

(i) $a_1 s_W > (a_1 + 2a_2)\alpha_N$

(ii) $k_W \leq a_1 F'/(a_1 + 2a_2)\alpha_N$

where $k_W = (h_W - c_W) - (h_N - c)$.

(The proof of Theorem 3 is complex, and is set forth in Appendix 1.)

To summarize the implications of these three theorems, when all consumers prefer warranties, the only possible competitive equilibrium occurs when the relevant product is sold with a warranty at the competitive price. Whether this equilibrium actually will occur is solely a function of the percentage of shoppers (a_2) in the market at issue; if a_2 is sufficiently large, only warranties at competitive prices are offered (Theorem 1). If a_2 is too small to

a competitive equilibrium, but firms have a comparative advantage at selling with warranties ($\alpha_W < \alpha_N$), consumers again see only warranties, but at noncompetitive prices (Theorem 2). Should firms have a comparative advantage at selling without warranties, they will both deteriorate warranty coverage and charge noncompetitive prices. Indeed, if the number of shoppers in this case is sufficiently small, the market for warranties will disappear altogether; consumers see only goods sold without warranties and at supracompetititve prices (Theorem 3).

The cases that Theorem 3 models are the least desirable normatively. The worst case, when no warranties are seen although all consumers prefer them, is less likely to arise if a fair number of consumers shop. If fewer consumers than this shop, firms have incentives to exploit them by deteriorating warranty coverage and by charging monopoly prices. Should firms have a comparative advantage, as defined here, at selling <u>with</u> warranties at the monopoly price, they would never have an incentive also to deteriorate warranty coverage; Theorem 3 cannot apply. The appropriate comparative advantage obtains when (1) it costs little more to sell with warranties than without them ($F' = 0$ or is small), and (2) consumers strongly prefer warranties. Respecting the rationale for these conditions, if consumers strongly prefer warranties, the highest price they would be willing to pay for the good with a warranty should significantly exceed the highest price they would be willing to pay for the good without a warranty; hence, a firm offering the good at its highest price would need fewer customers to break even when selling with warranties than when selling without them, unless it costs considerably more to make a warranty. Condition (1) rules this possibility out. Our model therefore yields the seemingly sensible result that warranties will be more common when they cost relatively little to make and are strongly preferred, even in environments characterized by considerable imperfect information.[10]

B. No Consumers Prefer Warranties

This case can be described more quickly. Let l_W be the consumers' willingness to pay for the good with a warranty and l_N be the consumers' willingness to pay for the good without a warranty. Also, we again define the "comparative advantage" to firms of selling with and without warranties in terms of willingness to pay and costs. Here β_N is the comparative advantage of selling without warranties:

$$\beta_N = F/(l_N - c).$$

Similarly, β_W is the comparative advantage at selling with warranties:

$$\beta_W = (F + F')/(l_W - c_W).$$

Theorem 4. When no consumers prefer warranties and enough consumers comparison shop, a competitive equilibrium can occur in which all firms sell without warranties and all prices are competitive. The necessary and sufficient conditions for this outcome to obtain are:

(i) $a_1 \leq \beta_N/s$

(ii) $a_1 \leq \beta_W/s_W$.

Proof of Theorem 4:

If all firms charge the competitive price p_N^*, a firm wishing to deviate but not offer warranties will charge l_N because it sells only to nonshoppers. Also, with all firms charging p_N^*, the consumer firm ratio, σ, must equal s. Then, for a deviation from the competitive price to yield nonpositive profits, it must be that $a_1 s(l_N - c) - F \leq 0$. This reduces to $a_1 \leq \beta_N/s$. If a firm wishes instead to deviate by offering a warranty, it will charge l_W since again it gets only nonshoppers. For this strategy to yield nonpositive profits, $a_1 s_W(l_W - c_W) - (F + F') \leq 0$. This reduces to $a_1 \leq \beta_W/s_W$. Q.E.D.

Whether or not the competitive equilibrium just described is unique depends on whether or not $l_W - p_W^*$ is greater or less than $l_N - p_N^*$. To understand the relevance of this condition, recall that if too few shoppers exist to generate a competitive equilibrium, the question is whether firms will exploit consumers by charging supracompetitive prices or by both charging these prices <u>and</u> offering unwanted warranty coverage. Which outcome occurs is a function of the relationship between the consumers' willingness to pay for a warranty and the expected marginal cost to firms of offering warranty coverage. A consumer's willingness to pay for a warranty may be conceptualized as the difference between the highest price that a consumer is willing to pay for the good with a warranty and the highest price that the consumer is willing to pay for the good without one. If this willingness to pay for warranty protection is less than the marginal cost to firms of offering warranties, it could never be profitable for a firm to force unwanted coverage on consumers. Imperfect information could be exploited only through charging excessive prices.

In our model, consumers buy one unit or none and firms sell up to a capacity constraint. These assumptions yield "step function" demand curves and nondifferentiable average cost curves. Hence it is possible, within the model, for the consumers' willingness to pay for warranty protection to exceed the marginal cost of providing warranties, even though all consumers when facing only competitive prices would eshew warranty protection. Appendix II characterizes the various equilibria that could arise in this event. If we make the more typical assumption that firms have differentiable, u-shaped average cost curves, it turns out that consumers would never be willing to pay for warranty protection when they do not prefer warranties. To see why, realize that the case when consumers would be

unwilling to pay for warranty protection occurs when

$$l_W - l_N < c_W - c_N.$$

With "normal" cost curves, price equals marginal cost in competitive equilibrium: $c_W = p_W^*$; $c_N = p_N^*$. Hence, we have

$$l_W - l_N < p_W^* - p_N^*.$$

The left side of this inequality is the willingness to pay for warranty coverage; the right side is the premium a firm will charge for offering a warranty when the product is sold with and without warranties but at the respective competitive prices. The inequality necessarily holds, because we have defined the preference against warranties in terms of the refusal of consumers to pay the premium for warranty coverage that the market must charge when all prices are competitive. Hence, under more realistic assumptions respecting costs, when consumers do not prefer warranties, firms have no incentive to offer them; warranties would not sell. If an insufficient number of consumers comparison shop to generate a competitive equilibrium, prices will be noncompetitive but consumers will get the contract terms they prefer. This last case occurs when $a_1 > \beta_N/s$ (see Theorem 4 (i)).

4. <u>Extensions, Implications and Limitations</u>

A. <u>Positive Extensions</u>

The model developed here is interpretable in "pure" quality terms. Suppose that a heterogeneous good is sold that is described by price and some non-price quality attribute respecting which all consumers prefer higher levels. Let the high quality version of this product be produced with greater fixed costs, no lesser marginal costs and a smaller firm capacity than the low quality version. Then, the market can support an inefficient quality level if but only if (i) an insufficient number of consumers shop to generate a competitive equilibrium in the market for the high quality product <u>and</u> (ii) firms have a comparative advantage, as defined above, in producing low quality goods. If the comparative advantage runs the other way, firms again exploit imperfect information only by noncompetitive pricing in the high quality market.

Also, our model rules out signalling, for it is pointless of firms to send quality signals when all consumers have perfect information respecting quality; the consumers, that is, know π, the failure probability. Nevertheless, the model sheds some light on signalling theories. It shows that when consumers have perfect quality information and unanimously prefer warranties, equilibria can exist in which few or no warranties are offered. This implies that insufficient consumer search reduces the incentive of firms to use warranties to convey information about product quality. Thus, signalling equalibria are unlikely to emerge unless consumers possess considerable information about prices and contract terms. The failure of signalling models to recognize this fact may partly explain their

inconsistency with the data. A model that integrates search and signalling behavior would significantly extend understanding of the relative role that these phenomena play in warranty markets, as well as in other markets in which information about product quality is costly to obtain.

B. <u>Normative Implications</u>

A decisionmaker concerned with imperfect information should want to know (a) whether insufficient consumer search has caused a given market to behave noncompetitively; (b) the form that noncompetitive behavior is likely to take -- whether it is excessive prices, the offering of less preferred contract terms, or both; and (c) how noncompetitive behavior is best remedied. These questions are seldom asked rigorously, largely because rigorous tools with which to answer them are lacking. The model set forth here represents an attempt to fill this gap. As an example of its possible utility in evaluating market outcomes, suppose that representative data for the comparative advantages to firms in a particular market of selling with and without warranties can be obtained. A rough estimate of how much shopping is necessary to yield a competitive outcome in that market may be derived. Measuring actual shopping behavior should then illuminate the question whether the market is performing well or badly. Similarly, if such data showed, when consumers preferred warranties, that firms had a comparative advantage at making them, then even seemingly "thin" warranty coverage may actually be the product of consumer preferences; for when the comparative advantage runs this way, firms exploit market power due to insufficient search only by charging noncompetitive prices. If consumers engage in relatively little search, however, these prices probably are being charged.

Respecting remedies to ease information problems, because desirable equilibria are in significant part a function of the amount of comparison shopping in which consumers engage, the model also implies that regulation, when called for, should attempt to reduce the costs to consumers of directly comparing warranty coverage across firms. Statutes that encourage firms to set forth the terms of coverage in "plain" language thus seem inferior to legislation requiring all firms to standardize the language in which warranties are quoted. Also, our model suggests that in many markets firms are more likely to exploit insufficient search by charging noncompetitive prices than by offering undesirable warranty coverage. Thus, the law's strong efforts to expand warranty coverage seem misconceived. Perhaps the state should instead encourage greater search in warranty markets and accept the outcomes that search produces.

C. Limitations

These policy implications should be regarded more as tentatively set forth than as firmly suggested. This is initially because it will be very difficult--perhaps impossible--to obtain the data our model requires to evaluate market outcomes rigorously (see also Schwartz and Wilde, 1979). Hence, its use will at best suggest rather than demonstrate how particular markets are performing.

Suggestive results based on theory, though, seem an advance over what now passes for judgments of market performance. The model's use for policy purposes, however, is also limited by the strong assumptions on which it rests. What is the possible impact of relaxing some of these assumptions?

We make the strong assumption that consumers know failure probabilities.[11] Suppose we substitute the weaker assumption that perceived failure probabilities are a function of actual ones but that mistakes are possible. Three possible outcomes could obtain: (a) Perceived failure probabilities cluster in an unbiased way about actual ones. Then if each firm's demand curve is representative of the consumer population, firms probably will be induced to provide the correct coverage; they will respond as if each consumer knew the failure probability.[12] (b) Perceived failure probabilities cluster around points higher than the actual ones; consumers are "pessimistic," in that they believe product performance to be worse than it is in fact and consequently demand excessive warranty coverage. If sufficient consumer search occurs, the outcomes in this second case nevertheless should be satisfying normatively. To see why, suppose that consumers are pessimistic and firms respond by offering broad warranty coverage at prices that would be justified only if consumer perceptions were correct. A firm could costlessly reduce the price for this coverage because its warranty promise need not be redeemed; the product actually works well. If even some consumers shopped, a firm that so reduced prices would increase its demand, and thereby earn positive profits. Hence, a market outcome in which all firms overcharged consumers for warranty coverage would not be an equilibrium; firms would have an incentive to reduce the price for the excessive coverage that consumers demand. No firm would reduce its price for a warranty below the level at which it could recover its cost; when price equals cost it will reflect actual failure probabilities. This is the competitive price, and if enough consumers shop it will also be the market price. Thus, consumers will pay the correct price for warranty coverage despite their misperception. If insufficient search occurs, prices will be too high, but this is only to say that the problem is insufficient search, not the misperception of failure probabilities, _if_ consumers generally are pessimistic. (c) Consumers are optimistic; they demand less warranty coverage than they should want because they believe failure probabilities to be lower than they actually are.

This third case is troublesome because firms have no incentive to expand warranty coverage without increasing prices, since this sort of warranty promise will have to be redeemed. The market will not correct for optimism. Therefore, the question is whether consumers in general are optimistic. The very sparse evidence that exists suggests not,[13] and consumers commonly are assumed to be risk averse, which is inconsistent with excessive optimism. In any event, this plainly is a question on which facts are more desirable than speculation. To the extent that optimism is common and providing data about correct failure probabilities very expensive, it may be wise to expand

warranty coverage through regulation.

We also assume that consumers cannot affect failure probabilities. Suppose they can, if only by using the product more or less intensively. In one sense, this ability is irrelevant to our analysis. That consumers can affect failure probabilities will influence their demand for warranty coverage, but we take this demand as given, asking only whether markets will respond adequately to it. For example, suppose that consumers are less careful when they have warranty protection. Should consumers be risk neutral, an arguably reasonable approximation here, such increased carelessness would cause s_W to fall, for $s_W = s(1 - \pi)$. Also, α_W would fall because warranties would become more important to consumers, and firms have a comparative advantage relative to consumers at making them (b_W rises faster than c_W). If it costs relatively little to make a warranty, s_W would fall faster than α_W; Theorem 1 then implies that a competitive equilibrium is more likely to obtain. In addition, firms will be less likely to deteriorate warranty coverage when moral hazard exists. This is because moral hazard causes α_W to fall but leaves α_N unaffected; since α_W necessarily falls relative to α_N, firms are more likely to have a comparative advantage at selling with warranties.[14] Hence, our analysis seemingly accommodates relaxation of the assumption that consumers cannot affect π. This response is too simple, however, because when consumers can influence failure probabilities, they may desire different warranty coverages. A consumer with ten children could want a stronger warranty on a washing machine than a consumer with no children. We instead assume that consumers are homogeneous; in our model, all want warranties or none do.

Suppose that some consumers in a given market prefer warranties but others do not (or some prefer stronger warranties than others). We have shown in an earlier paper (Schwartz and Wilde, 1982) that when consumer preferences are heterogeneous and firms can offer different products, the markets for the products will often segment; firms selling particular products will sell only to consumers who prefer those products. Applying that analysis here, unless consumers who prefer warranties will buy without warranties if their search reveals only dealers who refuse to warrant, and unless consumers who do not prefer warranties will buy with warranties if their search reveals only dealers who warrant, the two markets will segment: all consumers who prefer warranties will be in one market, and all consumers who do not will be in the other. In effect, these will be markets for homogeneous goods, a case we have previously analyzed (see Schwartz and Wilde, 1979). Warranty and nonwarranty markets will also segment unless the marginal cost to firms of offering warranties exceeds the marginal willingness to pay for them of consumers who do not prefer warranties but is less than the marginal willingness to pay for them of consumers who do prefer warranties. The rationale of this second condition for nonsegmentation is that if the marginal cost of making warranties is less than the marginal willingness to pay for them of both sets of consumers, all consumers would be in the warranty market; similarly, if the marginal cost of making warranties exceeds

the willingness to pay for them of both sets of consumers, all consumers would be in the nonwarranty market. When the second condition for nonsegmentation does not hold, the analysis made in this paper would apply, for then all consumers in a particular market would prefer warranties or none would. Since the two conditions for nonsegmentation just described are nontrivial, when consumer preferences for warranties are heterogeneous, our existing analyses often will suffice.

We also have shown that when markets for heterogeneous goods interact, a competitive equilibrium will occur in both if a sufficient number of consumers comparison shop. Hence, our recommendation here that the performance of badly behaved warranty markets will be improved by reducing the costs to consumers of comparison shopping holds even when consumer preferences for warranties are assumed to be heterogeneous. However, no one has yet described the kinds of noncompetitive equilibria that will exist in markets in which products and consumers are heterogeneous and insufficient comparison shopping occurs. The practical importance of this failing is that neither the response of firms to imperfect information nor the features that characterize bad equilibria in these cases are now known. Therefore, if consumer preferences for warranty coverage are heterogeneous and the resultant warranty markets do interact, we cannot now tell decisionmakers how to recognize poorly performing warranty markets. Nor can we provide policy suggestions, apart from the recommendation that increasing comparison shopping is likely to improve poor performance. How significant a limitation on our analysis consumer heterogeneity creates is an empirical question.

FOOTNOTES

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1. Murray v. Holliday Rambler, Inc., 83 Wis. 2d 406, 265 N.W. 2d 513 (1978).

2. Magnuson-Moss Act section 102(a).

3. Magnuson-Moss Act section 108(a).

4. Magnuson-Moss Act section 102(a).

5. If a consumer buys the good without a warranty and it fails, we assume for convenience that the consumer exits the market rather than repurchases.

6. The assumption that π is known to consumers is not strictly necessary to the positive aspect of our results, in that the equilibria we characterize do not depend on consumers knowing π; rather, we ask only how consumer preferences for warranty

coverage, whether "mistaken" or "correct," are reflected in market performance. The assumption that consumers know π is relevant to a normative evaluation of our results, in that if the market accurately reflects mistaken preferences, the resultant equilibrium cannot be pareto optimal. Part 4 later discusses the normative implications of relaxing the assumption that π is known.

7. This search strategy is consistent with Stigler's original work on the economics of information but somewhat at odds with more recent search-theoretic models of consumer behavior in markets for heterogeneous goods (e.g. Wilde). For a justification of this strategy in the present context, see Wilde and Schwartz; Schwartz and Wilde (1979).

8. See Magnuson-Moss Act section 103.

9. By "willingness to pay" we refer to the highest price that a consumer would be willing to pay for the good with a warranty, and the highest price that would be paid for the good without one. We assume $h_W > h_N$ because it would be irrational even of consumers who do not prefer warranties to be willing to pay more for the good without a warranty than with one, a warranty being a desirable product feature. In our model, a consumer does not prefer a warranty only if, when offered the opportunity to buy the product with and without a warranty at each item's competitive price, the consumer is unwilling to pay the premium

for warranty coverage that firms must charge to recover their costs. For convenience, we assume that the willingness to pay--h_W and h_N--are identical for all consumers; the model's results are qualitatively unchanged if this assumption is relaxed. See Wilde and Schwartz, 1979. The absolute magnitude of h_W and h_N are in part a function of consumer preferences; for example, h_W is likely to be higher, other things equal, if consumers prefer warranties.

10. This result is of normative interest if, as decisionmakers commonly assume, warranties are "good things" for consumers to have. Warranties could be good things because it may seem prudent of consumers to insure against significant harms, such as a major consumer durable being a lemon, and because, in a related vein, positive externalities to the making of warranties may be thought to exist. As to these, a consumer's family may also benefit from his or her purchase of warranty protection. Section 2-318 A of the Uniform Commercial Code, in pursuance of the belief that externalities of this sort exist, provides that "a seller's warranty . . . extends to any natural person who is in the family or household of his buyer or is a guest in his home if it is reasonable to expect that such person may use, consume or be affected by the goods and who is injured in person by breach of the warranty."

11. An additional assumption made above, that is standard in the economic literature, is that each firm sells a single product; here, each firm sells with warranties or without them, but cannot offer different coverages. In actual markets, firms sometimes sell with a standard warranty but offer consumers an optional warranty at extra cost that is either more extensive or of longer duration than the regular warranty. Our analysis does not permit a formal evaluation of this practice, but we suspect that its welfare effects are ambiguous. The gain to consumers is an increased likelihood of getting warranties they want at reduced search costs. If consumers search for desired warranty coverage as well as for low prices, however, the presence of "multicoverage firms" may in fact reduce search, which could cause prices to rise. A similar ambiguous welfare effect could attend regulation that requires firms to expand warranty coverage. If consumers would search less because they knew that every firm offers a good warranty, prices could rise. On the other hand, if regulation is limited to the case that Theorem 3 describes, where firms have a comparative advantage at selling without warranties and present search is insufficient to generate competitive equilibria, regulation may produce net welfare gains, administrative costs aside.

12. This is likely to be the case if the standard deviation of consumer estimates is relatively small.

13. Corville and Hausman report a 1975 survey conducted by the University of Michigan Survey Research Center, in which consumers perceived a need for repairs in home appliances that was considerably greater than the failures they had actually experienced in past periods. Strictly speaking, this survey only supports the hypothesis that consumers think things are getting worse, not that they are pessimistic relative to actual failure probabilities. The survey nevertheless seems more consistent with a pessimistic than with an optimistic attitude.

14. Also, suppose we relax the assumption that π is exogeneous to provide that (i) firms reduce π—make products more reliable—when they make warranties and (ii) consumers are risk neutral. Then if the willingness to pay (h_W) rises more rapidly than firm costs (c_W), a_W falls and warranty coverage is less likely to be a problem. Otherwise, coverage is more likely to be an issue. If consumers are risk averse, a reduction in failure probabilities by firms is more likely to make coverage a problem, but the welfare implications of this conclusion are ambiguous.

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APPENDIX I

Proof of Theorem 3:

The proof of this result is tedious and begins with a series of Lemmas.

Lemma 1: $G_N(p_N^*) = 0$.

Proof of Lemma: Suppose $G_N(p_N^*) > 0$. Then expected demand at p_N^* is s (by zero profits). A firm charging $\bar{p}_W \equiv h_W - h_N + p_N^*$ will also attract s consumers. But it's profits are then

$$s_W(\bar{p}_W - c_W) - (F + F') > s_W(p_W^* - c_W) - (F + F') = 0$$

since $\bar{p}_W > p_W^*$ (when $h_W - h_N > p_W^* - p_N^*$) and $s > s_W$. Q.E.D.

Lemma 2: If $n_N > 0$ and $n_W > 0$ the supports of the distributions $G_N(\cdot)$ and $G_W(\cdot)$ cannot have more than a single point in common in the sense that $p_N \in \text{supp}\, G_N$ and $h_W - h_W + p_N \in \text{supp}\, G_W$.

Proof of Lemma: Suppose two such prices exists, p_N and q_N. Now expected demand at p_N equals expected demand at $h_W - h_N + p_N$ and expected demand at q_N equals expected demand at $h_W - h_N + q_W$. Thus, using obvious notation, we have

$$D_N(p_N)(p_N - c) - F = D_W(h_W - h_N + p_N)(h_W - h_N + p_N - c_W) - (F + F') \quad \text{(A1)}$$

and

$$D_N(q_N)(q_N - c) - F = D_W(h_W - h_N + q_N)(h_W - h_N + q_N - c_W) - (F + F'). \quad \text{(A2)}$$

Solving (A1) and (A2) for $D_N(p_W)$ and $D_N(q_N)$ we get

$$D_N(p_N) = (F'/k_W) = D_N(q_N),$$

where $k_W = (h_W - c_W) - (h_N - c)$. Thus expected demand is equal at p_N and q_N which violates zero profits. Q.E.D.

We are now ready to consider the possible equilibrium configurations in more detail. Three possibilities exist. In each $n_N > 0$ and the distribution in the nonwarranties market covers noncompetitive prices only. However, the warranties market can be nonexistent, competitive or noncompetitive. We consider the possibilities in that order.

<u>Lemma 3</u>: Necessary and sufficient conditions for $n_W = 0$ are:

(i) $a_1 s_W \geq \alpha_N(a_1 + 2a_2)$

(ii) $a_1 F'/k_W \geq \alpha_N(a_1 + 2a_2)$

<u>Proof of Lemma</u>: If $n_W = 0$ then the maximum price in the nonwarranties market is h_N. Zero profits then implies $\sigma = \alpha_N/a_1$. Let q_N be the minimum price in the nonwarranties market. Zero profits implies $[(A_1/N) + (2A_2/N)](q_N - c) - F = 0$, or

$$q_N = c + [a_1 F/\alpha_N(a_1 + 2a_2)]$$

Now it must be the case that $q_N \geq c + (F/s_W)$ or firms could enter the warranties market at $h_W - h_N + q_N$ and earn positive profits. This

reduces to condition (i).

Consider a firm entering the warranties market. We need to know the form of $\pi_W(q)$ for $q \varepsilon [q_W, h_W]$ where $q_W = h_W - h_N + q_N$. In general

$$\pi_W(q) = \sigma\{a_1 + 2a_1[1 - G_N(h_N - h_W + q)]\}(q - c_W) - (F + F'). \quad \text{(A3)}$$

But zero profits implies

$$1 - G_N(p) = [F - a_1\sigma(p - c)]/2a_2(p - c)\sigma \quad \text{(A4)}$$

Substituting (A4) into (A3) gives

$$\pi_W(q) = [F(q - c_W)/(h_N - h_W + q - c)] - (F + F')$$

which is decreasing in q when $k_W = (h_W - c_W) - (h_N - c) > 0$ (which it is when $F' \geq 0$). Thus we need only ensure $\pi_W(q_W) \leq 0$ — condition (ii). Q.E.D.

<u>Lemma 4</u>: Necessary and sufficient conditions for $n_W > 0$ and $G_W(p_W^*) = 1$ are:

(i) $(a_1 + 2a_2)\alpha_N > a_1 s_W$

(ii) $a_1 s_W > \alpha_N$

(iii) $a_1 F'/k_W \geq 2a_1 s_W - (a_1 + 2a_2)\alpha_N$

<u>Proof of Lemma</u>: Again, the maximum price in the nonwarranties market is h_N and hence $\sigma = \alpha_N/a_1$.

In this case q_N is given by $\sigma(a_1 + 2a_2 n_N)(q_N - c) - F = 0$. Zero profits at p_W^* implies $\sigma\{a_1 + 2a_2[n_N + (n_W/2)]\} = s_W$. Solving for

n_N, n_W and q_N gives

$$q_N = c + \{a_1 F/[2a_1 s_W - (a_1 + 2a_2)\alpha_N]\},$$
$$n_N = (a_1 s_W - \alpha_N)/a_2\alpha_N,$$
$$n_W = [(a_1 + 2a_2)\alpha_N - a_1 s_W]/a_2\alpha_N.$$

Condition (i) is given by $n_W > 0$ and condition (ii) by $n_N > 0$. These conditions also guarantee $c + (F/s_N) < q_N < h_N$ (see the proof of Lemma 3).

Condition (iii) guarantees $\pi_N(q_W) \leq 0$ where $q_W = h_W - h_N + q_N$. This suffices to rule out entry in the warranties market above p_W^* since $\pi_W(p)$ is decreasing as in the proof of Lemma 3. Q.E.D.

<u>Lemma 5</u>: Necessary and sufficient conditions for $n_N > 0$ and $n_W > 0$ with both markets noncompetitive are:

(i) $s_W \geq F'/k_W$

(ii) $F'/k_W \geq \alpha_N$

(iii) $(1 + a_2)k_W\alpha_N \geq a_1 F'$

(iv) $a_1 F'/k_W \leq 2a_1 s_W - (1 + a_2)\alpha_N$

<u>Proof of Lemma</u>:

It is impossible for both $p_N^{max} = h_N$ and $p_W^{max} = h_W$ since zero profits for both would be violated. Suppose $p_w^{max} = h_W$ and $p_n^{max} < h_N$. Zero profits at h_W implies $\sigma = \alpha_W/a_1$. Nonentry at h_N then implies $\alpha_W \geq \alpha_N$. Since $p_N^{max} < h_N$, there must be firms located continuously on $h_W - h_N + p_N^{max}$ in the warranties market. As in the proof of Lemma 2, expected demand will be equal at p_N^{max} and $h_W - h_N + p_N^{max}$. Furthermore, zero profits implies

$$\sigma\{a_1 + 2a_2 n_W[1 - G_W(h_W - h_N + p_N^{max})]\}(h_W - h_N + p_N^{max} - c) - (F + F') = 0$$
$$\sigma\{a_1 + 2a_2 n_W[1 - G_W(h_W - h_N + p_N^{max})]\}(p_N^{max} - c) - F = 0$$

Hence $p_N^{max} = c + (Fk_W/F')$, a constant. Let $q_N \equiv c + (Fk_W/F')$ and $q_W = h_W - h_N + q_N$. Next, note that Lemma 2 also implies $G_W(h_W - h_N + p_N^{max}) \equiv G_W(q_W) = 0$ (otherwise the supports of G_N and G_W would have two points "in common"). Thus we have

$$\sigma(a_1 + 2a_2 n_W)(q_N - c) - F = 0$$
$$\sigma(a_1 + 2a_2 n_W)(q_W - c_W) - (F + F') = 0,$$

which implies $n_W = a_1(F' - k_W\alpha_N)/2a_2 k_W\alpha_N$. That $n_W > 0$ then implies $F' - k_W\alpha_N > 0$, or $\alpha_W < \alpha_N$, which contradicts $\alpha_W \geq \alpha_N$.

Thus when $n_W > 0$ and some warranties are offered at noncompetitive prices, $p_W^{max} = q_W$ and $p_N^{max} = h_N$. Zero profits at q_N and q_W, respectively, now imply

$$\sigma(a_1 + 2a_2 n_N)(q_N - c) - F = 0,$$
$$\sigma(a_1 + 2a_2 n_N)(q_W - c_W) - (F + F') = 0.$$

Following the above procedure, we now get

$$n_N = a_1(F' - k_W\alpha_N)/2a_2 k_W\alpha_N,$$
$$n_W = [(a_1 + 2a_2)k_W\alpha_N - a_1 F']/2a_2 k_W\alpha_N,$$
$$q_N = c + (Fk_W/F').$$

Condition (i) is given by $q_N > c + (F/s_W)$, condition (ii) by $n_N > 0$ and condition (iii) by $n_W > 0$. Condition (iv) guarantees $G_W(\cdot)$ is nondegenerate. To derive it we first calculate $G_W(p_W^*)$. Zero profits at p_W^* implies

$$(A_1/N) + (2A_2/N)\{(N_N/N) + (N_W/N)[(G_W(p_W^*)/2) + 1 - G_W(p_W^*)]\} = s_W.$$

Solving for $G_W(p_W^*)$ gives

$$G_W(p_W^*) = 2k_W[(1 + a_2)\alpha_N - a_1 s_W]/[(1 + a_2)k_W\alpha_N - a_1 F'].$$

Condition (iv) is just $G_W(p_W^*) < 1$. Q.E.D.

The proof of the theorem follows from these lemmas. That $a_1 s_W > \alpha_N$ and $\alpha_N < \alpha_W$ are necessary follows easily from noting the latter is equivalent to $\alpha_N < F'/k_W$.

Sufficiency is more troublesome. Label the three types of equilibrium considered in Lemmas 4, 5, 6 as type A, type B and type C, respectively. Eliminating the conditions for each which are directly implied by $a_1 s_W > \alpha_N$ or $\alpha_N < \alpha_W$, we have, for each case,

A:	(i)	$(a_1 + 2a_2)\alpha_N > a_1 s_W$	(a)
	(ii)	$a_1 F'/k_W \geq 2a_1 s_W - (a_1 + 2a_2)\alpha_N$	(b)
B:	(i)	$s_W > F'/k_W$	(c)
	(ii)	$(a_1 + 2a_2)\alpha_W > a_1 F'/k_W$	(d)
	(iii)	$2a_1 s_W - (a_1 + 2a_2)\alpha_N > a_1 F'/k_W$	(~b)
C:	(i)	$(a_1 + 2a_2)\alpha_N < a_1 s_W$	(~a)
	(ii)	$(a_1 + 2a_2)\alpha_N < a_1 F'/k_W$	(~d)

These conditions are closely related; in fact they reduce to four constraints or their negations. These are labeled a, b, c and d. We need to show that for any <u>consistent</u> combination of a, b, c, d or

their negations, one of the equilibria hold. Table 1 shows that each consistent combination yields an equilibrium, and also illustrates the nature of the inconsistency for the others.

Q.E.D.

APPENDIX II

This appendix considers the case in which consumer prefer not to buy warranties given competitive prices. Limit prices for the good with and without warranties are l_W and l_N respectively. That consumers prefer not to buy warranties means $l_W - l_N < p_W^* - p_N^*$. Further, we define

$$\beta_N = F/(l_N - c),$$
$$\beta_W = (F + F')/(l_W - c_W),$$
$$k_N = (l_W - c_W) - (l_N - c).$$

Unlike the case in which consumers prefer warranties, k_N can be either positive or negative. While $k_N < 0$ is the "natural" assumption given $l_N - p_N^* > l_W - p_W^*$, there is nothing to rule out $k_N > 0$. This causes some complications in the necessary and sufficient conditions for the various types of equilibria, but the techniques of proof are similar to those used in appendix I. Additional notation will either be defined as needed or will be obvious. In general $G_N(\cdot)$ will refer to the distribution of prices in the nonwarranties market and $G_W(\cdot)$ will refer to the distribution of prices in the warranties market.

<u>Theorem 1</u>: Necessary and sufficient conditions for <u>nonwarranties competitive</u> and <u>warranties nonexistent</u> are:

(i) $a_1 \leq \beta_N/s$

(ii) $a_1 \leq \beta_W/s_W$.

<u>Proof</u>: With all firms charging p_N^* it must be that $A_1/N = a_1$ or $\sigma = \beta_N/s$. A firm deviating will either charge l_N for nonwarranties or l_W for warranties. In either case it gets only nonshoppers, but in the warranties market capacity equals $s_W < s$. Thus we need $a_1\sigma(l_N - c) \leq F$ and $S(l_W - c_W) \leq F + F'$. These yield (i) and (ii) respectively. Q.E.D.

<u>Theorem 2</u>: Necessary and sufficient conditions for <u>nonwarranties nonexistent</u> and <u>warranties competitive</u> are:

(i) $a_1 \leq \beta_N/s_W$

(ii) $a_1 \leq \beta_W/s_W$

(iii) $k_N \geq [(a_1 + 2a_2)(F + F') - F]/s_W(a_1 + 2a_2)$.

<u>Proof</u>: Conditions (i) and (ii) are derived in a fashion analogous to Theorem 1. Condition (iii) guarantees that entering the nonwarranties market at prices arbitrarily close to $\bar{p}_N - \varepsilon$ is unprofitable. In this case expected profit is

$$\pi_N(\bar{p}_N - \varepsilon) = \sigma(a_1 + 2a_2)(\bar{p}_N - \varepsilon - c) - F.$$

But $\sigma = \beta_W/a_1$ and $\bar{p}_N = l_N - l_W + p_W^* = l_N - l_W + c_W + (F + F')/s_W$. Requiring that $\pi_N(\bar{p}_N - \varepsilon) \leq 0$ for all $\varepsilon > 0$ yields (iii). Q.E.D.

<u>Theorem 3</u>: Necessary and sufficient conditions for <u>both markets competitive</u> are:

(i) $a_2 s > s - s_W > a_2 s_W$

(ii) $a_1(s - s_W)/a_2 \leq \min(\beta_N, \beta_W)$

(iii) $k_N \geq [(F + F')/s_W] - \{a_2 F/[(a_1 + 2a_2)s_W - a_1 s]\}$.

Proof: Zero profits at p_N^* and p_W^* requires expected demand equal s and s_W respectively. Hence

$$\sigma\{a_1 + 2a_2[n_N(1/2) + n_W]\} = s$$

and

$$\sigma[a_1 + 2a_2 n_W(1/2)] = s_W.$$

Solving for σ, n_N and n_W (using $n_W + n_N = 1$) yields:

$$\sigma = (s - s_W)/a_2$$

$$n_N = [s - (a_1 + 2a_2)s_W]/a_2(s - s_W)$$

$$n_W = (s_W - a_1 s)/a_2(s - s_W).$$

Condition (i) comes from $n_N > 0$ and $n_W > 0$. Next consider deviant firms. Possible profit maximizing prices are $\bar{p}_N - \varepsilon$ (for $\varepsilon > 0$), 1_N, and 1_W. Nonpositive profits at the latter two of those yields condition (ii) and at the former yields condition (iii). Q.E.D.

Theorem 4: Necessary and sufficient conditions for nonwarranties nonexistent and warranties noncompetitive are:

(i) $\beta_W < a_1 s_W$

(ii) $\beta_W < \beta_N$

(iii) if $(a_1 + 2a_2)\beta_W > a_1 s_W$ then

$$[k_N \geq [(F + F')/s_W] - [a_1 F/\beta_W(a_1 + 2a_2)].$$

Proof: When the warranties market is noncompetitive and $n_N = 0$, zero profits at 1_W (necessarily the highest price in the warranties market!) implies $\sigma = \beta_W/a_1$.

First we need to guarantee that G_N is nondegenerate. Let G_W^* be the size of any potential mass point at p_W^*. Zero profits implies

$$\sigma\{a_1 + 2a_2[(G_W^*/2) + 1 - G_W^*]\} = s_W,$$

or,

$$G_W^* = [(a_1 + 2a_2)\sigma - s_W]/a_2\sigma.$$

Hence $G_W^* < 1$ iff $a_1 s_W > \beta_W$, condition (i).

Next we calculate profits for entry into the nonwarranties market above $\bar{p}_W$. In general zero profits implies

$$\sigma\{a_1 + 2a_2[1 - G_W(p)]\}(p - c_W) = F + F'.$$

Hence

$$1 - G_W(p) = [(F + F') - a_1\sigma(p - c_W)]/\sigma(p - c_W)2a_2.$$

But

$$\pi_N(q) = \sigma\{a_1 + 2a_2[1 - G_W(1_W - 1_N + q)]\}(q - c) - F,$$

since a firm entering the nonwarranties market at q will lose shoppers to those firms offering warranties at prices less than $1_W - 1_N + q$. Substituting the definition of G_W and simplifying gives

$$\pi_N(q) = (F + F')k_N/(1_W - 1_N + q - c_W)^2.$$

Thus if $k_N \geq 0$ we need $\pi_N(1_N) \leq 0$ which reduces to $\beta_W \leq \beta_N$. If $G_W^* > 0$

we also need to check $\pi_N(\bar{p}_N - \varepsilon)$ for all $\varepsilon > 0$. But $G_W^* > 0$ iff $(a_1 + 2a_2)\beta_W$ and $\pi_N(\bar{p}_N - \varepsilon) \le 0$ for all $\varepsilon > 0$ iff

$$k_N \ge [(F + F')/s_W] - [a_1F/\beta_W(a_1 + 2a_2)].$$

The latter condition implies $k_N \ge 0$ when

$$(F + F')/s_W > a_1F/\beta_W(a_1 + 2a_2).$$

As long as $F' \ge 0$ and $(a_1 + 2a_2)\beta_W > a_1 s_W$, this must always hold. Moreover, $\beta_W \le \beta_N$ is equivalent to $k_N \ge F'/\beta_N$ which also implies $k_N \ge 0$ as long as $F' \ge 0$. Thus conditions (ii) and (iii) suffice to characterize this case if we can show $k_N < 0$ is impossible. But this is trivial since $\pi_N(1_N) \le 0$ must always hold and this implies $k_N \ge 0$ as above. Q.E.D.

Theorem 5: Necessary and sufficient conditions for nonwarranties noncompetitive and warranties nonexistent are:

(i) $\beta_N < a_1 s$

(ii) $k_N < 0$ (which implies $\beta_N < \beta_W$).

(iiia) If $k_N > 0$ and $a_1 s \le (a_1 + 2a_2)\beta_N$ then

(1) $k_N \ge [(F + F')/s_W] - \{a_1F/[2a_2 s - (a_1 + 2a_2)\beta_N]\}$ implies

(α) $a_1(2s - s_W) < \beta_N(a_1 + 2a_2)$

(β) $k_N \le [(F + F')/s_W] - \{a_1F/[2a_1 s - (a_1 + 2a_2)\beta_N]\}$

(2) $k_N < [(F + F')/s_W] - \{a_1F/[2a_2 s - (a_1 + 2a_2)\beta_N]\}$ implies

(α) $k_N \le F'/s_W$.

(iiib) If $k_N > 0$ and $a_1 s > (a_1 + 2a_2)\beta_N$ then

(1) $k_N \ge [(F + F')/s_W] - [a_1F/(a_1 + 2a_2)\beta_N]$ implies

(α) $a_1 s_W > \beta_N(a_1 + 2a_2)$

(β) $k_N \ge a_1F'/(\beta_N(a_1 + 2a_2))$

(2) $k_N < [(F + F')/s_W] - [a_1F/(a_1 + 2a_2)\beta_N]$ implies

(α) $k_N \le F'/s_W$.

Proof: This case is somewhat complicated. First, note that $\sigma = \beta_N/a_1$ in the usual fashion. Also, if G_N^* is the size of any potential mass point at p_N^*, the usual argument for $G_N^* < 1$ yields $a_1 s > \beta_N$, condition (i).

Now consider entry into the warranties market. Using arguments similar to those found in the proof of Theorem 4, we have

$$1 - G_N(p) = [F - a_1\sigma(p - c)]/\sigma(p - c)2a_2$$

and

$$\pi_W(q) = -Fk_N/(1_N - 1_W + q - c)] - (F + F'),$$

whence

$$\pi_W'(q) = Fk_N/(1_N - 1_W + q - c)^2.$$

If $k_N < 0$ then $\pi_W' > 0$ so it matters not whether G_N has a mass point at p_N^*, we need only check $\pi_W(1_W) \le 0$. Expected demand at 1_W is $a_1\sigma$, or β_N. But $k_N < 0$ implies $\beta_N < \beta_W$. Hence $\beta_N < \beta_W < s_W$, and we need $\beta_N(1_W - c_W) \le F + P\_F'$, on $\beta_N < \beta_W$, which is already implied by $k_N < 0$.

Now, suppose $k_N > 0$ so that $\pi_W'(q) < 0$. First, consider a mass point at p_N^*; i.e., $a_1 s \le \beta_N(a_1 + 2a_2)$. Let r_N be the first price above p_W^* actually offered. Then zero profits gives

$$r_N = c + [a_1F/[2a_2s - (a_1 + 2a_2)\beta_N].$$

If $r_N \geq \bar{p}_N$, then one checks to see whether expected demand at r_N is greater or less than s_W, and then checks profits at $r_W = l_W - l_N + r_N$. If $r_N < \bar{p}_N$ then one does the same thing for $\bar{p}_N$ since $\bar{p}_W = l_W - l_N + \bar{p}_N < p_W^*$ in this case.

Consider first $r_N \geq \bar{p}_N$; i.e.,

$$k_N \geq [(F + F')/s_W] - \{[a_1F/[2a_2s - (a_1 + 2a_2)\beta_N]\}.$$

Expected demand at r_N is

$$ED_N(r_N) = [2a_2s - \beta_N(a_1 + 2a_2)]/a_1.$$

Thus $ED_W(r_W) = ED_N(r_N) < s_W$ iff

$$a_1(2s - s_W) < \beta_N(a_1 + 2a_2),$$

where $r_W = l_W - l_N + r_N$. This condition is necessary since $ED_W(r_W) \geq s_W$ means positive profits could be made at r_W. If it holds, then we still need $\pi_N(r_W) \leq 0$, or

$$k_N \leq a_1F'/[2a_2s - (a_1 + 2a_2)\beta_N]$$

If $r_N < \bar{p}_N$; i.e.,

$$k_N < [(F + F')/s_W] - \{a_1F/[2a_2s - (a_1 + 2a_2)\beta_N]\},$$

then we need $\pi_W(p_W^*) \leq 0$, or

$$k_N \leq F'/s_W.$$

Together these arguments give condition (iii).

Finally, suppose there is no mass point at p_N^*; i.e., $a_1s > \beta_N(a_1 + 2a_2)$. Let q_N be the first price above p_N^* actually offered. Then zero profits gives

$$q_N = c + [a_1F/\beta_N(a_1 + 2a_2)].$$

Proceeding as before, $q_N \geq \bar{p}_N$ iff

$$k_N \geq [(F + F')/s_W] - [a_1F/(a_1 + 2a_2)\beta_N].$$

Expected demand at q_N is

$$ED_N(q_N) = \beta_N(a_1 + 2a_2)/a_1.$$

Thus $ED_W(q_W) = ED_N(q_N) < s_W$ iff

$$a_1s_W > \beta_N(a_1 + 2a_2)$$

where $q_W = l_W - l_N + q_N$. Given this condition we need $\pi_W(q_W) \leq 0$, or

$$k_N \geq a_1F'/\beta_N(a_1 + 2a_2).$$

If $q_N < \bar{p}_N$; i.e.,

$$k_N < [(F + F')/s_W] - [a_1F/(a_1 + 2a_2/\beta_N],$$

then we need $\pi_W(p_W^*) \leq 0$ or

$$k_N \leq F'/s_W$$

as before. Q.E.D.

<u>Theorem 6</u>: Necessary and sufficient conditions for <u>nonwarranties competitive</u> and <u>warranties noncompetitive</u> are:

(i) $(a_1 + 2a_2)\beta_W > a_1 s > \beta_W$

(ii) $a_2\beta_W > a_1(s - s_W)$

(iii) $k_N \geq [(F + F')/s_W] - \{a_1 F/[2a_2 s - (a_1 + 2a_2)\beta_W]\}$

(iv) $\beta_W < \beta_N$

<u>Proof</u>: Since the warranties market is noncompetitive, $\sigma = \beta_W/a_1$. Zero profits at p_N^* implies

$$\sigma a_1 + 2a_2[(n_N/2) + n_W] = s,$$

or

$$n_N = [(a_1 + 2a_2)\sigma -]/a_2\sigma$$
$$n_W = (s - \sigma)/a_2\sigma.$$

Condition (i) is given by $n_N > 0$ and $n_W > 0$. Consider $G_W^* = G_W(p_W^*)$. Zero profits again implies

$$\sigma\{a_1 + 2a_2 n_W[G_W^*(1/2) + (1 - G_W^*)]\} = s_W,$$

or

$$G_W^* = [(2s - s_W) - (a_1 + 2a_2)\sigma]/(s - \sigma).$$

Condition (ii) is given by $G_W^* < 1$.

Finally, consider entry into the nonwarranties market above p_N^*. Prices arbitrarily close to $\bar{p}_N$ are always an issue. Following the standard procedure,

$$\pi_N(q) = \{[(F + F')(q - c)]/(l_W - l_N + q - c_W)\} - F$$

and

$$\pi_N'(q) = (F + F')k_N/(l_W - l_N + q - c_W)^2.$$

Now $\pi_N(\bar{p}_N - \varepsilon) \leq 0$ for all $\varepsilon > 0$ iff

$$k_N \geq [(F + F')/s_W] - \{a_1 F/[2a_2 s - (a_1 + 2a_2)\beta_W]\},$$

condition (iii). If $k_N > 0$ then $\pi_N'(q) > 0$ so we need $\pi_N(l_N) \leq 0$, or $\beta_W \leq \beta_N$. This is equivalent to $k_N \geq F'/\beta_N$. Hence $k_N < 0$ is impossible since $\pi_N(l_N) \leq 0$ is necessary whether l_N is the profit maximizing choice for a deviant firm or not! Note also, $k_N \geq F'/\beta_N$ is sufficient for $k_N \geq 0$. Q.E.D.

<u>Theorem 7a</u>: Necessary and sufficient conditions for <u>nonwarranties noncompetitive</u> and <u>warranties competitive</u> when $p_N^{max} = l_N$ are:

(i) $a_1(s - s_W) < a_2\beta_N$

(ii) If $(a_1 + 2a_2)\beta_N > a_1 s$ then:

(1) $(a_1 + 3a_2)\beta_N > a_1(2s - s_W) > (a_1 + 2a_2)\beta_N$

(2) $k_N \geq [(F + F')/s_W] - \{a_1 F/[2a_2 s - (a_1 + 2a_2)\beta_N]\}$

(3) $k_N \leq a_1 F'/[(a_1 + 2a_2)\beta_N - 2a_2(s - s_W)]$.

(iii) If $(a_1 + 2a_2)\beta_N \leq a_1 s$ then:

(1) $(a_1 + 2a_2)\beta_N > a_1 s_W > \beta_N$

(2) $k_N \geq [(F + F')/s_W] - [a_1 F/(a_1 + 2a_2)\beta_N]$

(3) $k_N \leq a_1 F'/[2a_2 s_W - \beta_N(a_1 + 2a_2)]$.

<u>Proof</u>: When the nonwarranties market is noncompetitive, the "normal"

case is for l_N to be the maximum price in that market.

Suppose first there is a mass point at p_N^*. Then it must be that $G_N(\bar{p}_N) = G_N(p_N^*)$ or zero profits would over-constrain the system (we would need zero profits at l_N, p_N^*, p_W^* and $\bar{p}_N - \varepsilon$ for all $\varepsilon > 0$). Setting expected demand equal to capacity at p_N^* and p_W^* gives

$$\sigma[a_1 + 2a_2[n_N(1 - G_N^*/2)) + n_W]] = s$$
$$\sigma[a_1 + 2a_2[n_N(1 - G_N^*) + (n_W/2)]] = s_W.$$

Solving for n_N, n_W and G_N^* yields

$$n_N = [(a_1 + 3a_2)\sigma - (2s - s_W)]/a_2\sigma$$
$$n_W = [(2s - s_W) - (a_1 + 2a_2)\sigma]/a_2\sigma$$
$$G_N^* = [(a_1 + 2a_2)\sigma - s]/[(a_1 + 3a_2)\sigma - (2s - s_W)].$$

Zero profits at l_N implies $\sigma = \beta_N/a_1$. That $G_N^* < 1$ yields condition (i), and $G_N^* > 0$ yields subcase (ii). Condition (ii)(1) is then a restatement of $n_N > 0$ and $n_W > 0$.

Let q_N be the first price above p_N^* actually offered. Then

$$\sigma[a_1 + 2a_2n_N(1 - G_N^*)](q_N - c) = F$$

or

$$q_N = c + \{a_1F/[(a_1 + 2a_2(s - s_W])\}.$$

It must be that $q_N > \bar{p}_N$ or the system would again be over-constrained. Hence we need

$$k_N \geq [(F + F')/s_W] - \{a_1F/[(a_1 + 2a_2)\beta_N - 2a_2(s - s_W)]\}.$$

But $\pi_N(\bar{p}_N - \varepsilon) \leq 0$ for $\varepsilon > 0$ must also hold. This reduces to

$$k_N \geq [(F + F')/s_W] - \{a_1F/[2a_2s - (a_1 + 2a_2)\beta_N]\}.$$

The latter, however, implies $q_N > \bar{p}_N$ when $n_W > 0$. This yield condition (ii) (2).

Finally consider profits in the warranties market above p_W^*. Let $q_W = l_W - l_N + q_N$. Then on $[q_W, l_W]$,

$$\pi_W(p) = [F(p - c_W)/(l_N - l_W + p - c)] - (F + F')$$

whence

$$\pi_W'(p) = -k_NF/(l_N - l_W + p - c)^2.$$

If $k_N \leq 0$ we need $\pi_W(l_W) \leq 0$. But $k_N \leq 0$ is impossible because $n_W > 0$ and $q_N > \bar{p}_N$ imply $k_N > 0$. If $k_N > 0$ then we need $\pi_W(q_W) \leq 0$. Since expected demand at q_W is also less than s_W this reduces to

$$k_N \leq a_1F'/[(a_1 + 2a_2)\beta_N - 2a_2(s - s_W)]..$$

These two constraints yield condition (ii)(3).

When $(a_1 + 2a_2)\beta_N \leq a_1s$ then no mass point appears at G_N^*. Let r_N be the first price above p_N^* actually offered. Then proceeding exactly as above, noting $r_N > \bar{p}_N$ is required, we get (since $G_N(\bar{p}_N) = G_N^*$)

$$n_W = [(a_1 + 2a_2)\sigma - s_W]/a_2\sigma$$
$$n_N = (s_W - \sigma)/a_2 \text{sigma}.$$

That $n_W > 0$ and $n_N > 0$ yields condition (iii)(1). That $\pi_N(\bar{p}_N - \varepsilon) \leq 0$

for all $\varepsilon > 0$ implies $r_N > \bar{p}_N$ and yields condition (iii)(2). Since the form of $\pi_W(p)$ is the same on $[q_W, 1_W]$ as above, and $k_N \leq 0$ is again impossible, condition (iii)(3) is yielded by $\pi_W(q_W) \leq 0$.

<u>Theorem 7b</u>: Necessary and sufficient conditions for <u>nonwarranties noncompetitive</u> and <u>warranties competitive</u> when p_N^{max} is arbitrarily close to $\bar{p}_N$ are:

(i) $F'/s_W < k_N < (F + 2F')/s_W$

(ii) $k_N \lessgtr [(F + F')/s_W] - \{a_2F/[s_W(a_1 + 2a_2) - a_1s]$ as $s_W(a_1 + 2a_2) \gtrless a_1s$

(iii) $s_W[F + 2(F' - k_Ns_W)]/(F + F' - k_Ns_W) < \min\{\beta_W, \beta_N\}$.

<u>Proof</u>: Zero profits at $\bar{p}_N - \varepsilon$ for all $\varepsilon > 0$ and at p_W^* imply

$$\sigma(a_1 + 2a_2n_W)(\bar{p}_N - c) = F$$
$$\sigma[a_1 + 2a_2n_W(1/2)] = s_W.$$

Solving for n_N, n_W and σ gives

$$n_N = [a_2F + (a_1 + 2a_2)(F' - k_Ns_W)]/a_2[F + 2(F' - k_Ns_W)]$$
$$n_W = a_1(k_Ns_W - F')/a_2[F + 2(F' - k_Ns_W)]$$
$$\sigma = s_W[F + 2(F' - k_Ns_W]/a_1(F + F' - k_Ns_W).$$

Now $\bar{p}_N - c > 0$ implies $F + F' - k_Ns_W > 0$. Hence $\sigma > 0$ iff

$$F + 2(F' - k_Ns_W) > 0.$$

This, in turn, implies $n_N > 0$ and $n_W > 0$ iff

$$a_2F > (a_1 + 2a_2)(F' - k_Ns_W)$$

$$k_Ns_W > F'.$$

These can be rewritten, respectively, as

$$k_N < (F + 2F')/2s_W$$
$$k_N > [(a_1 + 2a_2)F' - a_2F]/s_W(a_1 + 2a_2)$$
$$k_N > F'/s_W.$$

But $F'/s_W > [(a_1 + 2a_2)F' - a_2F]/s_W(a_1 + 2a_2)$, so $n_W > 0$ implies $n_N > 0$ when $\sigma > 0$. Thus the second constraint is redundant.

Next, let G_N^* be the size of any potential mass point at p_N^*. Then

$$\sigma\{a_1 + 2a_2[n_N(1 - (G_N^*/2)) + n_W]\} = s,$$

or

$$G_N^* = [(a_1 + 2a_2)\sigma - s]/a_2\sigma n_N.$$

Hence $G_N^* < 1$ iff $(a_1 + 2a_2)\sigma < a_2\sigma n_N$, or

$$k_N > [(F + F')/s_W] - \{a_2F/[s_W(a_1 + 2a_2) - a_1s]\} \quad \text{if } s_W(a_1 + 2a_2) > a_1s$$
$$k_N < [(F + F')/s_W] - \{a_2F/[s_W(a_1 + 2a_2) - a_1s]\} \quad \text{if } s_W(a_1 + 2a_2) < a_1s$$

Finally, we need $\pi_N(1_N) \leq 0$ and $\pi_W(1_W) \leq 0$. These reduce to $a_1\sigma < \beta_N$ and $a_1\sigma < \beta_W$ respectively, or

$$s_W[F + 2(F' - k_Ns_W)]/(F + F' - k_Ns_W) < \min\{\beta_W, \beta_{[illegible]}\} \qquad \text{Q.E.D.}$$

<u>Theorem 8a</u>: Necessary and sufficient conditions for <u>both markets noncompetitive</u> when $p_N^{max} = 1_N$ and $G_N^* = G_N(p_N^*) > 0$ are:

(i) $F'/\beta_N > k_N > F'/s_W$

(ii) $(a_1 + 2a_2)\beta_N > a_1 s$

(iii) $k_N[(a_1 + 4a_2)\beta_N - 2a_2 s] > a_1 F' > k_N[(a_1 + 2a_2)\beta_N - 2a_2 s$

(iv) $k_N[(a_1 + 2a_2)\beta_N - 2a_2(s - s_W)] > a_1 F'$

(v) $k_N \geq [(F + F')/s_W] - [a_1 F/[2a_2 s - (a_1 + 2a_2)\beta_N]]$.

Proof: Suppose $G_N(p_W^*) = G_W^* > 0$. Then the first price above p_N^* actually offered must be t_N where

$$t_N = c + (Fk_N/F').$$

To see this note first that the distributions cannot overlap in the sense that there exists no $p_N \varepsilon$ supp G_N such that $p_W \varepsilon$ supp G_W where $p_W = l_W - l_N + p_N$ except at $p_N = t_N$. Suppose there exists such a $p_N(\neq t_N)$. Then expected demand at p_N equals expected demand at p_W. Zero profits implies this expected demand equals F'/k_N. Since expected demand cannot be constant (or zero profits is violated), there is only one price which can satisfy equal expected demands and zero profits. The latter implies, however, that

$$(F'/k_N)(p_N - c) = F,$$

or $p_N = t_N$ as defined above. It then follows that $p_W = t_W$, where

$$t_W = c_W + [(F + F')k_N/F'].$$

It must be the case that $l_N > t_N > \overline{p}_N$ (the latter because $t_N \leq \overline{p}_W$ would imply $n_W = 0$). Hence we need

$$F'/\beta_N > k_N > F'/s_W.$$

Note the right-hand side of this constraint implies $k_N > 0$.

Next consider n_W, n_N and G_N^*. Zero profits at p_N^* and t_W, respectively, imply

$$\sigma[a_1 + 2a_2[n_N(1 - (G_N^*/2)) + n_W]] = s$$
$$\sigma[a_1 + 2a_2 n_N(1 - G_N^*)](t_N - c) = F.$$

Solving for n_W, n_N and G_N^* give

$$n_N = [(a_1 + 4a_2)\sigma k_N - 2k_N s + F']/2a_2\sigma k_N$$
$$n_W = [2k_N s - F' - (a_1 + 2a_2)\sigma k_N]/2a_2 k_N \sigma$$
$$G_N^* = 2k_N[(a_1 + 2a_2)\sigma - s]/[(a_1 + 4a_2)\sigma k_N - 2k_N s + F'].$$

We require $0 < G_N^* < 1$. Hence $F'/a_1 k_N > \sigma > s/(a_1 + 2a_2)$, the first inequality being equivalent to $l_N > t_N$. We also require $n_N > 0$ and $n_W > 0$, or

$$k_N[(a_1 + 4a_2)\sigma - 2s] > F' > k_N[(a_1 + 2a_2)\sigma - 2s]$$

Now consider the warranties market. Zero profits at p_W^* implies

$$\sigma[a_1 + 2a_2[n_N(1 - G_N^*) + n_W(1 - (G_W^*/2))]] = s_W,$$

or

$$G_W^* = 2k_N[2s - s_W - (a_1 + 2a_2)\sigma]/[2sk_N - F' - (a_1 + 2a_2)\sigma k_N].$$

That $G_W^* < 1$ requires

$$k_N[(a_1 + 2a_2)\sigma - 2(s - s_W)] > F',$$

a condition which implies $(a_1 + 2a_2)\sigma > 2(s - s_W)$ since $k_N > 0$ and $F' \geq 0$.

Now consider entry. Prices arbitrarily close to $\bar{p}_N$ must be ruled out. In the usual fashion this reduces to

$$k_N \geq [(F + F')/s_W] - \{F/[2s - (a_1 + 2a_2)\sigma]\}.$$

Two other prices need to be checked. One is l_W. That $\pi_N(l_W) \leq 0$, however, reduces to $\beta_N < \beta_W$ or $k_N < F'/\beta_N$, a constraint equivalent to $l_N > t_N$. Finally, suppose G_W has a mass point at p_W^* and let q_W be the first price above p_N^* actually offered. Then zero profits implies

$$q_W = c_W + \{(F + F')/(a_1 + 2a_2)\sigma - 2(s - s_W)]\}.$$

That $q_W > p_W^*$ is implied by $G_W^* > 0$ and that $q_W < t_W$ is implied by $G_W^* < 1$. Define $q_N = l_N - l_W + q_W$. Then $\pi_N(q_N) \leq 0$ reduces again to $G_W^* < 1$. A similar set of calculations applies when $G_W^* \geq 0$.

Substituting $\sigma = \beta_N/a_1$ into these various constraints gives the theorem. Q.E.D.

<u>Theorem 8b</u>: Necessary and sufficient conditions for <u>both markets noncompetitive</u> when $p_N^{max} = l_N$ and $G_N^* = G_N(p_N^*) = 0$ are:

(i) $F'/\beta_N > k_N > F'/s_W$

(ii) $(a_1 + 2a_2)\beta_N k_N > a_1 F' > a_2 \beta_N k_N$

(iii) $k_N[2a_2 s_W - (a_1 + 2a_2)\beta_N] > a_1 F'$

(iv) $k_N \geq [(F + F')/s_W] - [a_1 F/\beta_N(a_1 + 2a_2)]$.

<u>Proof</u>: The lowest price in the nonwarranties market is again t_N where t_N and t_W are defined as in the proof of Theorem 8a. Hence we again need $F'/\beta_N > k_N > F'/s_W$. Zero profits at t_N implies $\sigma(a_1 + 2a_2 n_N)(t_N - c) = F$, or

$$n_N = (F' - a_1\sigma k_N)/2a_2\sigma k_N$$
$$n_W = [(a_1 + 2a_2)\sigma k_N - F']/2a_2\sigma k_N.$$

Hence $n_N > 0$ and $n_W > 0$ imply $(a_1 + 2a_2)\sigma k_N > F' > a_2\sigma k_N$. Consider next $G_W(p_W^*) = G_W^*$. Zero profits implies

$$\sigma\{a_1 + 2a_2[n_N + n_W(1 - G_W^*/2)]\} = s_W$$

or

$$G_W^* = 2k_N[(a_1 + 2a_2)\sigma - s_W]/[(a_1 + 2a_2)\sigma k_N - F'].$$

Hence $G_W^* < 1$ iff

$$k_N[2s_W - (a_1 + 2a_2)\sigma] > F'.$$

Again, as in the proof of Theorem 8a, entry is only an issue at prices arbitrarily close to $\bar{p}_N$. This is unprofitable iff

$$k_N \geq [(F + F')/s_W] - [F/\sigma(a_1 + 2a_2)].$$

Substituting $\sigma = \beta_N/a_1$ into these constraints gives the theorem. Q.E.D.

<u>Theorem 8c</u>: It is impossible for <u>both markets to be noncompetitive</u> when $p_W^{max} = l_W$ and $p_N^{max} = t_N$.

<u>Proof</u>: In this equilibrium t_N and t_W remain the only prices at which

the supports of G_N and G_W "overlap", but now $G_N(t_N) = 1$ and $G_W(t_W) < 1$. We still need $\bar{p}_N < t_N < 1_N$, or $F'/\beta_N > t_N > F'/s_W$. We also need $\pi_N(1_N) \leq 0$, or $\beta_W \leq \beta_N$, which is equivalent to $F'/\beta_N \leq k_N$, a contradiction to $t_N < 1_N$. Q.E.D.

<u>Theorem 8d</u>: Necessary and sufficient conditions for <u>both markets noncompetitive</u> when $p_W^{max} = 1_W$ and p_N^{max} <u>is arbitrarily close to $\bar{p}_N$ are:</u>

(i) $[(F + F'))/s_W] - [a_1F/(a_1 + 2a_2)\beta_W] > k_N > [(F + F')/s_W] - (F/\beta_W)$

(ii) $k_N < [(F + F')/s_W] - \{a_1F/[2a_2s - (a_1 + 2a_2)\beta_W]\}$

(iii) $k_N < [(F + F')/s_W] - [F/(2s_W - \beta_W)]$

(iv) $\beta_W \leq \beta_N$.

<u>Proof</u>: Since $p_W^{max} = 1_W$, $\sigma = \beta_W/a_1$. Zero profits at $\bar{p}_N - \varepsilon$ for all $\varepsilon > 0$ implies that

$$\sigma(a_1 + 2a_2n_W)(\bar{p}_N - c) = F,$$

or

$$n_W = [Fs_W - a_1\sigma(F + F' - k_Ns_W)]/2a_2\sigma(F + F' - k_Ns_W)$$

$$n_N = [(a_1 + 2a_2)\sigma(F + F' - k_Ns_W) - Fs_W]/2a_2\sigma(F + F' - k_Ns_W).$$

Thus we need $n_W > 0$ and $n_N > 0$, or

$$(a_1 + 2a_2)\sigma(F + F' - k_Ns_W) > Fs_W > a_1\sigma(F + F' - k_Ns_W.$$

Next consider mass points. At p_N^*

$$\sigma\{a_1 + 2a_2[n_W + n_N(1 - (G_N^*/2))]\} = s,$$

or

$$G_N^* = [(a_1 + 2a_2)\sigma - s]/a_2\sigma n_N.$$

Thus $G_N^* < 1$ iff

$$(a_1 + 2a_2 - a_2n_N)\sigma < s,$$

which reduces to

$$k_N < [(F + F')/s_W] - \{F/[2s - (a_1 + 2a_2)\sigma]\}.$$

At p_W^*,

$$\sigma\{a_1 + 2a_2n_W[1 - (G_W^*/2)]\} = s_W,$$

or

$$G_W^* = 2s_W[k_Ns_W - F']/[Fs_W - a_1\sigma(F + F' - k_Ns_W)].$$

Hence $G_W^* < 1$ iff

$$k_N < [(F + F')/s_W] - [F/(2s_W - a_1\sigma)]$$

Finally, consider entry above $\bar{p}_N$. Following the usual procedure,

$$\pi_N(p) = [(F + F')(p - c)/(1_W - 1_N + p - c_W)] - F$$

whence

$$\pi_N'(p) = (F + F')k_N/(1_W - 1_N + p - c_W)^2.$$

Now we always need $\pi_N(1_N) \leq 0$, or $\beta_W \leq \beta_N$. This reduces to $k_N > F'/\beta_N$. Thus $k_N < 0$ is impossible and $\pi_N'(p) > 0$. Thus $\beta_W \leq \beta_N$ suffices to guarantee $\pi_N(p) \leq 0$ for all $p \geq \bar{p}_N$.

Q.E.D.

Corollary 1: If $k_N < 0$ then the warranties market can never exist. Furthermore,

(i) the nonwarranties market is competitive iff $a_1 \leq \beta_N/s$.

(ii) the nonwarranties market is noncompetitive iff $a_1 > \beta_N/s$.

Proof: This result follows from noting all cases exist 1 and 5 require $k_N > 0$, and when $k_N < 0$, $\beta_W > \beta_N$. This reduces the necessary and sufficient conditions of Theorems 1 and 5 to (i) and (ii) above.

Q.E.D.

Corollary 2: If $F' = 0$ and $k_N > 0$, then the following types of equilibria are possible (ignoring the "pathological" cases 7b and 8d):

(1) all the various competitive equilibria

(2) nonwarranties nonexistent and warranties noncompetitive

(3) nonwarranties competitive and warranties noncompetitive

Furthermore, the necessary and sufficient conditions for each are not mutually exclusive.

Corollary 1 is the interesting result here. Recall $k_N = (l_W - c_W) - (l_N - c)$. If we assumed differentiable, u-shaped average cost curves, then in a competitive equilibrium $p_N^* =$ marginal cost of nonwarranties $\equiv MC_N$ and $p_W^* =$ marginal cost of warranties $\equiv MC_W$. Thus $k_N = (l_W - MC_W) - (l_N - MC_N) = (l_W - p_W^*) - (l_N - p_N^*)$. Hence $l_W - l_N < p_W^* - p_N^*$ would imply $k_N < 0$. When $k_N < 0$ only two possible equilibrium configurations can occur, in neither of which any firms offer the good with a warranty; i.e., there is no coverage

problem. That $k_N > 0$ is possible in our model arises because of the discontinuous nature of marginal costs at s and $s_W = s(1 - \pi)$.